Random Trochoidal Images

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Abstract

Interesting images are generated by combining several randomly-generated randomly-colored trochoids. A simple code that generates these images is given.

Introduction

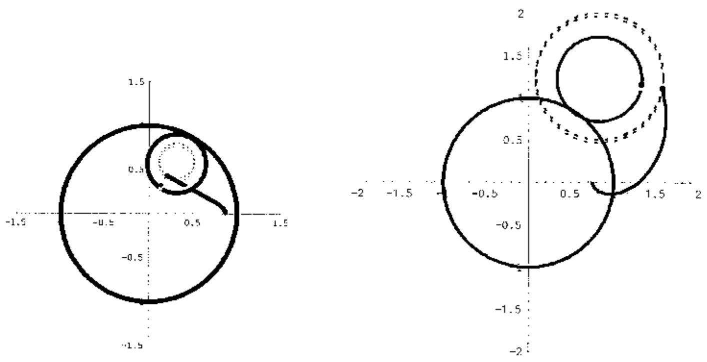
Two interesting roulettes are known by the names of hypotrochoids and epitrochoids (Figure 1). The former is a curve traced by a point attached at a distance h from the center of a circle of radius b rolling around the inside of a fixed circle of radius a. An epitrochoid, on the other hand, is generated when a circle rolls around the outside of a fixed circle, (Figure 1b). The parametric equations of hypotrochoids and epitrochoids are respectively given by

$$x = (a-b)\cos t + h\cos(a/b-1)t, \quad y = (a-b)\sin t - h\sin(a/b-1)t \tag{1}$$

and,

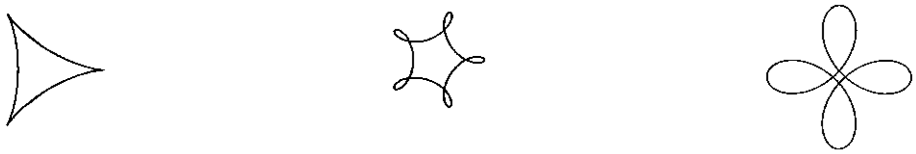
$$x = (a+b)\cos t - h\cos(a/b+1)t, \quad y = (a+b)\sin t - h\sin(a/b+1)t. \tag{2}$$

If $h<b$, the curve is known as curtate hypocycloid (or curtate epicycloid); if $h=b$ (the point is on the rim of the rolling circle), the curve is known as hypocycloid (or epicycloid); and if $h>b$, the curve is called prolate hypocycloid (or prolate epicycloid). Figure 2 and 3 show some typical curves.



(a) (b)

Figure 1: A hypotrochoid and an Epitrochoid



(1,1/3,1/3) (1,4/5,1/2) (1,1/4,2/3)

Figure 2: Some Hypotrochoids; the numbers shown are (a,b,h)

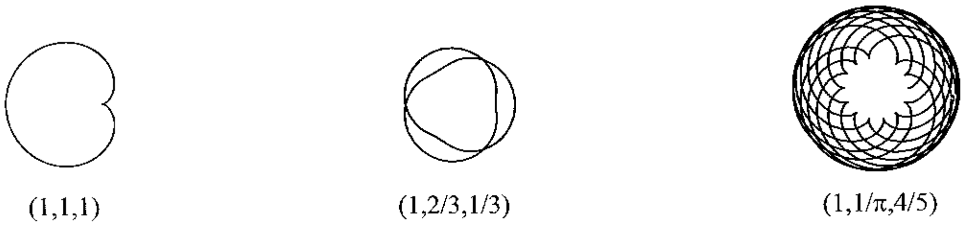


Figure 3: Some Epitrochoids; the numbers shown are (a, b, h)

The case when the radius, *b*, of the rolling circle is equal to ka, where $k(=n/m)$ is any rational number and a is the radius of the fixed circle, is of interest. This case indicates that the rolling circle rotates exactly *m* times to go exactly *n* trips around the fixed circle and comes back to its starting position. In the first row of figure 4, the rolling circle respectively rotates 4, 7, 30, and 100 times during one trip around the fixed circle and comes back to its starting position. In the second row, the rolling circle rotates the same number of times around itself as in the first row but this time, it respectively makes 3, 2, 13, and 67 trips around the fixed circle before it comes back to its starting position. One can notice that the number of loops (edges or vertices) in the hypotrochoids of Fig. 4, is exactly equal to m (the denominator of the irreducible rational radius of the rolling circle when taking the radius of the fixed circle as unity). For example, each of the cases (1,1/7,1/4) and (1,2/7,1/4) has seven loops. The difference is that the seven loops in the first case are traced out consecutively during one trip around the fixed circle. However, two trips around the fixed circle are needed in the second case, (1,2/7,1/4), to complete the seven loops. During the first trip, the odd loops (1, 3, 5, and 7) are generated, whereas the loops 2, 4, and 6 are generated during the second trip before the rolling circle goes back to its starting position at loop 1.

