



**UNIVERSITI PUTRA MALAYSIA**

**COMPUTATIONAL ANALYSIS OF INCOMPRESSIBLE  
VISCOUS FLOW OVER SINGLE AND MULTI-ELEMENT  
AIRFOILS**

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**COMPUTATIONAL ANALYSIS OF INCOMPRESSIBLE VISCOUS FLOW  
OVER SINGLE AND MULTI-ELEMENT AIRFOILS**

**By**

**OMER ALI EL-SAYED**

**Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia,  
in Fulfilment of the Requirement for the Degree of Master of Science**

**March 2003**



*DEDICATION*

*TO*

*My parents, Carmen, Karim , Ali and Rayan  
For all their love and understanding*



Abstract of thesis presented to senate of Universiti Putra Malaysia in fulfillment of the requirement to the degree of Master of Science

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**March 2003**

**Chairman: Dr. Ashraf Ali Omar**

**Faculty: Engineering**

The flow-field around a multi-element airfoil with leading-edge slat and trailing-edge flap in landing configuration was performed as well as the prediction of the time dependent flow over a NACA 0012 airfoil.

The two dimensional incompressible Navier–Stokes equations with a numerical method based on the pseudo-compressibility approach was developed to simulate viscous turbulent flow around single and multi-element airfoils. The algorithm uses upwind-biased scheme of third order accuracy for the calculation of the inviscid fluxes, while a second order central differencing is used for viscous fluxes, the equations are solved using Lower-Upper Symmetric Gauss Seidel (LU-SGS) scheme. The grids around multi-element airfoil are efficiently generated using a multi-block structure technique.



The Baldwin-Lomax algebraic turbulence model is used to consider the effect of turbulence. Computed results for the studied cases were compared with experimental data in terms of surface pressure and lift coefficients which show reasonable agreement.

**Key Words:** Multi-Element Airfoil, Pseudo-Compressibility, Confluent Boundary Layer, Flow Separation, Baldwin-Lomax Turbulence Model

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Master Sains

**ANALISIS PERKOMPUTERAN UNTUK ALIRAN LIKAT TIDAK MAMPAT  
MENGELILINGI AEROFOIL TUNGGAL DAN BERBILANG ELEMEN**

Oleh

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Kapal terbang pengangkut menggunakan sistem julang tinggi untuk memperolehi persembahan julangan yang maksima ketika pendaratan dan pecahan julangan kepada seretan yang optimum ketika fasa pelepasan. Lapangan aliran mengelilingi aerofoil berbilang elemen yang dilengkapi dengan bidai bahagian depan dan kibas bahagian belakang dalam konfigurasi pendaratan akan dinilai. Dalam pada itu, simulasi aliran bersandarkan masa mengelilingi “NACA 0012” aerofoil juga disiasat.

Persamaan “Navier-Stokes” tidak mampat dua dimensi dengan kaedah berangka berdasarkan cara kemampatan palsu akan dihasilkan untuk simulasi berangka bagi aliran likat gelora mengelilingi aerofoil tunggal dan berbilang elemen. Algoritma yang digunakan untuk perkiraan fluks tidak likat adalah berlandaskan kaedah berbentuk “upwind-biased” dengan tiga kali ketepatan. Sementara itu, “central-differencing” dengan dua kali ketepatan akan digunakan untuk fluks likatnya. Akhirnya, keseluruhan persamaan akan diselesaikan dengan menggunakan

kaedah Lower-Upper Symmetric Gauss Seidel (LU-SGS). Selain itu, grid untuk airfoil berbilang elemen dihasilkan dengan menggunakan teknik struktur berbilang blok.

Secara praktiks, padang aliran likat bagi aerofoil berbilang elemen adalah kompleks di mana alirannya beralih menjadi gelora. Dalam penyelidikan ini, model “Baldwin-Lomax algebraic turbulence” digunakan untuk menilai kesan-kesan fenomena gelora. Keputusan simulasi bagi kes-kes yang dikaji, dibandingkan dengan data eksperimen dari aspek tekanan di permukaan dan pekali julangan di mana satu persetujuan yang baik diperolehi.

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I certify that an Examination Committee met on 20<sup>th</sup> March 2003 to conduct the final examination of Omer Ali Elsayed on his Master of Science thesis entitled “Computational Analysis of Incompressible Viscous Flow Over Single and Multi-Element Airfoils” in accordance with Universiti Pertanian Malaysia (Higher Degree) Act 1980 and Universiti Pertanian Malaysia (Higher Degree) Regulations 1981. The Committee recommends that the candidate be award the relevant degree. Members of the Examination Committee are as follows:

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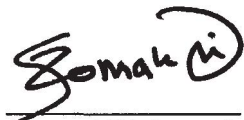


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## DECLARATION

I hereby declare that the thesis is based on my original work except for quotations and citations, which have been duly, acknowledged. I also declare that it has not been previously or concurrently submitted for any other degree at UPM or other institutions.



