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Bayes Estimator for Exponential Distribution with Extension of Jeffery Prior Information

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ABSTRACT

In this paper the extension of Jeffery prior information with new loss function for estimating the parameter of exponential distribution of life time is presented. Through simulation study the performance of this estimator was compared to the standard Bayes with Jeffery prior information with respect to the mean square error (MSE) and mean percentage error (MPE). We found the extension of Jeffery prior information gives the best estimator.

Keywords: Extension of Jeffery prior information, loss function, exponential distribution, Bayes method.

INTRODUCTION

One of the most useful and widely exploited models is the exponential distribution. Epstein,(1984) remarks that the exponential distribution plays as important role in life experiments as that played by the normal distribution in agricultural experiments.

Maximum likelihood estimation has been the widely used method to estimate the parameter of an exponential distribution. Lately Bayes method has begun to get the attention of researchers in the estimation procedure. The only statistical theory that combines modeling inherent uncertainty and statistical uncertainty is Bayesian statistics. The theorem of Bayes provides a solution on how to learn from data. Related to survival function and by using Bayes estimator, Elli and Rao,(1986), estimated the shape and scale parameters of the Weibull distribution by assuming a weighted squared error loss function. They minimized the corresponding expected loss with respect to a given posterior distribution. Sinha and Sloan,(1988), obtained Bayes estimator of three parameters Weibull distribution and compared the posterior standard deviation estimates with the corresponding asymptotic standard deviation of their maximum likelihood counterparts and numerical examples are given.

Elfessi,(2001) presented some thought provoking insights on the relationship between Bayesian and classical estimation using exponential distribution. He showed how the classical estimators can be obtained from various choices made within Bayesian framework. In 2002, Klaus Felsenstein developed Bayesian procedures for vague data. These data were assumed to be vague in the sense that the likelihood is a mixture of the model distribution with error distribution. In this case the standard updating procedure of the model prior would fail. Al-Bayyati,(2002) studied the problem of estimating parameters of Weibull distribution and reliability function in situation where there is no information on the parameters. He proposed a method based on the primary information with weighted Bayes. An extension of Jeffery prior information with square error loss function in exponential distribution was studied by Al-Kutubi,(2005). In this paper, we proposed an extension of Jeffery prior information with a new loss function.

BAYES ESTIMATOR

Let $t_1, t_2, ..., t_n$ be the life time of a random sample of size n with distribution function $F(t;\theta)$ and probability density function $f(t;\theta)$ .In the exponential case (Chiou,1993 and Elfessi and Reineke,2001), we assumed that the probability density function of the life time is given by

$$f(t;\theta) = \frac{1}{\theta}\exp(-\frac{t}{\theta}).$$

To obtain Bayes estimator, the following steps are needed.

1. A number of n items put to test and the life times of this random sample are recorded with the probability density function $f(t;\theta)$, where θ is real valued random variable.
2. The life time probability density function $f(t;\theta)$ is regarded as a conditional probability density function $f(t|\theta)$ where the marginal probability density function of θ is given by $g(\theta)$, the Jeffery prior information.

3. The joint probability density function is given by:

$$H\left(t_1,\ldots,t_n,\theta\right)=\prod_{i=1}^{n} f\left(t_i\left|\theta\right.\right)g(\theta)=L\left(t_1,\ldots,t_n\left|\theta\right.\right)g\left(\theta\right)$$

4. The marginal probability density function of $\left(t_1,\ldots,t_n,\theta\right)$ is given by:

$$p\left(t_1,\ldots,t_n\right)=\int H\left(t_1,\ldots t_n,\theta\right)d\theta$$

5. The conditional probability density function of θ given the data $\left(t_1,\ldots,t_n,\theta\right)$ is called posterior distribution of θ, given by

$$\Pi\left(\theta\left|t_1,\ldots,t_n\right.\right)=\frac{H\left(t_1,\ldots,t_n,\theta\right)}{p\left(t_1,\ldots,t_n\right)}$$

6. Bayes estimator of θ is given by using squared error loss function

$$E\left(\theta\left|t_1,\ldots,t_n\right.\right)=\int\theta\Pi\left(\theta\left|t_1\ldots,t_n\right.\right)d\theta$$

i. Jeffery Prior Information

Consider the one parameter exponential life time distribution

$$f(t;\theta)=\frac{1}{\theta}\exp(-\frac{t}{\theta})$$

We find Jeffery prior by taking $g(\theta)\propto\sqrt{I(\theta)}$, where

$$I\left(\theta\right)=-nE\left(\frac{\partial^2\ln f\left(t,\theta\right)}{\partial\theta^2}\right)=\frac{n}{\theta^2}.$$

Taking $g(\theta)\propto\frac{\sqrt{n}}{\theta}$, $g\left(\theta\right)=k\frac{\sqrt{n}}{\theta}$, with k a constant.

