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Dynamic Optimization for Controller Tuning with Embedded Safety and Response Quality Measures

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ABSTRACT

Controller tuning is needed to select the optimum response for the controlled process. This work presents a new tuning procedure of PID controllers with safety and response quality measures on a non-linear process model by optimization procedure, with a demonstration of two tanks in series. The model was developed to include safety constraints in the form of path constraints. The model was then solved with a new optimization solver, NLPOPT1, which uses a primal-dual interior point method with a novel non-monotone line search procedure with discretized penalty parameters. This procedure generated a grid of optimal PID tuning parameters for various switching of steady-states to be used as a predictor of PID tunings for arbitrary transitions. The interpolation of tuning parameters between the available parameters was found to be capable to produce state profiles with no violation on the safety measures, while maintaining the quality of the solution with the final set points targeted achievable.

Keywords: **Non-linear programming, optimal PID tuning parameters, path constraints, PID controller tuning, primal-dual interior point method**

NOMENCLATURE

Latin Symbols

A_1, A_2	cross-sectional areas of tank 1 and tank 2, respectively
a_{pipe}	cross-sectional area of the pipes
c_s	controller's actuating signal
$c(t)$	controller's output
F_{in}	low rate into tank 1
F_{max}	maximum flow rate
$f(x)$	objective function
h_1, h_2	liquid levels of tank 1 and tank 2, respectively
h_m	measured liquid level

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h_{sp} targetted liquid level
h_{ss} initial steady-state level
$h(x)$ equality constraints as defined in the context
K_C proportional gain
l total losses in pipe
l_∞ infinity norm of the Lagrange multiplier values
$l_{contraction}$ loss contributed by a sudden contraction at tank exit
l_{valve} loss contributed by a half open globe valve in pipe
t_f final time
t_s starting time
x primal variables
x^L lower bounds on variables x
x^U upper bounds on variables x

Greek symbols

ε_{ss} final steady-state satisfaction parameter
$\varepsilon(t)$ deviation
μ barrier parameter
Φ_B barrier function
τ_D derivative time constant
τ_I integral time constant

Abbreviations

AI artificial intelligent
CPU CPU time
FL fuzzy logic
GA genetic algorithm
IAE integral of the absolute value error
IFT Iterative Feedback Tuning
IMC Internal Model Control
ISE integral of the square error
ITAE integral of the time-weighted absolute error
MILP mixed integer linear programming problem
MINLP mixed integer nonlinear programming problem
MIQP mixed integer quadratic programming problem
MPC Model Predictive Control
NLP nonlinear programming problem
PI proportional-integral
PID proportional-integral-derivative
SIMC Simple Internal Model Control
SISO single-input single-output
SOC second order correction

INTRODUCTION

This paper focuses on a proportional-integral-derivative (PID) controller, which can be represented as:

$$c(t) = K_c \varepsilon(t) + \frac{K_c}{\tau_1}\int_{to}^{t} \varepsilon(t')\,dt' - K_c \tau_D \frac{d\varepsilon}{dt} + c_s \qquad (1)$$

In the model, $c(t)$ is the controller's output, K_c is the proportional gain of the controller, τ_I is the integral time constant, τ_D is the derivative time constant, and c_s is the controller's actuating signal at $\varepsilon(t) = 0$. These parameters vary across process models and must be tuned to get the optimal behaviour of the process.

The processes studied are non-linear dynamic processes. This is because most of the chemical processes are nonlinear and non-stationary. The characteristics of the process change over time. In general, the particular case of servo-control (steady-state switching) is a situation that is not easy to deal with by the local linearization of the dynamic model of the system.

For example, the controller must be targeted to satisfy the maximum product yield, stabilize the process, sustain product quality, and maintain the maximum feed throughput. Meanwhile, too tight control might produce large oscillations, low product yield, and process swing.

PID Controller Tuning

In tuning a controller, the general performance criteria used to determine good controller tuning parameters are that it is required to produce a response with minimum overshoot and oscillation, use minimum settling time to reach the new steady-state, and fulfil the steady-state performance criterion which needs the deviation error to be zero after a sufficient time. Different performance criteria chosen for the same process will give different optimal tuning of the controller.

There are two ways the controller can be tuned, manually or computationally, e.g. as described by Stephanopoulos (1984). Manually, for a process with a possibility of offline tuning, the system can be subjected to changes in input, where the response is then measured to determine the controller parameters. For online tuning, two common methods to choose from are the Cohen-Coon's process reaction curve tuning method and the Ziegler-Nichols tuning method. Basically, no process model is required for the Cohen-Coon method. The open control loop of the system is subjected to changes in its input, and the model of the recorded response is then approximated as a first-order system with a dead-time. From the approximated process model, the tuning parameters are then determined using the developed Cohen-Coon formula. Almost similar is the Ziegler-Nichols method, where some changes are introduced to the closed loop system, in the input or the disturbance, from which the Ziegler-Nichols recommended settings can be used to tune the controller.

Computationally, provided the complete mathematical model of the system is available, the optimization of time-integral performance criteria can be used to tune the parameters, such as those described by Stephanopoulos (1984). Here, the response of the system is optimized so that the deviation in the set point is minimized throughout the process. The solution generated computationally is expected to perform better than the tuning parameters produced through a manual procedure as these tuning parameters should comply all the control constraints required, while minimizing the performance criterion. Various criteria for the error of the response are the integral of the square error (ISE), integral of the absolute value of the error (IAE) and integral of the time-weighted absolute error (ITAE). Similarly, the optimizers used vary as well, across a whole range of state-of-the-art optimizers available. An example is an ant system algorithm as implemented by Tan *et al.* (2005) to obtain optimal PID tuning parameters in their PID controller. Another example is the tuning of a PID controller for the first-order plus time delay models by Tavakoli & Tavakoli

(2003), whereby dimensional analysis and curve fitting techniques are used to build the model, and a genetic algorithm optimization technique is used to find the optimal PID tuning parameters. It has been demonstrated that the proposed novel optimal design method performs better compared to traditional tuning procedures.

