Proof of Kochen["]CSpecker Theorem: conversion of product Rule to sum rule

Abstract

Valuation functions of observables in quantum mechanics are often expected to obey two constraints called the sum rule and product rule. However, the Kochen–Specker (KS) theorem shows that for a Hilbert space of quantum mechanics of dimension $d \ge 3$, these constraints contradict individually with the assumption of value definiteness. The two rules are not irrelated and Peres [Found. Phys. 26 (1996) 807] has conceived a method of converting the product rule into a sum rule for the case of two qubits. Here we apply this method to a proof provided by Mermin based on the product rule for a three-qubit system involving nine operators. We provide the conversion of this proof to one based on sum rule involving ten operators.

Keyword: Kochen-Specker Theorem; Hidden variables