The joint probability density function $f(t_{1,}t_2,\ldots,t_n,\theta)$ is given by

$$H\left(t_1,\ldots,t_n,\theta\right)=\prod_{i=1}^{n} f\left(t_i,\theta\right)g\left(\theta\right)=L\left(t_1,\ldots,t_n\left|\theta\right.\right)g\left(\theta\right),$$

where

$$L\left(t_1,\ldots,t_n\left|\theta\right.\right)=\prod_{i=1}^{n} f\left(t_i\left|\theta\right.\right)=\frac{1}{\theta^n}\exp\left(-\frac{\sum_{i=1}^{n} t_i}{\theta}\right)$$

$$H(t_1,\ldots,t_n,\theta)=\frac{1}{\theta^n}\exp\left(-\frac{\sum_{i=1}^{n} t_i}{\theta}\right)\frac{k\sqrt{n}}{\theta}=\frac{k\sqrt{n}}{\theta^{n+1}}\exp\left(-\frac{\sum_{i=1}^{n} t_t}{\theta}\right).$$

The marginal probability density function of θ given the data $(t_1,t_2,\ldots,t_n)$ is

$$p\left(t_1,\ldots,t_n\right)=\int H\left(t_1,\ldots,t_n,\theta\right)d\theta$$

$$=\int_0^{\infty}\frac{k\sqrt{n}}{\theta^{n+1}}\exp\left(-\frac{\sum_{i=1}^{n} t_i}{\theta}\right)d\theta=\frac{\left(k\sqrt{n}\right)(n-1)!}{\left(\sum_{i=1}^{n} t_i\right)^n},$$

where $\int_0^{\infty}\frac{1}{\theta^{n+1}}e^{\frac{-\sum t_i}{\theta}}d\theta=\frac{(n-1)!}{(\sum t_i)^n}$

The conditional probability density function of θ given the data $(t_1,t_2,\ldots,t_n)$ is given by $\Pi\left(\theta\left|t_1,\ldots,t_n\right.\right)=\frac{H\left(t_1,\ldots,t_n,\theta\right)}{p\left(t_1,\ldots,t_n\right)}$

$$=\frac{\frac{k\sqrt{n}}{\theta^{n+1}}\exp\left(-\frac{\sum_{i=1}^{n} t_i}{\theta}\right)}{\frac{k\sqrt{n}}{\left(\sum_{i=1}^{n} t_i\right)^n}(n-1)!}=\frac{\exp\left(-\frac{\sum_{i=1}^{n} t_i}{\theta}\right)}{\theta^{n+1}}\frac{\left(\sum_{i=1}^{n} t_i\right)^n}{(n-1)!}$$

By using squared error loss function $\ell\left(\hat{\theta}-\theta\right)=c\left(\hat{\theta}-\theta\right)^2$, we can obtain the Risk function, such that

$$R\left(\hat{\theta}-\theta\right)=\int_0^\infty \ell\left(\hat{\theta},\theta\right)\Pi\left(\theta|t_1,\ldots,t_n\right)d\theta$$

$$=c\hat{\theta}^2-2c\hat{\theta}\int_0^\infty \frac{\left(\sum_{i=1}^n t_i\right)}{(n-1)!}\theta^{-n}\exp\left(-\frac{\sum_{i=1}^n t_i}{\theta}\right)d\theta+\zeta(\theta)$$

where $\zeta(\theta)=\frac{c\,(\sum t_i)^2}{(n-1)\,(n-2)}$.

Let $\frac{\partial R\left(\hat{\theta},\theta\right)}{\partial\hat{\theta}}=0$, then the Bayes estimator is

$$\hat{\theta}_{B_1}=\frac{\left(\sum_{i=1}^n t_i\right)^n}{(n-1)!}\int_0^\infty \theta^{-n}\exp\left(-\frac{\sum_{i=1}^n t_i}{\theta}\right)d\theta=\frac{\sum_{i=1}^n t_i}{n-1} \tag{1}$$

ii. New Extension Of Jeffery Prior information

The new extension of Jeffery prior (Al-Kutubi 2005) is $g(\theta)\propto\left[I(\theta)\right]^{c_1}, c_1\in R^+$. With $g(\theta)\propto\left[\frac{n}{\theta^2}\right]^{c_1}$, then $g(\theta)=k\frac{n^{c_1}}{\theta^{2c_1}}$, k is a constant.