Another relatively new method in optimization is in the area of artificial intelligent (AI) where methods such as neural networks, fuzzy logic, and genetic algorithms are used to tune the controller parameters. Some research in the area, including a paper by Gadoue *et al.* (2005). In their study, the researchers compared an offline genetic algorithm (GA) strategy to online fuzzy logic (FL) tuning scheme in tuning a PI controller of an induction motor. They found that the GA strategy is better for the normal operation conditions, while FL performed better when there are variations in the system parameters. Another example is by Lin *et al.* (2004), where a real-time GA is used to search the optimal controller tuning parameters of a PI controller of a linear induction motor. Meanwhile, combinations between AI with the optimization methods have also been done, such as the work by Kao *et al.* (2006), in which a heuristic optimization algorithm, particle swam optimization method, is used to produce the optimal tuning parameters of a slider crank mechanism system at no load and full load conditions. With this information, a fuzzy rule is then used to tune the parameters online according to changes in the system.

Apart from optimization methods, direct synthesis (as discussed in the book by Smith & Corripio, 1985), is another tuning method suitable for processes with a complete process model. If there is a complete control loop, with the dynamics of each element in the control loop including the process dynamics as well as the desired form of the closed-loop response characteristic, direct substitution and rearrangement of the closed-loop system equations will then give the optimal controller tuning parameters for the system. Other related methods include the Internal Model Control applied to PID controller design (IMC-PID) by Rivera *et al*. (1986), and the Simple Control Internal Model Control on PID controller (SIMC-PID) by Skögestad (2003). With the availability of a process model, the IMC-PID and SIMC-PID design procedures will give controller parameters which are related straightforwardly to the model parameters, except for one tuning parameter, i.e. the closed-loop time constant. Meanwhile, the IMC-PID tuning rules are derived analytically for each process, SIMC-PID tuning rules simplify all the processes to an approximate first- or second-order process with time delay. Then, only the approximated model is subjected to direct synthesis to obtain the relationship between the controller tuning parameters and the process model.

Steady-state Switching and Existing Control Methodologies

Often, steady-state conditions are disturbed while operating a process. This may be due to some upsets or disturbances into the process, where a new steady-state must be obtained to get the optimal process output. An example is in a combustion system where the temperature of the air affects the efficiency of the combustion. Therefore, the inlet fuel/air ratio needs to be changed accordingly to maintain the efficiency of the system (Stephanopoulos, 1984). Another example is a process consisting of a methanol synthesis fixed-bed reactor (Shahrokhi & Baghmished, 2005), where changes in the input feed composition of the reactor will affect the reactor yield. Therefore, pressure of the boiling water in the reactor which is the manipulated variable, must be changed accordingly responding to the changes in the feed.

When the steady-state of a process switches, it is then natural to expect the controller to adapt itself to the changes. The controller implemented will be able to bring the process towards the required new steady-state provided enough time is given and “bad behaviour” is allowed in the controller response, such as oscillations, high overshoot, or long settling time, which will affect the product output.

However, as the performance of the controller is not guaranteed, more insights are needed into the tuning procedure to include steady-state switching. There is a number of existing control methodologies used to deal with the steady-state switching so that the final output is still under control, some of the techniques are to be discussed next.

Adaptive control

An adaptive control, as described by Stephanopoulos (1984), adapts itself automatically when it detects changes in the steady-state condition of the process. The extra control loop in the controller provides new tuning parameters based on an extra objective. The objective is set so that it applies one of the performance criteria and produces adjusted tuning parameters which are then fed to the controller. For a process with a complete model of the system available, programmed adaptive control can be used. The controller measures an auxiliary variable of the process, detects the changes in the chosen variable when there is a change of process steady-state, does a simple calculation based on the changes using the process model, and provides the adjusted tuning parameters to the controller. However, if the process model is not available, self-adaptive control can be used. The model-reference adaptive control tunes the controller parameters online based on the output from the process and compares the response produced to the reference model. From the deviation in the error of the response, new values for controller parameters are obtained. Another self-adaptive controller, the self-tuning regulator estimates new tuning parameters after a steady-state switching by assuming the process to behave like a first-order system with dead time. The tuning parameters produced from the estimated model are then adjusted accordingly to fulfil the design criteria before being fed back to the controller (Stephanopoulos, 1984).

Optimization with internal model control (IMC)

This is another method that has been used recently by Shahrokhi & Baghmisheh (2005) in a methanol synthesis fixed-bed reactor system. The method is based on online changes in the set-point, in this case the pressure of the boiling water. First, the pressure dynamics are modelled as an offline open loop response using the least squares method. Various magnitudes of pressure steady-state switching are introduced to the loop to collect the input-output data. These data are then used to estimate the process model by the least squares method. Next, the IMC technique is applied based on the model obtained to provide the required PID tuning parameters. This is a simple technique which calculates the suggested PID tuning parameters using the supplied equations. It is interesting to note that this technique produces a single set of tuning parameters that consider all the possible steady-state switchings. The shortcoming with this is that it will by necessity be slower in cases a more rapid response should be feasible, and in general desired.

Iterative feedback tuning (IFT)

Iterative feedback tuning technique has been first introduced by Hjalmarsson *et al.* (1994). It is a procedure used on a model-free process. The technique is based on optimizing a specific cost function which includes the system's output error and the control effort to obtain the optimal controller tuning parameters. Since this is a model-free process, the iterations in the minimization will be based on signal information on the closed-loop system of the process. The optimization technique used is based on Newton's method. However, since it is a model-free process, the gradient and the Hessian of the cost function is unavailable analytically. Here, the gradient of the cost function is an estimated gradient found from iterative external experiments done on the closed-loop system formed by the real plant and the actual controller. On the other hand, the Hessian can be estimated, for instance, using an identity matrix as the Hessian, or the Gauss-Newton approximation of the Hessian, as long as it is a positive definite matrix which will be able to give a descent direction.

