

# Binding bound variables in epistemic contexts\*

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It is now commonly acknowledged that much early theorising concerning modal notions suffered from various confusions and conflation. A major advance, at least in twentieth century philosophy, was Kripke’s work, which brought great clarity too the nature of—and varieties of—modality (e.g. Kripke 1963 and Kripke 1980). The background to much of Kripke’s work in this area concerns issues in the model-theoretic semantics for modal logic, especially quantified modal logic and the forceful Quinean objections to such an enterprise (e.g. Quine 1947 and Quine 1953b). Quine insisted that quantifying into modal contexts was incoherent, since the truth of a formula  $\exists x \Box Fx$  would require the satisfaction of the embedded open formula  $\Box Fx$  by some object. But Quine insisted that it didn’t make sense to say of an object that *it* must (or might) have some property, independently of how the object was described. For example, what could it mean for an object *a* to satisfy  $\Box(x > 7)$ ? It seems to matter whether *a* is described as “the number 8” or as “the number of planets”. Consider the contrast between (1) and (2):

- (1)  $\Box(\text{the number of planets} > 7)$
- (2)  $\Box(8 > 7)$

When giving this objection Quine understood “necessity” in terms of *semantical* necessity, or analyticity, or apriority (as did the modal logicians he was most immediately reacting to, e.g. Ruth Barcan (1946) and Rudolf Carnap (1947)). In those terms his point is that while it

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makes sense to say that “the inventor of bifocals is the inventor of bifocals” is semantically necessary (i.e. analytic), it makes little sense to say that the inventor of bifocals (i.e. Ben Franklin) is such that it is analytic that *he* is the inventor of bifocals. If it makes any sense at all to say of an object that ‘ $x$  is the inventor of bifocals’ is analytically true of it, it would seem to depend on how that object was described (e.g. “the inventor of bifocals” versus “the founder of the American Philosophical Society”).

After distinguishing various notions of modality—epistemic, semantical, and metaphysical—Kripke responds to the Quinean worry by shifting the focus to the objective or metaphysical understanding of modality, whereby he insists that it makes perfect sense talk about what properties certain objects *had to have* had or what properties they *could have* lacked (see Burgess 1998). And he insists that the person who objects to these common sense notions along the Quinean lines is being a *philosopher* in the pejorative sense:

Suppose that someone said, pointing to Nixon, ‘That’s the guy who might have lost’. Someone else says, ‘Oh no, if you describe him as *Nixon*, then he might have lost; but, of course, describing him as *the winner*, then it is not true that he might have lost’. Now which one is being the philosopher, here, the unintuitive man? (Kripke 1980: 41)

In the background Kripke has a model-theory for quantified modal logic whereby a formula such as ‘ $\Box Fx$ ’ is true of an object independently of how it is described or denoted. This is due to the fact that on this, now standard, “objectual” interpretation of quantified modal logic the value of a variable relative to an assignment is independent of the world parameter (variables are *rigid* de jure) and modals don’t meddle with the assignment parameter.

To see this consider a standard version of modal predicate logic with identity. In addition to brackets, the basic symbols consist of the following:

*Variables:*  $x, y, z, \dots$

*Predicates:*  $F, G, H, \dots$

*Constants:*  $\neg, \wedge, \exists, =, \Box$

For any sequence of variables,  $\alpha_1, \dots, \alpha_n$ , any  $n$ -place predicate  $\pi$ , and any variables  $\alpha$  and  $\beta$ , the sentences of the language are provided by the following definition:

$$\phi ::= \pi\alpha_1 \dots \alpha_n \mid \alpha = \beta \mid \neg\phi \mid (\phi \wedge \psi) \mid \exists\alpha\phi \mid \Box\phi$$

Let a model  $\mathfrak{M} = \langle W, R, D, I \rangle$ , where  $W$  is a (non-empty) set of worlds,  $R$  is a binary relation on  $W$  (an accessibility relation),  $D$  is a (non-empty) set of individuals, and  $I$  is the interpretation, which relative to a world assigns the predicates sets of tuples of individuals drawn from  $D$ . We can then provide the semantic clauses relative to a variable assignment, a world, and a model  $\mathfrak{M}$  as follows:

- $\llbracket \alpha \rrbracket^{g,w} = g(\alpha)$
- $\llbracket \pi\alpha_1 \dots \alpha_n \rrbracket^{g,w} = 1$  iff  $\langle \llbracket \alpha_1 \rrbracket^{g,w}, \dots, \llbracket \alpha_n \rrbracket^{g,w} \rangle \in I(\pi, w)$
- $\llbracket \alpha = \beta \rrbracket^{g,w} = 1$  iff  $\llbracket \alpha \rrbracket^{g,w} = \llbracket \beta \rrbracket^{g,w}$
- $\llbracket \neg\phi \rrbracket^{g,w} = 1$  iff  $\llbracket \phi \rrbracket^{g,w} = 0$
- $\llbracket \phi \wedge \psi \rrbracket^{g,w} = 1$  iff  $\llbracket \phi \rrbracket^{g,w} = 1$  and  $\llbracket \psi \rrbracket^{g,w} = 1$
- $\llbracket \exists\alpha\phi \rrbracket^{g,w} = 1$  iff for some  $g'$  that differs from  $g$  at most in that  $g'(\alpha) \neq g(\alpha)$ ,  $\llbracket \phi \rrbracket^{g',w} = 1$
- $\llbracket \Box\phi \rrbracket^{g,w} = 1$  iff for all  $w'$  such that  $wRw'$ ,  $\llbracket \phi \rrbracket^{g,w'} = 1$

Many have agreed with Kripke that this interpretation of quantified modal logic fits well with an “objective” understanding of the modals, e.g. the kinds of readings of modals that linguistics might label nomological, historical, or circumstantial. Kripke’s work ushered in intense philosophical and logical investigation of these objective modalities—with a special focus on the most general variety, which Kripke called *metaphysical* modality.

But, as is well known, there are other “flavors” of modality (see Kratzer 1977). The standard semantics for quantified modal logic may not be the most appropriate for certain varieties of modality—in particular, the *epistemic* modalities. Philosophical discussions have tended to echo Kripke’s stance on epistemic modality—it is often mentioned in order to distinguish it from genuine *metaphysical* modality, but is then quickly set aside as “merely epistemic”.<sup>1</sup> In an unpublished note from the late 1990s David Chalmers (unpublished)

<sup>1</sup>This attitude was probably more prevalent ten years ago than it is today. There has been a recent flurry of interest in epistemic modalities mostly stemming from issues in the contextualism/relativism literature, see Egan and Weatherson (2011). Even here quantified epistemic modality or epistemic modality *de re* has not been of primary focus—yet important explorations in this vein include Gerbrandy (1997), Gerbrandy (2000), Aloni (2005), Yalcin (2015), Swanson (2010), Chalmers (2011), and Ninan (forthcoming), “Quantification and Epistemic Modality”.

suggests that the post-Kripkean era has been complicit in a “tyranny of the subjunctive”. He insists that the discussion has been overly biased toward metaphysical modality, and that epistemic necessity deserves the same focus and study as metaphysical necessity (see also Chalmers 2011). Chalmers invites us to imagine an alternative universe in which Kripke instead focused primarily on epistemic modality. Chalmers suggests that Kripke would have written a completely different book, perhaps called *Naming and possibility*, within which he, among other things would:

- defend a link between necessity and apriority
- argue for a more descriptivist view of names
- argue against the necessity of identity, and
- have a different view about *de re* modality and quantified modal logic.

Even the actual Kripke admits that although identities are metaphysically necessary, they are not *epistemically* necessary. He maintains that “for all we knew in advance, Hesperus wasn’t Phosphorus” (Kripke 1980: 104), and that “we do not know *a priori* that Hesperus is Phosphorus” (*ibid.*).<sup>2</sup> When discussing the sense in which prior to empirical investigation it could turn out either way whether Hesperus is Phosphorus, he says

... there is one sense in which things might turn out either way. ... Obviously, the ‘might’ here is purely ‘epistemic’—it merely expresses our present state of ignorance, or uncertainty. (*ibid.*: 103)

Although Kripke is downplaying the “merely” epistemic possibility, he nevertheless seems to be committed to the claim that an agent could use an epistemic modal to truly say “Hesperus might not be Phosphorus”. Agents can be ignorant of metaphysical necessities and we can express such ignorance using epistemic modals. To get a solid example on the table consider this one:

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<sup>2</sup>Kripke in fact makes many statements about what *might* turn out (or might have turned out) in the epistemic sense, which are not possible in the metaphysical sense, e.g. he admits that Nixon might be an automaton (46), a certain wooden table might have been, given certain evidence, made of ice (145-146), and Gold might not to be an element (143), etc.: “If I say, ‘Gold *might* turn out not to be an element’, I speak correctly; ‘might’ here is *epistemic* and expresses the fact that the evidence does not justify *a priori* (Cartesian) certainly that gold is an element” (Kripke 1980: 143fn72). See also Kripke (1971), footnote 15.

An ancient astronomer has been investigating the moons of various celestial bodies. She has gathered various evidence from observation and geometrical calculation. In the mornings she has been investigating a celestial body that she calls 'Phosphorus'. The visibility in the morning is poor, but given certain evidence she suspects it has a moon. Also there is a celestial body that she calls 'Hesperus', which she has been investigating in the evenings. She has been able to collect an immense amount of evidence, and according to her best geometrical models it'd be inconsistent with the data for it to have a moon.