The likelihood function is

$$L\left(t_1,\ldots,t_n|\theta\right)=\prod_{i=1}^n f\left(t_i|\theta\right)$$

The joint probability density function is given by

$$H(t_1,\ldots,t_n,\theta) = \prod_{i=1}^{n} f\left(t_i \middle| \theta\right) g\left(\theta\right) = \frac{kn^{C_1}}{\theta^{n+2C_1}} \exp\left(-\frac{\sum_{i=1}^{n} t_i}{\theta}\right).$$

The marginal probability density function of $(t_1 t_2 \ldots, t_n)$ is given by

$$p(t_1,\ldots,t_n) = \int_0^\infty H\left(t_1,\ldots,t_n,\theta\right) d\theta = \frac{kn^{c_1}(n+2c_1-2)!}{(\sum_{i=1}^{n} t_i)^{n+2c_1-1}}.$$

The conditional probability density function of θ given the data $(t_1 t_2 \ldots, t_n)$ is

$$\Pi\left(\theta \middle| t_1,\ldots,t_n\right) = \frac{\frac{kn^{c_1}}{\theta^{n+2c_1}} \exp\left(-\frac{\sum_{i=1}^{n} t_i}{\theta}\right)}{\frac{kn^{c_1}\left(n+2c_1-2\right)!}{\left(\sum_{i=1}^{n} t_i\right)^{n+2c_1-1}}} = \frac{\theta^{-n-2c_1} \exp\left(-\frac{\sum_{i=1}^{n} t_i}{\theta}\right)}{\left(\sum_{i=1}^{n} t_i\right)^{1-n-2c_1} \left(n+2c_1-2\right)!}.$$

The Risk function,

$$R(\hat{\theta},\theta) = \int_0^\infty c(\hat{\theta}-\theta)^2 \frac{\theta^{-n-2c_1} \exp\left(-\frac{\sum_{i=1}^{n} t_i}{\theta}\right)}{\left(\sum_{i=1}^{n} t_i\right)^{1-n-2c_1} \left(n+2c_1-2\right)!} d\theta$$

Let $\frac{\partial R\left(\hat{\theta},\theta\right)}{\partial \hat{\theta}} = 0$, then $\hat{\theta}_{B_2} = \frac{\sum_{i=1}^{n} t_i}{n+2c_1-2}$ (2)

If we let $c_1 = \frac{1}{2}$, $\hat{\theta}_{B_1} = \frac{\sum_{i=1}^{n} t_i}{n-1}$ which is Jeffrey estimator and it is a special case of our proposed method .

iii. New Loss Function

Al-Bayyati,(2002) introduced a new loss function using Weibull distribution that is $\ell\left(\hat{\theta},\theta\right) = \theta^{c_2}\left(\hat{\theta}-\theta\right)^2$. Here we employ this loss function with exponential distribution.

(a) Jeffery Prior Information

Through the Risk function, the Bayes estimator is

$R\left(\hat{\theta},\theta\right) = \int_0^\infty \theta^{c_2}\left(\hat{\theta}-\theta\right)^2 \Pi\left(\theta \middle| t_1,...,t_n\right) d\theta$, and by $\frac{\partial R\left(\hat{\theta},\theta\right)}{\partial \hat{\theta}} = 0$,

then,

$$\hat{\theta}_{B_3} = \frac{\int_0^\infty \theta^{c_2+1} \Pi\left(\theta \middle| t_1,...,t_n\right) d\theta}{\int_0^\infty \theta^{c_2} \Pi\left(\theta \middle| t_1,...,t_n\right) d\theta}$$

$$\int_0^\infty \theta^{c_2+1} \Pi\left(\theta \middle| t_1,...,t_n\right) d\theta = \int_0^\infty \theta^{c_2+1} \frac{\left(\sum_{i=1}^{n} t_i\right)^n}{(n-1)!} \theta^{-n-1} \exp\left(-\frac{\sum_{i=1}^{n} t_i}{\theta}\right) d\theta$$

$$= \frac{\left(\sum_{i=1}^{n} t_i\right)^{c_2+1} (n-c_2-2)!}{(n-1)!}$$

$$\int_0^\infty \theta^{c_2} \Pi\left(\theta \middle| t_1,...,t_n\right) d\theta = \int_0^\infty \theta^{c_2} \frac{\left(\sum_{i=1}^{n} t_i\right)^n}{(n-1)!} \theta^{-n-1} \exp\left(-\frac{\sum_{i=1}^{n} t_i}{\theta}\right) d\theta$$

