



UNIVERSITI PUTRA MALAYSIA

**ROBUST DIAGNOSTICS AND ESTIMATION IN HETEROSCEDASTIC
REGRESSION MODEL IN THE PRESENCE OF OUTLIERS**

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**ROBUST DIAGNOSTICS AND ESTIMATION IN HETEROSCEDASTIC
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By

MD. SOHEL RANA

**Thesis Submitted to the School of Graduate Studies, Universiti Putra
Malaysia, in Fulfilment of the Requirements for the Degree of
Doctor of Philosophy**

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Dedication

- To the best memory of my parents who passed away and wanted to see my dreams come true
- To my family, having unconditional love for me
- To my one beloved teacher who uplifted my life



Abstract of thesis presented to the Senate of Universiti Putra Malaysia in
fulfilment of the requirement for the degree of Doctor of Philosophy

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Chairman: Habshah Midi, PhD

Faculty: Institute for Mathematical Research

The violation of the assumption of homoscedasticity in OLS method, usually called heteroscedasticity, gravely misleads the inferential statistics. The current study has considered the situation when outliers occur in heteroscedastic data. Hence, the main focus of this research is to take remedial measures on the violation of the assumption of homoscedasticity in the presence of outliers. This thesis also concerns on the normality assumption of the errors of regression model in the presence of outliers. It is now evident that outliers have great impact on the existing normality tests, heteroscedasticity tests, and the estimators for heteroscedastic model. We propose the Robust Rescaled Moment (RRM) test for testing the normality of the regression residuals when there is an evidence of outlier(s). The results of the study signify that the RRM test offers substantial improvements over other existing tests in the presence of outliers. For the detection of heteroscedasticity in the presence of outliers, a modified version of



the classical Goldfeld-Quandt (MGQ) test is proposed which is most powerful than the classical tests of heteroscedasticity. Most statistics practitioners assume that the forms of the heteroscedastic error structures are known which may lead to inefficient estimates if it is not correctly specified. In this respect, a Leverage Based Near-Neighbor (LBNN) method is proposed, where prior information on the structure of the heteroscedastic error is not required. The findings indicate that the LBNN is very efficient for correcting the problem of heteroscedastic errors with unknown structure. We also examine the effect of outliers on the existing remedial measures of heteroscedasticity. Hence, in this thesis, a one step M-type of Robust Weighted Least Squares Method (RWLS) and the Two-Step Robust Weighted Least Squares (TSRWLS) are developed. Finally, the new robust wild bootstrap techniques which are resistant to outliers are proposed. The proposed techniques are based on the weighted residuals which incorporated the MM estimator, robust location, robust scale and the bootstrap sampling schemes of Wu (1986) and Liu (1988). All procedures, in this thesis, are examined by using real data and Monte Carlo simulation studies. The comparative studies among the classical and proposed robust methods reveal that all the proposed robust methods outperform the classical methods.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia
sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

**DIAGNOSTIK TEGUH DAN PENGANGGARAN MODEL REGRESI BER
HETEROSKEDASTIK DENGAN KEHADIRAN TITIK TERPENCIL**

Oleh

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Andaian homoskedastik yang tidak dipatuhi oleh kaedah OLS, biasanya dinamakan heteroskedastik, memburukkan pentakbiran statistik. Kajian terkini mempertimbangkan situasi titik terpencil berada dalam data berheteroskedastik. Oleh itu, fokus utama dalam kajian ini adalah untuk mengambil langkah yang sewajarnya bagi menangani andaian homoskedastik yang tidak dipenuhi dengan kehadiran titik terpencil. Tesis ini juga mempertimbangkan andaian normal ralat bagi model regresi dengan kehadiran titik terpencil. Bukti terkini menunjukkan bahawa titik terpencil memberikan kesan buruk keatas ujian kenormalan, ujian heteroskedastik dan penganggar bagi model heteroskedastik. Kami mencadangkan ujian Teguh Momen Berskala (RRM) bagi menguji kenormalan reja regresi apabila terdapat bukti kehadiran titik terpencil. Keputusan kajian menunjukkan bahawa ujian RRM lebih baik daripada kaedah sedia ada dalam kehadiran titik terpencil. Bagi pengenalpastian heteroskedastik dengan kehadiran titik terpencil, pengubahsuaian ujian Goldfeld klasik Quandt (MGQ) dicadangkan yang

