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Yield criteria for glaciotectonically deformed deposits

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Abstract

Most glaciotectonically deformed deposits, including varved clays and glacial tills, are characterised by cracks and fissures. This paper presents a method for describing the yield criteria for glacitectonically deformed cohesive deposits using a model of cracked geomaterial with isotropic or anisotropic matrix. The general representation of the limit conditions for anisotropic materials in plane-strain is used to determine the yield criterion. The yield criterion represents a convex, piece-wise surface in the three-dimensional stress space revealing explicitly global, plastic properties of the materials considered. An example of using proposed yield criteria to solve a bearing capacity problem of a strip foundation constructed on a glaciotectonically cracked layer is presented. The lower and upper-bound estimates of limit loads on the strip footing are given. The limit state analysis presented in this paper can be used to solve many other geotechnical engineering problems, for example, the stability of slopes and reinforced walls or the bearing capacity of pile foundations.

Keywords

Shear strength; Theoretical analysis; Plasticity; Anisotropy; Limit state analysis; Glacial soils.

Highlights

- Most glaciotectonically deformed deposits are characterised by cracks and fissures
- The yield criteria of a cracked cohesive geomaterial are described
- These criteria are used to estimate the bearing capacity of a strip footing
- This method can be used to solve other geotechnical engineering problems

1. Introduction

Glaciotectonism is often referred to the processes of glacially-induced structural deformation of rock or sediment masses. Most glaciotectonic deformations of soil and rock occurred as a direct result of vertical (static) loading imposed by a glacier or its forward (dynamic) movement (Aber and Ber 2007). Glaciotectonic structures range in scale from microscopic to continental. Glacially-induced structures are very common and a complete listing of all them is practically impossible because of their wide variety in type, style and size. An overview of major types of glaciotectonic structures, their characteristics and analysis methods can be found in Aber and Ber (2007). For example, glaciotectonically deformed soils in Poland include Quaternary and Neogene deposits of various lacustrine clays and glacial tills (Strozyk 2015). Their mineralogy, physical and mechanical properties vary noticeably depending mainly on age, lithology and thickness of each strata.

A common feature of glaciotectonic clay deposits is their sedimentation in lacustrine environment, in which very fine clay colloids and coarser particles of silt or sand were subjected to very slow deposition processes resulting in a distinct lamellar structure. A lacustrine varved clay from the Miocene-Pliocene strata of variegated clays of Poznan formation is one of the most interesting examples of glaciotectonic deposits in Poland (Flieger-Szymańska and Machowiak 2011; Florkiewicz et al. 2014, Strozyk 2015). An accurate prediction of the behaviour of glaciotectonically deformed cohesive deposits is often affected by irregular geological stratification, including numerous folds and faults created by glacial and tectonic activities. Consequently, these deposits are anisotropic in nature and present significant geotechnical challenges in many countries around the world, both in terms of design and construction (e.g. De Groot and Lutenegger 2009; Florkiewicz et al. 2014).

Another type of postglacial deposit that needs to be carefully analysed in engineering design is glacial till, formed typically in the ground moraines at the base of glacier or deposited in the recessional moraine during glacier’s retreat (Aber and Ber 2007). Formation process of ground moraine results in a heterogeneous structure of glacial till, without significant planar and/or linear fabric (Derski et al. 1988). Because of extremely heterogeneous characteristics of glacial tills and their well-graded content, including practically all particle sizes, from clays to mixtures of clay, silt, sand, gravel, and boulders, their deformation is typically random and often depends on local stress conditions. Therefore, most glacial tills can be characterised by quasi-isotropic fabric. However, it is also possible that a specific dynamic action of moving glacier and the location of accumulated till may result in anisotropic fabric, e.g. flow till or lodgement till (Aber and Ber 2007).

Most glaciotectonically deformed deposits, including varved clays and glacial tills described above, are often characterised by secondary cracks and fissures. From engineering point of view, this means that mechanical properties of glaciotectonic deposits will be affected by two different processes. On one hand, glaciotectonic soils may have primary isotropic or anisotropic matrix, depending on their geological origin. On the other hand, such deposits will be characterised by secondary anisotropic properties because of cracks created later as a consequence of changing stress conditions.

The problem of accurate evaluation of limit load for a foundation resting on a cracked (or layered) geomaterial is of fundamental importance for engineering design. However, despite early solutions of this problem for a homogeneous soil (Prandtl 1920; Hill 1950), the effective and accurate methods of limit analysis for cracked geomaterials are not available yet. Therefore, the most appropriate mathematical form of the yield criterion for cracked and layered geomaterials is the subject of extensive ongoing research (e.g. Florkiewicz 1989; Mroz and Maciejewski 2002, Lydzba et al. 2003; Yu 2006).

The bearing capacity of foundations on cracked and layered geomaterials is also a subject of ongoing research (e.g. Florkiewicz 1989, 2013, Michalowski and Shi 1995; Al-Shamrani and Moghal 2015; Valore et al. 2017; Ziccarelli et al. 2017). For example, Michalowski and Shi (1995) analysed the bearing capacity of footing constructed on a layer of sand resting on cay using a kinematic approach and the Mohr-Coulomb yield criterion. Designed charts developed by Michalowski and Shi (1995) for different internal friction angles of sand can be used as a useful tool for calculation of the bearing capacity of strip footings over a layered soil system.

Most recently, Valore et al. (2017); Ziccarelli et al. (2017) demonstrated experimentally and numerically that even a thin discontinuity inside a soil strongly influences the failure mechanisms and the bearing capacity of a footing.

In this paper, the yield and strength criteria for a cracked cohesive geomaterial with isotropic and anisotropic matrix are proposed. These criteria can be very useful in applying limit state theory to solve bearing capacity problems of cracked or fissures geomaterials. An example of applying the proposed yield criteria to the bearing capacity problem of a shallow strip footing using upper and lower bound theorems is presented. Similar upper and lower bound solutions with the proposed yield functions can be derived for other practical problems with a wide range of geometrical configurations and loading conditions (e.g. Florkiewicz 1989).

2. Anisotropy of geomaterials

Shear strength is a fundamental soil property used in geotechnical design. Thus, it must be determined with reasonable accuracy. However, the stress-strain-strength behaviour of most sedimentary deposits is anisotropic. Soil strength is generally lower when the direction of major principal stress is farther away from the deposition direction. Hence, soil anisotropy has attracted long-lasting interest of geotechnical researchers and practitioners.

Numerous experimental studies on the soil anisotropy have been carried out in the past (e.g. Schokking 1998; Lade et al. 2008; Cai et al. 2013; Yang et al. 2015; Chu et al. 2016). Based on laboratory observations, a number of advanced constitutive models have been developed, e.g., bounding surface plasticity model (Li and Dafalias 2004), double shearing model (Zhu 2006), yield vertex and double shearing model (Yu 2006).

Although most sedimentary deposits are inherently anisotropic due to their natural deposition in horizontal layers, further anisotropy can be induced by the applied stresses or strains. A detailed analysis of glaciotectonically deformed deposits needs to include their anisotropic characteristics in both, elastic and plastic domains. However, elastic and plastic anisotropy should be considered separately because each type of anisotropy is described by different material parameters. Furthermore, coupling between elastic and plastic anisotropy of soil is not always necessary (Boehler and Sawczuk 1977; Derski et al. 1988; Pietruszczak and Mroz 2000, Mroz and Maciejewski 2002, Lydzba et al. 2003; Yu 2006, Yu et al. 2009).