When the ratio of the radius of the rolling circle to the radius of the fixed circle is irrational, the produced image is never periodic and eventually the image fills in the whole plane. Fig. 5 shows the case (1,1/π,1/2) after 1, 5, 15, 50, and 100 rotations around the fixed circle.

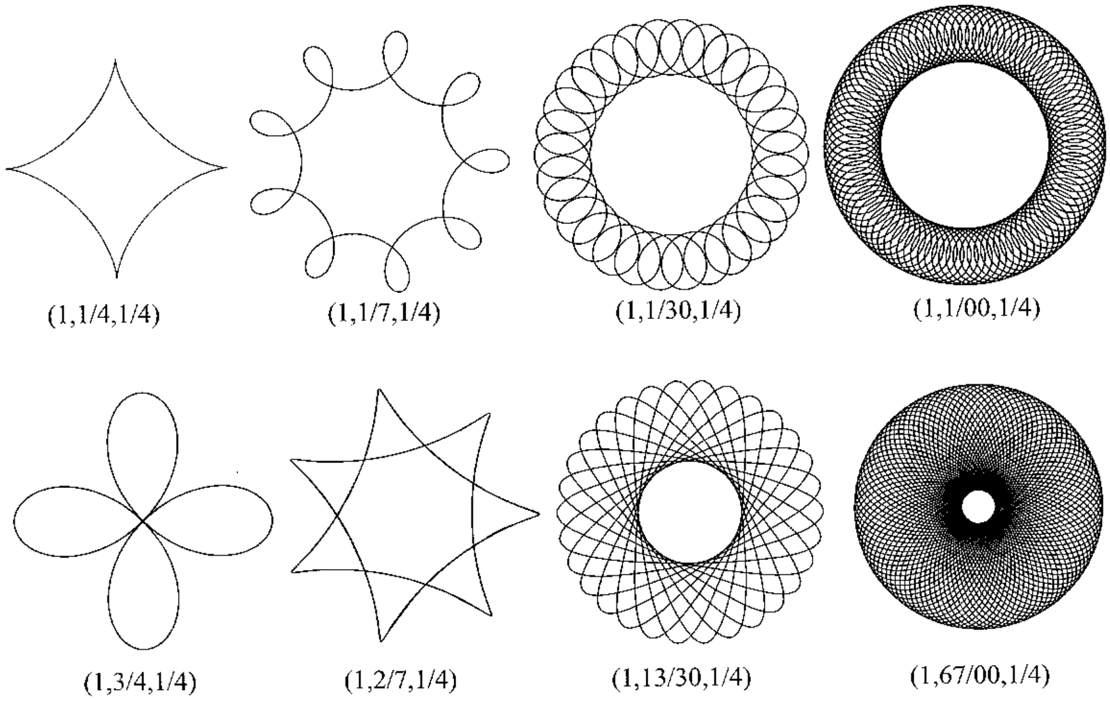


Figure 4: Some cases with rational circumference ratios

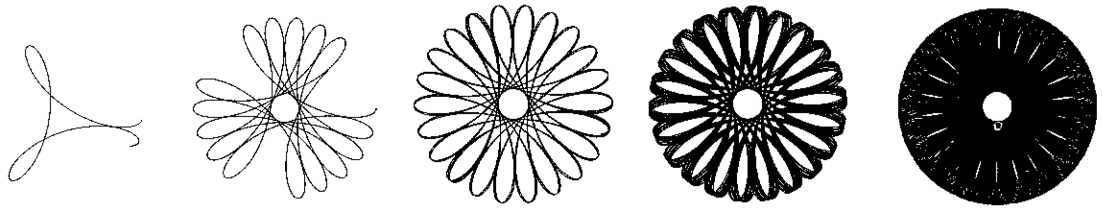


Figure 5: A case with irrational circumference ratio.

Combined randomly generated trochoids

A Spirograph produces the mathematical curves hypotrochoids and epitrochoids. The term has also been used to describe a variety of software applications that display similar curves. A classical spirograph consists of a set of plastic gears and other shapes such as rings, triangles, or straight bars. There are several sizes of gears and shapes, and all edges have teeth to engage any other piece. The gears fit inside or outside the rings and rotate around the inside or along the outside edge of the rings to produce colorful figures.

