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Effect of GPS Tropospheric Delay Neill Mapping Function Simplification

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ABSTRACT

The mathematical modeling on the mapping function models should be revised and also simplified to improve the calculation of the GPS tropospheric delay. The zenith tropospheric delay can be amplified by a coefficient factor called mapping function to form total tropospheric delay. There are many mapping functions have been established to calculate the scale factor which can affect the total tropospheric delay. Most of the modern models have separated mapping functions for the hydrostatic and the wet part. Recently, the developed tropospheric delay models use mapping functions in the form of continued fractions which is quite tedious in calculation. There are 26 mathematical operations for Neill Mapping Function (NMF) to be done before getting the mapping function scale factor. There is a need to simplify the mapping function models to allow faster calculation and also better understanding of the models. The mapping functions for NMF models for hydrostatic and wet components are given in a form of continued fraction, whereby the elevation angle is the variable. These mapping function models have been selected to be simplified, because of their ability to achieve mapping function scale factor, down to 3 degree of elevation angle.

Keywords: tropospheric, zenith, mapping function

INTRODUCTION

Recently, the developed tropospheric delay models use mapping functions in the form of continued fractions. The Saastamoinen model (Saastamoinen, 1972) does not use a mapping function in the same sense as the models with continued fractions. Most of the modern models have separated mapping functions for the hydrostatic and the wet component, in a form of continued fraction (Kleijer, 2004).

The calculation for finding the mapping function scale factor, which is in a form of continued fractions are quite tedious. There are many mathematical operations (26 operations for Neill Mapping Function, NMF) to be done before getting the mapping function scale factor.

There is a need to simplify the mapping function models to allow faster calculation and also better understanding of the models. The NMF models for hydrostatic and non hydrostatic components are given in a form of continued fraction, whereby the elevation angle is the variable as shown in Figure 1.

All mapping function graphs shown in Figure 1 are in a shape of a parabolic. These graphs give very close mapping function values when the elevation angles more than 10 degree, however, for the elevation angles less than 10 degrees, each mapping function model give difference scale factor. These mapping function models are very tedious in calculating the value of mapping function, due to its continued fraction form.

In this study, NMF for both components, either hydrostatic or non hydrostatic will be selected to be simplified, due to its ability to calculate mapping function value down to 3 degree of elevation angle. However, the graphs can also be obtained by using other form of equation which is simpler than the established equations. At 90 degree the mapping function should be normalized to unity, 1 (Guo, 2003). As a coefficient of zenith hydrostatic delay and also zenith non hydrostatic delay, the mapping function scale factor value plays an important role for getting the total tropospheric delay value.

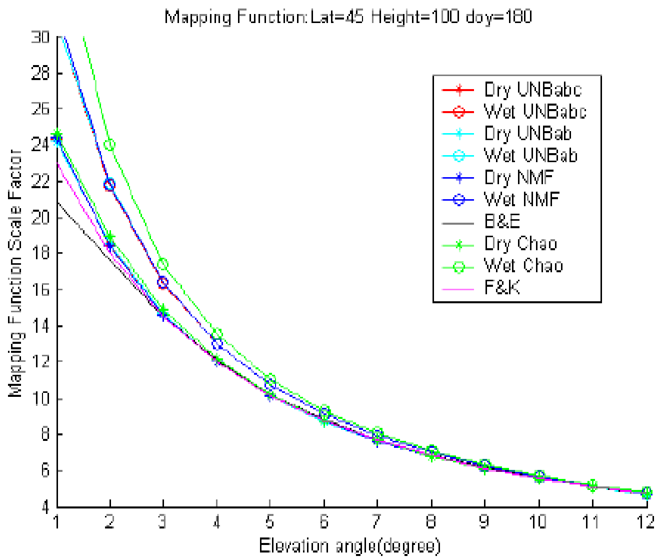

Figure 1: Comparison of mapping function values (Guo, 2003).

The mapping function depends on the elevation angles, whereby at 90 degree of elevation angle, the mapping function scale factor value is 1. So, this value will give minimum value for the tropospheric delay (TD) as given below (Schuler, 2001):

$$TD = ZHD.m_h(\varepsilon) + ZWD.m_w(\varepsilon) \tag{1}$$

where:

ZHD is zenith hydrostatic delay (m)
ZWD is zenith wet delay (m)
m_h (ε) is the hydrostatic mapping function (-)
m_w (ε) is the wet mapping function (-)

In this paper, Neill mapping function model for hydrostatic and wet components will be used to be simplified to a simpler equation. This model is selected due to its have many operations and also its ability to achieve mapping function scale factor value down to 3 degree of elevation angle. This simpler equation can be used to calculate the mapping function scale factor by varying the elevation angles in the model.