Given this situation, it would be felicitous for the astronomer to say each of the following:

- (3) Phosphorus might have a moon.
- (4) Hesperus must not have a moon.

And this is so even though Hesperus is identical to Phosphorus. But on the standard interpretation (3) and (4) cannot both be true, since the following sentences would have to be jointly satisfiable (where 'x' and 'y' are the two names of the planet, and 'M' is the predicate "having a moon"):

$$x = y \quad \Diamond Mx \quad \neg \Diamond My$$

But, of course, they can't be. To satisfy the latter two it must be that:

$$\llbracket \Diamond Mx \rrbracket^{g,w} \neq \llbracket \Diamond My \rrbracket^{g,w}$$

That can only be if  $g(x) \neq g(y)$ , but to satisfy the first it must be that  $g(x) = g(y)$ .<sup>3</sup>

Returning to the discussion of Quine's objection to quantified modal logic. It seems that Kripke is right that the following is confused:

[Winning the 1968 election] is a contingent property of Nixon only relative to our referring to him as 'Nixon'. But if we designate Nixon as 'the man who

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<sup>3</sup>Since we are presumably in a different epistemic state than this astronomer some might insist that we should not accept her utterances. This feature is well-known in the contextualism/relativism literature. To avoid this complication it may be better to consider an example that avoids it. Consider utterances of "Banksy might be Robert Del Naja" and "Banksy might be Robin Gunningham". It seems that both are true, but they can't be given and the standard semantic assumptions combined with the fact that Robert Del Naja is distinct from Robin Gunningham.

won the election in 1968', then it will be a [metaphysically] necessary truth, of course, that the man who won the election in 1968, won the election in 1968.  
(Kripke 1980: 40)

In other words, the relevant reading of

(5) The winner might not have been the winner

is perfectly fine. But altering it to epistemic modality gets the opposite result.

Winning the 1968 election is not epistemically necessary of Nixon relative to our referring to him as 'Nixon'. But if we designate Nixon as 'the man who won the election in 1968', then it will be epistemically necessary, of course, that the man who won the election in 1968, won the election in 1968.

Or in other words, as Yalcin (2015) has pointed out, there is no true reading of the following epistemic claim:

(6) The winner might not be the winner

Whether or not a property holds of an object by epistemic possibility (or necessity) seems to depend on how that object is described. But this, of course, is in direct conflict with the standard interpretation of quantified modal logic. These issues concerning *epistemic* modality *de re*, in particular, are not really settled by Kripke's discussion—they are ignored, and thus in the epistemic case there remain residual Quinean worries (cf. Burgess 1998 and Chalmers 2011: 89). These questions remain: What is the status of epistemic modality *de re* and "quantifying in" to epistemic contexts? What changes to the model-theory are required to accommodate quantified epistemic modality? And most directly how should we interpret variables under epistemic modals so that " $x = y$ " holds while " $\Box(x = y)$ " doesn't?

In one form or another these questions have been addressed since the mid-1900s, and I can't hope to answer them in a completely comprehensive and satisfactory way here. My modest aim here is to run a continuous thread through various existing strands that are already in the literature, highlight lessons along the way, and sketch out an appealing approach to quantified epistemic modality. I make no claim to novelty in detail—my contributions here involve summary, emphasis, and gestures toward new horizons.

Central to my discussion is the idea that certain modals ought to be understood as “assignment-shifting” devices: Various theorists have been toying with assignment-shifting treatments of *epistemic contexts* such as attitude verbs and epistemic modals (e.g., Cumming 2008, Santorio 2012, Ninan 2012, Pickel 2015, Rieppel 2017). On such views an epistemic “ $\Box$ ” ends up binding the  $x$  in  $\Box Fx$ . One might worry that this kind of binding yields the undesirable result that any attempt to “quantify in” to an epistemic environment is blocked, e.g.  $\exists x \Box Fx$  would be a case of vacuous quantification. If quantifying into the relevant constructions is vacuous, then such views would seem hopelessly misguided and empirically inadequate.<sup>4</sup>

Here there are enlightening, and perhaps surprising, connections to a famous alternative to Kripke’s semantics for quantified modal logic, namely Lewis’ counterpart semantics (Lewis 1968). In an important sense these assignment-shifting treatments of modals just are versions of a counterpart semantics—or the other way round: Lewis’ counterpart semantics treats the *boxes* and *diamonds* as assignment-shifting (in addition to worlds-shifting) devices. Thus, similar worries about quantifying in and vacuity arise for Lewis’ counterpart semantics—in the well-known footnote 13, just before giving the “Humphrey objection”, Kripke (1980) complains precisely about this by insisting that Lewis’ view “suffers from a purely formal difficulty”. But as I’ll demonstrate below the mere fact that a variable is bound is no obstacle to binding it. This provides a helpful lesson for those modelling *de re* epistemic contexts with assignment sensitivity, and perhaps leads the way toward the proper treatment of binding in both metaphysical and epistemic contexts: *Kripke for objective modality, Lewis for epistemic modality*.

## 1 Binding the bound

Let’s start with some folk wisdom concerning quantification and binding that turns out to be false—obviously false, perhaps, but the counterexample brings out the devices that

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<sup>4</sup>One might insist, on the contrary, that blocking quantification into epistemic contexts is a feature, not a bug. One might insist that *de re* epistemic modality doesn’t even make sense, or one might insist that there are certain constraints on the scopal interaction of quantifiers with epistemic modals which disallow quantifying in, see the Epistemic Containment Principle of Von Steup and Iatridou (2003). While there may be certain kinds of constructions that don’t allow for the quantifier to scope over the epistemic modal (“everyone inside might be outside”), it also seems clear that there are cases of quantifying in, e.g. after painting the ceiling I might warn you to walk carefully by saying: “Almost every square inch of the floor might have paint on it” (Swanson 2010; see also Yalcin 2015). Or discussing the lottery I might say “Every ticket is very likely to have lost” (cf. the cases pointed to in Lennertz 2015).

may prove useful for modelling epistemic contexts. Consider this formula of first-order logic:

$$(7) \quad \forall x \exists x Fx$$

This is a paradigm example of what is called “vacuous quantification”. The occurrence of the variable ‘ $x$ ’ in ‘ $Fx$ ’ is already bound by ‘ $\exists x$ ’ in the subformula ‘ $\exists x Fx$ ’, thus prefixing ‘ $\forall x$ ’ is idle—the universal quantifier is *vacuous*. For example, the following is a theorem concerning vacuous quantification from introductory logic texts (see, e.g., Kalish and Montague 1964: 164-65).

$$(8) \quad \forall x \exists x Fx \leftrightarrow \exists x Fx$$

In general, one might insist on the following principle concerning binding and vacuity.

THE PRINCIPLE OF VACUOUS QUANTIFICATION. If all the variables in a formula  $\phi$  are bound, then for any quantifier  $\Sigma$ ,  $\Sigma\phi \leftrightarrow \phi$ .

In slogan: *You can’t bind a bound variable!* This is the bit of folk wisdom that isn’t exactly correct. There is more going on with “vacuous quantification” than is commonly recognised.

In order to show that the the Principle of Vacuous Quantification is false I will introduce a simple language that sticks close to the syntax and semantics of first-order logic—this helps to demonstrate that there is nothing tricky going on in my counterexample.

Bear with me while I set out the formalities. First we define the syntax. In addition to parenthesis, the basic symbols consist of the following:

*Variables:*  $x, y, z, \dots$

*Predicates:*  $F, G, H, \dots$

*Connectives:*  $\neg, \wedge$

*Quantifiers:*  $\exists, \forall$

For any sequence of variables  $\alpha_1, \dots, \alpha_n$ , any  $n$ -place predicate  $\pi$ , and any variable  $\alpha$ , the sentences of the language are provided by the following grammar:

$$\phi ::= \pi\alpha_1 \dots \alpha_n \mid \neg\phi \mid (\phi \wedge \phi) \mid \exists\alpha\phi \mid \forall\alpha\phi$$



This language looks essentially like predicate logic, and we can define the other usual operators (e.g. ‘ $\vee$ ’, ‘ $\rightarrow$ ’, ‘ $\forall$ ’, etc.) as abbreviations in terms of our basic symbols. The only novel thing about the language, thus far, is the addition of a new quantifier symbol ‘ $\forall$ ’. There is nothing interesting about it syntactically, and although it will be given an interpretation that is different from ‘ $\exists$ ’ it is essentially a kind of existential quantifier.