There are many different ways for specifying colors. We use the system which combines red, green, and blue as the primary colors. Mathematica uses the graphics directive RGBColor [red, green, blue] which specifies the colors in which objects are to appear. The values assigned to red, green, and blue color intensities have to be in the range 0 to 1. The following simple Mathematica code generates a set of random-size randomly-colored hypotrochoids and superimposes them on each other. The random sizes and colors are obtained by generating random valued for *b*, *h*, red, blue, and green. The command Random [] is a built in Mathematica function. The number of hypotrochoids can be set by the user. Images of epitrochoids can be simply obtained by replacing the parametric equations by those given in (2). The remaining part of this article (Figure (6a) and (6b)) show some selected output of the code.

```
(*ncrv = number of "distinct" curves, nimg =
number of images, tf = time to stop which is the per
imit of the parameter t, npt = number of points to
use in ploting the curve*)
(*___________________________________________
_________________________________*)

ncrv=2;
nimg=50;
tf=500;
npt=500;
x = (a-b) Cos[t]+h Cos[(-1+a/b)t]; y = (a-b) Sin[t]-
h Sin[(-1+a/b)t];
Do[
s=Table[
bv=Random[];hv=Random[];
ParametricPlot[{x/.{a→1,b→bv,h→hv},y/.{a→1,b
→bv,h→hv}},{t,0,tf},PlotRange→{{-1.5,1.5} , {-
1.5,1.5}}, AspectRatio→Automatic,
PlotPoints→npt,
PlotStyle→{RGBColor[Random[], Random[],
Random[]], Thickness[0.001]},
DisplayFunction→Identity],
    {k,1,ncrv}];
Show[s,DisplayFunction→$DisplayFunction,Axes
→None]; ,{nimg}]
```

References

[1] Alassar, R.S. 2007. *"EpiXoids"*, *Mathematics and Computer Education*, fall edition, 219-229.

[2] Yates, R. C. 1952. *A Handbook on Curves and Their Properties*. Ann Arbor, MI: J. W. Edwards. (ISBN: 0-0665201-0-6)

