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On Properties of Modified Degree Six Chordal Rings Network

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ABSTRACT

Chordal rings are circulant graphs and have deserved significant attention in the last decade. Chordal rings were used to build interconnection networks for distributed and parallel systems. There are many of publications analyzing their networks properties. In this paper, we present the main properties of the latest method of chordal rings called Modified Chordal Rings Degree Six *(CHRm6).* The properties are connectivity, Hamiltonian cycle and asymmetry. We prove some lemmas and theorems for every property. All of these properties are useful for further works especially in developing a routing scheme.

Keywords: Chordal Rings, Circulant Graphs, Network Topology, Hamiltonian Cycle, Interconnection networks

INTRODUCTION

A large number of identical processing elements in parallel computer network are currently in various stages of development (Arden and Lee, (1981), Narayanan and Opatrny, (1999), Narayanan and Opatrny, (2001), Dubalski, Bujnowski, and Zabludowski, (2007), Dubalski, Zabludowski and Bujnowski, (2008)). The processing elements can communicate easily to each other. One of the main factors to be considered in the design of interconnection networks is their topology which is related to communication delay, throughput and routing. There are several network architectures proposed in a distributed system. Interprocessor communication is achieved by sending messages along routes or paths in the network.

A chordal ring is one of the important and common network topology (Liestman, Opatrny and Zaragoza, (1998)). Chordal ring is a circulant graph by adding more links or chord lengths into a ring in a uniform way. Adding more links is much better than a few links. A chordal ring is an undirected graph where the vertices correspond to processor or communication ports and the edges correspond to communication channel or links. In a chordal ring research area, most of key features of interest are degree, diameter, connectivity, structures, congestion, symmetry and routing. One can achieve the appropriate parameters such as low diameter, high connectivity and an efficient routing by choosing suitable chord lengths.

Many of chordal ring methods had been proposed and suggested in the literature to achieve better performance. Arden and Lee, (1981) had introduced the first Chordal Rings Degree 3 *(CR3)* by adding a chord into a ring. Doty, (1984), Browne and Hodgson, (1990), Arden and Lee, (1981), Narayanan and Opatrny, (1999, 2001), Bujnowski, Dubalski and Zabludowski, (2004) and many other authors had discussed about Chordal Rings Degree 4 *(CR4)*. Dubalski, Bujnowski and Zabludowski, (2007) had proposed the first modified structure called as Modified Degree 4 Chordal Rings *(CHRm4)*. Atheer and Othman, (2005) had discussed about the Uniqueness Property of Minima when diameter is given. Dubalski, Bujnowski and Zabludowski, (2008) also had discussed about the comparison of Chordal Rings Degree Six *(CR6)* and *CR4* in terms of structure, formulations, geometrical representation, diameter and average path lengths. Modified Degree Six Chordal Rings *(CHRm6)* is the latest method of chordal rings and were proposed by R.N. Farah, M. Othman, M.H. Selamat and Y.H. Peng, (2008). R.N. Farah, M. Othman, M.H. Selamat and Y.H. Peng, (2008) had analyzed the structure, tree visualization, paths, formulations and geometrical representation between *CHRm6* and *CR6*.

The purpose of this paper is to study the main properties (connectivity, Hamiltonian cycle and asymmetry) of *CHRm6*. We will focus on circulant graph of degree six, *CHRm6*. This topology has different properties compared to the previous traditional chordal rings. There are no researchers working on *CHRm6* properties until now. In this paper, we describe all the main properties in the following section.

PROPERTIES

Connectivity

Connectivity is one of the basic concepts of graph theory (Gross and Yellen, (2004)). It is closely related to the theory of network flow problem. The connectivity of a graph is an important measure of its robustness as a network. Definition 1 below describes how to construct *CHRm6* (R.N. Farah, M. Othman, M.H. Selamat and Y.H. Peng, (2008)). Messages can be easily sent from source node to destination node without intermediate node for common neighbor node (source node directly connected to destination node), and vice versa for not common neighbor node (source node not directly connected to destination node and must go through the intermediate node).