Omer Ali Elsayed

Date: 7/4/2003

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## NOMENCLATURE

A, B, C	Jacobian matrix of convective flux vectors
ADI	Alternating Direction Implicit
C	Chord of the Airfoil
CFL	Courant-Friedrichs - Lewy number
$C_l$	Lift coefficient
$C_p$	Pressure Coefficient
$C_y$	Side force coefficient
D	Numerical dissipation term
$D^+$	Backward difference operator
$D^-$	Forward difference operator
E,F	Inviscid flux vectors
$E_v, F_v$	Viscous flux vectors
I	Identity matrix
J	Jacobian
LU-SGS	Lower-Upper Symmetrical-Gauss-Seidel
M	Mach number
P	Pressure
Q	Vector of primitive variables
R	Residual
$R_e$	Reynolds number
$\Delta\tau$	Pseudo-time step
u, v, w	Velocity
U, V, W	Contra-variant velocity components
x,y	Cartesian coordinate



## Greek symbols

$\alpha$	Angle of attack
$\beta$	Pseudo-compressibility parameter
$\xi, \eta$	Generalized curvilinear coordinate
$\kappa$	Constant
$\lambda$	Eigenvalue
$\rho$	Density
$\tau$	Shear stress
$\nu$	Kinematic Viscosity

## Subscripts

$i, j$	Indices of body fitted coordinate system
lam.	Laminar
max.	Maximum
turbo.	Turbulent
$\infty$	Free stream condition

Note: other symbols are defined in the text.

# CHAPTER 1

## INTRODUCTION

### 1.1 Overview

Many technologies must be successfully integrated in the design of the next generation advanced subsonic transport. Among these are wing design, propulsion integration, design methodology and advanced high-lift systems. As subsonic transport designs get larger an issue such as airport tempo and noise abatement procedures become more important, the design of efficient high-lift systems becomes increasingly more important for improving the take-off and landing phase of the overall airplane mission. Additionally, improvements made in the design of the cruise wings also impact the design of the high-lift system. Recently developed wing design technology allows designers to develop more efficient wings than those that exist on current subsonic transports. The performance benefits gained by this technology can be used to perform trade studies to improve the overall aircraft system. One way designers exploit these benefits is to reduce the size of the wing (which can help reduce the cost of the aircraft). This reduced wing area means the high-lift system must work even harder to achieve the necessary levels of lift to meet takeoff and landing requirements. More efficient high-lift systems would allow designers to take advantage of these new cruise wing designs. Therefore, the understanding of and ability to analyze these multi element high-lift systems is a problem that must be solved in order to allow the aircraft designer to develop a high-lift system which meets the required performance levels while still designing a wing which is easily integrated into the airplane configuration.

## 1.2 Multi-element Airfoil

When an aircraft is landing or taking-off, high values of lift coefficient are required in order to maintain flight at the desired low speeds. It would be quite simple to design an airfoil with the much higher lift coefficient, for example by using much more camber than is commonly the case. Unfortunately, this would also greatly increase the drag of the airfoil, not only at high incidence (low speed), when extra drag is not necessarily a disadvantage, but also at low incidence (high speed), when it certainly is. The problem is solved by incorporating auxiliary devices which can be used to give increases in maximum lift coefficient when required at low speed operation, but which can be rendered ineffective at higher speed. These auxiliary devices fall broadly into two classes

- ❖ Those which alter the geometry of the airfoil (slat and flaps)
- ❖ Those which control the behavior of the boundary layer (boundary layer blowing, boundary layer suction ...etc.)

### 1.2.1 Leading-edge Slat

To appreciate qualitatively the effect of upstream element (the slat) on the immediate down stream element (the main airfoil) the former can be modeled by vortex. When one considers the component of velocity induced by vortex in the direction of the local tangent to the airfoil contour in the vicinity of the leading edge, the slat (vortex) acts to reduce the velocity along the edge of the boundary layer on the upper surface and has the opposite effects on the lower surface. Thus the effect of the slat is to reduce the

severity of the adverse pressure gradient on the main airfoil and the consequent reduction of the pressure on the upper surface is counter balanced by the rise in pressure on the lower surface. For the well designed slat/main airfoil combination it can be arranged that the latter effect predominates resulting in slight rise in lift coefficient. Leading edge slat Configuration and nomenclature for a three-element airfoil are shown in figure 1.1.

### **1.2.2 Trailing-edge Flap**

Trailing edge flap control the behavior of the boundary layer by allowing the passage of air flow through the carefully designed gap from the high pressure region below the wing to the low pressure region above it. Thus energy is added to the boundary layer on the upper surface, and any tendency of separation of the flow is reduced. In other respects trailing edge flap alter the geometry of the aerofoil by deflecting the air stream downwards to give an increase in the effective camber of the wing as well as sliding backwards which in turns increases the area of the airfoil. Thus the lift increases whenever this is required and returns to the neutral position when this lift increment is not needed.

### **1.3 Multi-element Airfoil Problems**

An accurate calculation of the flow over multi-element airfoils designed for use on transport airplanes is presently an unsolved problem, even though much progress has been made by code developers in industry and research centers. This may come as a surprise, since the flow is two dimensional and free-stream Mach numbers are low,



typically ranging from 0.1 to 0.4. Reynolds numbers of interest, based on velocity of an undisturbed uniform free-stream and airfoil reference chord, are usually between 1.2 to 40 million.

### 1.3.1 Physical Aspect

Butter and Williams (1980) studied the physics of multi-elements and showed the following list of flow features which are not found in cruise airfoils as in figure 1.2. The flow region surrounding multi-elements slotted airfoils is multiply connected, which complicate the topological laws governing viscous separated flows and even makes the calculation of inviscid flow a difficult task.

- ❖ Limited region of transonic flow may appear on the upper surface of the leading edge of highly loaded flapped airfoils, even though the free stream Reynolds number is low.
- ❖ Wakes of upstream airfoil elements often merge with boundary layers on the surfaces of down stream elements. The resulting turbulent shear layer is referred to as a confluent boundary layer.
- ❖ The region of viscous flow above the surface of trailing-edge flap is relatively thick, particularly in landing configurations, often resulting in flow separation even near normal operating conditions(i.e., well before maximum lift is attained).
- ❖ Stream line curvature and its effect on turbulent flow development are significant.