Malaysian Journal of Mathematical Sciences 5(1): 13-25 (2011)

Radiation Effects on Marangoni Convection over a Flat Surface with Suction and Injection

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ABSTRACT

The radiation effect on a steady two-dimensional Marangoni convection flow over a permeable flat surface is studied numerically. The general governing partial differential equations are transformed into a set of two nonlinear ordinary differential equations by using unique similarity transformation. Numerical solutions of the similarity equations are obtained using the shooting method. Numerical results are obtained for the interface velocity and the surface temperature gradient as well as the velocity and temperature profiles for some values of the governing parameters. The results indicate that the heat transfer rate at the surface decreases as the radiation parameter increases. The effects of suction or injection parameter on the flow and heat transfer characteristics are discussed.

Keywords: Marangoni convection, thermal radiation, similarity solutions, suction/injection.

INTRODUCTION

In recent times a good deal of attention has been devoted to the studies of Marangoni convection. This is mainly because it is important in crystal growth melts and greatly influences other industrial processes. Nishino *et al.* (2007) stated that Marangoni convection negatively affects the quality of silicon crystals for semiconductors and the convection also occurs in heat pipe for heat radiation devices of personal computers. The surface tension gradient variations along the interface may induce the Marangoni convection. In particular, the surface tension gradients that are responsible of Marangoni convection can be due to gradients of temperature (thermal convection) and/or concentration (solutal convection). Many researchers have investigated Marangoni convection in various geometries such as Okano *et al.* (1989), Christopher *et al.* (1998), Arafune *et al.* (1999), and

Pop *et al.* (2001). The most relevant paper to this work includes Christopher and Wang (2001) who studied the effects of Prandtl number for Marangoni convection over a flat surface. Meanwhile, Al-Mudhaf and Chamkha (2005) presented similarity solutions for MHD Marangoni convection in the presence of heat generation or absorption effects. Exact analytical solutions of thermosolutal Marangoni flows in the presence of temperature-dependent volumetric heat source/sinks as well as of a first order chemical reaction have been examined by Magyari and Chamkha (2007).

It was realized that the studies of thermal radiation and heat transfer are important in electrical power generation, astrophysical flows, solar power technology and other industrial areas. Recently, there is a lot of work on boundary layer flow involving radiation (Chamkha *et al.* (2001); Elbashbeshy and Dimian (2002); Chaudhary *et al.* (2006); Bataller (2008)). Ishak (2009) investigated the radiation effects on the flow and heat transfer over a moving plate in a parallel stream. He showed that the existence of thermal radiation is to reduce the heat transfer rate at the surface. Recently, Ishak (2010) considered the thermal boundary layer flow induced by a linearly stretching sheet in a micropolar fluid with radiation effect and he also found that the heat transfer rate at the surface decreases as the radiation parameter increases.

Motivated by these previous works, we aim to study the Marangoni flow over a permeable flat surface with the effect of thermal radiation. The present study may be regarded as the extension of the paper by Christopher and Wang (2001). Besides, the present results also study the effects of suction and injection parameter of the problem.

PROBLEM FORMULATION

Consider steady laminar boundary layer flow of an electrically-conducting fluid over a flat surface in the presence of surface tension due to temperature gradients at the wall. The surface is assumed to be permeable so as to allow for possible suction and injection at the wall. Unlike the Boussinesq effect in buoyancy-induced flow, the Marangoni effect acts as a boundary condition on the governing equations for the flow fields. Taking into account the thermal radiation term in the energy equation, the governing equations are (see Christopher and Wang (2001))

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}, \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y}, \tag{3}$$

subject to the boundary conditions

$$v(x,0)=v_0,\;\; T(x,0)=T(0,0)+Ax^{m+1},\;\; \mu\frac{\partial u}{\partial y}=-\frac{d\sigma}{dT}\frac{\partial T}{\partial x} \;\text{ at }\; y=0 \tag{4}$$

$$u(x,\infty)=0,\;\; T(x,\infty)=T_\infty \;\text{ as }\; y\to\infty$$

where u and v are the velocity components along the x and y axes, respectively, v is the kinematic viscosity, k is the thermal conductivity, c_p is the specific heat of the fluid at a constant pressure, ρ is the density, q_r is the radiative heat flux, T is the temperature across the thermal boundary layer, A is the temperature gradient coefficient, m is the constant exponent of the temperature, μ is the dynamic viscosity and σ is the surface tension.