Since then, this technique has been modified, and a comparison done by Lequin *et al.* (2003) showed that it outperforms other online techniques, namely the Ziegler-Nichols, IMC and the ISE methods.

Optimization with mathematical modelling

In their paper, Syrcos & Kookos (2005) suggested that the bilinear expression in their PID controller to be remodelled so that the available optimizer could be used to solve the model. In their model, the process is formulated as a single input single output (SISO) linear time variant system, while the PID controller was presented in the velocity form of the discrete approximation of an ideal PID controller. Included in the model are limit constraints on the state variables. This model is then presented as an optimization problem with the objective of minimizing the performance criterion of the controller. In their model, however, exists a bilinear term due to the formulation of the PID controller. On the other hand, the bilinear term occurs can be removed by mathematical programming formulation and reformulated into linear terms. The final model will then be either MINLP, MILP, or an MIQP optimization problem, depending on the objective function, and solved using GAMS interface to CPLEX and MINOS solvers.

An almost related method is by Kazantzis *et al.* (2005) who suggested optimization on the performance criteria to select the best tuning parameters. The performance criterion chosen is a quadratic function of the tracking error and the control effort, which is solved by Zubov's partial differential equations. The resulting model is an NLP model which is then optimized using the non-linear programming library of MAPLE. In their paper, the optimal control parameters were found to depend on the step-size of the set-point. This means, each steady state switching needs different control parameters to behave optimally. They also suggested finding a single set of optimal controller parameters for the process using minimax optimization to find the optimal solution by minimizing the effect from the worst case scenario, taken to be the biggest step up and biggest step down in the steady state changes.

By understanding the above procedures, an approach for a simpler but effective optimal tuning procedure for PID controllers for processes with steady state switching is introduced.

The New Approach

For the present work, a simple and effective tuning procedure for PID controller adaptation, with respect to steady-state switching operations, is formulated through an optimization procedure. The targets are to produce tunings that give high quality controller performance (rapid and stable transfer to new steady-state), and satisfy operational and state constraints (path constraints) throughout the transition. Here, the demonstration will be done on servo tuning of a system of two tanks in series with path constraints, which include the limitation on the liquid level in the tanks, the amount of inlet feed flow rate permitted and the requirement for a stable steady-state condition to be reached in reasonable time.

PROCESS MODEL

In the real world chemical engineering scenario, a linear, first-order, steady state process rarely occurs. Therefore, a first-order case study would be arbitrary, while a non-linear, higher order, and dynamic example will be more beneficial as it refers closely to the actual situation. In the example is a second order non-linear process consisting of two tanks arranged in series, as shown in *Fig. 1.* The control objective is to control the level of the liquid in tank 2, switched from one steady-state to another desired steady-state, without violating any safety measures while maintaining stability and fast response, in the presence of changes in the inlet feed flow rate.

To model the process, material balance was done on the two tanks, for which, for tank 1 is:

$$A_1\frac{dh_1}{dt} + a_{pipe}\sqrt{\frac{2gh_1}{l+1}} = F_{in(t)} \tag{2a}$$

$$h_1(0) = h_{ss} \tag{2b}$$

and, for tank 2:

$$A_2\frac{dh_2}{dt} + a_{pipe}\sqrt{\frac{2gh_2}{l+1}} = a_{pipe}\sqrt{\frac{2gh_1}{l+1}} \tag{3a}$$

$$h_2(0) = h_{ss`} \tag{3b}$$

h_1 and h_2 are the liquid levels of tank 1 and tank 2, respectively. At the beginning of the process, h_1 and h_2 were at the same initial steady-state level h_{ss} *as* both tanks were assumed to be at atmospheric pressure. A_1 and A_2 are the cross-sectional areas of tank 1 and tank 2, respectively, for which, in this case $A_1 = 3.80$ m^2 and $A_2 = 2.54$ m^2. a_{pipe} is the cross-sectional area of the pipes with $a_{pipe} = 7.85$ ⑨ 10^{-5}m^2 in the system. F_{in} is the flow rate of the liquid into tank 1 and h_{ss} is the initial steady-state of the system, which is the initial liquid level in tank 2. l ($l = 10.0$) is the total losses in the pipe contributed by a sudden contraction at tank exit ($l_{contraction} = 0.5$) with a half open globe valve used in the pipe ($l_{valve} = 9.5$) (Perry *et al.*, 1963).

The liquid levels in tank 1 and tank 2 were restricted to an upper and lower limit to prevent overflow and dry condition in the tanks. The liquid level limit for tank 1 was such that:

$$1.0 \leq h_1(t) \leq 3.0 \tag{4}$$

and the liquid level limit for tank 2 was:

$$1.0 \leq h_2(t) \leq 3.0 \tag{5}$$

To measure the liquid level, a measurement device with the first-order dynamics was installed for tank 2. The differential equation is such that:

$$10\frac{dh_m}{dt} + h_m = h_2 \tag{6a}$$

$$h_m(0) = h_{ss`} \tag{6b}$$

where h_m is the measured liquid level in tank 2.

In the system, a PID controller was used to achieve the desired output value. Here, h_{sp} was let to be the set point, i.e. the targeted liquid level in tank 2. Then, the difference of the measured and the desired liquid level was recorded as the deviation $\varepsilon(t)$ where:

$$\varepsilon(t) = h_{sp} - h_m(t) \tag{7}$$

The response of the PID controller was given by:

$$F_{in}(t) = K_c \varepsilon(t) + \frac{K_c}{\tau_1}\int_{to}^{t} \varepsilon(t')dt' - K_c \tau_D \frac{d\varepsilon}{dt} + c_s \tag{8}$$

For this case study, the controller's actuating signal was set to $c_s = h_{ss}$. Here, the manipulated variable F_{in} was related directly to the PID controller output, although this was not actually the case in real models. The final control element's response to changes was assumed to be very fast which is true for small or medium-size valves (Stephanopoulos, 1984) that the dynamics could be neglected. The remaining constant gain term was set as 1. Therefore, the dynamics of the actuator was not included in the equations.

