**UNIVERSITI PUTRA MALAYSIA**

# RUNGE-KUTTA-NYSTROM METHODS FOR SOLVING OSCILLATORY PROBLEMS

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**DOCTOR OF PHILOSOPHY**
**UNIVERSITI PUTRA MALAYSIA**

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# RUNGE-KUTTA-NYSTRÖM METHODS FOR SOLVING OSCILLATORY PROBLEMS

**By**

**NORAZAK BIN SENU**

**Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirements for the Degree of Doctor of Philosophy**

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**RUNGE-KUTTA-NYSTROM METHODS FOR SOLVING OSCILLATORY PROBLEMS**

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**February 2010**

**Chairman : Professor Dato' Mohamed bin Suleiman, PhD**

**Faculty : Science**

New Runge-Kutta-Nyström (RKN) methods are derived for solving system of second-order Ordinary Differential Equations (ODEs) in which the solutions are in the oscillatory form. The dispersion and dissipation relations are imposed to get methods with the highest possible order of dispersion and dissipation. The derivation of Embedded Explicit RKN (ERKN) methods for variable step size codes are also given. The strategies in choosing the free parameters are also discussed. We analyze the numerical behavior of the RKN and ERKN methods both theoretically and experimentally and comparisons are made over the existing methods.

In the second part of this thesis, a Block Embedded Explicit RKN (BERKN) method are developed. The implementation of BERKN method is discussed. The numerical results are compared with non block method. We find that the new code on Block Embedded Explicit RKN (BERKN) method is more efficient for solving system of second-order ODEs directly.

Next, we discussed the derivation of Diagonally Implicit RKN (DIRKN) methods for solving stiff second order ODEs in which the solutions are oscillating functions. The dispersion and

dissipation relations are developed and again are imposed in the derivation of the methods. For solving oscillatory problems with high frequency, method with P-stability property is discussed. We also derive the Embedded Diagonally Implicit RKN (EDIRKN) methods for variable step size codes. To see the preciseness and effectiveness of the methods, the constant and variable step size codes are developed and numerical results are compared with current methods given in the literature.

Finally, the Parallel Embedded Explicit RKN (PERKN) method is developed. The parallel implementation of PERKN on the parallel machine is discussed. The performance of the PERKN algorithm for solving large system of ODEs are presented. We observe that the PERKN gives the better performance when solving large system of ODEs.

In conclusion, the new codes developed in this thesis are suitable for solving system of second-order ODEs in which the solutions are in the oscillatory form.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

**KAEDAH RUNGE-KUTTA-NYSTROM BAGI MENYELESAIKAN MASALAH BERAYUNAN**

Oleh

**NORAZAK BIN SENU**

**Februari 2010**

**Pengerusi : Professor Dato' Mohamed bin Suleiman, PhD**

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Kaedah baharu Runge-Kutta-Nyström (RKN) diterbitkan bagi menyelesaikan Persamaan Pembezaan Biasa (PPB) peringkat dua yang mana penyelesaiannya adalah dalam bentuk berayunan. Hubungan serakan dan lesapan dikenakan bagi mendapatkan kaedah dengan peringkat serakan dan lesapan setinggi yang mungkin. Penerbitan kaedah Benaman Tak Tersirat RKN (BTRKN) untuk kod panjang langkah berubah turut diberikan. Strategi pemilihan parameter bebas juga dibincangkan. Kami menganalisa kelakuan berangka bagi kaedah RKN dan BTRKN secara teori dan eksperimen serta perbandingan dibuat terhadap kaedah sedia ada.

Di dalam bahagian kedua tesis, kaedah Blok Benaman Tak Tersirat RKN (BBRKN) dibincangkan. Implimentasi ke atas kaedah BBRKN turut dibincangkan. Keputusan berangka dibandingkan dengan kaedah bukan blok. Kami perolehi bahawa kod baharu Blok Benaman Tak Tersirat RKN (BBRKN) adalah lebih efisien bagi menyelesaikan sistem PPB peringkat dua.

Seterusnya, kami membincangkan penerbitan kaedah Pepenjuru Tersirat (PTRKN) bagi menyelesaikan PPB kaku peringkat dua yang penyelesaiannya berbentuk berkala. Hubungan serakan dan lesapan dibangunkan dan sekali lagi diaplikasikan dalam penerbitan kaedah. Untuk menyelesaikan masalah berkala dengan frekuensi tinggi, kaedah dengan sifat P-kestabilan dibincangkan. Kami juga menerbitkan kaedah Benaman Pepenjuru Tersirat RKN (BPTRKN) bagi kod panjang langkah berubah. Untuk melihat kejituan dan keefisienan kaedah, kod panjang langkah tetap dan berubah dibangunkan serta keputusan berangka dibandingkan terhadap kaedah sedia ada.

Akhir sekali, kaedah Selari Benaman Tak Tersirat RKN (SBTRKN) dibangunkan. Implimentasi SBTRKN ke atas mesin selari dibincangkan. Prestasi algoritma SBTRKN bagi menyelesaikan sistem PPB berdimensi besar diberikan. Kami perolehi SBTRKN memberikan prestasi yang baik bila dilaksanakan terhadap sistem PPB berdimensi besar.

Kesimpulannya, kod baharu yang dibangunkan di dalam tesis ini sesuai untuk sistem PPB peringkat dua yang mana penyelesaian adalah dalam bentuk berayunan.

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I certify that a Thesis Examination Committee has met on 22 February 2010 to conduct the final examination of Norazak bin Senu on his thesis entitled "Runge-Kutta-Nyström Methods for Solving Oscillatory Problems" in accordance with Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the Doctor of Philosophy.

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This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfilment of the requirement for the degree of Doctor of Philosophy. The members of the Supervisory Committee were as follows:

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## DECLARATION

I hereby declare that the thesis is based on my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously or concurrently submitted for any other degree at UPM or other institutions.

**NORAZAK BIN SENU**

Date: 22 February 2010

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