Definition 1: The modified degree six chordal rings called *CHRm6* is an undirected circulant graphs. *CHRm6* is denoted by $CHRm6(N, s, h_1, h_2, h_3)$ where N is the number of nodes, s is a ring edge with length 1, while h_1, h_2 and h_3 are chords by even lengths where $h_1 < h_2 < h_3$. *CHRm6* consists of one ring with N nodes, where N is positive even number of nodes. Each even node, i_{2k} and odd node, i_{2k+1} are additionally connected to four nodes for $0 \le k < N/2$. i_{2k} is connected to $i_{(2k-h_1)(\bmod N)}, i_{(2k+h_1)(\bmod N)}, i_{(2k-h_3)(\bmod N)}$ and $i_{(2k+h_3)(\bmod N)}$. i_{2k+1} is connected to $i_{(2k+1-h_2)(\bmod N)}, i_{(2k+1+h_2)(\bmod N)}$, $i_{(2k+1-h_3)(\bmod N)}$ and $i_{(2k+1+h_3)(\bmod N)}$. The values of N and h_1, N and h_2, N and h_3 must have $\gcd(N, h_1, h_2, h_3) = 2$.

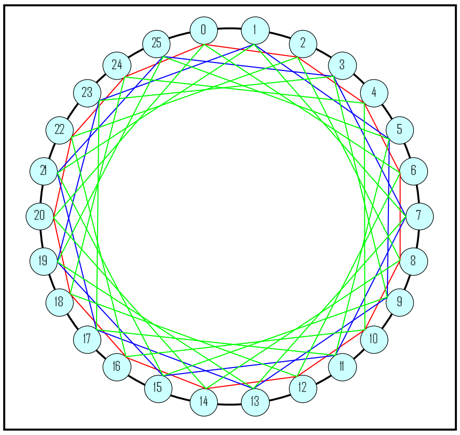

Figure 1: Shows the example of *CHRm6* structure.

This paper studies the connectivity between source node, i and destination node, j. For a connected graph *CHRm6*, the distance $d(i, j)$ between two vertices i and j is defined as the minimum of the lengths of the $i-j$ paths of *CHRm6*. Postulate 1 describes how *CHRm6* fulfills the distance functions.

Postulate 1: A *CHRm6* obeys all distance functions of:

1) $d(i,j) \geq 0$ for all pairs i, j of vertices of *CHRm6* and $d(i,j) = 0$ if and only $i = j$.

2) $d(i,j) = d(j,i)$ for all pairs i, j of vertices of *CHRm6*.

3) $d(i,k) \leq d(i,j) + d(j,k)$ for all i, j, k .

Theorem 1 below describes about all connected internodes in *CHRm6*. This asserts that no disjoint between nodes and messages can simply reach destination node via many paths in $m6$.

Theorem 1: $CHRm6(N, s, h_1, h_2, h_3)$ is all connected if and only if $\gcd(N, s, h_1, h_2, h_3) = 1$.

Proof.
Necessity : If $\gcd(N, s, h_1, h_2, h_3) = d < 1$, then node i_{2k} can only reach node $i_{(2k \pm h_1)}$ if $i_{2k} \equiv i_{(2k \pm h_1)(\bmod d)}$ and reach node $i_{(2k \pm h_3)}$ if $i_{2k} \equiv i_{(2k \pm h_3)(\bmod d)}$ while node $i_{(2k+1)}$ can only reach $i_{(2k+1 \pm h_2)}$ if $i_{(2k+1)} \equiv i_{(2k+1 \pm h_2)(\bmod d)}$ and reach node $i_{(2k+1 \pm h_3)}$ if $i_{(2k+1)} \equiv i_{(2k+1 \pm h_3)(\bmod d)}$ where $0 \leq k < \frac{N}{2}$.

Sufficiency: Suppose $\gcd(N, s, h_1, h_2, h_3) = 1$. Then there exist p, q, r and t such that $ps + qh_1 + rh_2 + th_3 = \alpha$, where $\alpha = \gcd(s, h_1, h_2, h_3)$. Since $\gcd(N, s, h_1, h_2, h_3) = 1$, i.e. $\gcd(N, \alpha) = 1$, there exist β such that $\beta(ps + qh_1 + rh_2 + th_3) = \alpha \cdot \beta \equiv 1(\bmod N)$.

Let
1) $|\beta p|, s$ - steps $((-s)$ steps if $p < 0)$,
2) $|\beta q|$, h_1 - steps $((-h_1)$ steps if $q < 0)$,
3) $|\beta r|$, h_2 - steps $((-h_2)$ steps if $r < 0)$ and
4) $|\beta t|$, h_3 - steps $((-h_3)$ steps if $t < 0)$.