Thus, it is important to get a high quality and stable solution. To guarantee this, the final steady state was set to:

$$\int_{ts}^{tf} \left(\frac{dh_2}{dt}\right)^2 dt \le \varepsilon_{ss'} \tag{9}$$

where ε_{ss} was the final steady-state satisfaction parameter such that $\varepsilon_{ss} < 1$ and t_s and t_f were the starting and end time of the integration. In this case study, $\varepsilon_{ss} = 10^{-6}$ and $t_s = 0.7t_f$ were used. Here, the final time, t_f, was set as a free variable. This was done to allow the final time to vary, so that the tank could have a longer time for a difficult switching with the objective being to minimize the time needed to satisfy all the requirements (t_f).

Apart from guaranteeing the quality of the solution, the operability and safety measures needed to be satisfied as well. The level of the liquid was controlled such as not to exceed the operable limit as in equations (4) and (5). The inlet flow rate was also controlled as the operating capacity was restricted to be below a limit to follow some safety measures such that:

$$0 \le F_{in}(t) \le F_{\max} \tag{10}$$

Where, F_{max} was the maximum allowable flow rate in the pipe. Alternatively, such constraints might be imposed by the pumping capacity available. Here, $F_{max} = 0.04m^3/s$ was used.

To conclude, the objective was defined as to find the optimal response in getting to the required liquid level in tank 2. It can be formulated as:

$$min\,imize \int_{to}^{tf} |h_2(t) - h_{sp}|\, dt \tag{11}$$

Where, the criterion was based on the integral of the absolute value of the error (IAE) (Stephanopoulos, 1984).

METHODS AND PROCEDURES

The model produced was then converted into general non-linear equations to be used in a non-linear optimization solver. All the integral equations were reformulated into differential equations, from which, the differential equations were then discretized using backward finite differences to produce a set of general nonlinear equations such that:

$$\min_{x\, f\,(x)} \tag{12a}$$

subject to:

$$h\,(x) = 0 \tag{12b}$$

$$x^L \leq x \leq x^U \tag{12c}$$

where $x \in R^{nx}$. Inequality constraints were reformulated into equalities with appropriately bounded slack variables. The sequence of equality constrained problems with bounded variables was solved with a primal-dual interior point method by solving:

$$\min_{\mathrm{T}}[(\downarrow B(x) = f(x) - \mu \sum \downarrow (i = 1)^{\uparrow}(n \downarrow x) \quad [(x \downarrow i^{\uparrow} U - x \downarrow i) + 1\mathrm{n}(x \downarrow i - x \downarrow) + 1\mathrm{n}(x \downarrow i - x \downarrow i^{\uparrow} L \tag{13a}$$

subject to:

$$h\,(x) = 0 \tag{13b}$$

where $\mu > 0$ was the barrier parameter and μ must be chosen such as $\mu \to 0$ so that $x^*(\mu) \to x^*$, x^*, was the optimal and feasible solution of the NLP problem of equations (12a) to (12c).

The above model was then solved through an optimization procedure using the NLPOPT1 solver, developed by Vassiliadis *et al.* (2006). In short, the NLPOPT1 solver is a primal-dual interior point algorithm using logarithmic barrier method. It solves the Lagrangian of the barrier equation (13a) and (13b) using the Newton method with line search and backtracking procedure to obtain the optimal solution.

In NLPOPT1, a novel non-monotone line search procedure with a standard l_1-penalty function for the definition of the merit function as a measure of accepting or rejecting the updated points from the line search is used. However, one of the major problems with the l_1-penalty function is such that the penalty parameter is not known *a priori*. Another problem with the mentioned merit function is the well-known Maratos effect (Maratos, 1978).

To overcome these issues, a coarse-grained non-monotone line search scheme had been implemented by Vassiliadis *et al.* (2006) in their line search technique. Here, the penalty parameter is made up from discretization of a whole range of meaningful numbers (10^{+16} to 10^{-16}) to produce penalty parameter levels, with an additional provision of a memory list for each level to maintain the non-monotonicity. In this way, there is no need to specify a single penalty parameter, which is hard to guess in the first instance, and may not be correct until the iteration is very close to the optimal solution. At each iteration, the operating merit function selected will be based on the active operating penalty parameter level, which is obtained through a comparison with the infinity norm of the Lagrange multiplier value, l_∞. Acceptance of the search direction is then determined by an Armijo non-monotone acceptance criterion (Armijo, 1966). Otherwise, a backtracking procedure will be applied.

As a safety measure, a Levenberg-Marquardt scheme is implemented in the solver, to be activated in the cases where a descent direction is not obtainable. Second-order correction (SOC) steps (Wächter & Biegler, 2004) are also implemented as an extra caution measure to counter the Maratos effect.

NLPOPT1 was programmed in Mathematica™ version 5.0, with a customised equation-based interface and able to produce exact first and second-order sparse derivative information. All the

To demonstrate the application of the PID controller tuning parameter grid produced, two situations were selected where the initial steady states and the final set points were not in the grid.

simulations were done on a Pentium IV computer with 224 MB RAM and 1.19 GHz CPU clock. Each optimization model, obtained using 30 backward finite difference steps, consisted of 322 variables, 165 equality constraints, 123 equality constraints, 168 lower bounds, 168 upper bounds, 65 linear constraints, and 223 non-linear constraints. The sparsity of the problem involved 1421 non-zeros in the constraint Jacobian, 1044 non-zeros in the Lagrangian Hessian, and 3886 non-zeros in the overall Lagrange-Newton matrix. The number of the variables, lower and upper bounds, non-zeros in the Jacobian, the Hessian and the Lagrange-Newton overall matrix, includes the automatically generated slack variables for inequality constraints. On average, each steady-state switching optimization run required 15 CPU seconds for symbolic parsing and 114 CPU seconds for solving.