Turning to the semantics, let a model  $\mathfrak{A} = \langle D, I \rangle$ , where  $D$  is a (non-empty) set of individuals and  $I$  is an interpretation function, which assigns values to the predicates. Since our language has variable-binding operators we relativise to an *assignment*, which assigns values to the variables. An assignment  $g$  is a function from the set of variables to set of individuals  $D$ . We provide the following recursive semantics, in the style of Tarski, by recursively defining 1 (truth or satisfaction) relative to an assignment  $g$ :<sup>5</sup>

- $\llbracket \alpha \rrbracket^g = g(\alpha)$
- $\llbracket \pi \alpha_1 \dots \alpha_n \rrbracket^g = 1$  iff  $\langle \llbracket \alpha_1 \rrbracket^g, \dots, \llbracket \alpha_n \rrbracket^g \rangle \in I(\pi)$
- $\llbracket \neg \phi \rrbracket^g = 1$  iff  $\llbracket \phi \rrbracket^g = 0$
- $\llbracket \phi \wedge \psi \rrbracket^g = 1$  iff  $\llbracket \phi \rrbracket^g = 1$  and  $\llbracket \psi \rrbracket^g = 1$
- $\llbracket \exists \alpha \phi \rrbracket^g = 1$  iff for some  $g'$  that differs from  $g$  at most in that  $g'(\alpha) \neq g(\alpha)$ ,  $\llbracket \phi \rrbracket^{g'} = 1$
- $\llbracket \forall \alpha \phi \rrbracket^g = 1$  iff for some  $g'$  that differs from  $g$  at most in that  $g'(\alpha) > g(\alpha)$ ,  $\llbracket \phi \rrbracket^{g'} = 1$

The last clause deserves comment, since it appeals to the *greater than* relation. For this to make sense, of course, the individuals in  $D$  have to be ordered: we could impose an ordering on any domain, but let’s instead just assume that  $D$  is the set of natural numbers with their natural ordering. It is not essential to my argument that we use this particular relation, nor that we order the domain. But a nice relation like this helps to keep the initial set up simple, and then one can generalise after seeing the key point. Notice that ‘ $\forall$ ’ is very similar to ‘ $\exists$ ’ in that it is an existential quantifier, but it only “looks” at a subset of the assignments that ‘ $\exists$ ’ looks at. ‘ $\exists$ ’ is the “for some” quantifier, while ‘ $\forall$ ’ is the “for some greater” quantifier.

Consider the following sentence of our language:

(9)  $\forall x Fx$

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<sup>5</sup>Here we will not worry about the distinction between “satisfaction by a sequence” and “truth”—of course, Tarski reserves *truth* for formulae that are satisfied by all sequences.

Certainly, in (9) the variable in the embedded formula ' $Fx$ ' is bound by the quantifier ' $\forall x$ '.

CLAIM 1. All the variables in ' $\forall xFx$ ' are bound.

This claim seems innocent enough, but some may suspect some kind of trickery: some sleight of hand with the operative notion of “binding” or some variable up my sleeve. Are all the variables in ' $\forall xFx$ ' really bound? Standardly, an occurrence of a variable  $\alpha$  is said to be *bound* in a formula just in case it is immediately attached to the quantifier or within the scope of a quantifier that is indexed with  $\alpha$  (and free otherwise). So in a formula such as ' $(\exists xFx \wedge Gy)$ ' both occurrences of ' $x$ ' are bound, while the occurrence of ' $y$ ' is free. Clearly, given this standard definition, all the occurrences of variables in ' $\forall xFx$ ' are bound.

Thus, if we can establish that prefixing a quantifier, such as ' $\exists x$ ', to (9) is not idle, then we will have a counterexample to the Principle of Vacuous Quantification. More precisely, we will demonstrate the following (for some model  $\mathfrak{M}$ ):

CLAIM 2.  $\llbracket \exists x \forall x Fx \rrbracket \neq \llbracket \forall x Fx \rrbracket$

We are assuming that  $D$  is the set of natural numbers, and let's also assume a particular interpretation for ' $F$ ', namely  $I(F) = \{8\}$ . So ' $F$ ' is only true of 8. Provided this model, consider the truth conditions of (9) relative to some assignment  $g$ , which are calculated as follows:

$$\begin{aligned} \llbracket \forall x Fx \rrbracket^g = 1 & \text{ iff for some } g' \text{ that differs from } g \text{ at most in that } g'(x) > g(x), \llbracket Fx \rrbracket^{g'} = 1 \\ & \text{ iff for some } g' \text{ that differs from } g \text{ at most in that } g'(x) > g(x), g'(x) \in I(F) \\ & \text{ iff for some } n \geq g(x), n = 8. \end{aligned}$$

Assume  $g(x) = 10$ , then since, of course, there isn't an  $n \geq 10$  such that  $n = 8$ , it follows that  $\llbracket \forall x Fx \rrbracket^g = 0$ . Now let's drop the hammer: embed ' $\forall x Fx$ ' under ' $\exists x$ ':

$$(10) \quad \exists x \forall x Fx$$

By calculating the truth conditions (relative to the same model) we see that the outer quantifier is not vacuous:

$\llbracket \exists x \forall x Fx \rrbracket^g = 1$  iff for some  $g'$  that differs from  $g$  at most in that  $g'(x) \neq g(x)$ ,  $\llbracket \forall x Fx \rrbracket^{g'} = 1$   
 iff for some  $g'$  that differs from  $g$  at most in that  $g'(x) \neq g(x)$ ,  
     for some  $g''$  that differs from  $g'$  at most in that  $g''(x) > g'(x)$ ,  $g''(x) \in I(F)$   
 iff for some  $m$ , for some  $n \geq m$ ,  $n = 8$ .

Since there is an  $m$  and an  $n$  such that  $n \geq m$  and  $n = 8$ , it follows that  $\llbracket \exists x \forall x Fx \rrbracket^g = 1$ . Thus, relative to this model and assignment  $g$ ,  $\llbracket \forall x Fx \rrbracket^g = 0$ , while  $\llbracket \exists x \forall x Fx \rrbracket^g = 1$ . This completes the proof that  $\llbracket \exists x \forall x Fx \rrbracket \neq \llbracket \forall x Fx \rrbracket$ , and thus that the following biconditional is not valid (it is false at  $g$ ):

$$(11) \quad \exists x \forall x Fx \leftrightarrow \forall x Fx$$

This provides a simple counterexample to the Principle of Vacuous Quantification. Thus, even when all the variables in a formula  $\phi$  are bound, prefixing a quantifier  $\Sigma$  to  $\phi$  can be non-vacuous.<sup>6</sup>

Now let's take stock. If we restrict our focus to standard first-order logic, then the Principle of Vacuous Binding clearly holds, so what is the essential difference introduced by the ' $\forall$ ' quantifier? First-order quantifiers (e.g. ' $\exists$ ' and ' $\forall$ ') are standardly appointed a special kind of stability—one that is not essential to their status as variable-binding operators. The set of  $\alpha$ -variants that ' $\exists$ ' looks to when assessing its embedded formula are not at all constrained by what the initial variable assignment assigns to  $\alpha$ , so shifting what the input assignment assigns to  $\alpha$  will be idle. Whereas, the set of  $\alpha$ -variants that ' $\forall$ ' looks to when assessing its embedded formula *are* constrained by what the initial variable assignment assigns to  $\alpha$ —they have to assign something greater than (or equal to) what

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<sup>6</sup>But what about the claim that bound variables can be re-bound? Does prefixing ' $\exists x$ ' to ' $\forall x Fx$ ' result in ' $x$ ' being rebound? I think so, but I'm not so interested in explicitly defending it here, since this turns on some subtle terminological issues. We'd need a definition of when a quantifier *binds* a particular occurrence of a variable in a formula. There is a standard definition of this in terms of syntax which stipulates that a bound variable can't be re-bound (see e.g. Heim and Kratzer 1998: 120). So, even though a natural way to describe the counterexample above would be to say that ' $\exists x$ ' rebinds the last occurrence of ' $x$ ', this is ruled out by a standard definition. What's going on? Under the stress of the counterexample the syntactic definition of binding is pulling apart from the background semantic understanding of binding. The class of "variables" and "quantifiers" (or variables-binders in general) are grouped together due to their interesting *semantic* properties, not their syntactic properties. Thus, in this more fundamental semantic sense we should say that a quantifier binds an occurrence of a variable in a formula when the sensitivity of the variable is affected by the shifting induced by the quantifier. It is in the semantic sense that you can bind a bound variable.

the initial variable assignment assigns to  $\alpha$ —thus shifting the input assignment can make a difference. In this way, ‘ $\exists$ ’ standardly has a certain indifference to the input assignment, whereas for ‘ $\forall$ ’ the input assignment genuinely matters—‘ $\forall$ ’ is “context” sensitive.<sup>7</sup>

This suggests that the essential difference between ‘ $\exists$ ’ and ‘ $\forall$ ’ concerns the *accessibility relations* involved. Thus, it can be illuminating at this point to view first-order logic as a modal logic in the way associated with Amsterdam (see Van Benthem 1977 and especially Blackburn et al. 2002, §7.5 on reverse correspondence theory). On this way of viewing things, the assignments are the “worlds”, and the model includes a stock of binary accessibility relations  $R^\alpha$  that hold between assignments (relative to a variable  $\alpha$ ). Standard first-order logic is only concerned with a special subset of all the possible such models for first-order languages—that is, it constrains itself to a particular accessibility relation:

$$g \equiv^\alpha g' \text{ iff } g' \text{ differs from } g \text{ at most in that } g'(\alpha) \neq g(\alpha)$$

Notice that  $\equiv^\alpha$  is reflexive, transitive, and symmetric. So given standard assumptions the accessibility relation between assignments is an equivalence relation, and thus will validate the relevant formulae corresponding to S5, which will include the following theorems concerning vacuous quantification:

$$\forall x \forall x \phi \leftrightarrow \forall x \phi$$

$$\exists x \forall x \phi \leftrightarrow \forall x \phi$$

$$\exists x \exists x \phi \leftrightarrow \exists x \phi$$

$$\forall x \exists x \phi \leftrightarrow \exists x \phi$$

Prefixing further quantifiers to a closed sentence in standard first-order logic is “vacuous”—just as adding further boxes and diamonds is vacuous in an S5 modal logic. But—just as in modal logic—the relevant equivalencies only hold given particular restrictions on the accessibility relation. For example,  $\Box \phi \rightarrow \Box \Box \phi$  is invalid unless the frame is transitive. Likewise, if  $R^x$  is not transitive, then  $\forall x \phi \rightarrow \forall x \forall x \phi$  will be invalid.