menunjukkan bahawa ujian MGQ paling berkuasa daripada ujian heteroskedastik klasik. Kebanyakan pengamal statistik mengandaikan bentuk taburan ralat berheteroskedastik diketahui, yang akan menghasilkan penganggran yang tidak cekap sekiranya andaian ini tidak dipatuhi. Maka, kaedah Tuasan Berdasarkan Penghampiran Kejiranan (LBNN) telah dicadangkan, di mana maklumat asal berkenaan struktur ralat berheteroskedastik tidak diperlukan. Dapatan kajian menunjukkan bahawa kaedah LBNN sangat cekap dalam memperbaiki masalah taburan ralat berheteroskedastik dengan struktur yang tidak diketahui. Kami juga menguji kesan titik terpencil ke atas pengukuran perbaikan bagi heteroscedastik yang sedia ada. Maka dalam tesis ini, kami membangunkan Kaedah Satu Langkah jenis M-bagi kaedah Kuasdua Terkecil Berpemberat Teguh (RWLS) dan Dua langkah Kuasdua Terkecil Berpemberat Teguh (TSRWLS). Akhir sekali, teknik baru *wild bootstrap* teguh yang kalis titik terpencil dicadangkan. Teknik yang dicadangkan berdasarkan reja berpemberat yang menggabungkan penganggar MM, lokasi teguh, skala teguh dan skema persampelan bootsrap Wu (1986) dan Liu (1988). Semua prosedur dalam tesis ini dikaji dengan menggunakan data sebenar dan simulasi Monte Carlo. Perbandingan antara kaedah klasik dan kaedah teguh yang dicadangkan, menunjukkan bahawa semua kaedah teguh yang dicadangkan lebih baik daripada kaedah klasik.

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I certify that a thesis Examination Committee has met on 22-10-2010 to conduct the final examination of Md. Sohel Rana on his thesis entitled “Robust Diagnostics and Estimation in Heteroscedastic Regression Model in the Presence of Outliers” in accordance with Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the Doctor of Philosophy of Statistics.

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DECLARATION

I declare that the thesis is my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously, and is not concurrently, submitted for any other degree at Universiti Putra Malaysia or at any other institution.

MD. SOHEL RANA

Date: 22 October 2010

CONTENTS

	Page
DEDICATIONS	ii
ABSTRACT	iii
ABSTRAK	v
ACKNOWLEDGEMENTS	vii
APPROVAL	ix
DECLARATION	xi
LIST OF TABLES	xvi
LIST OF FIGURES	xix
LIST OF APPENDICES	xxiii
LIST OF ABBREVIATIONS	xxiv
CHAPTER	
1 INTRODUCTION	
1.1 Background of the Study	1
1.2 Importance and Motivation of the Study	2
1.3 Objectives of the Study	6
1.4 Plan of the Study	6
2 REVIEW OF LITERATURE	
2.1 Introduction	10
2.2 Outliers in Linear Regression	11
2.2.1 Identification of Outliers	12
2.3 Least Squares Regression and the Violation of its Homogeneity Assumption	17
2.4 Overview of the Detection of Heteroscedasticity and Test of Normality	21
2.5 Overview of the Remedial Measures of Heteroscedasticity	25
2.5.1 Weighted Least Squares (WLS) Regression	26
2.5.2 Variance Stabilizing Transformation	28
2.5.3 Heteroscedasticity Consistence Covariance Matrix (HCCM)	30
2.5.4 Near-Neighbor(NN) Estimators	32
2.5.5 Wild Bootstrap Estimators	33
2.6 Robust Regression Estimators	35
2.7 Review of Robust Test of Normality and Robust Heteroscedastic Regression (Test and Remedial Measures)	43
2.8 Conclusion	47