Every even node, i_{2k} and odd node, i_{2k+1} was connected by induction. □

Hamiltonian Cycle

A Hamiltonian cycle of a graph *CHRm6* visits every vertex exactly once (Gross, Jonathan L. and Yellen, Jay., (2004)). All *CHRm6* contains Hamiltonian cycle because it is a complete graph where each node is connected to everyother node. Lemma 1 shows how the Hamiltonian cycle occurs.

Lemma 1: A Hamiltonian cycle of *CHRm6* cannot contain:

1) $\{\overrightarrow{h_1},\overleftarrow{h_1}\}$,

2) $\{-\overrightarrow{h_1},-\overleftarrow{h_1}\}$,

3) $\{\overrightarrow{h_2},\overleftarrow{h_2}\}$,

4) $\{-\overrightarrow{h_2},-\overleftarrow{h_2}\}$,

5) $\{\overrightarrow{h_3},\overleftarrow{h_3}\}$ and

6) $\{-\overrightarrow{h_3},-\overleftarrow{h_3}\}$

Proof:

The nodes in Figure 2 represented as a part of 3D geometrical representation of *CHRm6*. s represent ring edges, h_1 represent a chord length for even nodes, h_2 represent a chord length for odd nodes and h_3 represent a chord length for both even and odd nodes. For $0 \le k < \frac{N}{2}$, the chord links for even nodes are $i_{2k} \to i_{2k+h_1}, i_{2k} \to i_{2k-h_1}, i_{2k} \to i_{2k+h_3}$ and $i_{2k} \to i_{2k-h_3}$, the chord links for odd number are $i_{2k+s} \to i_{2k+s+h_2}, i_{2k+s} \to i_{2k+s-h_2}, i_{2k+s} \to i_{2k+s+h_3}$ and i_{2k+s-h_3}. The edges links for even nodes are $i_{2k} \to i_{2k+s}$ and $i_{2k} \to i_{2k-s}$ and for odd nodes are $i_{2k+s} \to i_{2k+2s}$ and $i_{2k+s} \to i_{2k}$. Let H be a Hamiltonian cycle of $CHRm6(N,s,h_1,h_2,h_3)$. Suppose to the contrary that there exist $\overrightarrow{h_1}$ chords and $\overleftarrow{h_1}$ chords or $-\overrightarrow{h_1}$ chords and $-\overleftarrow{h_1}$ chords for even nodes, $\overrightarrow{h_2}$ chords and $\overleftarrow{h_2}$ chords or $-\overrightarrow{h_2}$ chords and $-\overleftarrow{h_2}$ chords for odd nodes and $\overrightarrow{h_3}$ chords and $\overleftarrow{h_3}$ chords or $-\overrightarrow{h_3}$ chords and $-\overleftarrow{h_3}$ chords for both even and odd nodes. In that case H can be represented as a sequence of $\{s,\overrightarrow{h_1},\overleftarrow{h_1}\}$ or $\{s,-\overrightarrow{h_1},-\overleftarrow{h_1}\}$ or $\{s,\overrightarrow{h_2},\overleftarrow{h_2}\}$ or $\{s,-\overrightarrow{h_2},-\overleftarrow{h_2}\}$ or $\{s,\overrightarrow{h_3},\overleftarrow{h_3}\}$ or $\{s,-\overrightarrow{h_3},-\overleftarrow{h_3}\}$ or

$\{\overrightarrow{h_1},\overleftarrow{h_1},s\}$ or $\{-\overrightarrow{h_1},-\overleftarrow{h_1},s\}$ or $\{\overrightarrow{h_2},\overleftarrow{h_2},s\}$ or $\{-\overrightarrow{h_2},-\overleftarrow{h_2},s\}$ or $\{\overrightarrow{h_3},\overleftarrow{h_3},s\}$ or $\{-\overrightarrow{h_3},-\overleftarrow{h_3},s\}$. It is not possible for H to contain $\{\overrightarrow{h_1},\overleftarrow{h_1}\}$, $\{-\overrightarrow{h_1},-\overleftarrow{h_1}\}$, $\{\overrightarrow{h_2},\overleftarrow{h_2}\}$ or $\{-\overrightarrow{h_2},-\overleftarrow{h_2}\}$ or $\{\overrightarrow{h_3},\overleftarrow{h_3}\}$ or $\{-\overrightarrow{h_3},-\overleftarrow{h_3}\}$ as a subsequence. There are many possibilities for combining $s,h_1,h_2,h_3,-h_1,-h_2,-h_3$ except $\{...,\overrightarrow{h_1},\overleftarrow{h_1},...\}$,$\{...,-\overrightarrow{h_1},-\overleftarrow{h_1},...\}$,$\{...,\overrightarrow{h_2},\overleftarrow{h_2},...\}$,$\{...,-\overrightarrow{h_2},-\overleftarrow{h_2},...\}$,$\{...,\overrightarrow{h_3},\overleftarrow{h_3},...\}$ or $\{...,-\overrightarrow{h_3},-\overleftarrow{h_3},...\}$. Figure 2 shows H can consists of $i_{2k} \to i_{2k+s} \to ...$, $i_{2k} \to i_{2k-s} \to ...$, $i_{2k} \to i_{2k+h_1} \to ...$, $i_{2k} \to i_{2k-h_1} \to ...$, $i_{2k} \to i_{2k+h_3} \to ...$ or $i_{2k} \to i_{2k-h_3} \to ...$ for even nodes, while $i_{2k+s} \to i_{2k+2s} \to ...$, $i_{2k+s} \to i_{2k} \to ...$, $i_{2k+s} \to i_{2k+s+h_2} \to ...$, $i_{2k+s} \to i_{2k+s-h_2} \to ...$, $i_{2k+s} \to i_{2k+s+h_3} \to ...$ or $i_{2k+s} \to i_{2k+s-h_3} \to ...$ for odd nodes. □