Using the Rosseland approximation for radiation (see Brewster (1992)), the radiative heat flux is simplified as

$$q_r=-\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \tag{5}$$

where $\sigma*$ is the Stefan-Boltzmann constant and $k*$ is the mean absorption coefficient. It is assumed that the temperature differences within the flow such the term T^4 may be expressed as a linear function of temperature. This is accomplished by expanding T^4 in a Taylor's series about T_∞ and neglecting higher-order terms, thus

$$T^4\approx 4T_\infty^3 T-3T_\infty^4 \tag{6}$$

Using (5) and (6), equation (3) reduces to (see Ishak (2009))

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha(1+N_R)\frac{\partial^2 T}{\partial y^2} \tag{7}$$

where $\alpha = k/(\rho c_p)$ is the thermal diffusivity and $N_R = 16\sigma^* T_\infty^3/(3kk^*)$ is the radiation parameter.

In order to obtain a solution of Equations (1), (2) and (7), the following similarity variables are introduced (see Christopher and Wang (2001)):

$$\eta = C_1 x^{\frac{m-1}{3}} y$$

$$f(\eta) = C_2 x^{\frac{-2-m}{3}} \psi(x,y) \tag{8}$$

$$\theta(\eta) = \frac{(T(x,y)-T(0,0))x^{-1-m}}{A}$$

where m, A, C_1 and C_2 are constants with A, C_1 and C_2 given by

$$A=\frac{\Delta T}{L^{m+1}},\ C_1=\sqrt[3]{\frac{\left(d\sigma/dT\right)A\rho}{\mu^2}},\ C_2=\sqrt[3]{\frac{\rho^2}{\left(d\sigma/dT\right)A\mu}} \tag{9}$$

with L being the length of the surface and ΔT is the constant characteristic temperature. Substituting (8) into equations (2) and (7), we obtain the following ordinary differential equations:

$$f''' - \left(\frac{2m+1}{3}\right)f'^2 + \left(\frac{2+m}{3}\right)ff'' = 0 \tag{10}$$

$$\left(\frac{1+N_R}{\Pr}\right)\theta'' + \left(\frac{2+m}{3}\right)f\theta' - (1+m)f'\theta = 0 \tag{11}$$

where primes denote differentiation with respect to η and $\Pr = \nu/\alpha$ is the Prandtl number. The boundary conditions (4) now become

$$f(0) = f_0,\ f''(0) = -1,\ \theta(0) = 1 \tag{12}$$

$$f'(\infty) = 0,\ \theta(\infty) = 0$$

with f_0 is the dimensionless suction or injection velocity.

The surface velocity is given by (see Christopher and Wang (2001))

$$u(x,0) = \sqrt[3]{\frac{((d\sigma / dT)A)^2}{\rho\mu}} f'(0) x^{(2m+1)/3}. \tag{13}$$

The temperature gradient coefficient can be defined in terms of the total temperature difference along a surface of length L as $A = \Delta T / L^{m+1}$, so the Marangoni number can then be defined for a general temperature profile as

$$Ma_L = \frac{(d\sigma / dT)(\Delta T / L^{m+1}) L^{m+2}}{\mu a}$$

$$= \frac{(d\sigma / dT)\Delta T L}{\mu a} \tag{14}$$

The Reynolds number defined in terms of the surface velocity is then related to the Marangoni number as

$$\mathrm{Re}_L = \frac{u(x,0)L}{\nu} = f'(0) Ma_L^{2/3} \mathrm{Pr}^{-2/3} \tag{15}$$