NEILL MAPPING FUNCTION MODEL

The mapping functions derived by Arthur Neill in 1996, are the most widely used, and are known to be the most accurate and easily-implemented functions (Ahn, 2005). He proposed the new mapping function (NMF) based on temporal changes and geographic location rather than on surface meteorological parameters. He argued that all previously available mapping functions have been limited in their accuracy by the dependence on surface temperature, which causes three dilemmas. All of these are because there is more variability in temperature in the atmospheric boundary layer, from the Earth's surface up to 2000 m.

First, diurnal alterations in surface temperature cause much smaller variations than those calculated from the mapping functions. Second, seasonal changes in surface temperature are normally larger than upper atmosphere changes (but the computed mapping function yields artificially large seasonal variations). Third, the computed mapping function for cold summer days may not significantly differ from warm winter days. For example, actual mapping functions are quite different than computed values because of the difference in lapse rates and heights of the troposphere.

The new mapping functions have been derived from temperature and relative humidity profiles, which are in some sense averages over broadly varying geographical regions. Niell compared NMF and ray traces calculated from radiosonde data spanning about one year or more covering a wide range of latitude and various heights above sea level.

Such comparison was to ascertain the validity and applicability of the mapping function NMF. Through the least-square fit of four different latitude data sets, Niell showed that the temporal variation of the hydrostatic mapping function is sinusoidal within the scatter of the data.

Neill Mapping Function, NMF as given in equation (2) and (3) below state that;

For hydrostatic component;

$$NMF_h(\varepsilon) = \frac{1+\frac{a}{1+\frac{b}{1+c}}}{\sin\varepsilon + \frac{a}{\sin\varepsilon + \frac{b}{\sin\varepsilon + c}}} + \underbrace{\left[\frac{1}{\sin\varepsilon} - \left(\frac{1+\frac{a_{ht}}{1+\frac{b_{ht}}{1+c_{ht}}}}{\sin\varepsilon + \frac{a_{ht}}{\sin\varepsilon + \frac{b_{ht}}{\sin\varepsilon + c_{ht}}}}\right)\right]}_{\text{(height correction terms)}} \quad (2)$$

and for wet component:

$$NMF_w(\varepsilon) = \frac{1+\frac{a_{wet}}{1+\frac{b_{wet}}{1+c_{wet}}}}{\sin\varepsilon + \frac{a_{wet}}{\sin\varepsilon + \frac{b_{wet}}{\sin\varepsilon + c_{wet}}}} \quad (3)$$

where:

ε - elevation angle
$NMF_h(\varepsilon)$ - hydrostatic mapping function
$NMF_w(\varepsilon)$ - wet mapping function
H - station height above sea level (km).

For the hydrostatic NMF mapping function, the parameter a at tabular latitude ϕ_i at time t from January 0.0 (in UT days) is given as:

$$a(\phi, t) = a_{avg}(\phi) + a_{amp}(\phi)\cos\left(\frac{t - DOY}{365.25}2\pi\right) \quad (4)$$

where DOY (day of year) is the adopted phase, DOY = 28 for Northern hemisphere and DOY = 211 for Southern hemisphere. The linear interpolation between the nearest $a(\phi, t)$ is used to obtain the value of parameter $a(\phi, t)$ which is stated as parameter a in equation [2]. For parameters b and c, the same procedure can be applied. Height correction coefficients are given as a_{ht}, b_{ht} and c_{ht} were determined by a leastsquares fit to the height correction at nine elevation angles. However, for the wet NMF mapping function coefficients which are stated as a_{wet}, b_{wet} and c_{wet}, no temporal dependence is included in the wet NMF mapping function. Therefore, only an interpolation in latitude for each parameter is required as described in (Neill, 1996).

Mendes (1999) analyzed the large number of mapping functions by comparing against radiosonde profiles from 50 stations distributed worldwide (32,467 benchmark values). The models that meet the high standards of modern space geodetic data analysis are Ifadis, Lanyi, MTT, and NMF. He found that for elevation angle above 15 degrees, the models Lanyi, MTT, and NMF yield identical mean biases and the best total error performance. At lower elevation angles, Ifadis and NMF are superior.