In plasticity theory, anisotropy defined as a directionally dependent yield criterion, can be interpreted by two approaches. In classical plasticity theories of anisotropic materials, proposed for example by von Mises (1928) and Hill (1950), anisotropic yield criteria were derived by generalizing existing isotropic yield criteria. The other approach is based on the anisotropic plastic tensor theory (Boehler and Sawczuk 1977; Nova 1980), in which a tensor function theory allows one to derive both, the yield criteria and constitutive relations of anisotropic materials. Detailed reviews of yield and failure criteria derived for various types of soil and rock can be found in Izbicki and Mroz (1976), Yu (2006), Florkiewicz (2013), Yang (2015) and many others. The analytical formulations of yield criteria can be expressed in several ways. For example, the representation in terms of invariants of stress and structure tensors was used by Boehler and Sawczuk (1977) and Nova (1980). Hoek and Brown (1980) derived yield criteria by applying the critical plane approach requiring the yield condition to be satisfied on a potential yield plane. A simplified approach to formulation of anisotropic failure criteria using the microstructure tensor was also discussed by Pietruszczak and Mroz (2000) and Mroz and Maciejewski (2002). In these publications, the critical plane approach incorporating spatial distribution of microcracks was applied in quantitative description of the variation of compressive strength with orientation of principal stress axis relative to anisotropy axes. More recently, Lydzba et al. (2003) used two conceptually different methodologies for specification of the conditions at failure in sedimentary rock formations. The first approach was based on a homogenization technique and the second one was a phenomenological framework, which incorporated the notion of a microstructure/fabric tensor. It was demonstrated that both methods yield similar results in compression and tension regimes.

It is clear from the literature that the formulation of appropriate yield (or failure criteria) for anisotropic geomaterials is of paramount importance for their constitutive modelling and potential applications in geotechnical practice. The formulation of the yield function proposed by

Booker and Davis (1972) and its application to the analysis of glaciotectonically deformed soils is discussed in the subsequent section.

3. Yield criteria of glaciotectonically deformed soils

A general representation of the yield function for a homogeneous perfectly plastic anisotropic medium in the plane-strain conditions was proposed by Booker and Davis (1972) in the form:

$$f(\sigma_{ij}) = R - g(p, \psi) = 0 \tag{1}$$

where

$$R = \frac{1}{2}\left[\left(\sigma_x - \sigma_y\right)^2 + 4\tau_{xy}^2\right]^{\frac{1}{2}} \tag{2}$$

$$p = \frac{1}{2}\left(\sigma_x + \sigma_y\right) \tag{3}$$

$$tan2\psi = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \tag{4}$$

and g is a known function of p (mean pressure) and ψ (direction of the major principal stress). This leads to the following relations:

$$\sigma_x = p + R\,cos2\psi \tag{5a}$$

$$\sigma_y = p - R\,cos2\psi \tag{5b}$$

$$\tau_{xy} = R\,sin2\psi \tag{5c}$$

In this paper, the yield function proposed for an anisotropic material in plane-strain conditions by Booker and Davis (1972) is used to derive the yield function for two cases of a cracked geomaterial: (1) with isotropic matrix, and (2) with anisotropic matrix. Plane-strain conditions are very common in most geotechnical problems (Wanatowski et al. 2010, Wanatowski and Chu 2012; Chu et al. 2015, Yuan et al. 2018). Thus, the yield function proposed by Booker and Davis (1972) can be suitably adapted for the use in geotechnical practice (e.g. Yuan et al. 2018).

A classical rigid-perfectly plastic model (without any work hardening or softening) is used for the description of sliding along the cracks. This model depends only on the stress state and describes an immediate transition from the rigid state into the plastic flow characterised by the horizontal plateau on the stress-strain curve. Thus, elastic deformation is neglected in this model. The global yield criterion for the cracked geomaterial is represented by a convex surface in the stress space. Classical interpretation of the yield criterion dictates that none of the combinations of stress tensor components (σ_{ij}) inside this convex surface can result in immediate sliding of the cracked medium. Furthermore, the incremental plastic strains are normal to the yield surface, i.e. the flow rule is associated. Despite its simplicity, the rigid-perfectly plastic model has been used successfully for solving many practical problems in soil and rock mechanics (e.g. Yu 2006).

3.1 Cracked geomaterial with isotropic matrix

A cracked soil (or rock) with a parallel set of densely distributed cracks is quite common. This set imposes an anisotropic structure on soil with a specific weak orientation along which the preferential slip may occur. The contact limit condition along the crack (or discontinuity) direction can be described in a form

$$\tau_n = \sigma_n \tan \varphi^* + c^* \tag{6}$$

where σ_n and τ_n, are the normal and tangential contact stresses on the discontinuity surface, c* and φ* are cohesion and the friction angle along the crack direction.

First, let us consider the properties of the soil with one parallel system of cracks inclined at the angle χ to the horizontal axis, as shown in Fig. 1. The contact limit condition given by Eq. 6 is satisfied along the crack direction. The soil is assumed to have isotropic matrix and is defined by its specific weight γ, cohesion c, and the angle of internal friction φ. It is also assumed that the yield criterion for the material is governed by the Coulomb condition in a form

$$(\sigma_x - \sigma_y)^2 + 4\tau^2_{xy} = (\sigma_x + \sigma_y + 2c \cot\varphi)^2 \sin^2\varphi \tag{7}$$

where c ≥ c*, φ ≥ φ* (Fig. 1).

The criterion in Eq. (7) is illustrated in Fig. 1(a) where $X = ½ (\sigma_x + \sigma_y)$, $Y = ½ (\sigma_x - \sigma_y)$, $Z = \tau_{xy}$, Eq. 7 can be illustrated as a conical surface with the axis coinciding with the hydrostatic pressure axis (X), as shown in Fig. 2a.

The normal (σ_n) and tangential (τ_n) contact stresses can be expressed using the stress transformation equations

$$\sigma_n = \frac{1}{2}(\sigma_x + \sigma_y) - \frac{1}{2}(\sigma_x - \sigma_y)cos2\chi - \tau_{xy}sin2\chi \tag{8a}$$

$$\tau_n = \tau_{xy} cos2\chi - \frac{1}{2}(\sigma_x - \sigma_y) sin2\chi \quad (8b)$$

Using Eqs. (8a) and (8b) we can transform Eq. (6) into Eq. (9)

$$\frac{1}{2}(\sigma_x - \sigma_y) sin(2\chi \pm \varphi^*) - \tau_{xy} cos(2\chi \pm \varphi^*) \pm \frac{1}{2}(\sigma_x + \sigma_y) sin\varphi^* \pm c^* cos\varphi^* = 0 \quad (9)$$

Thus, in the stress space X-Y-Z, the condition expressed by Eq. (6) can now be represented by two planes according to Eq. (9). The line of cross-section of the two planes intersects the X-axis at the distance $c^*\cot\varphi^*$ from the origin and is inclined to the X-axis at the angle $\pi/4$, as shown in Fig. 2b. The yield criterion (Eq. 9) in the cross-section X = const is also presented in Fig. 2a. The interpretation of R and ψ, used earlier in Eqs. 5a-c, is also shown in Fig. 2.