To round off this point, consider again the the quantifier ‘ $\forall$ ’ and the accessibility relation that it appeals to:

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<sup>7</sup>An alternative way to put the difference here is to say that standard first-order quantifiers take an “external” perspective, whereas a quantifier like ‘ $\forall$ ’ must take an “internal” perspective on the relevant relational structures (see Recanati 2007, 65-71 and Blackburn et al. 2002, xi-x).

$$g \geq^\alpha g' \text{ iff } g' \text{ differs from } g \text{ at most in that } g'(\alpha) > g(\alpha)$$

This relation is reflexive and transitive, but it is not symmetric. And our counterexample to the vacuous quantification principle implicitly exploited the fact that the relation was not symmetric. Notice that, in general, if the accessibility relation between assignments is not assumed to be symmetric then the paradigm example of vacuous quantification mentioned at the outset becomes invalid.

$$(8) \quad \forall x \exists x Fx \leftrightarrow \exists x Fx$$

Consider the right-to-left direction: If someone taller than me is happy, it doesn't follow that everyone taller than me is such that there is someone taller than them that is happy.<sup>8</sup> One can imagine that by generalising and playing around with the first-order accessibility relations there are things of intrinsic interest to metalogic, e.g. concerning decidability (see discussion in Blackburn et al. 2002: 466-469), but the application we are concerned with is modeling epistemic contexts—and the key insight here is that assignment-shifting operators can non-trivially stack.

## 2 Variables in counterpart semantics

As I mentioned at the outset the discussion above has illuminating connections to Lewis' counterpart semantics (Lewis 1968). The connection is that in counterpart semantics the falsity of the Principle of Vacuous Binding is presupposed. Let me explain.

Lewis provides a first-order translation for formulae of quantified modal logic such as the following (where ' $W$ ' is the property of being a world, ' $T$ ' is the relation that holds between an object  $z$  and a world  $y$  when  $z$  is *in*  $y$ , and ' $C$ ' is the counterpart relation):

$$\Box Fx \approx \forall y \forall z ((Wy \wedge Izy \wedge Czx) \rightarrow Fz)$$

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<sup>8</sup>There is a tight connection here between ' $\exists$ ' and the kinds of quantifiers used for knowledge representation by description logics, e.g. ' $\exists R$ ' (see Blackburn 2006 and Baader et al. 2003): they both restrict to the set of individuals that bear a relation to the input individual. The sentences of description logic function much like the sentences of Prior's Egocentric logic such as "Someone-more-perfect standing", which is true at an individual  $a$  iff there is an  $x$  such that  $x$  is more perfect than  $a$  and  $x$  is standing (see Prior 1968).

This says (roughly) that every counterpart of  $x$ , in any world, is an  $F$ . And such a translation generalises for any modalised open sentence  $\lceil \Box \pi \alpha_1 \dots \alpha_n \rceil$ . Lewis demonstrates how we can follow the translation procedure, and then provide the resulting first-order formulae with their standard first-order interpretation, thereby endowing the modal formulae with truth-conditions.

But the intermediate translation is not required. We can instead directly provide a model-theoretic semantics for the language of quantified modal logic that corresponds to Lewis' translation rules (See Hazen 1979, Cresswell and Hughes (1996: 353-358), and especially Schwarz 2012). And this is where things get interesting. Since we are only concerned with the model-theoretic semantics—not with certain metaphysical commitments—we will follow the advice of Schwarz (2012), who says:

... we should dissociate counterpart semantics from various Lewisian doctrines that are commonly lumped together under the heading of “counterpart theory”. (Schwarz 2012: 9)

So, in particular, we are not here concerned with modal realism, world-bound agents (i.e. Postulate 2), or Lewis' specific commitments on the similarity relation and counterpart-hood.<sup>9</sup> We are instead concerned with model-theoretic *counterpart semantics*.

For our purposes the interesting action is going to be with modalised sentences such as  $\Box Fx$  and binding into such formulae. First, think about how we might interpret a modalised formulae such as  $\Box Fx$ . This says, roughly, that every counterpart of  $x$ , in any world, is an  $F$ , so we need to evaluate the embedded open formula  $Fx$  relative to every counterpart of the individual assigned to  $x$  at every accessible world. Thus, if  $\Box Fx$  is evaluated at a variable assignment  $g$  and world  $w$ , then  $Fx$  must be evaluated at assignments  $g'$  where  $g'(x)$  in a world is a “counterpart” of  $g(x)$  in  $w$ . In this way, the modal shifts the assignment and effectively binds all the variables in its scope. In general, the clause for  $\Box$  will yield:

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<sup>9</sup>Postulate 2 stipulates that “Nothing is in two worlds”. In footnote 2, Lewis (1968) entertains the idea of allowing identities across worlds by giving up Postulate 2, but later clarifies that he is firmly committed to Postulate 2: “Footnote 2 has given some readers the impression that I regard Postulate 2 as a mere convention, and that we could just as truly say that some things are identical with their otherworldly counterparts after all. Not so. I was alluding to the possibility of a hybrid theory—a theory opposed to my own, a theory which I take to be false—according to which there are identities across worlds, but we use the counterpart relation anyway” (Lewis 1983: 46). We are setting aside Lewis' particular metaphysical commitments that lead him to oppose the hybrid theory.

$\ulcorner \Box \phi \urcorner$  is true relative to a world  $w$  and assignment  $g$  iff  $\phi$  is true relative to all  $w$ -accessible worlds  $w'$  and assignments  $g'$  that assign to each free variable  $\alpha$  in  $\phi$  a counterpart  $g'(\alpha)$  at  $w'$  of  $g(\alpha)$  at  $w$ .

The model, then, should include a relation on world-assignment pairs, which encodes the *counterpart relations*.

The counterpart relation and the assignment meddling of the modals is the most interesting departure from the basic Kripke semantics. The rest of the semantics is essentially the same. But just to have a full model on display let's fill out the rest. Let the language be modal predicate logic with identity, as defined above. Let a model  $\mathfrak{M} = \langle W, R, D, I \rangle$ , where  $W$  is a (non-empty) set of worlds,  $R$  is a binary relation on  $D^{\mathbb{N}} \times W$  (an accessibility relation incorporating the counterpart relation),  $D$  is a (non-empty) set of individuals, and  $I$  is the interpretation, which relative to a world assigns the predicates sets of tuples of individuals drawn from  $D$ .<sup>10</sup> We can then provide the semantic clauses relative to a model  $\mathfrak{M}$  as follows:

- $\llbracket \alpha \rrbracket^{g,w} = g(\alpha)$
- $\llbracket \pi \alpha_1 \dots \alpha_n \rrbracket^{g,w} = 1$  iff  $\langle \llbracket \alpha_1 \rrbracket^{g,w}, \dots, \llbracket \alpha_n \rrbracket^{g,w} \rangle \in I(\pi, w)$
- $\llbracket \alpha = \beta \rrbracket^{g,w} = 1$  iff  $\llbracket \alpha \rrbracket^{g,w} = \llbracket \beta \rrbracket^{g,w}$
- $\llbracket \neg \phi \rrbracket^{g,w} = 1$  iff  $\llbracket \phi \rrbracket^{g,w} = 0$
- $\llbracket \phi \wedge \psi \rrbracket^{g,w} = 1$  iff  $\llbracket \phi \rrbracket^{g,w} = 1$  and  $\llbracket \psi \rrbracket^{g,w} = 1$

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<sup>10</sup>Here I am loosely following Schwarz (2012), but there are a few differences in my presentation. Schwarz has the counterpart relation hold between individual-world pairs, and then uses this to construct the required alternative sequences (see p. 13ff), whereas I have a counterpart relation on sequence of individuals and world pairs. Both strategies allow for multiple counterparts at one world, but there are issues concerning multiply *de re* modality that I must gloss over here (see Hazen 1979, 328-330, and Lewis 1983, 44-45). I also assume a constant domain, while Schwarz doesn't. This is just for ease of presentation. The counterpart semantics can make all the same extra sophistications as the Kripke semantics can, e.g. with variable domains or inner/outer domains, etc. In fact, there are good reasons to add such sophistications. Here is one for the epistemic case: agents can think that there are more individuals than there in fact are. For example, assume the domain has two elements: {Pythagorus, Venus}. If Pythagorus can truthfully say "Hesperus might not be Phosphorus", then there is a world where a Hesperus-counterpart is distinct from a Phosphorus-counterpart. But Pythagorus knows that he is distinct from both Hesperus and Phosphorus, so the only candidates for Hesperus- and Phosphorus-counterparts is Venus. Thus, the "might"-claim isn't true unless we allow the domains to vary with worlds or we do some trick with inner/outer domains. See how to do it in Schwarz (2012).