Simplification of hydrostatic Neill mapping function, $NMF_h(\varepsilon)$

Regression method is used to find the same type of graph for the original NMF. However there is a slight difference for some points of the graph. From the statistical analysis, the difference between the original and the simplified model is small and not significant as described below.

$NMF_h(\varepsilon)$ model has been named as Y, while the simplified models have been named as Y1, Y2 and Y3. These four mapping function models give a graph of parabolic shape. However there is a slight difference between the Y model and the simplified models.

The simplified models (Y1, Y2 and Y3) have been generated using regression method, which give the model in a form of:

$$Y1 = AX^{B} \qquad (5)$$

where

$Y1$: simplified $NMF_h(\varepsilon)$

A, B : constant

X : elevation angle (independent variable).

This model is simpler than the original Y mapping function. By using these simplified models, we can reduce the computation time from 26 operations to only 2 operations. Model Y1 has been generated from regression method, whereby model Y2 and Y3 have been generated based on Y1 model. Model Y2 is formed by fixing the value of constant B and changing the value of constant A; while model Y3 is formed by fixing the value of constant A and changing the value of constant B. Model Y2 and Y3 are formed when they give unity when X is 90 degree.

Sum of Error Calculation For hydrostatic $NMF_h(\varepsilon)$

Sum of error method can be used to show how the simplified models deviate from the original model. The smaller deviation is better, which shows that the simplified model is closer to the original model.

TABLE 1: Sum of error for $NMF_h(\varepsilon)$, Y and simplified models (Y1,Y2,Y3)

X	Y = NMF(h)	Y1 = 33.748X^ (-0.8144)	Y2 = 33.748X^ (-0.782)	Y3 = 39.042*X^ (-0.8144)	(Y - Y1)^2	(Y - Y2)^2	(Y - Y3)^2
2	18.581	19.191	19.626	22.201	0.372	1.093	13.104
5	10.151	9.099	9.586	10.527	1.106	0.319	0.141
10	5.556	5.174	5.575	5.986	0.145	0.000	0.185
15	3.802	3.719	4.060	4.303	0.007	0.067	0.251
20	2.898	2.942	3.242	3.404	0.002	0.119	0.256
25	2.353	2.453	2.723	2.838	0.010	0.137	0.235
30	1.993	2.115	2.361	2.447	0.015	0.136	0.206
35	1.739	1.865	2.093	2.158	0.016	0.125	0.175
40	1.553	1.673	1.886	1.936	0.014	0.111	0.146
45	1.413	1.520	1.720	1.759	0.012	0.094	0.120
50	1.304	1.395	1.584	1.614	0.008	0.078	0.096
55	1.220	1.291	1.470	1.493	0.005	0.062	0.075
60	1.154	1.203	1.373	1.391	0.002	0.048	0.056
65	1.103	1.127	1.290	1.303	0.001	0.035	0.040
70	1.064	1.061	1.217	1.227	0.000	0.023	0.027
75	1.035	1.003	1.153	1.160	0.001	0.014	0.016
80	1.015	0.951	1.097	1.101	0.004	0.007	0.007
85	1.004	0.906	1.046	1.048	0.010	0.002	0.002
90	1.000	0.864	1.000	1.000	0.018	0.000	0.000
Sum of error					**1.748**	**2.469**	**15.138**

From the Table 1 above, although the sum of error is small (1.748), the Y1 model has not been selected due to it does not meet the constraint requirement (0.864), which is at 90 degree the mapping function scale factor should be unity. That is the constraint used in finding the mapping function model. Although the Y3 model meets the requirements, whereby the model gives big value of sum of error (15.138), which is most of the points are scattered quite far from the original $NMF_h(\varepsilon)$ mapping function model.

So, Y2 = 33.748X$^{(-0.782)}$ model has been selected as the simplification mapping function model for $NMF_h(\varepsilon)$ due the smallest sum of error (2.469) compared to the others as given in Figure 2 below.