$$=\frac{\left(\sum_{i=1}^{n} t_i\right)^{c_2}(n-c_2-1)!}{(n-1)!}.$$

Finally,

$$\hat{\theta}_{B_3}=\left[\frac{\left(\sum_{i=1}^{n} t_i\right)^{c_2+1}(n-c_2-2)!}{(n-1)!}\right]\left[\frac{(n-1)!}{\left(\sum_{i=1}^{n} t_i\right)^{c_2}(n-c_2-1)!}\right]=\frac{\sum_{i=1}^{n} t_i}{(n-c_2-1)} \qquad (3)$$

(b) Extension of Jeffery Prior Information

Taking the posterior distribution as the extension of Jeffery prior, such that $\prod_2(\theta \mid t_1,\ldots,t_n)=\dfrac{(\sum t_i)^{n+2c_1-1}\, e^{\frac{-\sum t_i}{\theta}}}{(n+2c_1-2)!\,\theta^{n+2c_1}}$ the Risk function is given by $R(\hat{\theta},\theta)=\int_0^{\infty}\theta^{c_2}(\hat{\theta}-\theta)^2\prod(\theta \mid t_1,..,t_n)d\theta$

$$=\int_0^{\infty}\theta^{c_2}\hat{\theta}^2\Pi\left(\theta \mid t_1,\ldots,t_n\right)d\theta-2\int_0^{\infty}\hat{\theta}\theta^{c_2+1}\Pi\left(\theta \mid t_1,\ldots,t_n\right)d\theta+\zeta(\theta)$$

where $\zeta(\theta)=\dfrac{(\sum t_i)^{c_2+2}\,(n+2c_1-c_2-\varphi)!}{(n+2c_1-1)!}$.

Let $\dfrac{\partial R(\hat{\theta},\theta)}{\partial\hat{\theta}}=0$, then the Bayes estimator is

$$\hat{\theta}_{B_4}=\frac{\int_0^{\infty}\theta^{c_2+1}\Pi\left(\theta \mid t_1,\ldots,t_n\right)d\theta}{\int_0^{\infty}\theta^{c_2}\Pi\left(\theta \mid t_1,\ldots,t_n\right)d\theta},$$

with

$$\int_0^\infty \theta^{c_2+1}\Pi\left(\theta\,|t_i,\ldots,t_n\right)d\theta = \int_0^\infty \theta^{c_2+1}\frac{\theta^{-n-2c_1}\exp\left(-\frac{\sum_{i=1}^{n}t_i}{\theta}\right)}{\left(\sum_{i=1}^{n}t_i\right)^{1-n-2c_1}\left(n+2c_1-2\right)!}d\theta$$

$$=\frac{\left(\sum_{i=1}^{n}t_i\right)^{c_2+1}\left(n+2c_1-c_2-3\right)!}{\left(n+2c_1-2\right)!}$$

and

$$\int_0^\infty \theta^{c_2}\Pi\left(\theta\,|t_1,\ldots,t_n\right)d\theta$$

$$=\int_0^\infty \theta^{c_2}\frac{\theta^{-n-2c_1}\exp\left(-\frac{\sum_{i=1}^{\infty}t_y}{\theta}\right)}{\left(\sum_{i=1}^{n}t_i\right)^{1-n-2c_1}\left(n+2c_1-2\right)!}d\theta$$

$$=\frac{\left(\sum_{i=1}^{n}t_i\right)^{c_2}\left(n+2c_1-c_2-2\right)!}{\left(n+2c_1-2\right)!},$$

$$\hat{\theta}_{B_4}=\left[\frac{\left(\sum_{i=1}^{n}t_i\right)^{c_2+21}\left(n+2c_1-c_2\,\_\,3\right)!}{\left(n+2c_1-2\right)!}\right]\left[\frac{\left(n+2c_1-2\right)!}{\left(\sum_{i=1}^{n}t_i\right)^{c_2}\left(n+2c_1-c_2-2\right)!}\right] \quad (4)$$

$$=\frac{\left(\sum_{i=1}^{n}t_i\right)}{\left(n+2c_1-c_2-2\right)}$$

(c) New Loss Function with Jeffery Prior Information

By letting $c_2 = 2$, the Risk function is,

$$R\left(\hat{\theta},\theta\right) = \int_0^\infty \theta^2 \left(\hat{\theta} - \theta\right)^2 \Pi\left(\theta \middle| t_1 \ldots, t_n\right) d\theta \ .$$