The total mass flow in the boundary layer per unit width is given by

$$\dot{m} = \int_0^\infty \rho u dy = \sqrt[3]{\frac{d\sigma}{dT} A\rho\mu} x^{(m+2)/3} f(\infty) \tag{16}$$

which can be written in dimensionless form as

$$\overline{\mathrm{Re}}_x = \frac{\rho\bar{u}\delta}{\mu} = f(\infty) Ma_x^{1/3} \mathrm{Pr}^{-1/3} \tag{17}$$

Analysis of the similarity transformation shows that both are true if

$$C_1 L^{(m+2)/3} = \sqrt[3]{\frac{(d\sigma / dT) A\rho L^{m+2}}{\mu^2}}$$
$$= Ma_L^{1/3}\,\mathrm{Pr}^{-1/3} \gg 1 \tag{18}$$

The numerical results can be used to show that the boundary layer assumptions hold within the momentum boundary layer for $Ma_L / \mathrm{Pr} > 10^6$ and for $m \le 2$.

RESULTS AND DISCUSSION

The nonlinear ordinary differential equations (10) and (11) subject to the boundary conditions (12) are solved numerically using the shooting method. In order to discuss the problem under consideration, the results of numerical computations are presented in the form of velocity and temperature profiles. Figures 1 to 5 illustrate the influence of the suction or injection parameter f_0 on the surface velocity, velocity and temperature profiles. Figure 1 shows the variations of the reduced surface velocity $f'(0)$ with m, where $m = 0$ refers to a linear variation of the surface temperature with the distance x measured along the flat plate, $m = 1$ refers to a quadratic variation of the surface temperature with x, while $m = -0.5$ refers to a surface temperature variation relative to the square root of x. Figures 2 and 3 present the distribution of the dimensionless velocity $f'(\eta)$ and temperature $\theta(\eta)$ profiles for several values of the Prandtl number Pr when $m = 0$ and $N_R = 0$. The thickness of the thermal boundary layer increases with the decreasing Prandtl number as shown in Figure 3. In addition, fluid wall suction decreases as Prandlt number increases while fluid wall injection produces the opposite effect. It should be also noticed that for $f_0 = 0$ (impermeable surface) in Figures 1 to 3, the results obtained by Christopher and Wang (2001) are reproduced. We found that, imposition of fluid suction $(f_0 > 0)$ at the wall has the tendency to decrease the fluid velocity and temperature. However, fluid injection $(f_0 < 0)$ increases in the fluid velocity and temperature.

Effect of radiation parameter N_R on temperature of the fluid with the influence of the suction or injection parameter f_0 is presented in Figures 6 to 9. Figure 6 displays the variation with N_R of the reduced temperature gradient, $-\theta'(0)$ with different values of f_0. One can see that the reduced temperature gradient, $-\theta'(0)$ decreases as N_R increases. Thus, the heat transfer rate at the surface decreases in the presence of radiation. This result qualitatively agrees with expectations, since the effect of radiation is to decrease the rate of energy transport to the fluid, thereby decreasing the temperature of the fluid. From Figures 7 to 9, we observe that the temperature profiles increase as radiation parameter N_R increases. Thus, radiation can be used to control the thermal boundary layers quite effectively.

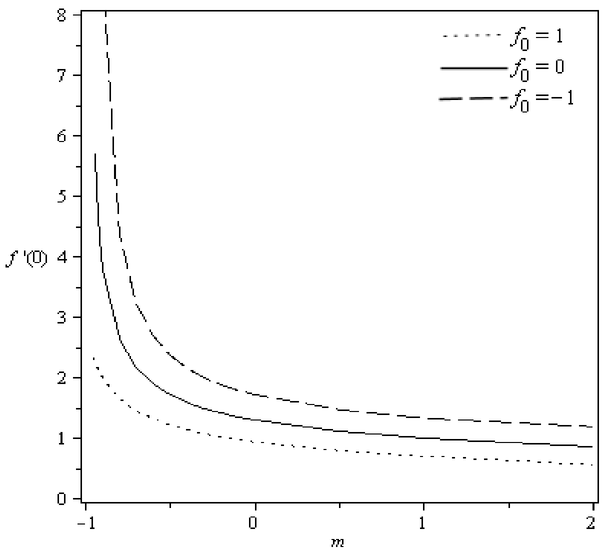

Figure 1: Variations of surface velocity with temperature gradient exponent m for various f_0.