RESULTS AND APPLICATIONS

Through the optimization runs on the model of equations (2a) to (11), the optimal and feasible PID controller tuning parameters (K_c, τ_I and τ_D) of the process model from various selected initial steady states to corresponding final set points were obtained. The points for the steady state transitions were purposely selected so that the points were more concentrated at the crucial liquid level limits of the tanks, which were the lower and upper bound constraints on liquid level. The data obtained were given on a grid as in Table 1, and were to be stored into the memory of the controller. When needed, the memory would be evoked and interpolated to give the tuning parameters for cases not available in the memory storage.

TABLE 1
PID controller tuning parameters (K_c, τ_I and τ_D) for selected steady states transitions

Initial steady states, h_{ss}		Set points, h_{sp}						
		1.001	1.25	1.5	2	2.5	2.75	2.999
1.001	K_c		0.118453	0.059108	0.029524	0.019676	0.016864	0.014762
	τ_I		4027.76	3157.04	2999.35	2788.9	2345.38	4087.46
	τ_D		117.769	143.828	204.778	237.655	203.971	605.321
1.25	K_c	0.047146		0.113043	0.037681	0.022609	0.01884	0.016158
	τ_I	5173.23		4410.86	3700.75	3069.18	3083.95	4587.48
	τ_D	450.097		122.913	189.934	237.257	317.504	633.4
1.5	K_c	0.025771	0.051439		0.05428	0.02714	0.021712	0.018106
	τ_I	3461.32	2795.39		5301.39	2556.77	3956.14	5227.07
	τ_D	428.673	129.482		178.271	226.117	396.227	671.511
2	K_c	0.014864	0.019799	0.029699		0.050302	0.01782	0.025176
	τ_I	2597.15	2165.05	2551.8		5497.2	3515.92	7329.34
	τ_D	397.103	145.938	139.702		182.273	250	727.219
2.5	K_c	0.010754	0.013282	0.016602	0.033204		0.075292	0.034034
	τ_I	2283.56	2022.37	2417.89	3495.57		8503.02	10000
	τ_D	378.248	166.968	167.788	157.294		249.086	754.203
2.75	K_c	0.009956	0.011608	0.01393	0.023216	0.069649		0.034297
	τ_I	2100.65	1715	1505.02	3445.76	5177.42		10000
	τ_D	291.138	105.612	8.87E-06	185.944	157.306		762.564

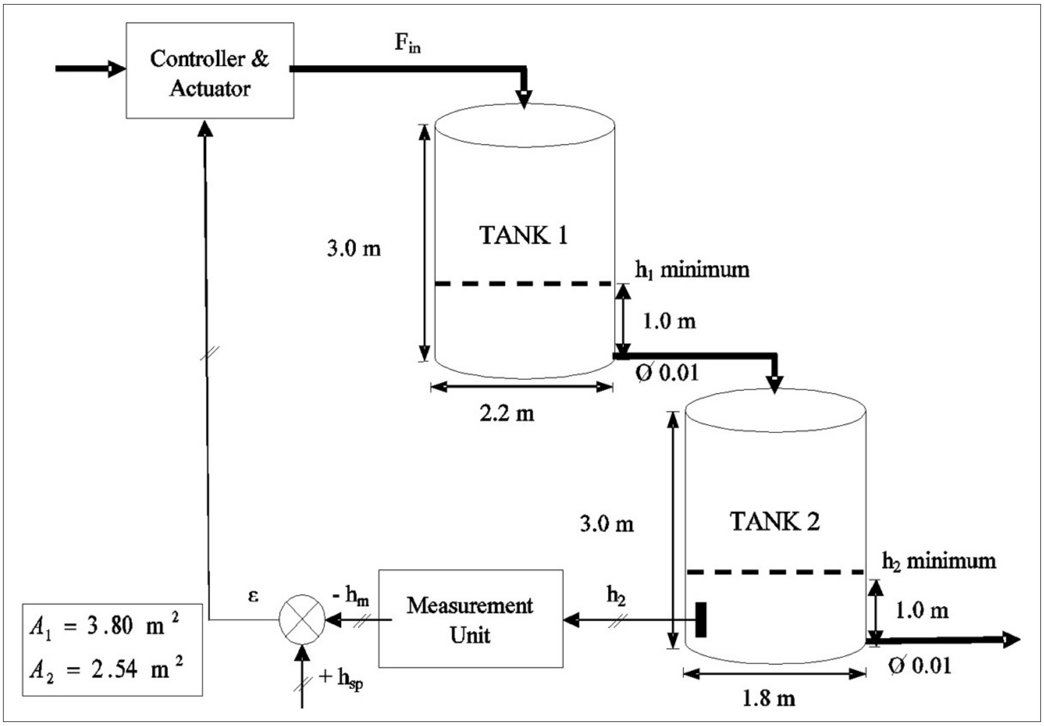

Fig. 1: Process diagram of the two tanks in series with controller loop

From the interpolation done on the neighbouring steady state transitions, the tuning parameters obtained were used in the simulation to produce the profiles in *Figs. 2* and *3*. The integrations had been set to simulate the real situation, where the controller would be saturated at both ends, with the flow rate only allowed to vary between 0 and 0.04 m^3/s. The profiles of the steady-state switching from 1.3 m to 2.9 m obtained show no violation on the safety measure and quality measures with the final set points targeted achievable. However, in the case study of steady-state switching from 2.6 m to 1.4 m, one of the constraints, the lower level limit on tank 1, had been violated.

The assumption for this violation was that more steps for the discretization of the differential equations were needed. The model was then reoptimized with the differential equations discretized to 100 steps, with 88% of the steps concentrated at the earlier 30% of the total time used, to reproduce the optimal tuning parameters for the steady-state switching around the desired steady state. The modified optimal tuning parameter sets were as shown in Table 2. Re-interpolating and re-integrating the model gave the profiles as in *Fig. 4*. The new profiles obtained showed no violation on the safety and quality measures with the final set points targeted achievable. The conclusion from this was that the assumption was correct, and tight handling of path constraints was necessary to produce a reliable grid of optimal pre-tuned PID settings.