Letting $\frac{\partial R\left(\hat{\theta},\theta\right)}{\partial \hat{\theta}} = 0$, $\quad \hat{\theta}_{B_5} = \frac{\int_0^\infty \theta^3 \Pi\left(\theta \middle| t_1,\ldots,t_n\right) d\theta}{\int_0^\infty \theta^2 \Pi\left(\theta \middle| t_1,\ldots,t_n\right) d\theta}$,

with $$\int_0^\infty \theta^3 \Pi\left(\theta \middle| t_1,\ldots,t_n\right) = \int_0^\infty \theta^3 \frac{\left(\sum_{i=1}^n t_i\right)}{(n-1)!} \theta^{-n-1} \exp\left(-\frac{\sum_{i=1}^n t_i}{\theta}\right) d\theta$$

$$= \frac{\left(\sum_{i=1}^n t_i\right)^3 (n-4)!}{(n-1)!},$$

and $$\int_0^\infty \theta^2 \Pi\left(\theta \middle| t_1,\ldots,t_n\right) = \int_0^\infty \theta^2 \frac{\left(\sum_{i=1}^n t_i\right)}{(n-1)!} \theta^{-n-1} \exp\left(-\frac{\sum_{i=1}^n t_i}{\theta}\right) d\theta$$

$$= \frac{\left(\sum_{i=1}^n t_i\right)^2 (n-3)!}{(n-1)!}$$

The new Bayes estimator with new loss function and Jeffery prior information is

$$\hat{\theta}_{B_5} = \left[\frac{\left(\sum_{i=1}^{n} t_i \right)^3 (n-4)!}{(n-1)!} \right] \left[\frac{(n-1)!}{\left(\sum_{i=1}^{n} t_i \right)^2 (n-3)!} \right] = \frac{\sum_{i=1}^{n} t_i}{(n-3)} \tag{5}$$

(d) New Loss Function with Extension of Jeffery Prior Information

By letting $c_2 = 2$, the Risk function is,

$$R\left(\hat{\theta},\theta\right) = \int_0^\infty \theta^2 \left(\hat{\theta}-\theta\right)^2 \Pi\left(\theta \middle| t_1 \ldots, t_n\right) d\theta$$

$$= \int_0^\infty \theta^2 \hat{\theta}^2 \Pi\left(\theta \middle| t_1, \ldots, t_n\right) d\theta - 2\int_0^\infty \hat{\theta}\theta^3 \Pi\left(\theta \middle| t_1, \ldots, t_n\right) d\theta + \zeta(\theta)$$

where $\zeta(\theta) = \dfrac{(\sum t_i)^4 \, (n+2c_1-\varphi)!}{(n+2c_1-2)!}$.

Letting $\dfrac{\partial R\left(\hat{\theta},\theta\right)}{\partial \hat{\theta}} = 0$, then $\hat{\theta}_{B_6} = \dfrac{\int_0^\infty \theta^3 \Pi\left(\theta \middle| t_1, \ldots, t_n\right) d\theta}{\int_0^\infty \theta^2 \Pi\left(\theta \middle| t_1, \ldots, t_n\right) d\theta}$,

with $\int_0^\infty \theta^3 \Pi\left(\theta \middle| t_i, \ldots, t_n\right) d\theta = \int_0^\infty \theta^3 \dfrac{\theta^{-n-2c_1} \exp\left(-\dfrac{\sum_{i=1}^{n} t_i}{\theta} \right)}{\left(\sum_{i=1}^{n} t_i \right)^{1-n-2c_1} (n+2c_1-2)!} d\theta$

$$= \frac{\left(\sum_{i=1}^{n} t_i \right)^3 (n+2c_1-5)!}{(n+2c_1-2)!}$$

and $$\int_0^\infty \theta^2 \Pi(\theta|t_1,...,t_n)\,d\theta = \int_0^\infty \theta^2 \frac{\theta^{-n-2c_1}\exp\left(-\frac{\sum_{i=1}^{\infty} t_y}{\theta}\right)}{\left(\sum_{i=1}^{n} t_i\right)^{1-n-2c_1}(n+2c_1-2)!}\,d\theta$$

$$= \frac{\left(\sum_{i=1}^{n} t_i\right)^2 (n+2c_1-4)!}{(n+2c_1-2)!}$$

Then the new Bayes estimator with new loss function and extension of Jeffery prior information is

$$\hat{\theta}_{B_6} = \left[\frac{\left(\sum_{i=1}^{n} t_i\right)^3 (n-2c_1-5)!}{(n-2c_1-2)!}\right]\left[\frac{(n-2c_1-2)!}{\left(\sum_{i=1}^{n} t_i\right)^2 (n-2c_1-4)!}\right] = \frac{\sum_{i=1}^{n} t_i}{(n-2c_1-4)}. \quad (6)$$