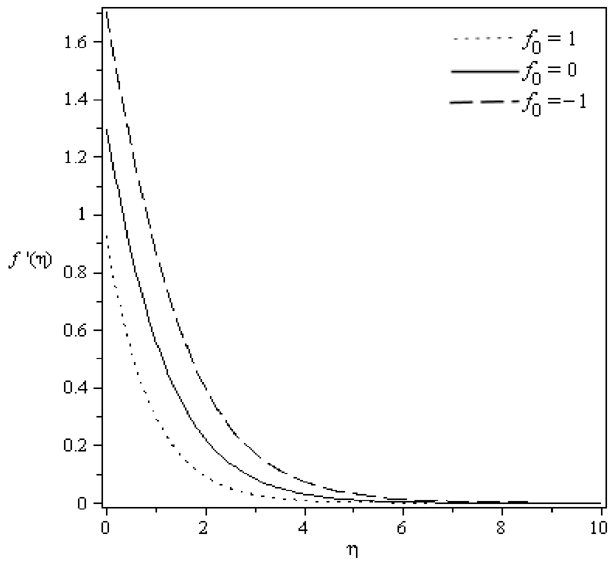

Figure 2: Velocity profiles for different values of f_0 when $m = 0$.

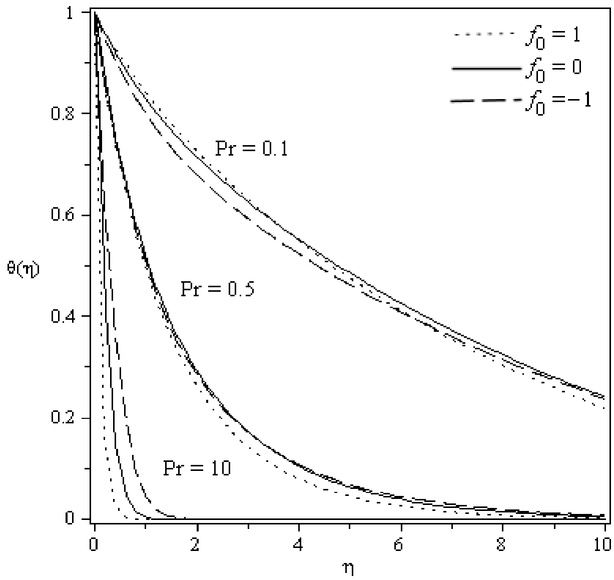

Figure 3: Temperature profiles for various values of Pr and f_0 when $m = 0$ and $N_R = 0$.

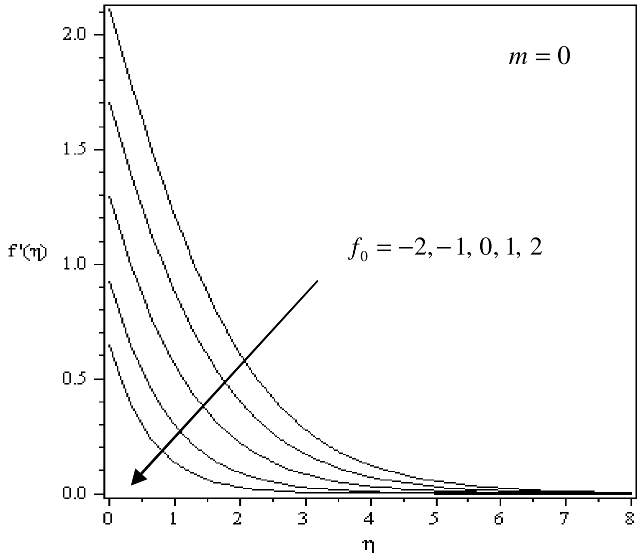

Figure 4: Effects of f_0 on velocity profiles when $m = 0$.