The global yield criterion of the cracked geomaterial with isotropic matrix in the X-Y-Z space can now be obtained by the superposition of conditions (Eq. 7) and (Eq. 9). This condition can be represented graphically by a convex surface, as shown in Fig. 3. All the points inside this surface will belong to a set

$$B = \{(R, X, \psi): R_{min} [g(X, \psi), g^*(X, \psi)]\} \quad (10)$$

where g and g* are known functions derived based on Eqs. (5), (7) and (9). The functions g and g* can be written as Eq. 11 and Eq. 12 for the isotopic soil matrix and the cracks, respectively

$$g = X \sin\varphi + c \cos\varphi \quad (11)$$

$$g^* = \frac{\pm X sin\varphi^* \pm c^* cos\varphi^*}{\sin(2\chi \pm \varphi^* - 2\psi)} \quad (12)$$

It should be mentioned that the yield condition described by Eq. (10) can also be adapted for a Tresca type material ($c=2c^*$ and $\varphi = \varphi^* = 0$), as shown in Fig. 4. Such interpretation of the yield criterion for cracked and layered geomaterials was discussed in more detail by Florkiewicz (1986) and Florkiewicz and Mroz (1989).

3.2 Cracked geomaterials with anisotropic matrix

Let us now consider the case of the cracked geomaterial with anisotropic matrix of Tresca's type ($\varphi = 0$, $c = s_u \neq 0$). Furthermore, let us assume that the geomaterial is orthotropic, so it can be defined by the yield stress T and the degree of orthotropy d (Fig. 5) and the yield criterion is defined by Hill's condition (Hill 1950). The Hill yield criterion depends only on the deviatoric stresses and is pressure independent. Although Hill's criterion was initially formulated for metals, it has been successful extended for composites (Tsai and Wu 1971) and soils and rocks (Lo, 1965, Pariseau 1972, Aubeny et al. 2003).

Hill's condition in the plane strain conditions with respect to the principal axes of orthotropy x ,y (Fig. 5), can be written as

$$\frac{(\sigma_x - \sigma_y)^2}{4(1-d)} + \tau_{xy}^2 = T^2 \tag{13}$$

where

d is material constant determined by the degree of orthotropy ($-\infty<d<1$) and T is the yield stress in shear with respect to the axes of orthotropy (x, y).

In the stress space X-Y-Z, the criterion (Eq. 13) can be represented by the sides of infinitely long elliptical cylinders, with their axes of symmetry coinciding with the X-axis, as shown in Fig. 6. It should be noted that in the X-Y plane, the criterion (Eq. 13) is represented by two lines parallel to X-axis, which confirms that Hill's condition does not depend on the magnitude of mean stress.

Let us now introduce a set of cracks described earlier by Eq. 9 and shown in Fig. 2b to the orthotropic soil matrix with the yield criterion described by Eq. 13 and shown in Fig. 6. The global yield criterion for the cracked geomaterial with orthotropic matrix can be derived by superposition of Eqs. (9) and (13).

Similarly to the cracked geomaterial with isotropic matrix analysed earlier, the yield condition for the cracked geomaterial with orthotropic matrix can be presented in the X-Y-Z stress space as a surface, every point of which belongs to the set described by Eq. 10. However, the functions g and g* are now described by Eqs. (14) and (15), for the orthotropic soil matrix and the cracks, respectively.

$$g = T\sqrt{\frac{1-d}{1-d sin^2 2\psi}} \quad (14)$$

$$g^* = \frac{\pm X sin\varphi^* \pm c^* cos\varphi^*}{\sin(2\chi \pm \varphi^* - 2\psi)} \quad (15)$$

Examples of graphical visualisation of the above yield condition for two different pairs of cracked geomaterial parameters, (1) $c^* \neq 0$, $\varphi^* \neq 0$ (Coulomb's model), and (2) $c^* \neq 0$, $\varphi^* = 0$ (Tresca's model), are shown in Fig. 7 and Fig. 8, respectively. It should be mentioned that the global yield conditions for glaciotectonically deformed soils can be derived for any arbitrary number of discontinuity (crack) systems (Florkiewicz 1986, 1989, 2013).

4. Material parameters

It can be noticed that all the material parameters required for the yield criteria of graciotectonically deformed deposits analysed in this paper (c, φ, c*, φ* for the cracked geomaterial with isotropic matrix and T, d, c*, φ* for the cracked geomaterial with orthotropic matrix) can be determined using standard triaxial or shear box tests. In the case of soil model of Tresca type ($\varphi = \varphi^* = 0$) characterised by the undrained shear strength ($c = s_u \neq 0$), it is possible to determine material parameters using an uniaxial compression test. However, such a test should allow defining the unconfined compressive strength for different orientations of soil samples in the ground so that the effect of inherent anisotropy in the form of cracks, layers or discontinuities can be investigated. Modern soil sampling and rock coring techniques (e.g., block samples, large diameter oil-operated samplers, rotary sidewall coring) and subsequent coring in the laboratory must be used in order to achieve very good quality samples for subsequent laboratory testing (Agarwal et al. 2014; ASTM D2113-14, 2014; Bo et al. 2017; BS EN ISO 22475-1, 2006; Chung et al. 2014; Gylland et al. 2014; Long et al. 2010).

The interpretations of the unconfined compression tests for the cracked samples with isotropic matrix of Tresca type (the yield condition is shown in Fig. 4) and orthotropic matrix of Hill type (the yield condition is shown Fig. 8), are shown in Fig. 9 and Fig. 10, respectively. Both samples are characterised by the cracks of Tresca type ($c^* \neq 0$ and $\varphi^* = 0$). Figs. 9 and 10 are presented in a similar way and illustrate (a) orientation of the compressed soil element in the ground, (b) Mohr's circle of stress, (c) effect of the angle θ on the yield stress (σ_c). The samples are inclined at the angle θ to the horizontal plane with the assumption that $\theta + \chi = \pi/4$, as shown in Fig. 9a and Fig. 10a. It can be observed from Figs. 9c and 10c that the minimum value of σ_c is obtained for the angle $\theta = \chi - \pi/4$, where χ is the inclination angle of cracks inclined to the horizontal plane (see Fig. 9a and 10a).

5. Bearing capacity example

The yield criteria described in the preceding sections can be applied to solving various geotechnical design problems related to cracked or layered geomaterials (Florkiewicz 1989; Florkiewicz and Mroz 1989, Florkiewicz 2013, Florkiewicz and Kubzdela 2013). For example, let us consider a strip foundation on a cracked stratum with both isotropic matrix and cracks of Tresca type ($c = s_u \neq 0$, $\varphi = 0$, and $c^* \neq 0$, $\varphi^* = 0$), shown in Fig. 11. One of the yield criteria described in this paper (see Fig. 4) can be used to estimate the bearing capacity of this foundation.

Obviously, one of existing bearing capacity methods, for example the solutions proposed by Chen (1975), is also required to solve this problem. A method for application of Chen's solution to a soil stratum with directional crack systems is presented in Florkiewicz and Mroz (1989). The upper bound (p_k) and lower bound (p_s) bearing capacity solutions obtained for two examples of the shear strength reduction due to the cracks developed in a perfectly plastic soil of Tresca type (see Fig. 4), are shown in Fig. 12. The solutions are plotted in the form of relationships between the normalised bearing pressure $\overline{p}/c$ and the direction of cracks χ (see Fig. 11) taking into account the weakening effect of the cracks along the direction χ on the bearing capacity of the soil matrix. This weakening effect reduces soil's cohesion along the crack direction, χ, and can be described by the reduction ratio, c^*/c. Fig. 12 shows two examples of $c^*/c = 0.2$ and $c^*/c = 0.4$. It can be observed that the lower c^*/c ratio results in the lower bearing capacity of the cracked geomaterial (i.e. the larger reduction in the shear strength of the cracked geomaterial).

The closed-form bearing capacity solution for the isotropic cohesive soil without any cracks is also indicated by the dashed line in Fig. 12. It can be seen from Fig. 12 that the bearing capacity solutions calculated for the cracked anisotropic strata are significantly lower than that obtained for the uncracked isotropic stratum.