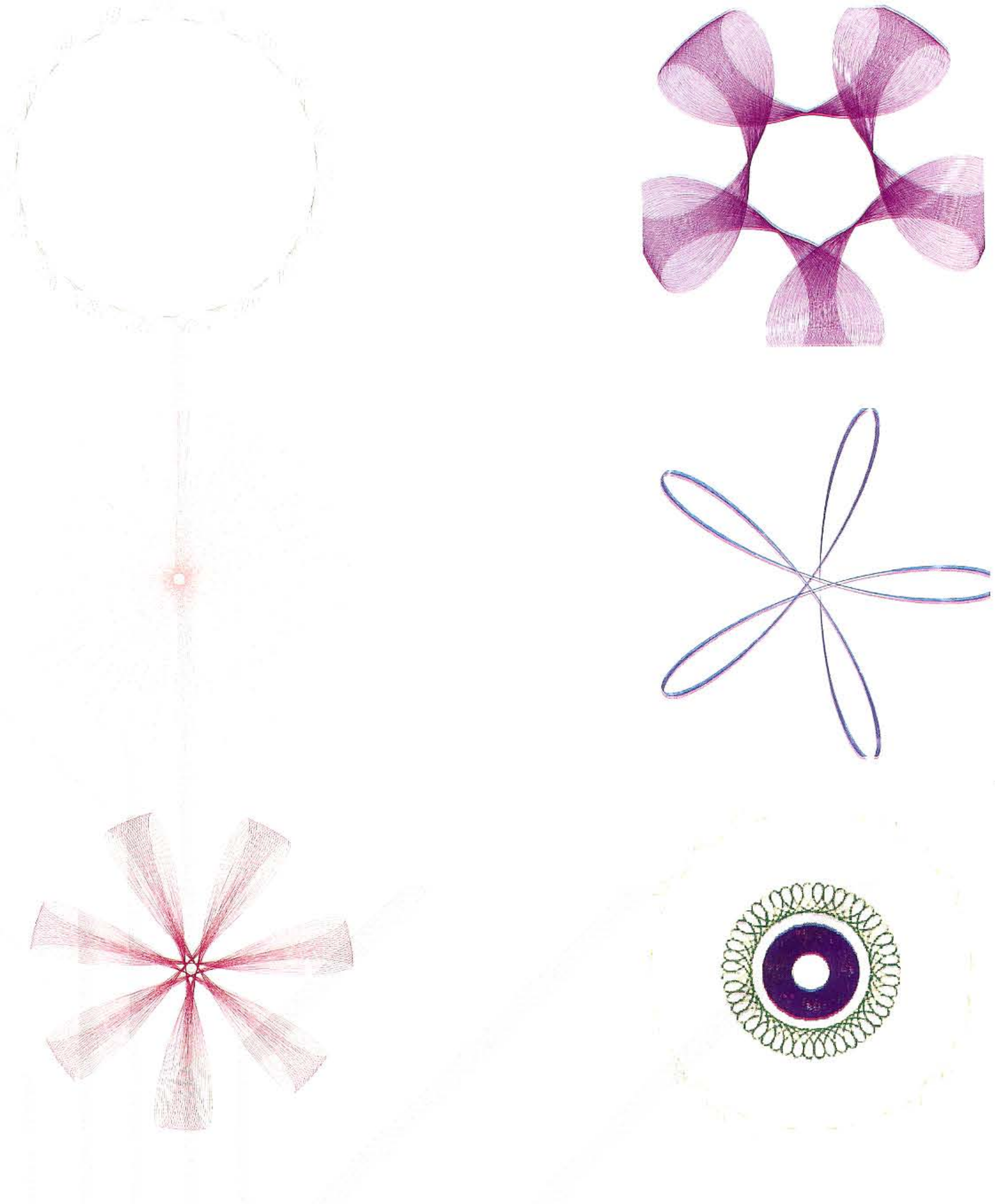


Figure 6(a): Some randomly generated trochoidal images

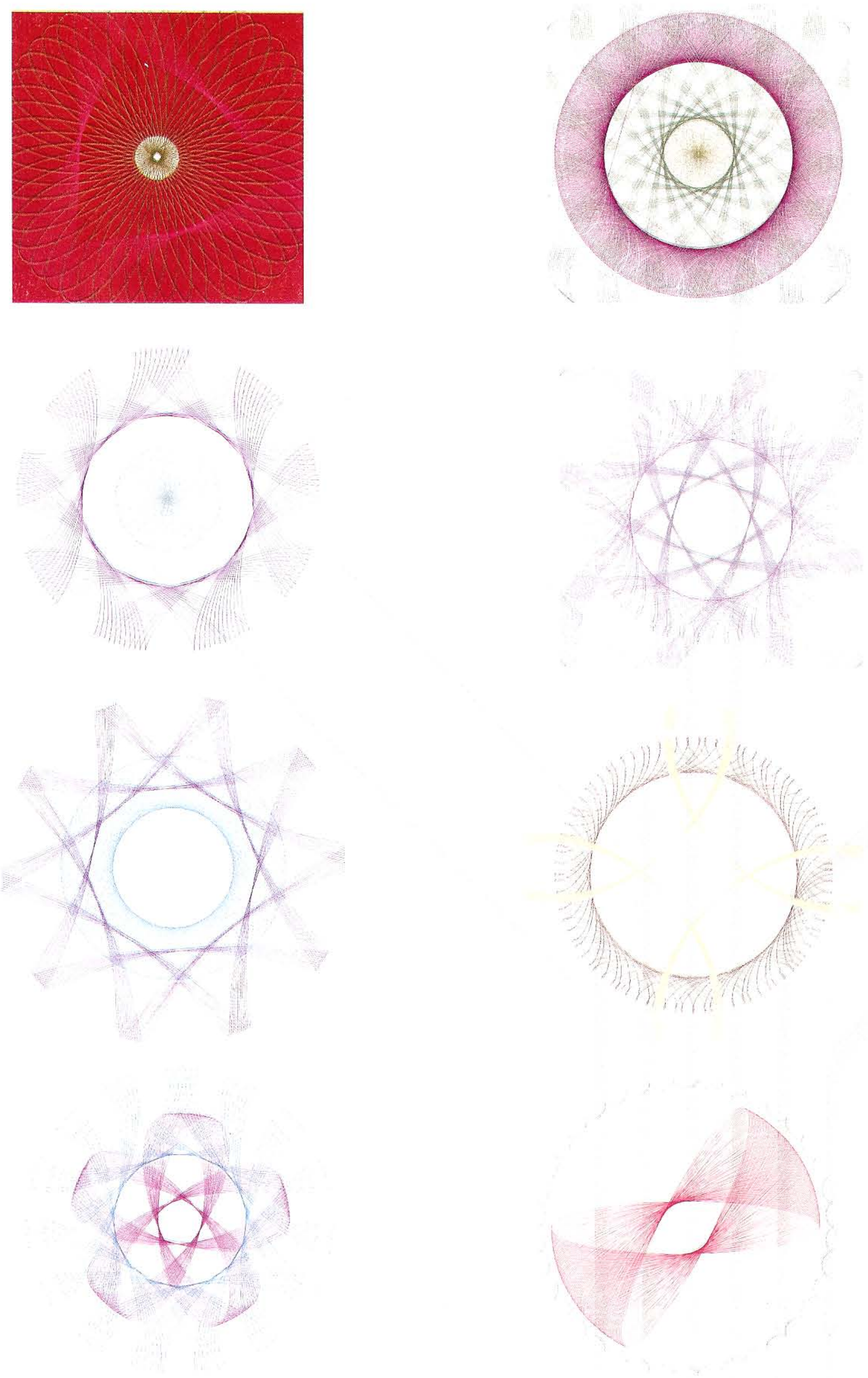


Figure 6(b): Some randomly generated trochoidal images