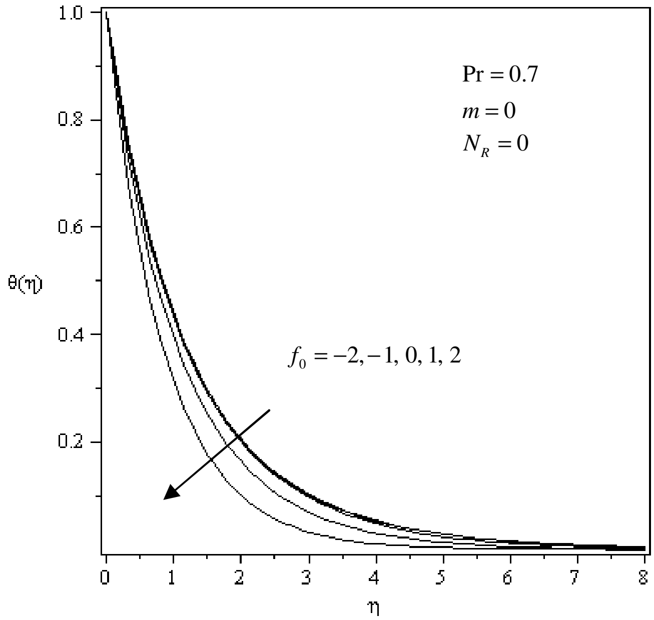

Figure 5: Effects of f_0 on temperature profiles when $m = 0$, and $\Pr = 0.7$.

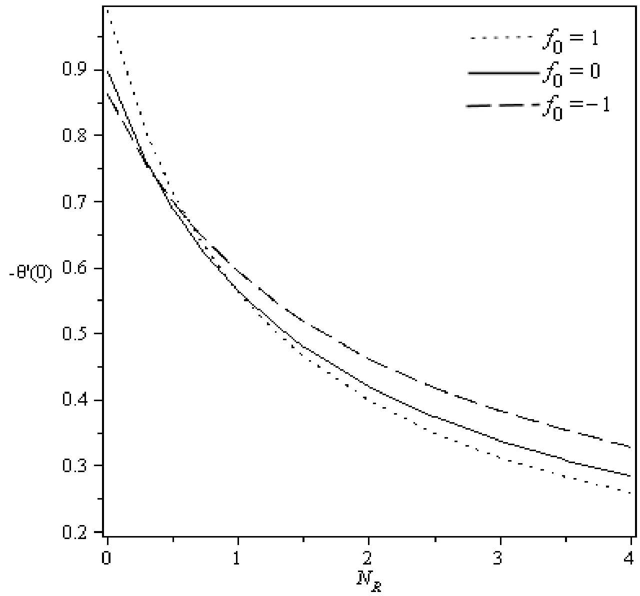

Figure 6: Variation of $-\theta'(0)$ with N_R for various f_0

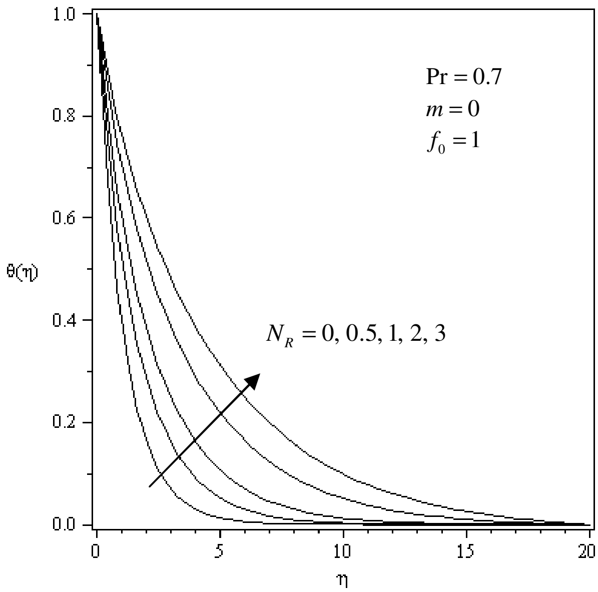

Figure 7: Effects of N_R on temperature profiles when $f_0 = 1$ (suction)