SIMULATION AND RESULTS

In this simulation study, we have chosen $n = 25,50,100$ to represent small, moderate and large sample size, several values of parameter θ=0.5, 1, 1.5, four values of Jeffery extension, c_1=0.4,1.4 and four values of new loss function c_2=0.3,1.3. The number of replication used was R=1000. The simulation program was written by using Matlab program. After the parameter was estimated , mean square error (MSE) and mean percentage error (MPE) were calculated to compare the methods of estimation, where

$$\text{MSE}\left(\hat{\theta}\right)=\frac{\sum_{i=1}^{1000}\left(\hat{\theta}-\theta\right)^2}{R} \quad \text{and} \quad \text{MPE}\left(\hat{\theta}\right)=\frac{\left[\sum_{i=1}^{1000}\frac{\left|\hat{\theta}_i-\theta\right|}{\theta}\right]}{R}$$

The results of the simulation study are summarized and tabulated in Table 1 and Table 2 for the MSE and the MPE of the six estimators for all sample sizes and θ values respectively. It is obvious from these tables, Bayes estimator with the extension of Jeffery prior information, $\hat{\theta}_{B_2}$ is the best estimator. In most of the cases, it is apparent that Bayes estimator with new loss function $\hat{\theta}_{B_5}$, is the next best estimator. Bayes with Jeffery prior information $\hat{\theta}_{B_1}$ in most case has the largest MSE and MPE.

The effect of sample size on the MSE and MPE of all six estimators are depicted in Figure 1 and Figure 2. In these two figures, the identity number of scenario ($x-axis$) indicates the increased of sample size from $n=25$ to 100 (considering all factors). In Figure 1, for sample size $n=25$ (indicated by identity number of scenario from 1 to 61), the trend is the same for $n=50$ and $n=100$. But overall, the MSE decreases as n increases. For the MPE of the estimators, they also decreases as n increases (Figure 2). As shown in Figure 2, the variability of MPE within the sample sets are smaller as sample increases.

TABLE 1: MSE of estimated parameter of exponential distribution

Size	**Theta**	**C1**	**C2**	$\hat{\theta}_{B_1}$	$\hat{\theta}_{B_2}$	$\hat{\theta}_{B_3}$	$\hat{\theta}_{B_4}$	$\hat{\theta}_{B_5}$	$\hat{\theta}_{B_6}$
25	0.5	0.4	0.3	0.0121	0.0120	0.0130	0.0130	0.0136	0.0122
			1.3	0.0149	0.0128	0.0157	0.0157	0.0129	0.0113
		1.4	0.3	0.0114	0.0095	0.0101	0.0101	0.0102	0.0130
			1.3	0.0134	0.0086	0.0108	0.0108	0.0102	0.0124
	1	0.4	0.3	0.0479	0.0418	0.0494	0.0476	0.0514	0.0494
			1.3	0.0586	0.0468	0.0626	0.0597	0.0522	0.0501
		1.4	0.3	0.0456	0.0377	0.0404	0.0481	0.0407	0.0498
			1.3	0.0586	0.0393	0.0431	0.0588	0.0406	0.0499
	1.5	0.4	0.3	0.1059	0.1041	0.1100	0.1059	0.1144	0.1100
			1.3	0.1242	0.0986	0.1305	0.1242	0.1084	0.1042
		1.4	0.3	0.1052	0.0870	0.0805	0.0980	0.0812	0.1019
			1.3	0.1433	0.0872	0.0919	0.1292	0.0856	0.1081