It is worth noting that the plots of the normalised bearing pressure $\overline{p}/c$ given in Fig. 12 represent the lower ($\overline{p}_s$) and upper ($\overline{p}_k$) bound solutions of bearing pressures derived based on the theorems of limit analysis, described in detail by Chen (1975). The lower bound bearing pressure $\overline{p}_s$ is determined by constructing statically admissible stress fields, explained in detail by Chen (1975). The construction of these fields is identical for the angles χ and $\pi/2 - \chi$ (e.g. $\chi = 30°$ and $\pi/2 - \chi = 60°$), which results in the mirror reflection with respect to the foundation axis of symmetry. The upper bound bearing pressure $\overline{p}_k$ is determined by constructing kinematically admissible velocity fields composed of rigid blocks dividing the soil stratum underneath the foundation. Constructions of these fields for the isotropic soil media are described by Chen (1975). The construction of the fields for the cracked geomaterial with isotropic matrix can be found in Florkiewicz and Mroz (1989). The symmetry of the $\overline{p}_k/c$ plots with respect to $\chi = 45°$ is related to the limit state mechanisms assumed by Florkiewicz and Mroz (1989).

Finally, it should be pointed out that the charts plotted in Fig. 12 can be used to estimate the bearing capacity of the cracked strata for a specific χ direction. Fig. 12 shows that the lowest bearing capacity values in both examples are obtained for the crack directions χ of around 30^{o} and 60^{o}, which is qualitatively consistent with the solutions published by Al-Shamrani and Moghal (2015).

6. Conclusions

Glaciotectonically deformed soil and rock strata are commonly present in many countries of the Northern Hemisphere, including large areas of Poland and other countries in the Baltic region. Design and construction of geotechnical structures on these strata is often challenging because of their anisotropic characteristics. This paper presents a method for describing the yield criteria of glaciotectonically disturbed cohesive soils using a model of the cracked geomaterial with isotropic or anisotropic matrix. The general representation of the limit conditions for anisotropic materials in plane-strain is used to determine the yield criteria. The yield criteria analysed in the paper can be represented in the three-dimensional convex surfaces, which help reveal explicitly global plastic properties of the models considered. Furthermore, the model can be used as a basis for a classical interpretation of interrelation between statics and kinematics leading to effective solutions of limit analysis problems. The approach proposed in the paper was applied to solve successfully a bearing capacity problem of shallow strip footing constructed on cracked strata. The example solved in the paper demonstrates that the modified formulation of yield criteria for glaciotectonically deformed soils is essential for computing the bearing capacity of foundations on such soils. An accurate prediction of the bearing capacity of foundations on cracked or layered strata is not possible without the detailed knowledge of these yield criteria. The limit state analysis presented in this paper can be used to solve many other geotechnical engineering problems, for example, the stability of slopes and reinforced walls or the bearing capacity of pile foundations (Florkiewicz 1989, Florkiewicz and Kubzdela 2013).

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References

Aber JS and Ber A (2007) Glaciotectonism. Developments in Quaternary sciences Vol. 6, Elsevier, Amsterdam.

Agarwal A, Laronga R, Walker L (2014) Rotary sidewall coring – size matters, Oilfield Review, 25(4), 30-39.

Al-Shamrani MA, Moghal AAB (2015) Closed-form solutions for bearing capacity of footing on anisotropic cohesive soils, Revista Ingeniería de Construcción 30(2), 109-125.

ASTM D 2113-14 (2014). Standard Practice for Rock Core Drilling and Sampling of Rock for Site Investigation, vol. 04.08. American Society of Testing Materials, West Conshohocken, Pa.

Aubeny C, Han S-W, Murff JD (2003) Suction caisson capacity in anisotropic, purely cohesive soil, International Journal of Geomechanics, 3(2), 225-235.

Bo MW, Arulrajah A, Choa V, Horpibulsuk S, Samingthong W (2017) Research-oriented ground investigation projects at Changi, Singapore. Geotechnical Research 4(1), 30-46.

Boehler JR and Sawczuk A (1977) On yielding of oriented solids. Acta Mechanica, 27, 185-206.

Booker IR and Davis AR (1972) A general treatment of plastic anisotropy under conditions of plain strain. J Mech Phys Solids 20, 239-250.

BS EN ISO 22475-1 (2006) Geotechnical investigation and testing - Sampling methods and groundwater measurements - Part 1: Technical principles for execution, British Standards Institution, London, UK.

Cai Y, Yu HS, Wanatowski D and Li X (2013), Non-coaxial behavior of sand under various stress paths, Journal of Geotechnical and Geoenvironmental Engineering, ASCE, 139(8), 1381-1395, doi: 10.1061/(ASCE)GT.1943-5606.0000854

Chen WY (1975) Limit analysis and soil plasticity, Elsevier, New York.

Chu J, Wanatowski D, Leong WK, Loke WL, He J (2015) Instability of dilative sand, Geotechnical Research, 2(1), 35-48. DOI: 10.1680/gr.14.00015.

Chung SG, Lee JM, Jang WY (2014) Comparing the quality of samples obtained by three types of fixed-piston samplers for soft sensitive clay, Engineering Geology 179, 50-58.

Derski W, Izbicki RJ, Kisiel I and Mroz Z (1988) Rock and Soil Mechanics, Elsevier, Oxford-New York-Tokyo.

De Groot DJ and Lutenegger AJ (2009) Geology and engineering behavior of Connecticut Valley Varved Clay, Geo-Strata, Geo Institute ASCE 13(4): 16-19.

Flieger-Szymanska M and Machowiak K (2011) Basic engineering-geological parameters of varved clays in Junikowski Stream Valley. Science Nature Technology 5(5): 1-11.

Florkiewicz A (1986) Representation of a limit criterion for a jointed rock-like material. Studia Geotechnica et Mechanica 8(4): 17-24.

Florkiewicz A (1989) Upper bound to bearing capacity of layered soils, Canadian Geotechnical Journal 26, 730-736.

Florkiewicz A and Mroz Z (1989) Limit analysis for cracked and layered soils, Proc. 12th Int Conf Soil Mech Found Eng, Rio de Janerio, Vol. 1, 515-518.

Florkiewicz A (2013) Limit states in soil and rock media with periodic layered microstructure (in Polish). Wydawnictwo Politechniki Poznanskiej, Poznan, Poland.

Florkiewicz A and Kubzdela A (2013) Factor of safety in limit analysis of slopes. Geomechanics and Engineering 5(5), 485-497, doi: 10.12989/gae.2013.5.5.485.

Florkiewicz A, Flieger-Szymanska M, Machowiak K and Wanatowski D (2014), Engineering properties of varved clays from the Junikowski Stream Valley in Poland. In: Geotechnics for Catastrophic Flooding Events, S. Iai (Ed.), pp. 267-273.

Gylland A, Long M, Emdal A, Sandven R (2013) Characterisation and engineering properties of Tiller clay, Engineering Geology 164, 86-100.

Hill R (1950) The mathematical theory of plasticity, Clarendon Press, Oxford, United Kingdom.

Hoek E and Brown ET (1980). Empirical strength criterion for rock masses. Journal of Geotechnical Engineering Division, 106 1013-1035.

Izbicki JR and Mroz Z (1976) Methods of limit analysis in soil and rock mechanics (in Polish). Polish Scientific Publishers, Warsaw, Poland.

Lade PV, Nam J and Hong WP (2008). Shear banding and cross-anisotropic behaviour observed in laboratory sand tests with stress rotation. Canadian Geotechnical Journal 45, 74-84.