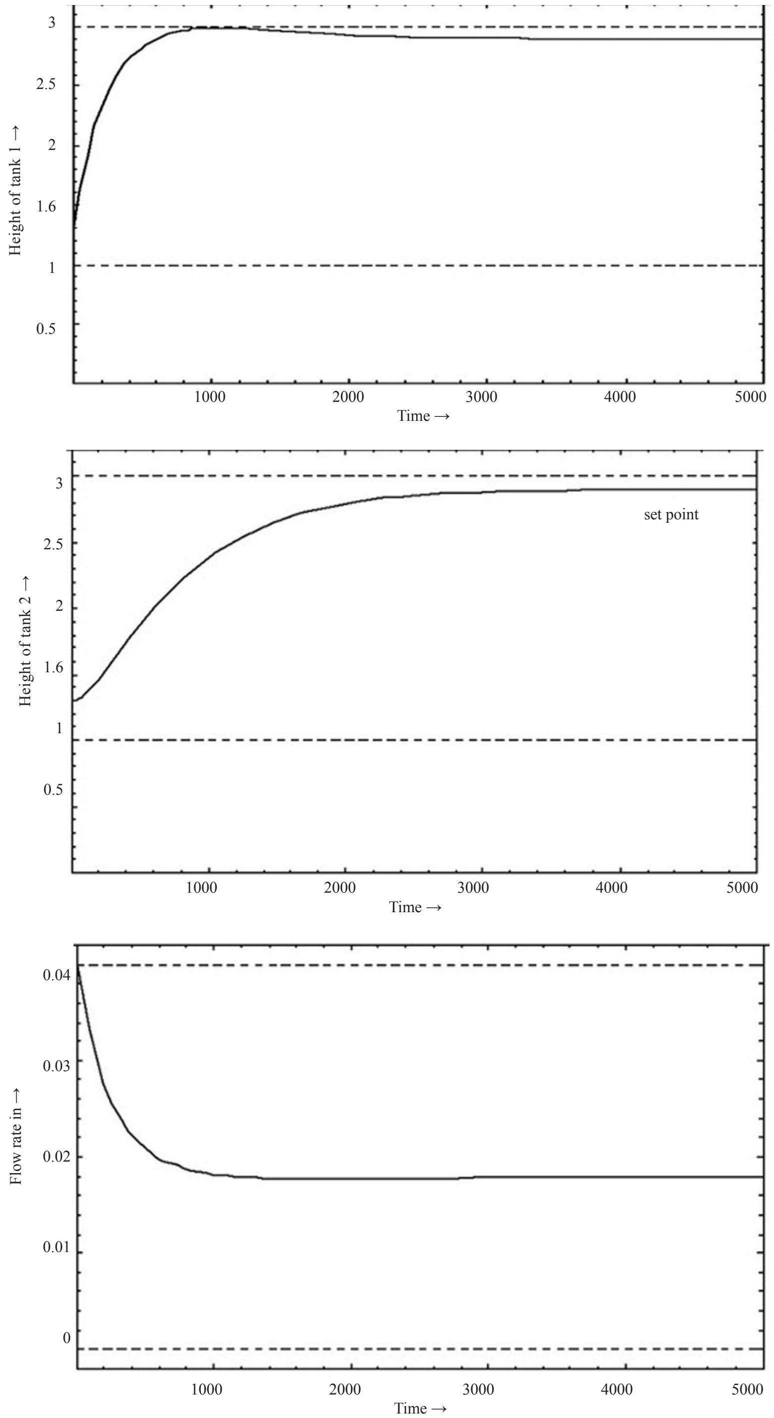

Fig. 2: Height of tank 1, height of tank 2 and inlet flow rate profiles for the steady state transitions from $h_{ss} = 1.3$ to $h_{sp} = 2.9$