TABLE 1 (continued): MSE of estimated parameter of exponential distribution

Size	Theta	C1	C2	$\hat{\theta}_{B_1}$	$\hat{\theta}_{B_2}$	$\hat{\theta}_{B_3}$	$\hat{\theta}_{B_4}$	$\hat{\theta}_{B_5}$	$\hat{\theta}_{B_6}$
50	0.5	0.4	0.3	0.0057	0.0055	0.0126	0.0191	0.0139	0.0102
			1.3	0.0063	0.0055	0.0151	0.0105	0.0191	0.0154
		1.4	0.3	0.0055	0.0048	0.0047	0.0052	0.0046	0.0053
			1.3	0.0061	0.0049	0.0052	0.0061	0.0051	0.0056
	1	0.4	0.3	0.0220	0.0218	0.0208	0.0205	0.0212	0.0208
			1.3	0.0246	0.0218	0.0248	0.0241	0.0225	0.0221
		1.4	0.3	0.0212	0.0195	0.0202	0.0228	0.0203	0.0232
			1.3	0.0252	0.0207	0.0204	0.0240	0.0298	0.0220
	1.5	0.4	0.3	0.0503	0.0498	0.0512	0.0503	0.0521	0.0512
			1.3	0.0547	0.0492	0.0543	0.0529	0.0493	0.0483
		1.4	0.3	0.0478	0.0424	0.0415	0.0460	0.0417	0.0469
			1.3	0.0566	0.0433	0.0434	0.0525	0.0416	0.0475
100	0.5	0.4	0.3	0.0027	0.0027	0.0025	0.0025	0.0026	0.0024
			1.3	0.0027	0.0025	0.0027	0.0026	0.0025	0.0023
		1.4	0.3	0.0025	0.0024	0.0025	0.0027	0.0025	0.0027
			1.3	0.0026	0.0021	0.0023	0.0025	0.0023	0.0024
	1	0.4	0.3	0.0110	0.0109	0.0099	0.0098	0.0099	0.0099
			1.3	0.0101	0.0095	0.0111	0.0110	0.0107	0.0106
		1.4	0.3	0.0106	0.0103	0.0106	0.0114	0.0106	0.0116
			1.3	0.0111	0.0096	0.0099	0.0108	0.0097	0.0103
	1.5	0.4	0.3	0.0227	0.0226	0.0245	0.0243	0.0248	0.0245
			1.3	0.0238	0.0224	0.0242	0.0239	0.0232	0.0230
		1.4	0.3	0.0258	0.0241	0.0218	0.0231	0.0219	0.0233
			1.3	0.0241	0.0205	0.0208	0.0224	0.0206	0.0215

TABLE 2: MPE of estimated parameter of exponential distribution

Size	Theta	C1	C2	$\hat{\theta}_{B_1}$	$\hat{\theta}_{B_2}$	$\hat{\theta}_{B_3}$	$\hat{\theta}_{B_4}$	$\hat{\theta}_{B_5}$	$\hat{\theta}_{B_6}$
25	0.5	0.4	0.3	0.1650	0.1637	0.1775	0.1745	0.1807	0.1775
			1.3	0.1893	0.1687	0.1868	0.1825	0.1718	0.1689
		1.4	0.3	0.1734	0.1589	0.1617	0.1749	0.1621	0.1780
			1.3	0.1897	0.1554	0.1621	0.1848	0.1598	0.1715
	1	0.4	0.3	0.1723	0.1708	0.1755	0.1723	0.1790	0.1755
			1.3	0.1936	0.1724	0.1962	0.1917	0.1800	0.1767
		1.4	0.3	0.1651	0.1543	0.1600	0.1724	0.1602	0.1750
			1.3	0.1830	0.1527	0.1633	0.1889	0.1598	0.1743
	1.5	0.4	0.3	0.1692	0.1679	0.1728	0.1700	0.1758	0.1728
			1.3	0.1906	0.1712	0.1833	0.1787	0.1672	0.1641
		1.4	0.3	0.1719	0.1586	0.1506	0.1626	0.1507	0.1655
			1.3	0.1919	0.1576	0.1609	0.1839	0.1590	0.1750