Li XS and Dafalias YF (2004) A constitutive framework for anisotropic sand including non-proportional loading. Geotechnique 54(1), 41-51.

Lo KY (1965) Stability of slopes in anisotropic soils, Journal of Soil Mechanics and Foundations Division ASCE, 91(4), 85-106.

Long M, Gudjonsson G, Donohue S, Hagberg K (2010) Engineering characterisation of Norwegian glaciomarine silt, Engineering Geology 110, 51-65

Lydzba D, Pietruszczak S and Shao JF (2003) On anisotropy of stratified rocks: homogenization and fabric tensor approach. Computers and Geotechnics 30, 289-302.

Michalowski RL and Shi L (1995) Bearing capacity of footings over two-layer foundation soils. Journal of Geotechnical Engineering ASCE 121(5), 421-428.

Mroz Z and Maciejewski J (2002) Failure criteria of anisotropically damaged materials based on the critical plane concept. International Journal for Numerical and Analytical Methods in Geomechanics, 26, 407-431.

Nova R (1980) The failure of transverally anisotropic rocks in triaxial compression. International Journal of Rock Mechanics Mineral Science and Geomechanic Abstracts, 17, 325-332.

Pariseau WG. (1972) Plasticity theory for anisotropic rocks and soils. In: Proceedings 10th Symposium on Rock Mechanics (AIME), 267-295.

Pietruszczak S and Mroz Z (2000). Formulation of anisotropic failure criteria incorporating a microstructure tensor. Computers and Geotechnics 26, 105-122.

Prandtl L (1920). Uber die Härte plastischer Körper. Göettingen Nachrichten, Math.-Phys. Kl. 74–85.

Schokking. F (1998) Anisotropic strength behaviour of a fissured overconsolidated clay in relation to Saalian glacial directions, Engineering Geology 49(1), 31-51.

Strozyk J (2015) The Overconsolidation Ratio of the Poznan Clays from the area of SW Poland, Procedia Earth and Planetary Science 15 (2015) 293-298, doi:10.1016/j.proeps.2015.08.071

Tsai SW and Wu E (1971) A general theory of strength of anisotropic materials. Journal of Composite Materials, 5, 58-80.

Tsutsumi S and Hashiguchi K (2005). General non-proportional loading behaviour of soils. International Journal of Plasticity 22, 826-857.

Valore C, Ziccarelli M., Muscolino SR (2017) The bearing capacity of footings on sand with a weak layer. Geotechnical Research 4(1), 12–29 http://dx.doi.org/10.1680/jgere.16.00020

Von Mises RV (1928) Mechanik der plastischen Formänderungen von Kristallen, ZAMM 8, s. 161-185.

Yang LT, Li X, Yu HS and Wanatowski D (2016). A laboratory study of anisotropic geomaterials incorporating recent micromechanical understanding, Acta Geotechnica 11(5), 1111-1129, doi: 10.1007/s11440-015-0423-7

Yang SQ (2015) Strength failure and crack evolution behaviour of rock materials containing pre-existing fissures, Science Press, Beijing and Springer-Verlag Berlin Heldelberg.

Yu HS (2006) Plasticity and Geotechnics, Springer.

Yu M, Xia G and Kolupaev VA (2009) Basic characteristics and development of yield criteria for geomaterials, Journal of Rock Mechanics and Geotechnical Engineering 1(1), 71-88.

Yuan R, Yu HS, Hu N, He Y (2018) Non-coaxial soil model with an anisotropic yield criterion and its application to the analysis of strip footing problems. Computers and Geotechnics 99, 80-92.

Wanatowski D and Chu J (2012) Factors affecting pre-failure instability of sand under plane-strain conditions, Géotechnique 62(2), 121-135, doi: 10.1680/geot.9.P.111

Wanatowski D, Chu J and Loke WL (2010) Drained instability of sand in plane-strain, Canadian Geotechnical Journal, 47(4), 400-412.

Zhu HN, Mehrabadi MM and Massoudi M (2006). Incorporating the effects of fabric in the dilatant double shearing model for planar deformation of granular materials. International Journal of Plasticity 22, 628-653.

Ziccarelli M, Valore C, Muscolino SR, Fioravante V (2017) Centrifuge tests on strip footing on sand with a weak layer. Geotechnical Research 4(1), 47–64, http://dx.doi.org/10.1680/jgere.16.00021

Figure captions

Figure 1. A cracked geomaterial with isotropic matrix and cracks of Coulomb type ($c \neq 0$, $\varphi \neq 0$ and $c^* \neq 0$ and $\varphi^* \neq 0$).

Figure 2. Yield criteria for (a) an isotropic geomaterial of Coulomb type, (b) a cracked geomaterial along the crack direction.

Figure 3. Yield criterion for the cracked geomaterial with isotropic matrix of Coulomb type in (a) X-Y-Z space, (b) Y-Z plane.

Figure 4. Yield criterion for the cracked geomaterial with isotropic matrix of Tresca type in X-Y-Z space.

Figure 5. A cracked geomaterial with orthotropic matrix of Hill type (T, d) and cracks of Coulomb type ($c^* \neq 0$ and $\varphi^* \neq 0$).

Figure 6. Hill's criterion for an orthotropic geomaterial in (a) X-Y-Z space, (b) Y-Z plane.

Figure 7. Yield criterion for the cracked geomaterial with orthotropic Hill's matrix (T, d) with cracks of Coulomb type ($c^* \neq 0$ and $\varphi^* \neq 0$).

Figure 8. Yield criterion for the cracked geomaterial with orthotropic Hill's matrix (T, d) with cracks of Tresca type ($c^* \neq 0$, $\varphi^* = 0$).

Figure 9. Analysis of the uniaxial compression for the cracked geomaterial with isotropic matrix and cracks of Tresca type ($c \neq 0$, $\varphi = 0$ and $c^* \neq 0$, $\varphi^* = 0$): (a) the orientation of compressed element in the ground, (b) Mohr's circle of stress, (c) effect of the angle θ on the uniaxial compressive strength.

Figure 10. Analysis of the uniaxial compression for the the cracked geomaterial with orthotropic matrix of Hill type (T, d) with cracks of Tresca type ($c^* \neq 0$, $\varphi^* = 0$): (a) the orientation of compressed element in the ground, (b) Mohr's circle of stress, (c) effect of the angle θ on the uniaxial compressive strength.

Figure 11. An example of strip foundation resting on the cracked stratum with isotropic matrix and cracks of Tresca type ($c \neq 0$, $\varphi = 0$ and $c^* \neq 0$, $\varphi^* = 0$).

Figure 12. Upper and lower bound solutions for ultimate bearning capacity of strip foundation resting on the cracked stratum with isotropic matrix and cracks of Tresca type ($c \neq 0$, $\varphi = 0$ and $c^* \neq 0$, $\varphi^* = 0$).