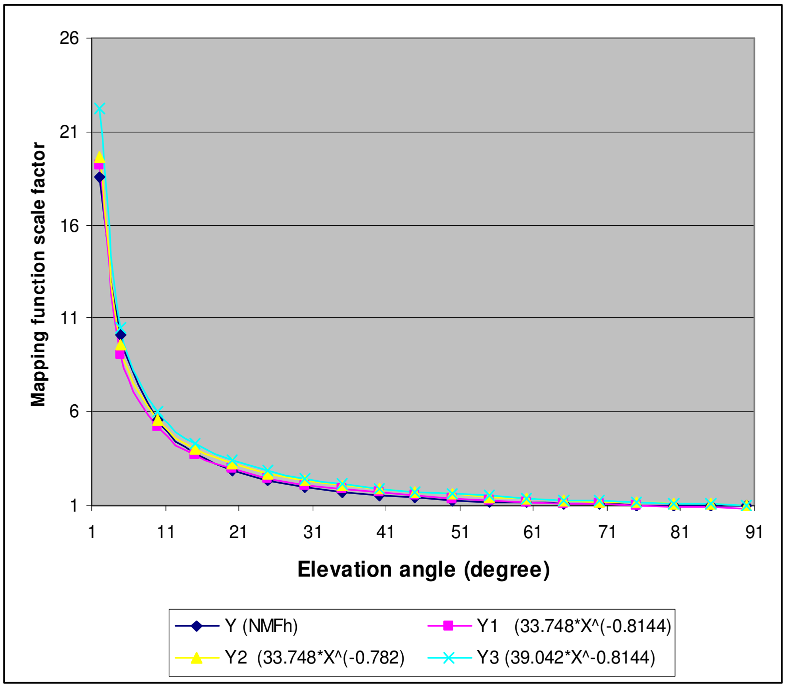

Figure 2: Graph of $NMF_h(\varepsilon)$ mapping function by regression

Simplification of wet Neill mapping function, $NMF_w(\varepsilon)$

$NMF_w(\varepsilon)$ mapping function model has been named as Z, while the simplified models have been named as Z1, Z2 and Z3. These four models give a shape of parabola graph. However there is a slight difference between the Z model and the three simplified models. The simplified models (Z1, Z2 and Z3) have been generated using regression method, which give the model in a form of:

$$Z1 = AX^{B} \qquad (6)$$

where

$Z1$: simplified $NMF_w(\varepsilon)$

A, B : constant

X : elevation angle (independent variable).

These simplified models are simpler than the original $NMF_w(\varepsilon)$ mapping function. By using these simplified models, we can reduce the computation time from 11 operations to only 2 operations. Model Z1 has been generated from regression method, whereby model Z2 and Z3 have been generated based on Z1 model. Model Z2 is formed by fixing the value of constant B and changing the value of constant A, while model Z3 is formed by fixing the value of constant A and changing the value of constant B. As a constraint for Model Z2 and Z3, they give unity values when X is 90 degree.

Calculation of Sum of Error For $NMF_w(\varepsilon)$

Sum of error method can be used to show how the simplified models deviate from the original model. Smaller deviation is better, which shows that the simplified model is closer to the original model.

TABLE 2: Sum of error for $NMF_w(\varepsilon)$, Z and simplified models (Z1,Z2, Z3)

X	Z	Z1= 38.079X^ (-0.8452)	Z2 = 38.079X^ (-0.8088)	Z3 = 44.846X^ (-0.8452)	(Z - Z1)^2	(Z - Z2)^2	(Z - Z3)^2
2	21.854	21.196	21.738	24.963	0.433	0.014	9.663
5	10.751	9.770	10.360	11.507	0.961	0.153	0.571
10	5.657	5.439	5.914	6.405	0.048	0.066	0.559
15	3.833	3.861	4.261	4.547	0.001	0.182	0.509
20	2.911	3.027	3.376	3.565	0.013	0.216	0.428
25	2.360	2.507	2.819	2.952	0.022	0.210	0.351
30	1.997	2.149	2.432	2.531	0.023	0.190	0.285
35	1.741	1.886	2.147	2.222	0.021	0.165	0.231
40	1.554	1.685	1.927	1.985	0.017	0.139	0.185
45	1.413	1.525	1.752	1.797	0.013	0.115	0.147
50	1.305	1.395	1.609	1.643	0.008	0.092	0.115
55	1.220	1.287	1.490	1.516	0.004	0.072	0.087
60	1.154	1.196	1.388	1.409	0.002	0.055	0.065
65	1.103	1.118	1.301	1.317	0.000	0.039	0.046
70	1.064	1.050	1.226	1.237	0.000	0.026	0.030

TABLE 2 (continued): Sum of error for $NMF_w(\varepsilon)$, Z and simplified models (Z1,Z2, Z3)

X	Z	Z1= 38.079X^ (-0.8452)	Z2 = 38.079X^ (-0.8088)	Z3 = 44.846X^ (-0.8452)	(Z - Z1)^2	(Z - Z2)^2	(Z - Z3)^2
75	1.035	0.991	1.159	1.167	0.002	0.015	0.017
80	1.015	0.938	1.100	1.105	0.006	0.007	0.008
85	1.004	0.891	1.048	1.049	0.013	0.002	0.002
90	1.000	0.849	1.000	1.000	0.023	0.000	0.000
Sum of error					**1.610**	**1.759**	**13.299**

From the Table 2 above, the Z1 model has not been selected due to it does not meet the constraint requirement (0.849), which is at 90 degree the mapping function scale factor should be unity, although the sum of error for S1 is small (1.610). Although the Z3 model meets the requirements, but the model gives big value for sum of error (13.299), which is most of the points are scattered quite far from the original $NMF_w(\varepsilon)$ model.