- $\llbracket \exists \alpha \phi \rrbracket^{g,w} = 1$  iff for some  $g'$  that differs from  $g$  at most in that  $g'(\alpha) \neq g(\alpha)$ ,  $\llbracket \phi \rrbracket^{g',w} = 1$
- $\llbracket \Box \phi \rrbracket^{g,w} = 1$  iff for all  $w'$  and  $g'$  such that  $\langle w, g \rangle R \langle w', g' \rangle$ ,  $\llbracket \phi \rrbracket^{g',w'} = 1$

Notice, again, in this last clause that the modal operator binds all the variables in its scope. But since the assignments that ' $\Box$ ' looks to when assessing its embedded formula  $\phi$  are constrained by what the initial variable assignment assigns to the variables in  $\phi$ , prefixing a quantifier to ' $\Box \phi$ ' needn't be vacuous. So although the modal operator binds the variables, they can be "re-bound".

In fact, when addressing Kripke's "purely formal objection" (1980, footnote 13), Lewis (1983)—the postscript to the 1968 article—explicitly points out that his modal operators bind the variables in their scope. The objection is that Lewis' theory invalidates certain cherished principles of the logic of identity and quantification. The way Lewis puts the objection is that his counterpart theory seems to invalidate Leibniz's Law, ' $\ulcorner \forall x \forall y (x = y \rightarrow (\phi_x \leftrightarrow \phi_y)) \urcorner$ ', since the following is not valid:

$$(12) \quad \forall x \forall y (x = y \rightarrow (\Diamond x \neq y \leftrightarrow \Diamond y \neq y))$$

But Lewis pleads "not guilty". He insists that on his view (12) is not really an instance of Leibniz's Law for much the same reason that (13) isn't an instance.

$$(13) \quad \forall x \forall y (x = y \rightarrow (\exists y x \neq y \leftrightarrow \exists y y \neq y))$$

Clearly, to think that (13) is an instance of Leibniz's Law is, as Lewis says, "to commit a fallacy of confusing bound variables". Lewis insists that the same holds for (12), since the modals bind the variables in their scope.<sup>11</sup>

The abbreviated notation of quantified modal logic conceals the true pattern of binding . . . The diamonds conceal quantifiers that bind the occurrences of ' $x$ ' and ' $y$ ' that follow. . . . So counterpart theory is no threat to standard logic. It is only a threat to simplistic methods of keeping track of variable-binding and instancehood when we are dealing with the perversely abbreviated language of quantified modal logic. (Lewis 1983: 46)

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<sup>11</sup>See Schwarz (2012: 17-18) for discussion of the appropriate restrictions on substitution in a counterpart semantics.



Thus, this formal complaint against counterpart theory does not hold up to scrutiny.

The counterpart framework remains controversial. Some think that it has various attractive metaphysical applications, e.g. it avoids the problem of accidental intrinsics, it allows for paradox-resolving flexibility in the attribution of modal properties, and allows for contingent identities, etc. While others think it is based on a confusion, or inherits various implausible metaphysical commitments, or falls victim to the “Humphrey objection”. My purpose is not to weigh in on these ongoing debates over the counterpart framework. But I do want to highlight that most of this controversy concerns whether Kripke’s framework or Lewis’ counterpart framework provides the most plausible analysis of *metaphysical* modality. The debate, however, looks very different in the context of epistemic modality. As I outlined already Kripke’s framework seems ill-suited for epistemic modality *de re*, but moreover the standard complaints against Lewis’ framework lose much of their force in the epistemic setting.

Consider the Humphrey objection. Kripke complains,

... if we say ‘Humphrey might have won the election (if only he had done such-and-such)’, we are not talking about something that might have happened to Humphrey but to someone else, a “counterpart”. Probably, however, Humphrey could not care less whether someone else, no matter how much resembling him, would have been victorious in another possible world. (Kripke 1980: 45)

Even if one finds this appeal convincing in the case of metaphysical possibility, it seems to miss the mark when aimed at a counterpart-theoretic analysis of *epistemic* modality. Kripke himself reaches for counterpart-theoretic devices when explaining the epistemic sense in which it might have turned out that Hesperus wasn’t Phosphorus.

And so it’s true that given the evidence that someone has antecedent to his empirical investigation, he can be placed in a sense in exactly the same situation, that is a qualitatively identical epistemic situation, and call two heavenly bodies ‘Hesperus’ and ‘Phosphorus’, without their being identical. So in that sense we can say that it might have turned out either way. (Kripke 1980: 103-104)

Thus the epistemic modal claim is true in virtue of “counterparts” of Hesperus and Phosphorus which are distinct in other worlds, not in virtue of Hesperus and Phosphorus themselves being distinct in other worlds. With the counterpart framework in mind,

one might read Kripke as putting forward the following suggestion: an epistemic use of “Hesperus might not be Phosphorus” is true iff there is a world  $w$  compatible with the speaker’s qualitative evidence where an epistemic counterpart of Hesperus in  $w$  is distinct from an epistemic counterpart of Phosphorus in  $w$ .<sup>12</sup> In fact, Kripke encourages this counterpart-theoretic construal in the following passages:<sup>13</sup>

Here, then, the notion of ‘counterpart’ comes into its own. For it is not the table, but an epistemic ‘counterpart’, which was hewn from ice; not Hesperus-Phosphorus-Venus, but two distinct counterparts thereof, in two of the roles Venus actually plays (that of Evening Star and Morning Star), which are different. . . if someone confuses the epistemological and the metaphysical problems, he will be well on the way to the counterpart theory of Lewis and others have advocated. (Kripke 1971, footnote 15)

. . . I (or some conscious being) could have been *qualitatively in the same epistemic situation* that in fact obtains, I could have the same sensory evidence that I in fact have, about *a table* which was made of ice. The situation is thus akin to the one which inspired the counterpart theorists (Kripke 1980: 333)

The intuition that a certain table might have turned out to be made of ice concerns “epistemic counterparts” of the table, the intuition that Hesperus could have turned out to be distinct from Phosphorus concerns distinct counterparts of Venus. Of course, Kripke is alluding to counterpart theory in a denigrating way: Lewis’ counterpart theory makes sense for metaphysical modality only when you confuse it with epistemic modality. But this also suggests a positive position: Lewis’ counterpart framework is well-suited for an analysis of epistemic modality.

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<sup>12</sup>The following passage also strongly suggests this reading: “So two things are true: first, that we do not know a priori that Hesperus is Phosphorus, and are in no position to find out the answer except empirically. Second, this is so because we could have evidence qualitatively indistinguishable from the evidence we have and determine the referents of the two names be positions of the two planets in the sky, without the planets being the same.” (Kripke 1980: 104)

<sup>13</sup>Although one can read Kripke as suggesting this counterpart treatment of epistemic modality, I’m not claiming that this is Kripke’s considered view. At times Kripke seems to suggest instead that talk of epistemic modality always involves a kind of “loose speak”. On this reading, an epistemic use of “Hesperus might not be Phosphorus” is strictly speaking false, but there is a *rephrasal* of it such as “There is a world compatible with the speaker’s qualitative evidence where an epistemic counterpart of Hesperus is distinct from an epistemic counterpart of Phosphorus”, which is true. See Bealer (2002), 81-83 for a nice discussion of this point.

### 3 Variables in epistemic contexts

Recently theorists have been appealing to “assignment-shifting operators” in treatments of certain natural language constructions such as attitude verbs and epistemic modals, see, e.g., Cumming (2008), Santorio (2012), and Ninan (2012), Ninan (forthcoming). These views all fall within the family of counterpart semantics, broadly construed, and share the feature that the modals or attitude verbs bind all the variables in their scope. But there are some differences in terms of motivation and implementation that are worth highlighting.

Cumming (2008) is concerned to provide a semantic view where belief attributions such as “Biron thinks that Hesperus is visible” and “Biron thinks that Phosphorus is visible” can differ in truth value. Cumming argues that both the Millian and the descriptivists views are untenable, even though they each harbour a half-truth: (i) Millianism is correct in maintaining that the referent of a name is not sensitive to the world parameter, (ii) Descriptivism is correct that the referent of a name is shiftable in epistemic contexts. We need an alternative that accommodates both if these features.

Cumming insists on two innovations. The first is that names should be semantically represented as variables, so that particular uses of “Hesperus is visible” and “Phosphorus is visible” will be equivalent to something like (14) and (15) respectively:

(14)  $x$  is visible

(15)  $y$  is visible

If variables are treated in the standard way then they retain their status as insensitive to the world parameter. But they are sensitive to the assignment parameter, and this allows for the second innovation. The second innovation is that attitude verbs quantify over alternative assignments in addition to worlds. In Cumming’s terminology attitudes verbs operate on *open propositions*, which are true and false with respect to a world and assignment. The justification for quantification over alternative assignments is that doxastic possibilities for an agent encode both information about the world and about the reference relation:

... verbs that create hyperintensional contexts, like ‘think’, are treated as operators that simultaneously shift the world and assignment parameters. ... This conforms to the intuition that the content of attitude ascriptions encapsulates *referential uncertainty*.