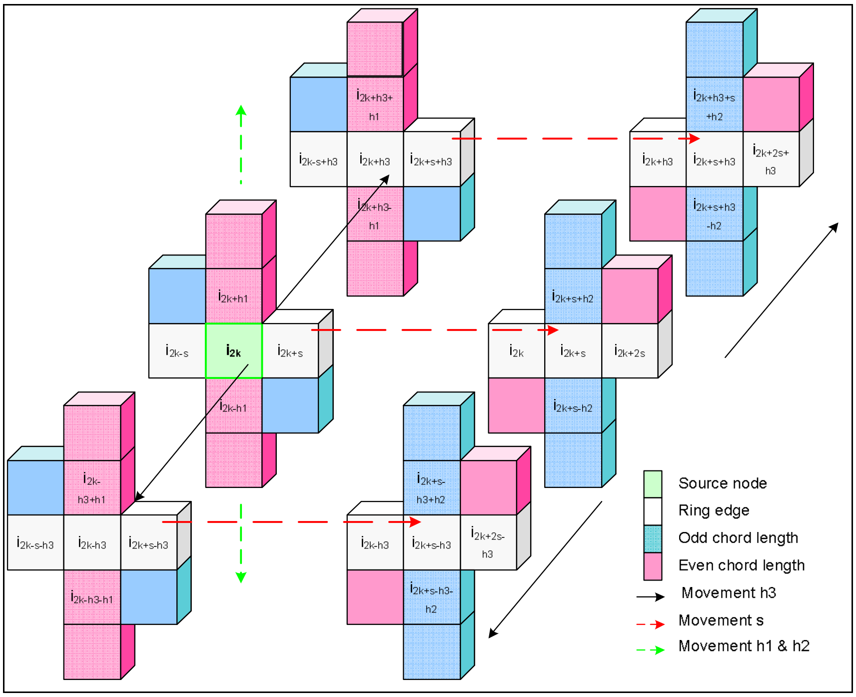

Figure 2: 3D Geometrical Representation of *CHRm6*

Corollary 1: A *CHRm6* must be consists of $s = 1$. All *CHRm6* is a circulant graphs that consists of ring edges 1.

The following lemma describes a Hamiltonian cycle that is constructed by ring edges.

Lemma 2: A *CHRm6* contains a Hamiltonian cycle with at least one of *s*-edges of length 1.

Proof:
A geometrical representation of ring edges are shown in Figure 3. Let H be a Hamiltonian cycle of $CHRm6\ (N, s, h_1, h_2, h_3)$. Let $0 \le k < \frac{N}{2}$, even node, i_{2k} will go to its right and continue with another right blocks or go to its left and continue with another left blocks. H can consist of $i_{2k} \to i_{2k+s} \to i_{2k+2s} \to ... \to i_{2k}$ or $i_{2k} \to i_{2k-s} \to i_{2k-2s} \to ... \to i_{2k}$. This basic concept of ring edges connection is applied to the entire of ring nodes.