3	ROBUST TEST FOR NORMALITY IN REGRESSION	
3.1	Introduction	49
3.2	The Robust Rescaled Moment Test	52
3.3	Numerical Results	58
3.3.1	Belgian Road Accident data	58
3.3.2	Shelf-Stocking Data	60
3.3.3	Housing Expenditures Data: A Heteroscedastic Data Set	62
3.4	Power Simulations	65
3.5	Conclusion	73
4	DETECTION OF HETEROSCEDASTICITY IN THE PRESENCE OF OUTLIERS	
4.1	Introduction	74
4.2	Methods of Detecting Heteroscedasticity	76
4.2.1	Graphical Methods	77
4.2.2	Analytical Methods	79
4.3	Weakness of Classical Heteroscedasticity Tests	86
4.4	Robust Tests of Heteroscedasticity	86
4.4.1	Robust Graphical Test: The Deletion Residuals-Deletion Fits (DR-DF) Plot	86
4.4.2	Modified Goldfeld-Quandt Test: A Proposed Robust Detection Technique of Heteroscedasticity	87
4.4.3	Why the Modification of Breusch-Pagan Test and White Test are Not Helpful?	89
4.5	Examples	90
4.5.1	Housing Expenditures Data	90
4.5.2	Consumption Expenditure Data	92
4.5.3	Restaurant Food Sales Data	94
4.6	Simulation Results	96
4.7	Conclusion	99
5	LEVERAGE BASED NEAR-NEIGHBORS METHOD: A REMEDIAL MEASURE OF HETEROSCEDASTIC MULTIPLE REGRESSION MODEL	
5.1	Introduction	100
5.2	Remedial Measures of Heteroscedasticity	104
5.2.1	Error Variances σ_i^2 is Known	105
5.2.2	Error Variances σ_i^2 is Unknown	106
5.3	The Leverage Based Near-Neighbor (LBNN) Method	110
5.4	Examples	114
5.4.1	Restaurant Food Sales Data	114



	5.4.2	Education Expenditure Data	117
	5.5	Monte Carlo Simulation	121
	5.6	Conclusion	125
6		ROBUST WEIGHTED LEAST SQUARES FOR SIMPLE LINEAR HETEROSCEDASTIC MODEL IN THE PRESENCE OF OUTLIERS	
	6.1	Introduction	127
	6.2	Robust Weighted Least Squares (RWLS)	128
	6.3	Numerical Examples	131
	6.3.1	Restaurant Food Sales Data	131
	6.3.2	Simulated High Leverage Data with Heterogeneous Variances	138
	6.4	Monte Carlo Simulation Results	142
	6.5	Conclusion	151
7		TWO-STEP ROBUST ESTIMATOR: A ROBUST REMEDIAL MEASURE FOR MULTIPLE REGRESSION MODEL WITH HETEROSCEDASTIC ERRORS IN THE PRESENCE OF OUTLIERS	
	7.1	Introduction	152
	7.2	Two-Step Robust Weighted Least Squares (TSRWLS)	154
	7.3	Numerical Evaluation	157
	7.4	Simulations	163
	7.5	Further Evaluation of TSRWS Estimator Based on Real Data	183
	7.5.1	Bootstrap and Monte Carlo Simulation	188
	7.6	Conclusion	195
8		ROBUST WILD BOOTSTRAP FOR STABILIZING THE VARIANCE IN HETEROSCEDASTIC REGRESSION MODEL	
	8.1	Introduction	197
	8.2	Classical Wild Bootstrap Techniques	199
	8.2.1	Limitations of Classical Wild Bootstraps	203
	8.3	Newly Proposed Robust Wild Bootstrap Techniques	203
	8.4	Numerical Example	207
	8.5	Simulation Study	214
	8.6	Concluding Remarks	219



9	SUMMARY, CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER STUDIES	
9.1	Introduction	220
9.2	Summary	220
9.2.1	Robust Rescaled Moments (RRM) Test for Testing the Normality of Regression Residuals in the Presence of Outliers	221
9.2.2	Modified Goldfeld-Quandt (MGQ) Test for Detecting Heteroscedasticity in the Presence of Outliers	221
9.2.3	Leverage Based Near-Neighbors (LBNN) Method for Remedial Measures of Heteroscedasticity in Multiple Linear Regression	222
9.2.4	Robust Weighted Least Squares (RWLS) for Simple Linear Heteroscedastic Model in the Presence of Outliers	223
9.2.5	Two-Step Robust Weighted Least Squares (TSRWLS) for Heteroscedastic Multiple Regression Model in the Presence of Outliers	224
9.2.6	Robust Wild Bootstrap for Stabilizing the Variance in Heteroscedastic Regression Model	224
9.3	Conclusions	225
9.4	Areas of Further Research	226
	REFERENCES	228
	APPENDICES	240
	BIODATA OF STUDENT	262
	LIST OF PUBLICATIONS	263
	AWARDS	265