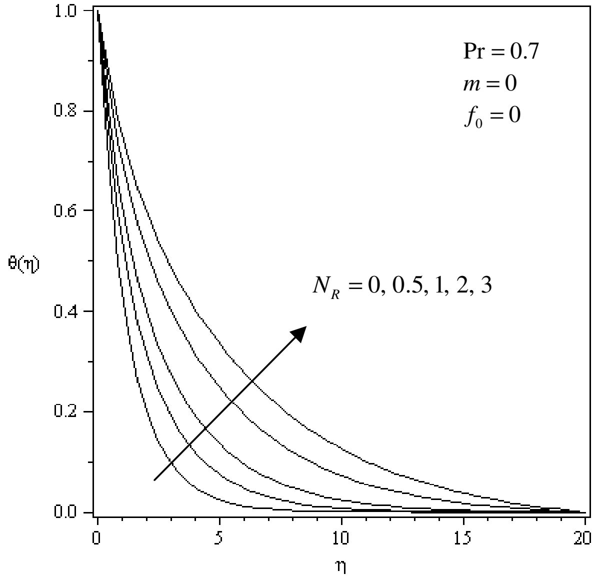

Figure 8: Effects of N_R on temperature profiles when $f_0 = 0$ (impermeable surface)

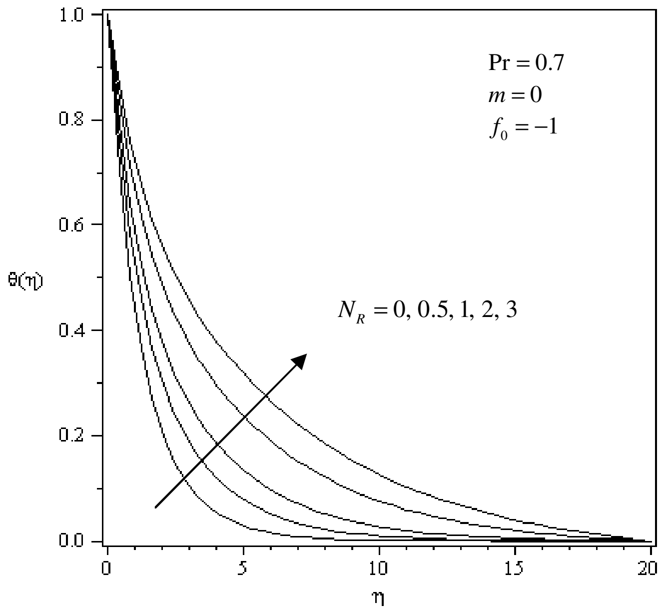

Figure 9: Effects of N_R on temperature profiles when $f_0 = -1$ (injection)

CONCLUSIONS

Marangoni convection flow along a flat surface in the presence of thermal radiation and suction or injection parameter was analyzed numerically. The governing equations are transformed into ordinary differential equations using appropriate transformations, and are then solved numerically by the shooting method. Comparison with previously published work was performed and the results were found to be in excellent agreement. The effects of thermal radiation parameter and the suction or injection parameter on the velocity and temperature profiles were presented in graphical form and discussed. It can be drawn from the present results that when the radiation parameter increases, the heat transfer rate at the surface $-\theta'(0)$ decreases. Meanwhile, the imposition of suction is to decrease the fluid velocity and temperature profiles, whereas injection shows the opposite effects.

ACKNOWLEDGEMENTS

The financial support received in the form of a fundamental research grant from the Ministry of Higher Education Malaysia is gratefully acknowledged.

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