My treatment of attitude verbs as operators that shift the assignment tallies with the reflection that attitude ascriptions can convey things about how an agent conceives of the reference relation (in addition to how they conceive of the world).

One needn't construe the epistemic possibilities involved in this overtly meta-linguistic way. Formally, it amounts to the same thing but it's perhaps better to construe the uncertainty involved as the agent's uncertainty about which individual  $x$  is, instead of the agent's uncertainty about which individual " $x$ " refers to. In any case, a belief report will have truth-conditions such as the following:

"Biron believes  $x$  is visible" is true iff for each assignment-world pair  $\langle g, w \rangle$  in Biron's belief set,  $g$  maps ' $x$ ' onto an object that is visible at  $w$

The official way that Cumming implements this in the model theory is as follows, where " $\Box_\tau$ " is an agent relativised belief operator and doxastic accessibility for each agent  $\tau$  is given by a relation  $\text{dox}_\tau$  between  $W$  and  $D^\mathbb{N} \times W$ :

$$\llbracket \Box_\tau \phi \rrbracket^{g,w} = 1 \text{ iff for all } g' \text{ and } w' \text{ such that } \text{dox}_\tau(w, \langle g', w' \rangle), \llbracket \phi \rrbracket^{g',w'} = 1$$

On this approach the truth of " $\Box_\tau Fx$ " does not depend on what the input assignment assigns to " $x$ ", thus we get the result that straightforward quantifying in with objectual binders is blocked. That is, for Cumming " $\Box$ " not only unselectively binds all the variables in its scope it does so in a way that renders the resulting formula insensitive to the input assignment. Seemingly aware of this, Cumming (2008) instead appeals to substitutional quantification as a means of quantifying into attitude contexts—the machinery employed is inspired by Kaplan's (1968) treatment of *de re* attitude ascriptions. But Pickel (2015) has shown that the insensitivity of " $\Box_\tau Fx$ " to the input assignment combined with the substitutional quantifiers nevertheless lead to undesirable results (see Pickel 2015: 339-341 and Rieppel 2017: 248-250).<sup>14</sup>

<sup>14</sup>Pickel instead provides a two-factor view where the truth of " $\Box Fx$ " depends on both (i) the truth of " $Fx$ " relative to shifted assignments (at the shifted worlds) and (ii) relative to the *input* assignment (at the shifted worlds). The latter conjunct gives rise to the arguably bad result that utterances of "Olivia believes that Hesperus is distinct from Phosphorus" (and presumably "Hesperus might not be Phosphorus") cannot be true—attitude reports have become *overly* sensitive to the input assignment. In terms of epistemic counterparts, Pickel's strategy allows for some of  $x$ 's counterparts in other worlds to be distinct from  $x$ , but nevertheless requires that  $x$  itself is always among  $x$ 's counterparts in other worlds.

Santorio (2012) points to close cousins of the problems with names in attitude reports, namely cases where indexicals occurring under epistemic modals seem to shift. Consider the case of mad Heimson.<sup>15</sup>

Heimson is a bit crazy, and takes himself to be a philosopher of the Scottish Enlightenment, but he's uncertain which one he is: Stewart, Hume, or Smith. Alone in his study, he says to himself, "I might be Hume".

Since for all Heimson knows he is Hume, it seems that Heimson's utterance of (16) is true (or at least, true relative to his epistemic state).

(16) I might be Hume.

But on a standard treatment of indexicals and modals Heimson's utterance would be false. The pronoun gets its value from the contextually determined assignment function—and thus refers to Heimson—while the modal quantifies over worlds compatible with Heimson's evidence. Thus Heimson's utterance of (16) is true only if there is a world where Heimson is identical to Hume. But there is no such world, so on the standard view (16) is false. Santorio makes the following suggestion:

When 'I' occurs under an informational modal, it refers not to the actual speaker, but rather to representatives of the actual speaker in the relevant information state. Think of an information state as a set of possible worlds, namely, the worlds that are compatible with the relevant attitude... Roughly, [the] representatives are individuals that, for all the subject knows, the speaker might be. (Santorio 2012: 373)

This is formally carried out by making a certain adjustment to the semantics of modals.

On the standard view, informational modals are, in essence, quantifiers over possible worlds. On the view I'm advocating, they also encode in their meaning an apparatus that locates real-world individuals within the set of worlds quantified over. Thus on the new picture these modals manipulate a greater amount of information. . . [This] is implemented by letting epistemic operators manipulate the assignment parameter. . . (Santorio 2012: 376)

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<sup>15</sup>This is a somewhat simplified version of the kinds of examples that Santorio (2012) actually uses.

Santorio builds contextually variable *counterpart functions* into the semantics, which “specify a way in which the subject of the epistemic state is acquainted with elements of the context” (*ibid.*). Formally, a counterpart function  $f$  is a function from worlds to individuals (i.e. an individual concept). For each variable  $x_1, x_2, \dots$  the context supplies both a value  $g(1), g(2), \dots$  for the variable and a counterpart function  $f_1, f_2, \dots$  associated with the variable. Then relative to an assignment  $g$ , a world  $w$ , and a sequence of counterpart functions  $F = f_1, f_2, \dots$  the clause for epistemic *might* and *must* are as follows:

$\llbracket \text{might } \phi \rrbracket^{g,w,F} = 1$  iff

- (i)  $[f_1(w) = g(1)] \wedge [f_2(w) = g(2)] \wedge [f_3(w) = g(3)] \wedge \dots$  and,
- (ii) for some  $\langle g', w' \rangle$  such that  $w' R w$  and  $g' = \langle f_1(w'), f_2(w'), \dots \rangle$ ,  $\llbracket \phi \rrbracket^{g',w'} = 1$

$\llbracket \text{must } \phi \rrbracket^{g,w,F} = 1$  iff

- (i)  $[f_1(w) = g(1)] \wedge [f_2(w) = g(2)] \wedge [f_3(w) = g(3)] \wedge \dots$  and,
- (ii) for all  $\langle g', w' \rangle$  such that  $w' R w$  and  $g' = \langle f_1(w'), f_2(w'), \dots \rangle$ ,  $\llbracket \phi \rrbracket^{g',w'} = 1$

Thus, (16), which is regimented as “ $\Diamond x = h$ ” can come out true in a context.<sup>16</sup> It is true just in case the counterpart function  $f$  associated with ‘ $x$ ’ picks out Heimson in the actual world and there is a world  $w'$  compatible with Heimson’s information state such that “ $x = h$ ” is satisfied by  $w'$  and a shifted assignment  $g'$ , where  $g'(x) = f(w')$  – that is, just in case there is a world compatible with Heimson’s information state where there is an “epistemic counterpart” of Heimson who is Hume.<sup>17</sup>

Santorio’s picture shares with Cumming’s the feature that the epistemic operators unselectively bind all the variables in thier scope. Santorio (2012) doesn’t mention quantifying in, but prefixing an objectual quantifier to an epistemically modalised sentence won’t end up being vacuous on his account. This is due to the “check” on the counterpart functions in the first conjunct, which ensures that the counterpart function applied to the input world aligns with the initial assignment. Given this check, a modal sentence such as “might  $\phi$ ” is still sensitive in the requisite way to the initial variable assignment.

Dilip Ninan has a series of papers (Ninan 2012, Ninan 2013, and Ninan forthcoming), where he makes similar use of assignment-sensitive content and “multi-centered worlds”.

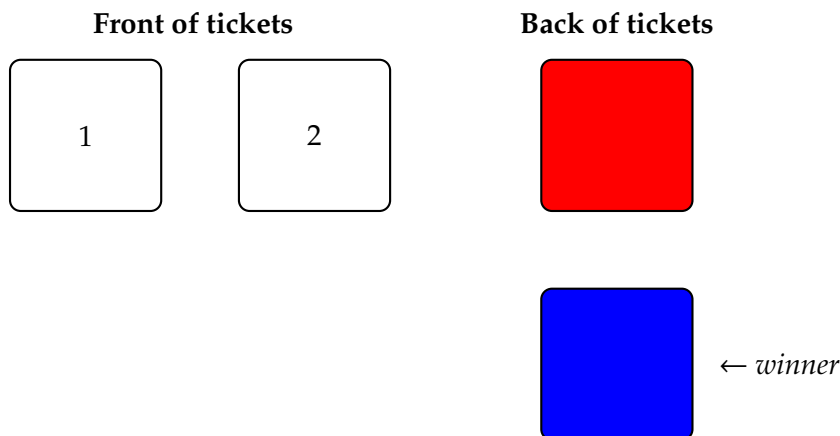
<sup>16</sup>Here we treat ‘Hume’ as a constant  $h$ , but this is just for ease of illustration. On a fuller account, all names, pronouns, and variables will be rendered as assignment-sensitive terms.

<sup>17</sup>On this view, like the ones discussed above, an identity “ $x = y$ ” will not be epistemically necessary, since the necessity claim will fail as long as there is an accessible point  $\langle g, w \rangle$  where  $g(x) \neq g(y)$ .