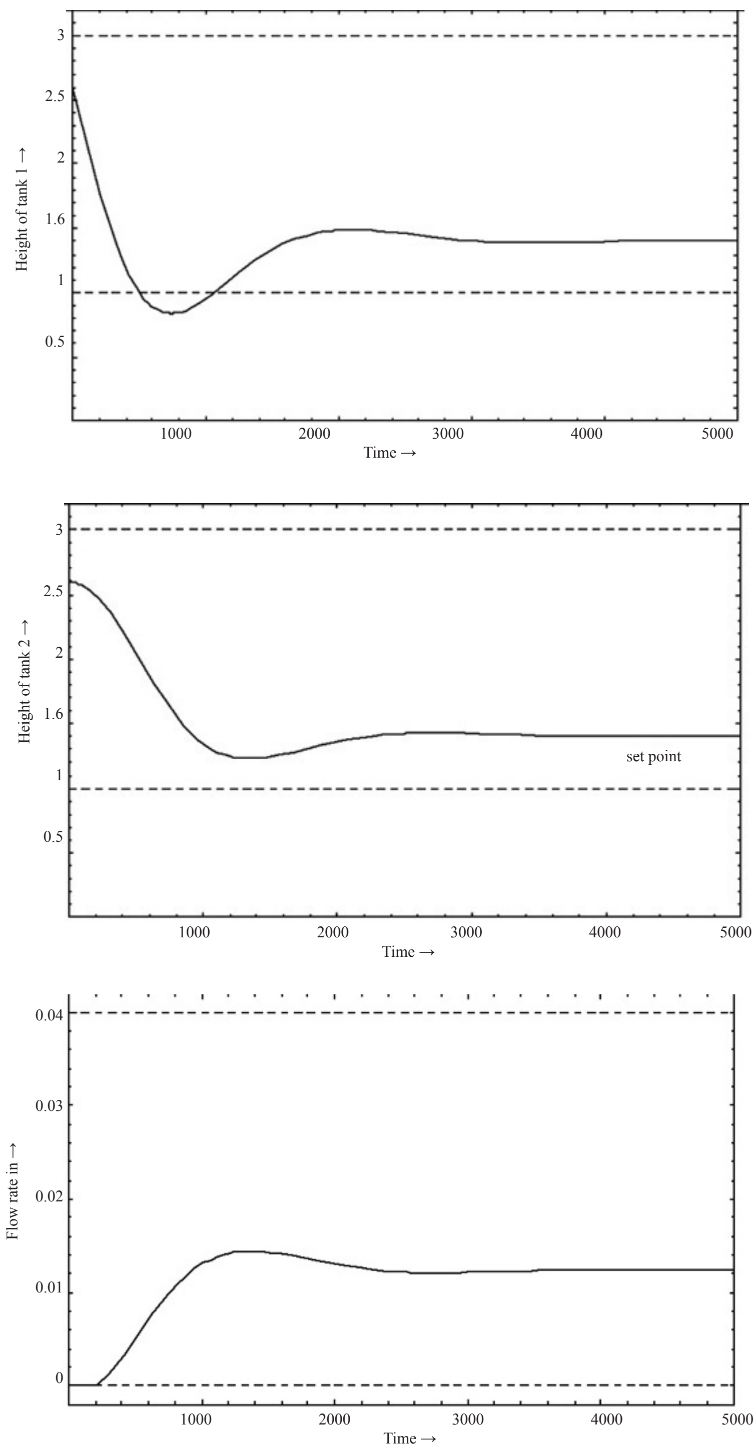

Fig. 3: Height of tank 1, height of tank 2 and inlet flow rate profiles for the steady state transitions from h_{ss} = 2.6 to h_{sp} = 1.4

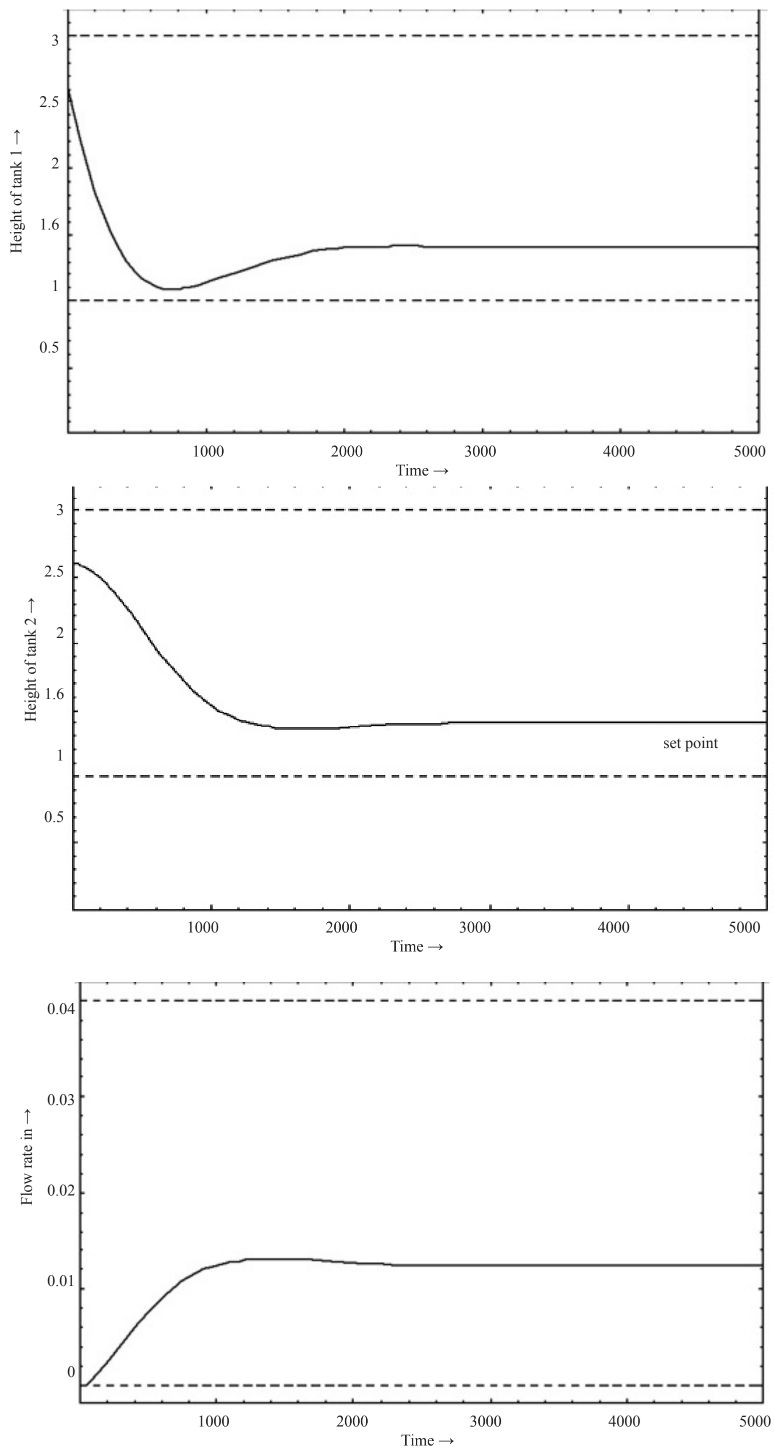

Fig. 4: Height of tank 1, height of tank 2 and inlet flow rate profiles for the steady state transitions from h_{ss} = 1.3 to h_{sp} = 2.9, re-integrated with 100 steps in the finite difference discretization

TABLE 2
New PID controller tuning parameters (K_c, τ_I and τ_D) for 4 steady states transitions after being re-optimized with 100 steps in the finite difference discretization

Initial steady states, h_{ss}		Set points, h_{sp}	
		1.25	1. 5
2.5	K_c	0.013282	0.016602
	τ_I	2149.03	2462.84
	τ_D	202.159	197.489
2.75	K_c	0.011608	0.01393
	τ_I	2100.60	2357.52
	τ_D	209.995	205.981

On the contrary, using non-optimal parameter tunings would give infeasible and/or worse objective solution. To show this, a steady state transition of 1.25 to 2.999 was simulated twice; one with the correct tuning parameters as produced by the optimizer and another with parameters produced by the optimizer, but for a different steady state transition (1.25 to 2.5). Again, the simulations done had been set to represent the real situation, with the controller saturated at both ends, where the flow rate was only allowed to vary between 0 and 0.04 m^3/s. The set with non-optimal parameters violated both the safety and operability measures, as shown in *Fig. 5*.