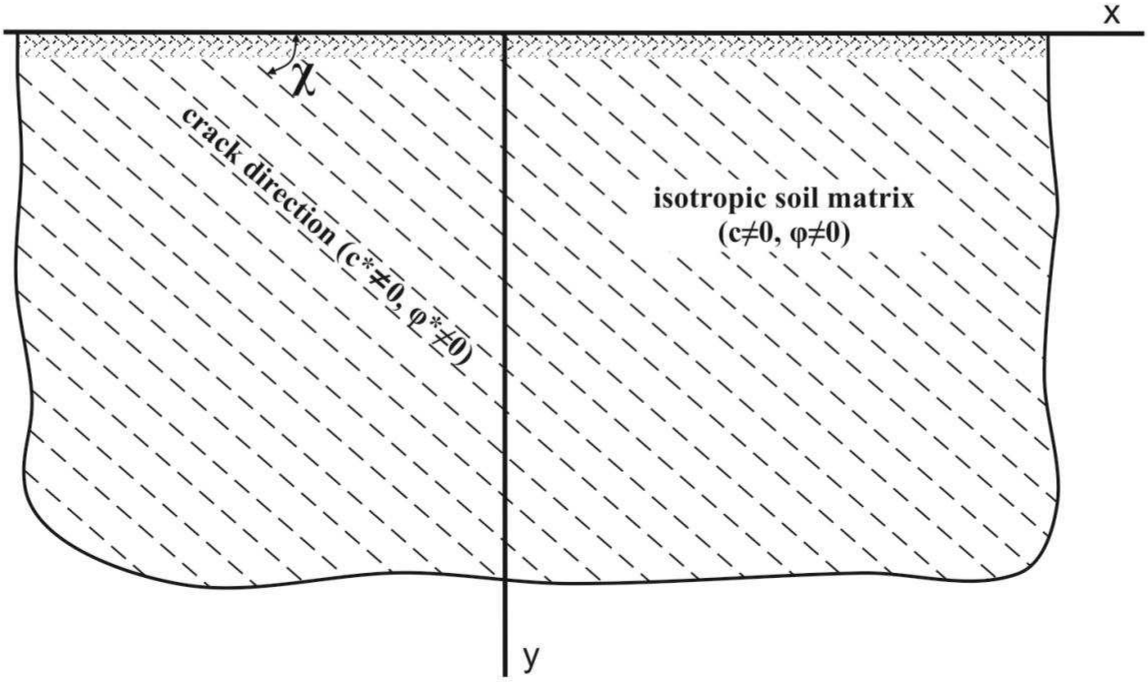
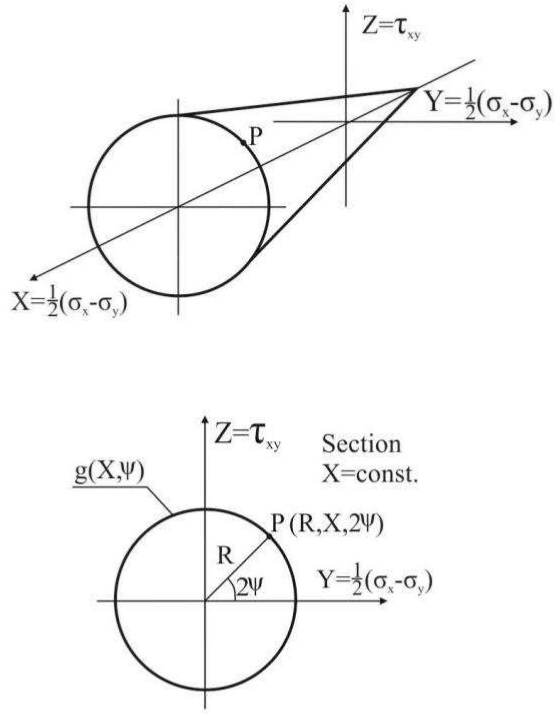
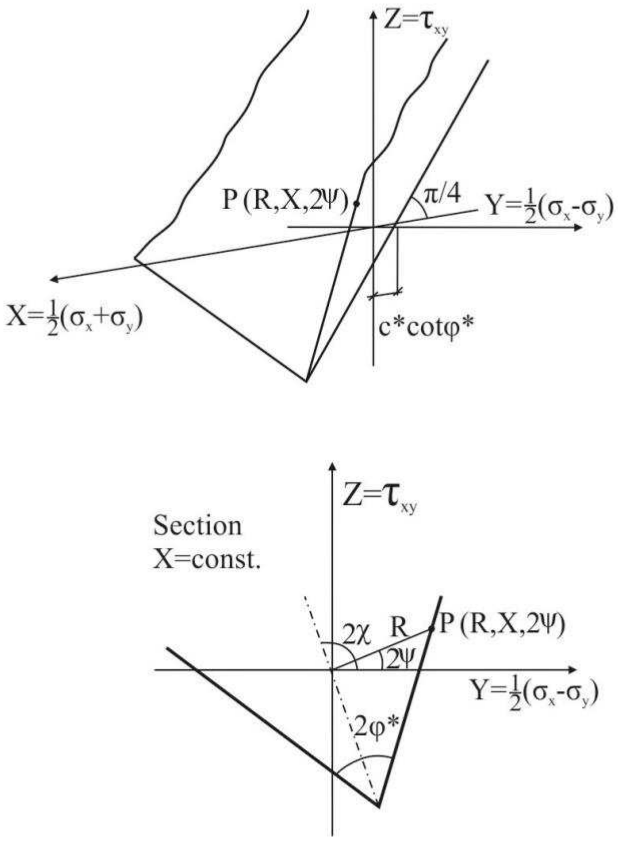


Fig. 1



Z=τxy
Y=½(σx-σy)
P
X=½(σx-σy)
Z=τxy
Section
X=const.
g(X,ψ)
P (R,X,2ψ)
R
2ψ
Y=½(σx-σy)

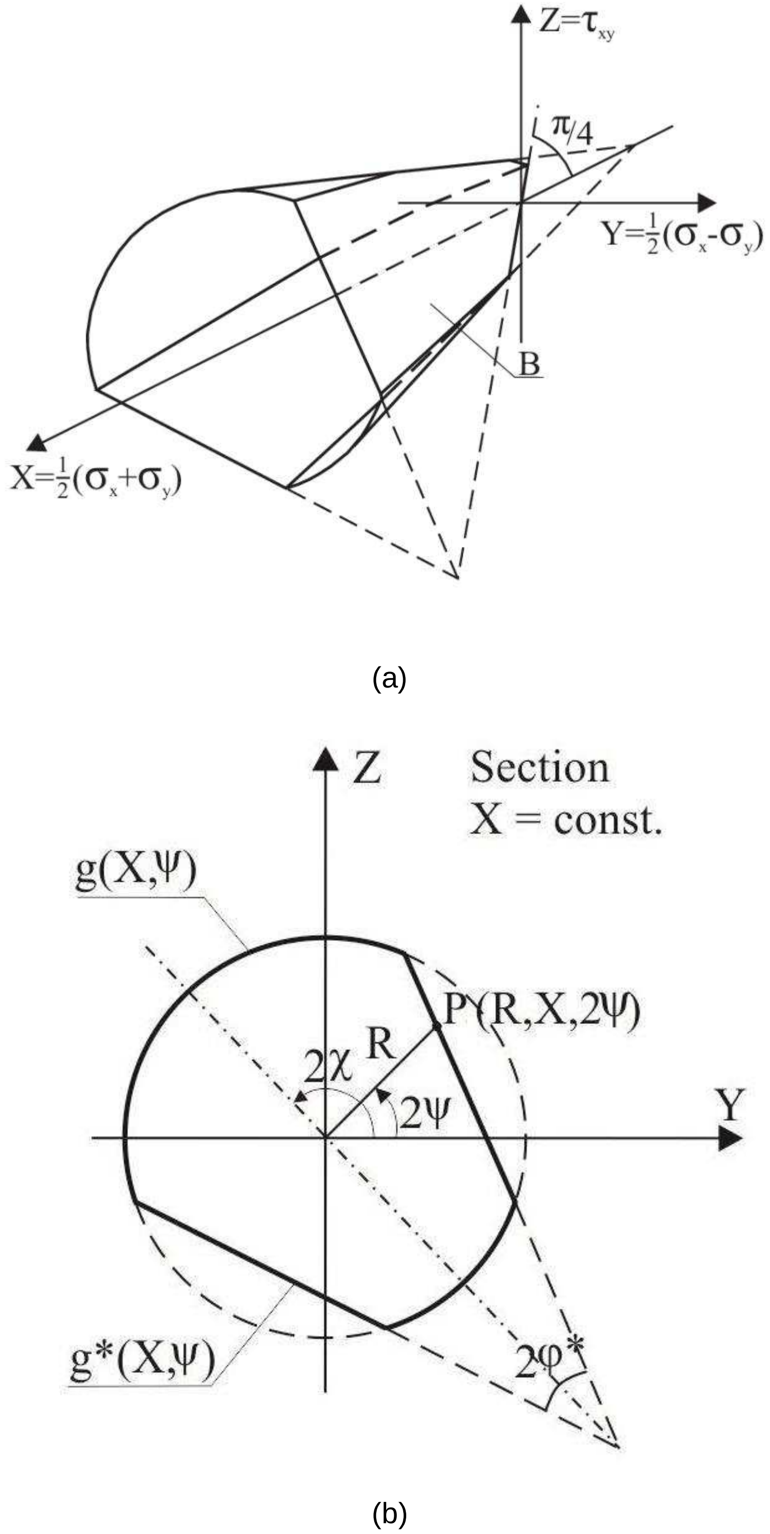
(a)