Ninan (2012) closely resembles the counterpart semantics described in the previous section, though it has additional bells and whistles.<sup>18</sup> In a more recent paper Ninan forthcoming presents a puzzle, which can be used to motivate epistemic counterpart semantics.<sup>19</sup> The puzzle begins with this scenario:

There is a lottery with only two tickets, a blue ticket and a red ticket. The tickets are also numbered 1 and 2, but we don't know which colour goes with which number. (Assume the number of the ticket is printed on one side of the ticket and the colour on the back.) The winner has been drawn, and we know that the blue ticket won. But since we don't know whether the blue ticket is ticket 1 or ticket 2, we don't know the number of the winning ticket.

We can present the front and back of the tickets, in no particular order, as follows (with the blue ticket indicated as the winner):



Ninan's puzzle proceeds by the following reasoning about this scenario:

- (A) Ticket 1 is such that it might be the winning ticket.
- (B) Ticket 2 is such that it might be the winning ticket.

<sup>18</sup>An earlier version of this paper surveyed Ninan (2012) and compared it to the views of Cumming and Santorio, but when doing the final revision for this paper Ninan sent me his new paper "Quantification and Epistemic Modality", which I focus on here since it provides a strong argument for the epistemic counterpart semantics I outlined above.

<sup>19</sup>Closely related puzzles are discussed in Aloni (2005) and Gerbrandy (2000). Of course, epistemic puzzles in this general vicinity have been discussed for years under the heading of puzzles of *de re* beliefs.

Since those are the only tickets, it follows that any ticket is such that it might be the winning ticket. But the red ticket is a ticket, so it follows that:

(C) The red ticket is such that it might be the winning ticket.

Thus, the puzzle is that from the apparently acceptable (A) and (B) we seem to be able to conclude the apparently unacceptable (C). (A) and (B) seem acceptable since we don't know the number of the winning ticket. But (C) seems unacceptable since we know that the red ticket lost. The reasoning from (A) and (B) to (C) seems valid, and given the standard Kripke-style semantics, it is valid. As Ninan notes the puzzle specifically concerns *epistemic* modals—there is no analogous puzzle with metaphysical modals (e.g. “The red ticket might have been the winning ticket” is perfectly acceptable). As an account of epistemic modality, the standard view is faced with the choice of denying the conjunction of (A) and (B) or accepting (C). Neither option looks plausible.

This provides motivation for seeking an alternative account—of course, it is more-or-less the same kind of motivation I've been harping on at various points in this paper, but Ninan's puzzle affords a clear and concise way of making the problem vivid. Ninan outlines two alternatives, which solve the puzzle. Importantly both alternatives abandon the “objectual” aspect of standard Kripke semantics, whereby variables are both rigid and immune to shifting by modals. Ninan insists that the way forward is to take on board the *Quinean insight* that “being necessarily or possibly thus and so is not a trait of the object concerned, but depends on the manner of referring to the object” (Quine 1953a: 148).

The first alternative Ninan develops builds on the dynamic proposal of Yalcin (2015) by supplementing it with Carnapian individual concepts, while the second alternative generalises the standard semantics with counterpart relations. The key to both approaches is that they allow us to say that object  $o'$  in world  $w'$  represents object  $o$  in world  $w$  even if  $o$  is not identical to  $o'$ . To implement the counterpart version of this Ninan adopts the kinds of models we defined above, where  $\mathfrak{M} = \{W, R, D, I\}$ . And then introduces counterpart relations which are binary relations on  $D \times W$ . Second, he construes the counterpart relation as a parameter of the index (instead of as fixed in the model).<sup>20</sup> Thus he defines

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<sup>20</sup>Schwarz (2012) and Cresswell and Hughes (1996) provide essentially the same semantics as Ninan, but they define the counterpart relation as an element of the model. The difference here doesn't matter much in terms of the pure semantics, but may lead to differences in implementation with the post-semantics definition of truth, under the assumption that context supplies the counterpart relation. Presumably Ninan is anticipating a standard Kaplanian definition of truth-in-a-context, and he includes the counterpart relation as a parameter of the index in order to secure the contextual variability of the counterpart relation. That is, it



the truth of a formula relative to a counterpart relation  $K$  in addition to a world  $w$  and a variable assignment  $g$  (and a model  $\mathfrak{M}$ ). All clauses ignore the counterpart parameter and are thus essentially the same as on the Kripke-semantics, save for epistemic “must” and “might”, which are defined as follows:

- $\llbracket \Box \phi \rrbracket^{g,w,K} = 1$  iff  $\llbracket \phi \rrbracket^{g',w',K} = 1$ ,

for all  $w'$  such that  $wRw'$  and for all  $g'$  such that for each variable  $i$ ,  $\langle g(i), w \rangle K \langle g'(i), w' \rangle$ .

- $\llbracket \Diamond \phi \rrbracket^{g,w,K} = 1$  iff  $\llbracket \phi \rrbracket^{g',w',K} = 1$ ,

for some  $w'$  such that  $wRw'$  and for some  $g'$  such that for each variable  $i$ ,  $\langle g(i), w \rangle K \langle g'(i), w' \rangle$ .

We can now model the lottery scenario. Let ‘ $W$ ’ be the predicate for “winning ticket”, let ‘ $t_1$ ’ go proxy for ‘ticket 1’, and let ‘ $t_2$ ’ go proxy for ‘ticket 2’. Then (A) and (B) will be rendered in our language as follows:<sup>21</sup>

(A')  $\Diamond Wt_1$

(B')  $\Diamond Wt_2$

The truth-conditions for each of these are as follows:

A':  $\llbracket \Diamond Wt_1 \rrbracket^{g,w,K} = 1$  iff  $\llbracket Wt_1 \rrbracket^{g',w',K} = 1$ ,

for some  $w'$  such that  $wRw'$  and for some  $g'$  such that  $\langle g(t_1), w \rangle K \langle g'(t_1), w' \rangle$ .

B':  $\llbracket \Diamond Wt_2 \rrbracket^{g,w,K} = 1$  iff  $\llbracket Wt_2 \rrbracket^{g',w',K} = 1$ ,

for some  $w'$  such that  $wRw'$  and for some  $g'$  such that  $\langle g(t_2), w \rangle K \langle g'(t_2), w' \rangle$ .

is an important aspect of this approach that which object-in-a-world represents another object-in-a-world is something that can vary with the utterance context.

<sup>21</sup>Ninan symbolises these with lambda binding— $(\lambda x. \Diamond Wx)(t_1)$  and  $(\lambda x. \Diamond Wx)(t_2)$ —in order to ensure that the modal predication are read *de re*. After all the stilted “is such that” locution is employed so that the sentences are read *de re*. We could easily add lambdas to our language to do this as well, or we could fake it with constructions such as “ $t_1 = x \wedge \Diamond Wx$ ”. But we needn’t do this—Ninan does because he is being neutral on whether or not  $t_1$  and  $t_2$  are descriptions or names. Since we are assuming that  $t_1$  and  $t_2$  are variables these further complications are actually unnecessary.

To make things concrete assume the domain only has two tickets so that  $D = \{a, b\}$ , and assume that that  $g(t_1) = a$  and  $g(t_2) = b$ . Then we can see that (A') is true just in case there is an accessible world where a counterpart of  $a$  under relation  $K$  is  $W$ , and (B') is true just in case there is an accessible world where a counterpart of  $b$  under relation  $K$  is  $W$ . Thus the truth of the conjunction of (A') and (B') entails that anything might be  $W$ :

$\llbracket \forall x \Diamond Wx \rrbracket^{g,w,K} = 1$  iff for all  $g'$  that differ from  $g$  at most in that  $g'(x) \neq g(x)$ ,  $\llbracket Wx \rrbracket^{g'',w',K} = 1$ ,  
for some  $w'$  such that  $wRw'$  and for some  $g''$  such that  $\langle g'(x), w \rangle K \langle g''(x), w' \rangle$ .

That is, anything might be  $W$  iff for each individual  $x$  in the domain there is an accessible world where a counterpart of  $x$  under  $K$  is  $W$ . And that follows if:

- (i)  $a$  and  $b$  are the only members of the domain and
- (ii) there is an accessible world where a counterpart of  $a$  under  $K$  is  $W$  and
- (iii) there is an accessible world where a counterpart of  $b$  under  $K$  is  $W$ .

But now what about the crucial sentence (C): "The red ticket is such that it might be the winner". Let ' $r$ ' go proxy for "The red ticket" (where again we are reading the sentence *de re* so we are assuming that ' $r$ ' is a name or variable.) So we have:

(C')  $\Diamond Wr$

And its truth-conditions are given as follows:

$C': \llbracket \Diamond Wr \rrbracket^{g,w,K} = 1$  iff  $\llbracket Wr \rrbracket^{g',w',K} = 1$ ,

for some  $w'$  such that  $wRw'$  and for some  $g'$  such that  $\langle g(r), w \rangle K \langle g'(r), w' \rangle$ .

Given that  $a$  and  $b$  are the only things in the domain,  $g(r) = a$  or  $g(r) = b$ . And on either choice we can see that (C') follows from (A') and (B'). That is, (C') is true just in case there is an accessible world where a counterpart of  $g(r)$  under  $K$  is  $W$ . And that is guaranteed given the truth of both (A') and (B') (plus the assumptions about the domain and that  $g(t_1) \neq g(t_2)$ ).

