Geometric Structures in Quantum Hall Systems*

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Introduction

Quantization being an ad-hoc procedure would require tailoring to suit the intrinsic properties of the physical system studied particularly the geometrical and topological properties. One particular quantization technique that does so is group-theoretic quantization initiated by Isham et al.(1984) for which the geometrical and topological properties are intertwined and encoded with some underlying group structure. This technique was later extended to include systems with external fields by Zainuddin et al. (1989). In this project, this technique is specifically applied to Hall systems of particle moving on a nonlinear configuration space in an external homogeneous magnetic field.

Materials and Methods

The first phase of the project is to study integer quantum Hall systems modelled by a particle moving on the two-torus and the two-sphere. The canonical group describing the kinematical symmetries of the systems are sought for and eventually the classification of the groups' representations are studied. The second phase is to study fractional quantum Hall systems, which are tougher. Punctures are introduced into the configuration space of the two-sphere to simulate the many-body effect required for the fractional Hall effect. The effect of these punctures on the (local) parental symmetry is studied together with their exchange symmetries in the hope of retrieving the canonical group structure.

Results and Discussion

The canonical group for the integer quantum Hall systems on the two-torus and two-sphere are simply found to be the universal cover and extension of the canonical group of the respective systems without the external magnetic field. The classification of their representations seem to reproduce the intger quantum Hall states labeled by integral 'magnetic monopole charge' (modulo geometrical factors) (Zainuddin et al, 1998). The case of the two-torus seem to display a richer set of possibilities than the case of the two-sphere which is studied in a related research (Zainuddin et al., 2000).

The punctured case of the two-sphere proves to be a very difficult problem. It was shown that the SU(2) parental symmetry still exists locally by examining the related Lie algebras, and on top of this there is the braid group $B_n(S^2)$ giving the exchange symmetries of the punctures with even the possibility of additional generators through lifting $B_n(S^2)$ over a line bundle (Bouketir et al. & Zainuddin et al., 2000). There is the difficulty of incorporating the braid group stucture into a wholesome canonical group and was left for further research. A simpler manageable approach is to use uniformization theorem and pass to the universal cover of the punctured surface and to perform the quantization on this covering space namely the upper half plane. The canonical (sub) group of SL(2, R) can be found but the classification of the group's representation does not reproduce the wanted fractional feature . There is a slight hint of how fractional filling fractions can arise from the study of triply punctured 2-sphere which involves the modular group as the homotopy group but this is left for future research.

Conclusions

Group-theoretic quantization of Hall systems reproduces quantum states labelled by integral magnetic charge needed for the usual integral quantum Hall states. The case of the fractional quantum Hall system can be studied by introducing punctures for which non-trivial modification of the kinematical symmetries. A more tractable problem can be obtained by using the covering space of the punctured surface and it is hoped that the fractional filling features can arise in a future study of the role of the homotopy group of punctured surfaces in the quantization of such systems.

Benefits from the study

A coherent explanation of the topological character of the integer Hall effect is given. The study also opens up a relatively new area of research on punctured surfaces.

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Project Publications in Conference Proceedings

None.

Graduate Research

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