CONCLUSIONS AND FUTURE WORK

The objective of this work had been dedicated in exploring a grid of PID controller tuning methodologies of a nonlinear process model for different steady-state switching. The tuning parameters produced were optimal and strictly satisfying all the response quality and safety constraints. The model produced was then reformulated into a standard nonlinear optimization model and the novel interior point method implemented in the NLPOPT1 solver was applied to solve them. It was observed that through interpolation of the available optimal tuning parameters in the steady-state switching tuning parameters grid, near-optimal tuning parameters for any steady-state switching between the ones considered could be obtained. The concept was demonstrated through two examples and the results seemed to be satisfactory. No constraints were violated and the new steady states were achievable.

This work is different from the reviewed literature in the sense that the solution found from this method did not only seek some optimal tuning solution, but also included the operational constraints. In realistic applications, these constraints are important and need to be accounted for in order to optimize the tuning of the controller parameters. Furthermore, this grid method is more refined to enable us to find a better solution as a process with large system changes should not only rely to optimal solutions found at only the two ends of the variations, at no load and at full load conditions. With a finer mesh of optimal and feasible solutions, it is more likely to find a good, feasible solution to begin with.

In the future, it is planned to replace the simple interpolation used in finding the controller pre-tuned settings for steady state changes with a better representation of interpolation by fuzzy-logic and neural network methodologies. Another challenging avenue to explore is the coupling of this approach with Model Predictive Control (MPC), particularly tying in model adaptation when parameter drift occurs. Although the pre-tuned settings might be adequate under certain conditions, the re-tuning of the grid will be inevitable. In that case, it is expected that the previous grid values

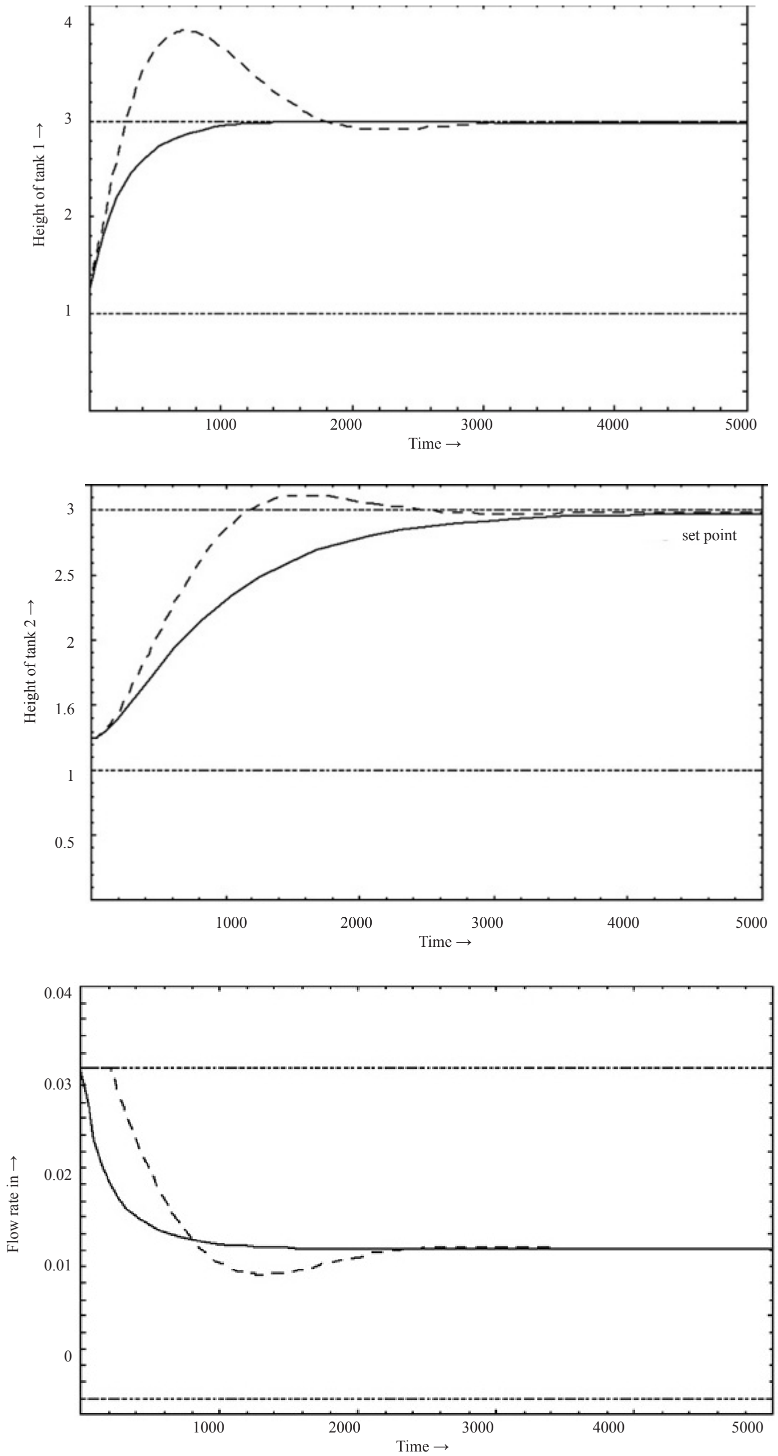

Fig. 5: Height of tank 1, height of tank 2 and inlet flow rate profiles for h_{ss} = 1.25 to h_{sp} = 2.999 simulations with correct (solid lines) and non-optimal (dashed lines) tuning parameters used

will provide close starting points so that the re-tuning of the grid will require for less effort in solving optimal control problems than the base case.

Other things to be done are to examine further applications in terms of regulatory control, extending the present work on servo control and to investigate the implementation of these concepts into more complex nonlinear systems. Finally, experimental testing of an on-line implementation of this approach will be investigated.

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