UNIVERSITI PUTRA MALAYSIA

CONVERGENCE OF INTERVAL SYMMETRIC SINGLE-STEP METHOD FOR SIMULTANEOUS INCLUSION OF REAL POLYNOMIAL ZEROS

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CONVERGENCE OF INTERVAL SYMMETRIC SINGLE-STEP METHOD FOR SIMULTANEOUS INCLUSION OF REAL POLYNOMIAL ZEROS

By

NUR RAIDAH BINTI SALIM

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Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment if Requirements for the Degree of Master of Science

January 2012

DEDICATIONS

I would like to dedicate this thesis to my father, Mr.Salim bin Abdullah and mother, Mrs.Hamidah binti Hashim and the rest of my family.

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Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Master of Science

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By

NUR RAIDAH BINTI SALIM

January 2012

Chairman: Mansor Bin Monsi, PhD

Faculty: Science

The purpose of this thesis is to find the inclusion of polynomial zeros by using interval analysis approach. We will focus on interval single-step method in order to gain the fastest speed of convergence for bounding simple polynomial zeros simultaneously. Firstly, we will generally describe on some basic mathematical background on interval analysis approach. Then, we will briefly discuss the procedure given in the literature which has been proved by other researchers. We present some information on interval single-step IS method together with the algorithm and the analysis on the rate of convergence.

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In order to improve IS method, we made several modifications using interval analysis approaches whereby it has been proved that these procedures not only including intervals for roots, but also convergent under a few assumptions. We have new modification namely ISS, IZSS and IZMSS methods which are describe precisely in this thesis. The processing time (CPU) of the algorithm of the modified methods may be done using Matlab 2007a associated with Intlab. Nevertheless, we will also present the theoretical analyses of the convergence rate of the modified procedure.

This thesis will cover the algorithms, theoretical analysis and numerical results for each modification. Based on the analysis that has been done, we finally found the rate of convergence for ISS is at least 9, for IZSS is at least 13 and for IMZSS is at least 16 while the rate of convergence of IS is at least $2(1+\tau) > 3$.

Finally, we conclude our thesis by comparing all the factors needed in a table and we give some possible extensions for future works.

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Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Master Sains

KADAR PENUMPUAN KAEDAH SELANG TUNGGAL SIMMETRIK UNTUK MERANGKUMI PENSIFAR NYATA SECARA SERENTAK BAGI POLINOMIAL

Oleh

NUR RAIDAH BINTI SALIM

January 2012

Pengerusi: Mansor Bin Monsi, PhD

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Objektif utama tesis ini adalah untuk merangkumi pensifar nyata bagi suatu polinomial menggunakan pendekatan analisis selang. Kami akan fokus kepada suatu kaedah yang dikenali sebagai langkah selang tunggal yang digunakan untuk mendapatkan kadar penumpuannya yang lebih tinggi semasa memerangkap pensifar secara serentak. Pertama sekali, kami akan menjelaskan secara kasar tentang latar belakang matematik bagi pendekatan analisis selang. Kemudian, kami akan membincangkan tentang kaedah-kaedah terdahulu yang telah dibuktikan benar oleh penyelidik lain. Selepas itu,

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kami akan membentangkan sedikit maklumat asas kaedah IS beserta dengan algoritma dan analisis ke atas kadar penumpuannya.

Untuk memperbaiki kaedah IS, kami telah melakukan beberapa pengubahsuaian dengan menggunakan analisis selang di mana ianya telah dibuktikan benar. Kaedah-kaedah ini bukan sahaja memberikan selang bagi setiap pensifar, malah ia akan sentiasa menumpu di bawah beberapa andaian. Pengubahsuaian baru yang telah dibuat adalah seperti kaedah ISS, IZSS dan IMZSS yang mana akan diperincikan dengan lebih lanjut di dalam tesis ini. Kami juga mempertimbangkan masa pemprosesan (CPU) algoritma bagi kaedah yang telah diubahsuai yang mana kesemuanya dijalankan menggunakan perisian Matlab R2007a dan dibantu oleh perisian Intlab. Analisis teori bagi kadar penumpuan untuk kesemua kaedah yang telah diubahsuai juga dibentangkan.

Tesis ini akan merangkumi aspek algoritma, teori analisis dan analisis berangka untuk setiap pengubahsuaian. Berdasarkan kepada analisis yang dilakukan, kadar penumpuan bagi ISS adalah sekurang-kurangnya 9, bagi IZSS kadar penumpuan adalah sekurang-kurangnya 13 dan IMZSS adalah sekurang-kurangnya 16. Sementara kadar penumpuan bagi kaedah IS adalah sekurang-kurangnya 3 atau $2(1+\tau)>3$

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Akhir sekali, kami simpulkan tesis ini dengan membuat perbandingan di antara kesemua faktor dan dibentangkan dalam bentuk jadual dan graf. Di samping itu, beberapa cadangan kajian lanjut turut disertakan.

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I certify that a Thesis Examination Committee has met on **16 January 2012** to conduct the final examination of **Nur Raidah Salim** on his (or her) thesis entitled "**On The Convergence Speed Of The Modified Interval Single-Step Methods For Simultaneous Inclusion Of Real Zeros Of Polynomials**" in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the **Master of Science**.

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DECLARATION

I declare that the thesis is my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously, and is not concurrently, submitted for any other degree at Universiti Putra Malaysia or any other institution.

_____

NUR RAIDAH BINTI SALIM.

Date:

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