So, Z2 = 38.079X$^{(-0.8088)}$ model has been selected as the simplification mapping function model for $NMF_w(\varepsilon)$ due the smallest sum of error (1.759) compared to the others and it's mapping function gives unity at 90 degree elevation angle as given in Figure 3 below:

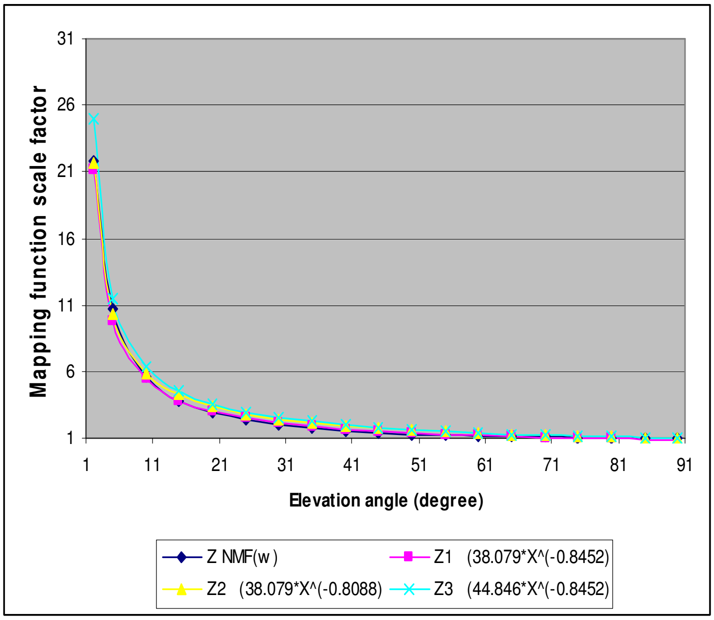

Figure 3: Graph of NMF(w) mapping function (Z), Z1, Z2 and Z3 by regression

DISCUSSION

The original Neill mapping function (NMF) as given in equation (1) and (2) are in a form of continued fraction. By using regression method, the NMF either for hydrostatic and also non hydrostatic components can be simplified to a simpler form as given in equation (4) and (5), which have only 2 operations. The simplification of NMF can reduce the computation time and also can give simpler parabolic equation model. The simplified equations give similar result with the original NMF, for both hydrostatic and also wet components. The result for the mapping function simplifications are given below.

Hydrostatic Neill Mapping Function, $NMF_h(\varepsilon)$

For hydrostatic component, the regression method gives a simpler parabolic model, Y1 in a form of equation (4), which has 2 constants namely as A and B. The Y2 equation has been formed by changing A value while B is unchanged. The Y3 equation has been formed by changing B value while A is unchanged. The calculation of sum of error shows that Y3 model can give smaller value than Y2 value. So, Y3 is similar to the original hydrostatic Neill mapping function model, Y.

Wet Neill Mapping Function, $NMF_w(\varepsilon)$

For wet component, the regression method gives a simpler parabolic model, Z1, which has 2 constants namely as A and B. The Z2 equation has been formed by changing A value while B is unchanged. The Z3 equation has been formed by changing B value while A is unchanged. The calculation of sum of error shows that Z2 model can give smaller value than Z3 value. So, Z2 is similar to the original wet NMF model, Z.

CONCLUSION

In this study, the simplification of hydrostatic and wet component for NMF can reduce the number of operations by using regression method. The models reduction percentage can be shown in Table 3.

TABLE 3 : Reduction percentage of model operation

Model	Operations (Current method)	Operations (Regression method)	Reduction	% Reduction
$NMF_h(\varepsilon)$	26	2	24	92.3
$NMF_w(\varepsilon)$	11	2	9	81.8

Table 3 shows that the operation for $NMF_h(\varepsilon)$ model can be reduced up 92 percent of reduction. The operation reduction can reduce the computing time and also can give better understanding of the models.

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