Z=τxy
P (R,X,2ψ)
π/4
Y=½(σx-σy)
X=½(σx+σy)
c*cotφ*
Z=τxy
Section
X=const.
2χ
R
P (R,X,2ψ)
2ψ
Y=½(σx-σy)
2φ*

(b)

Fig. 2


Z=τ_xy
π/4
Y=½(σ_x-σ_y)
B
X=½(σ_x+σ_y)
Z
Section
X = const.
g(X,ψ)
P(R,X,2ψ)
R
2χ
2ψ
Y
g*(X,ψ)
2φ*

(a)

(b)

Fig. 3

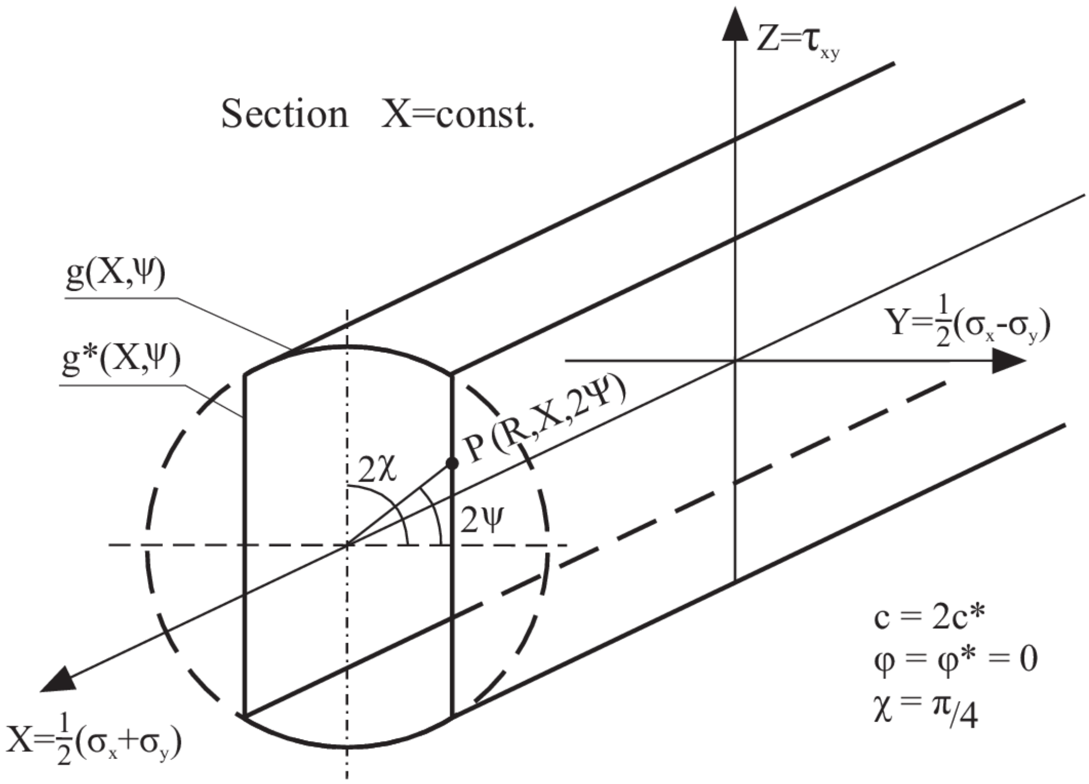

Z=τxy
Section X=const.
g(X,ψ)
g*(X,ψ)
Y=½(σx-σy)
P (R,X,2Ψ)
2χ
2ψ
c = 2c*
φ = φ* = 0
χ = π/4
X=½(σx+σy)