At this point one might think the counterpart theory has failed to diagnose the puzzle, since the argument comes out as valid! The whole problem was that we want to accept (A')

and (B') without accepting (C'). So how is this any help? While the argument is formally valid according to the counterpart semantics, the story I've told so far hasn't made full use of the counterpart-theoretic resources. In giving the formal argument we've just assumed some generic counterpart relation that we've held fixed throughout. Yet Ninan's lottery scenario is specifically designed to make two counterpart relations salient:

$K^n$  = the *number counterpart relation*, which holds between ticket  $t$  in world  $w$  and ticket  $t'$  in world  $w'$  iff  $t$  in  $w$  and  $t'$  in  $w'$  have the same number.

$K^c$  = the *colour counterpart relation*, which holds between ticket  $t$  in world  $w$  and ticket  $t'$  in world  $w'$  iff  $t$  in  $w$  and  $t'$  in  $w'$  are the same colour.

The counterpart-theoretic diagnosis of the puzzle insists that we want to accept (A') and (B') relative to counterpart relation  $K^n$  and we want to deny (C') relative to counterpart relation  $K^c$ . Although (C') is true relative to  $K^n$  it is practically impossible to utter "The red ticket is such that it might be the winner" without making the the colour counterpart relation salient. Likewise although (A') and (B') are false relative to  $K^c$  it is difficult, if not impossible, for that counterpart relation to be salient in a context in which  $a$  and  $b$  are referred to as *Ticket 1* and *Ticket 2*. Thus, the solution in terms of epistemic counterpart semantics is a contextualist solution—there is contextual variability of the counterpart relation (cf. Lewis 1983: 43-43; Lewis 1986: 251-263), and the contexts in which we want to accept the premises are different from the contexts in which we want to deny the conclusion.

This proposal is similar in spirit to various proposals that have been around forever. In particular, epistemic counterpart semantics bears an affinity to both a Carnapian semantics in terms of individual concepts (Carnap 1947 and Aloni 2005) and epistemic two-dimensionalism (Chalmers 2004). In fact, some of the formal devices that are employed by these approaches are more-or-less the same. There are many choice points within these frameworks and given the right tweaks on both ends we might even get to the same mathematical structure.<sup>22</sup>

Even though the picture shares some features with descriptivist views, it doesn't seem threatened by familiar anti-descriptivist arguments. It is important to notice that the epistemic counterpart relations that are associated with terms are not lexicalized, they are

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<sup>22</sup>One obvious difference is that individual concepts and primary intensions are functional, whereas counterpart relations are not so constrained.

instead determined by the context. (For example, the number relation and the colour relation are not somehow lexically attached to “Ticket 1” and “The red ticket”, respectively.) Thus, there is no commitment to the idea that names (or variables) are synonymous with descriptions, nor to the idea that the referent of a name is determined via description. Kripke’s (1980) arguments, then, can’t seem to get off the ground, if aimed at the epistemic counterpart approach. For example consider the argument from ignorance. If in a given context *c* the subject thinks of Cicero simply as “a Roman orator”, then “Cicero” in that context still manages to refer to Cicero because the way the subject thinks of Cicero is irrelevant to the determination of the referent of “Cicero”. Of course, the way the subject thinks of Cicero, in a sense, determines the referent of “Cicero” in the agent’s “epistemic worlds”—in every world compatible with the subject’s knowledge the epistemic counterparts of Cicero are Roman orators. But it’s not clear what the objection to this could be. Whereas with standard descriptivism, Carnapian individual concepts, or epistemic two-dimensionalism the function from worlds to individuals is lexically associated with the singular term, and it is this assumption in particular that is leveraged against those views. Of course, such views could be modified so that the mode of presentation is detached from reference determination and so that there is contextual variability of the relevant intensions. There has yet to be a detailed survey of these various accounts comparing them formally, and mapping out where there are genuine differences and where the differences are merely notational. This should be done. But in any case, I predict that the flexibility and generality of the epistemic counterpart approach outlined above will be seen as a virtue.

## Appendix: Quantified metaphysical and epistemic modality

I present a simple language incorporating the lessons from above. The language will be a mixed modal language with both objective modals and epistemic modals, and it will also have individual quantifiers.<sup>23</sup> Interestingly, we can model some version of Kripke’s cases of the *aposteriori* necessities and *apriori* contingencies, in way that (prima facie) differs from the two-dimensionalist models (e.g. Davies and Humberstone 1980). Here’s

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<sup>23</sup>It is not strictly necessary to lexically distinguish the metaphysical and epistemic modals, I do that here just for perspicuity. One could easily make this in line with the proposal of Kratzer (1977), by letting the context fix the accessibility relation between assignment-world pairs, where metaphysical contexts would turn out to set the counterpart relation to identity.

the language.

*Variables:*  $x, y, z, \dots$

*Predicates:*  $F, G, H, \dots$

*Constants:*  $\neg, \wedge, \exists, =, \Box, \blacksquare$

For any sequence of variables,  $\alpha_1, \dots, \alpha_n$ , any  $n$ -place predicate  $\pi$ , and any variables  $\alpha$  and  $\beta$ , the sentences of the language are provided by the following grammar:

$$\phi ::= \pi\alpha_1 \dots \alpha_n \mid \alpha = \beta \mid \neg\phi \mid (\phi \wedge \phi) \mid \exists\alpha\phi \mid \Box\phi \mid \blacksquare\phi$$

Let a model  $\mathfrak{M} = \langle W, R, K, D, I \rangle$ , where  $W$  is a (non-empty) set of worlds,  $R$  is a binary relation on  $W$  (a metaphysical accessibility relation),  $K$  is a binary relation on  $D^N \times W$  (an epistemic accessibility relation),  $D$  is a (non-empty) set of individuals, and  $I$  is the interpretation, which relative to a world assigns the predicates sets of tuples of individuals drawn from  $D$ . (Again we present a constant domain semantics for ease, while we really prefer a variable domain.) We then provide the semantic clauses relative to a model  $\mathfrak{M}$  as follows:

- $\llbracket \alpha \rrbracket^{g,w} = g(\alpha)$
- $\llbracket \pi\alpha_1 \dots \alpha_n \rrbracket^{g,w} = 1$  iff  $\langle \llbracket \alpha_1 \rrbracket^{g,w}, \dots, \llbracket \alpha_n \rrbracket^{g,w} \rangle \in I(\pi, w)$
- $\llbracket \alpha = \beta \rrbracket^{g,w} = 1$  iff  $\llbracket \alpha \rrbracket^{g,w} = \llbracket \beta \rrbracket^{g,w}$
- $\llbracket \neg\phi \rrbracket^{g,w} = 1$  iff  $\llbracket \phi \rrbracket^{g,w} = 0$
- $\llbracket \phi \wedge \psi \rrbracket^{g,w} = 1$  iff  $\llbracket \phi \rrbracket^{g,w} = 1$  and  $\llbracket \psi \rrbracket^{g,w} = 1$
- $\llbracket \exists\alpha\phi \rrbracket^{g,w} = 1$  iff for some  $g'$  that differs from  $g$  at most in that  $g'(\alpha) \neq g(\alpha)$ ,  $\llbracket \phi \rrbracket^{g',w} = 1$
- $\llbracket \Box\phi \rrbracket^{g,w} = 1$  iff for all  $w'$  such that  $wRw'$ ,  $\llbracket \phi \rrbracket^{g,w'} = 1$
- $\llbracket \blacksquare\phi \rrbracket^{g,w} = 1$  iff for all  $w'$  and  $g'$  such that  $\langle w, g \rangle K \langle w', g' \rangle$ ,  $\llbracket \phi \rrbracket^{g',w'} = 1$

This kind of model can deal with the puzzles concerning epistemic modality *de re*, that we have covered in the paper.<sup>24</sup> It also affords a kind of diagnosis of the Kripkean necessary *a posteriori* and contingent *apriori*. Here is a brief sketch. First, *a posteriori* necessities:

(17) Hesperus couldn't have failed to be Phosphorus.

$$\therefore \Box h = p$$

(18) Hesperus might not be Phosphorus.

$$\therefore \neg \blacksquare h = p$$

The metaphysical claim (17) is straightforwardly true, assuming  $h = p$ ; while the epistemic *might*-claim (18) is also true under certain epistemic counterpart relations, e.g. the counterpart relations that would be relevant in a context in which Pythagorus utters it. Next, cases of the contingent *apriori*:

(19) Julius has to be the inventor of the zip

$$\therefore \blacksquare Zj$$

(20) Julius could have failed to invent the zip

$$\therefore \neg \Box Zj$$

Of course the truth of (19) depends on which counterpart relation is contextually salient. And it is plausible that in a context where we have just introduced the name "Julius" as a name of the inventor of the zip, every epistemic counterpart of Julius will be a zip inventor. Nevertheless, (20) is true for the familiar reason that zip-inventing is a contingent property of Julius.<sup>25</sup>

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<sup>24</sup>I have included the counterpart relation as a part of the model, but for the reasons mentioned in footnote 20, if this type of model is employed in a natural language setting where there is contextual variability we ought to construe the counterpart relation as an index in the point of reference.

<sup>25</sup>An open question is how this system fares in terms of what Chalmers and Rabern (2014) call *the generalized nesting problem*, although it appears promising since it would seem to invalidate (A2).

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