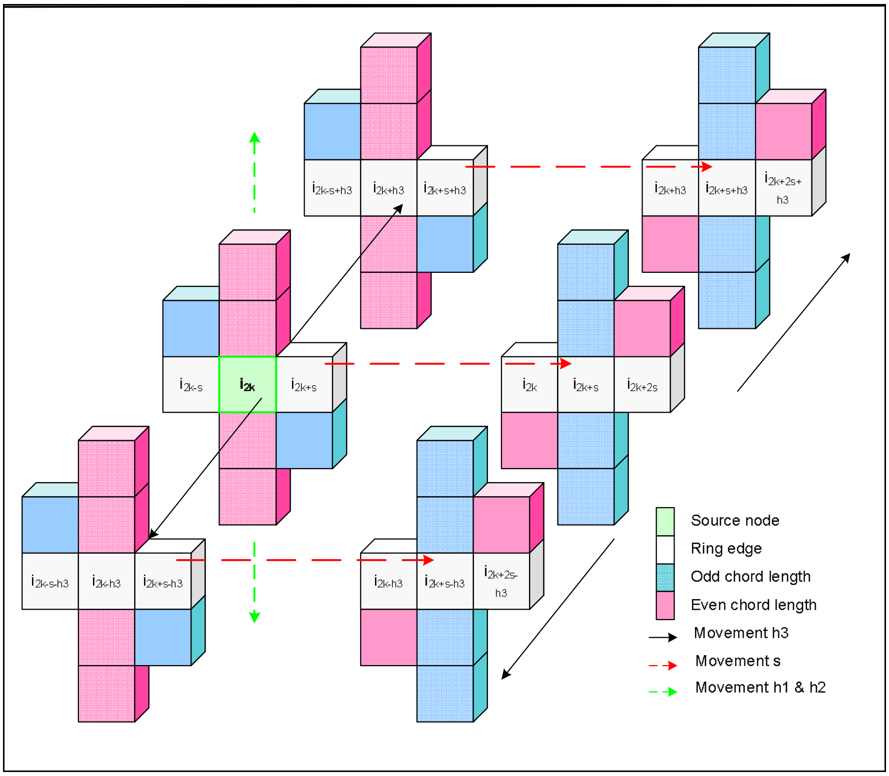

Figure 3: 3D Geometrical Representation of Ring Edges *CHRm6*

We can prove whether a *CHRm6* contains Hamiltonian cycle or not by the following theorem. This theorem shows the relation between Hamiltonian cycle and all connected graph for coprime $(\gcd = 1)$.

Theorem 2: A $CHRm6(N, s, h_1, h_2, h_3)$ contains a Hamiltonian cycle if an only if $\gcd(N, s, h_1, h_2, h_3) = 1$.

Proof:
Let H be a Hamiltonian cycle of CHRm6. $CHRm6(N, s, h_1, h_2, h_3)$ is H if $\gcd(N, s, h_1) = 1, \gcd(N, s, h_2) = 1$ and $\gcd(N, s, h_3) = 1$ and it is true conversely.

Corollary: A *CHRm6* must be consists of $s = 1$. All *CHRm6* is a circulant graphs that consists of ring edges 1.

Asymmetric Property

Asymmetry interconnection network is to be constrasted from symmetry interconnection network which has the property that the network viewed from any vertex of the network looks the same. In this paper, we propose a new approach of asymmetry for *CHRm6* where a small asymmetry can be seen clearly between even and odd nodes for the entire graph. However this $CHRm6$ will be asymmetric if we divide into two classes of nodes. The following definition describes the asymmetry in *CHRm6*.

Definition 2: Two nodes that is between even source node, (i_{2k}) and its destination node, $(i_{2k\pm1})$ or $i_{2k\pm h_1}$ or $i_{2k\pm h_3}$ in a *CHRm6* are not similar with odd source node, (i_{2k+1}) and its destination node, $(i_{2k+1\pm1})$ or $(i_{2k+1\pm h_2})$ or $(i_{2k+1\pm h_3})$, if for some automorphism α for i_{2k} case of *CHRm6*, $\alpha((i_{2k})) = i_{2k\pm1}$, $\alpha((i_{2k})) = i_{2k\pm h_1}$, $\alpha((i_{2k})) = i_{2k\pm h_3}$ and for (i_{2k+1}) case of *CHRm6*, $\alpha((i_{2k+1})) = i_{2k+1\pm1}$, $\alpha((i_{2k+1})) = i_{2k+1\pm h_2}$, $\alpha((i_{2k+1})) = i_{2k+1\pm h_3}$ with (i_{2k}), $(i_{2k\pm1})$, $(i_{2k\pm h_1})$, $(i_{2k\pm h_3})$, (i_{2k+1}), $(i_{2k+1\pm1})$, $(i_{2k+1\pm h_2})$, $(i_{2k+1\pm h_3}) \in N$. Two edges between even case $(i_{2k}, i_{2k\pm h_1})$ and odd case $(i_{2k+1}, i_{2k+1\pm h_2})$ in a *CHRm6* are not similar, if for some automorphism α for $\alpha\left(\left(i_{2k}, i_{2k\pm h_1}\right)\right) \neq \left(i_{2k+1}, i_{2k+1\pm h_2}\right)$ with $(i_{2k}, i_{2k\pm h_1})$, $(i_{2k+1}, i_{2k+1\pm h_2}) \in h$.