TABLE 2 (continued): MPE of estimated parameter of exponential distribution

Size	Theta	C1	C2	$\hat{\theta}_{B_1}$	$\hat{\theta}_{B_2}$	$\hat{\theta}_{B_3}$	$\hat{\theta}_{B_4}$	$\hat{\theta}_{B_5}$	$\hat{\theta}_{B_6}$
50	0.5	0.4	0.3	0.1190	0.1186	0.1178	0.1168	0.1189	0.1178
			1.3	0.1206	0.1143	0.1250	0.1235	0.1196	0.1185
		1.4	0.3	0.1170	0.1128	0.1189	0.1140	0.1191	0.1150
			1.3	0.1170	0.1094	0.1132	0.1217	0.1122	0.1168
	1	0.4	0.3	0.1095	0.1091	0.1140	0.1132	0.1149	0.1140
			1.3	0.1235	0.1159	0.1219	0.1205	0.1170	0.1161
		1.4	0.3	0.1149	0.1103	0.1132	0.1191	0.1134	0.1201
			1.3	0.1980	0.1113	0.1209	0.1201	0.1120	0.1154
	1.5	0.4	0.3	0.1205	0.1201	0.0077	0.1169	0.1186	0.1177
			1.3	0.1202	0.1163	0.1206	0.0091	0.1153	0.1143
		1.4	0.3	0.1169	0.1119	0.1075	0.1117	0.1073	0.1128
			1.3	0.1233	0.1134	0.1098	0.1194	0.1084	0.1140
100	0.5	0.4	0.3	0.0800	0.0790	0.0794	0.0792	0.0795	0.0794
			1.3	0.0788	0.0773	0.0826	0.0806	0.0806	0.0802
		1.4	0.3	0.0847	0.0839	0.0801	0.0804	0.0804	0.0814
			1.3	0.0802	0.0771	0.0769	0.0769	0.0769	0.0776
	1	0.4	0.3	0.0820	0.0818	0.0786	0.0783	0.0790	0.0781
			1.3	0.0834	0.0816	0.0824	0.0820	0.0809	0.0807
		1.4	0.3	0.0813	0.0791	0.0829	0.0857	0.0831	0.0861
			1.3	0.0839	0.0801	0.0790	0.0821	0.0784	0.0803
	1.5	0.4	0.3	0.0780	0.0779	0.0831	0.0827	0.0835	0.0831
			1.3	0.0809	0.0788	0.0825	0.0821	0.0810	0.0808
		1.4	0.3	0.0783	0.0784	0.0796	0.0813	0.0796	0.0816
			1.3	0.0848	0.0812	0.0774	0.0798	0.0772	0.0784

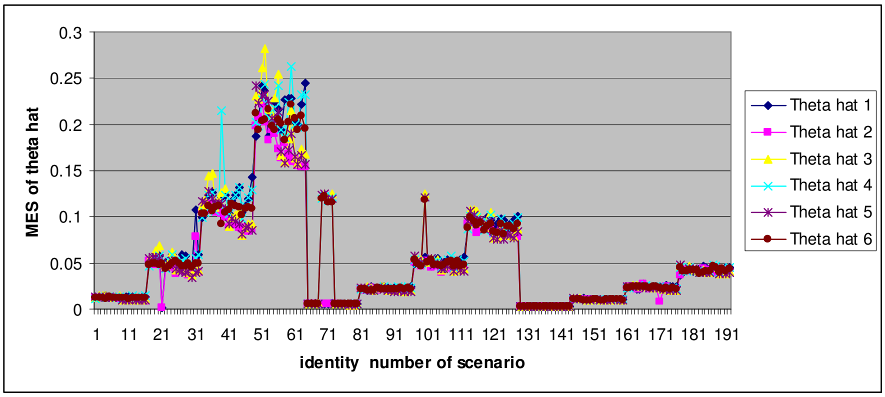

Figure 1: Comparison estimators distributed in different sample size using MSE

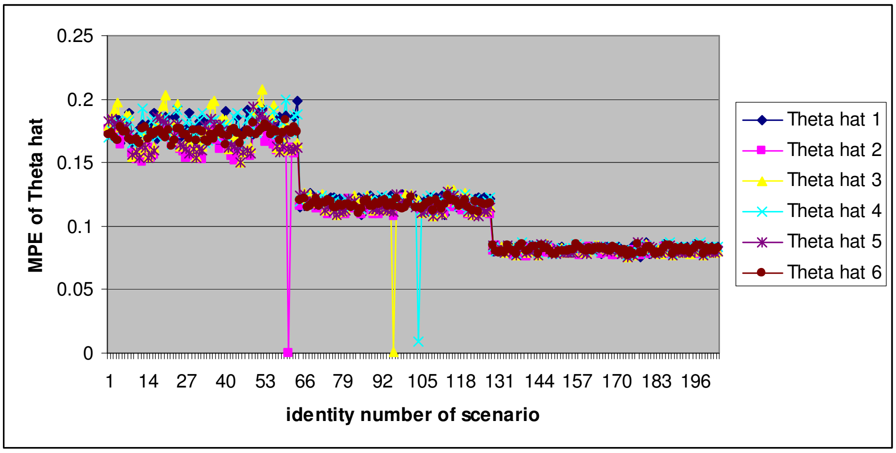

Figure 2: Comparison estimators distributed in different sample size using MPE

CONCLUSION

The new estimator with extension of Jeffery prior information $\hat{\theta}_{B_2}$ is the best estimator when compared to standard Bayes and other estimators. We can easily conclude that MSE and MPE of Bayes estimators decrease with an increased of sample size.

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