1) *CHRm6* is called not node-symmetric if not every pair of nodes is similar
2) *CHRm6* is called not edge-symmetric if not every pair of edges is similar
3) *CHRm6* is called asymmetric if it is not node-symmetric or not edge-symmetric or both.

Clearly, we can see that not every pair of nodes in *CHRm6* is similar between even and odd nodes.

Theorem 3: A *CHRm6* is not node symmetric.

Proof:
The connection between two nodes for even case, $(i_{2k}) \to (i_{2k\pm1})$, $(i_{2k}) \to (i_{2k\pm h_1})$, $(i_{2k}) \to (i_{2k\pm h_1})$, $(i_{2k}) \to (i_{2k\pm h_3})$ are not similar with the connection between two nodes for odd case, $(i_{2k+1}) \to (i_{2k+1\pm1})$, $(i_{2k+1}) \to (i_{2k+1\pm h_2})$, $(i_{2k+1}) \to (i_{2k+1\pm h_3})$ in a *CHRm6* with $(i_{2k}),(i_{2k\pm1})$, $(i_{2k\pm h_1}),(i_{2k\pm h_3}),(i_{2k+1}),(i_{2k+1\pm h_2}),(i_{2k+1\pm h_3}) \in N$. Hence the proof.

Not every pair of edges in *CHRm6* is similar.

Theorem 4: A *CHRm6* is not edge symmetric.

Proof:
Let be two edges $(i_{2k}, i_{2k\pm1})$ and $(i_{2+k}, i_{2k+1\pm1})$ are similar, if for some automorphism α of $CHRm6$, $\alpha((i_{2k}, i_{2k\pm1})) + (i_{2k+1}, i_{2k+1\pm1})$, also for $(i_{2k}, i_{2k\pm h_3})$ and $(i_{2k+1}, i_{2k+1\pm h_3})$ are similar if for some automorphism α of $CHRm6$, $\alpha((i_{2k}, i_{2k\pm h_3})) = (i_{2k+1}, i_{2k+1\pm h_3})$ except $(i_{2k}, i_{2k\pm h_1})$ and $(i_{2k+1}, i_{2k+1\pm h_2})$ are not similar if for some automorphism α of $CHRm6$, $\alpha((i_{2k}, i_{2k\pm h_1})) \neq (i_{2k+1}, i_{2k+1\pm h_2})$ with $(i_{2k}, i_{2k\pm1}),(i_{2k}, i_{2k\pm h_1}),(i_{2k}, i_{2k\pm h_3})$, $(i_{2k+1}, i_{2k+1\pm1}),(i_{2k}, i_{2k\pm1}),(i_{2k}, i_{2k\pm h_1}),(i_{2k}, i_{2k\pm h_3}),(i_{2k+1}, i_{2k+1\pm1})$, $(i_{2k+1}, i_{2k+1\pm h_2})$, $(i_{2k+1}, i_{2k+1\pm h_3}) \in h$. Hence every pair of edges for even case and odd case is not similar.

CONCLUSION

In this paper, we present some useful and important properties of *CHRm6*. All the nodes are connected to each other by Definition 1. We proved that a *CHRm6* contains at least one Hamiltonian cycle constructed by ring edges. The Hamiltonian cycle is important especially for updating messages and reducing delay. For example, an update message will be send to each of the node in a $CHRm6$ exactly once and back to itself. Then any updated node can send the message to the other node without resorting to the first source node that sends the updated message. The structure of *CHRm6* is very different from traditional chordal rings (*CR3, CR4* and *CR6*). This is clearly shown by asymmetric property but *CHRm6* can be symmetric if we divide into two classes. The first class is for even nodes and the second class is for odd nodes. It makes *CHRm6* more interesting. Thus the paths or routes can be examined by two cases of source node. One should explore the optimum routing scheme for *CHRm6* and referred to all these properties.

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