Fig. 4

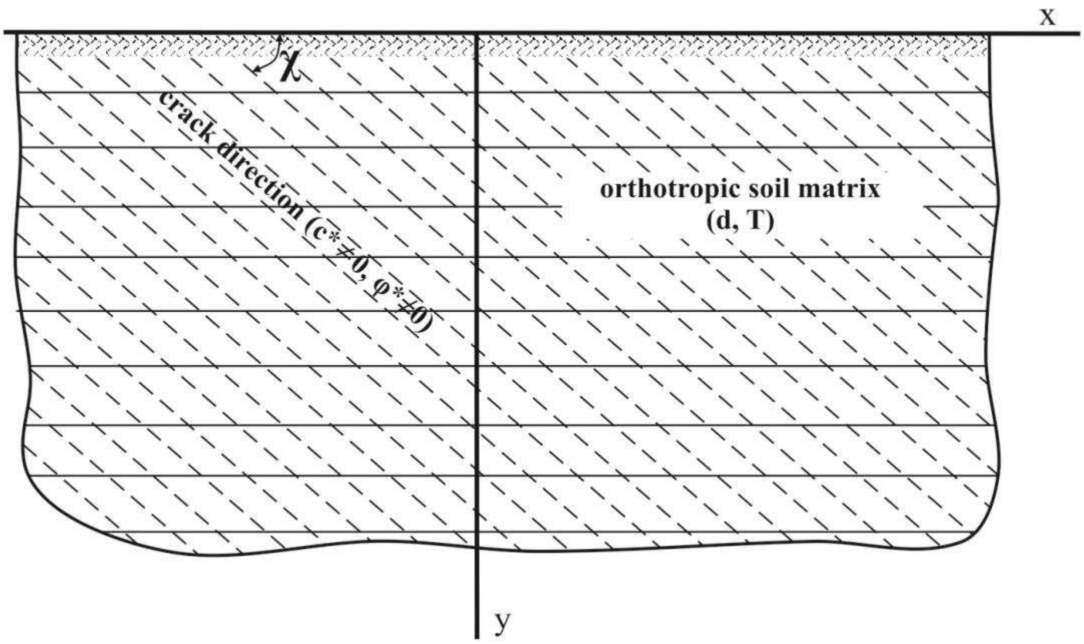
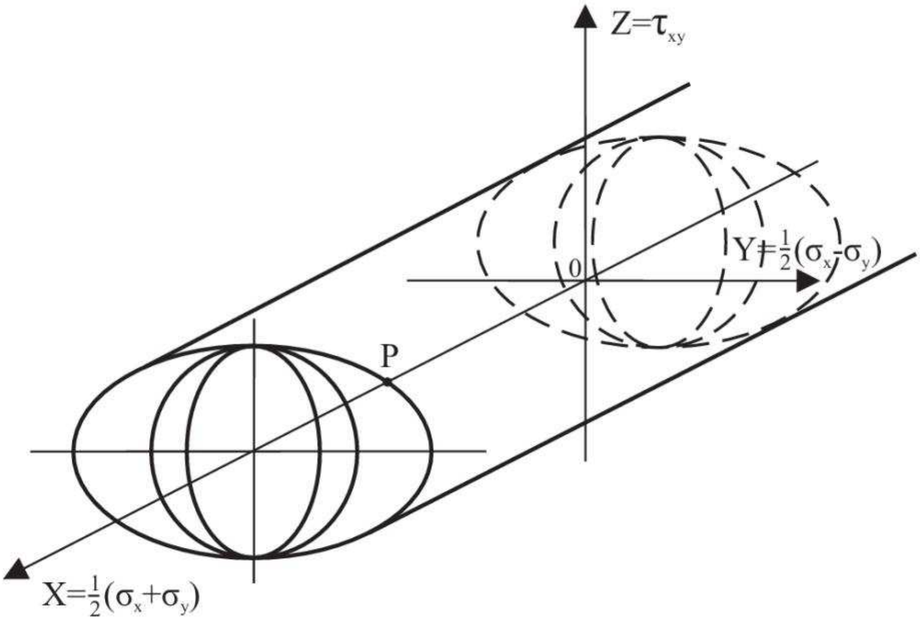
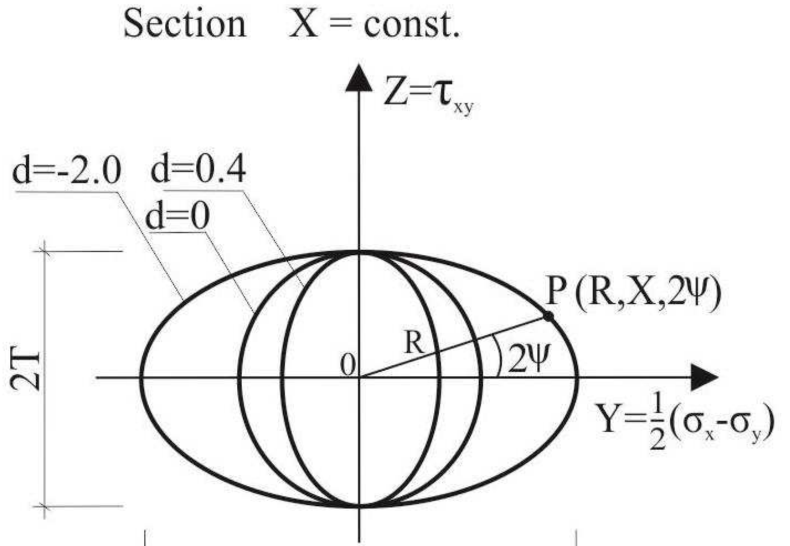


Fig. 5



$Z=\tau_{xy}$
$Y=\frac{1}{2}(\sigma_x-\sigma_y)$
0
P
$X=\frac{1}{2}(\sigma_x+\sigma_y)$

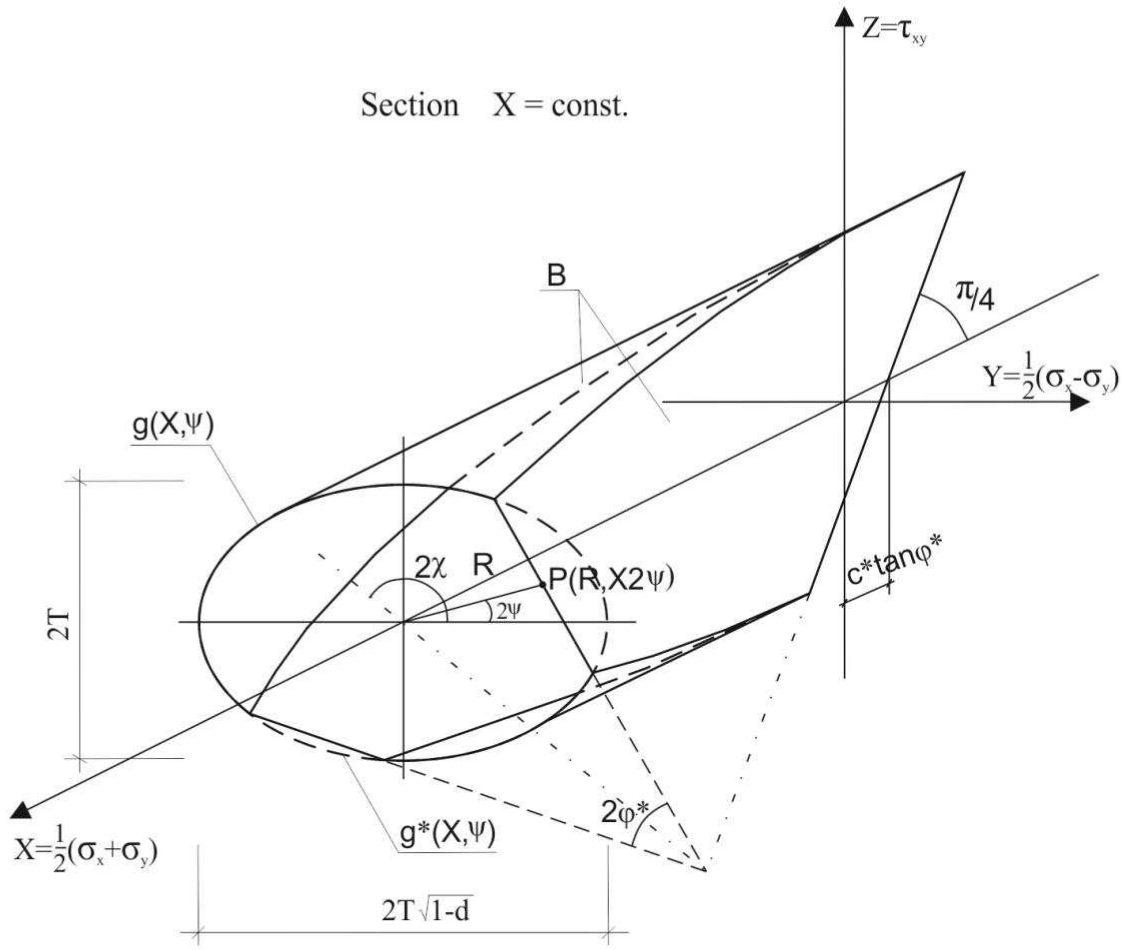
(a)



Section X = const.
$Z=\tau_{xy}$
d=-2.0
d=0.4
d=0
P (R,X,2Ψ)
2T
R
0
2Ψ
$Y=\frac{1}{2}(\sigma_x-\sigma_y)$
$2T\sqrt{1-d}$

(b)

Fig. 6



Section X = const.
$Z=\tau_{xy}$
B
$\pi/4$
$Y=\frac{1}{2}(\sigma_x-\sigma_y)$
$g(X,\psi)$
2χ
R
$P(R,X2\psi)$
2ψ
2T
$c^*\tan\varphi^*$
$2\varphi^*$
$g^*(X,\psi)$
$X=\frac{1}{2}(\sigma_x+\sigma_y)$
$2T\sqrt{1-d}$

Fig. 7

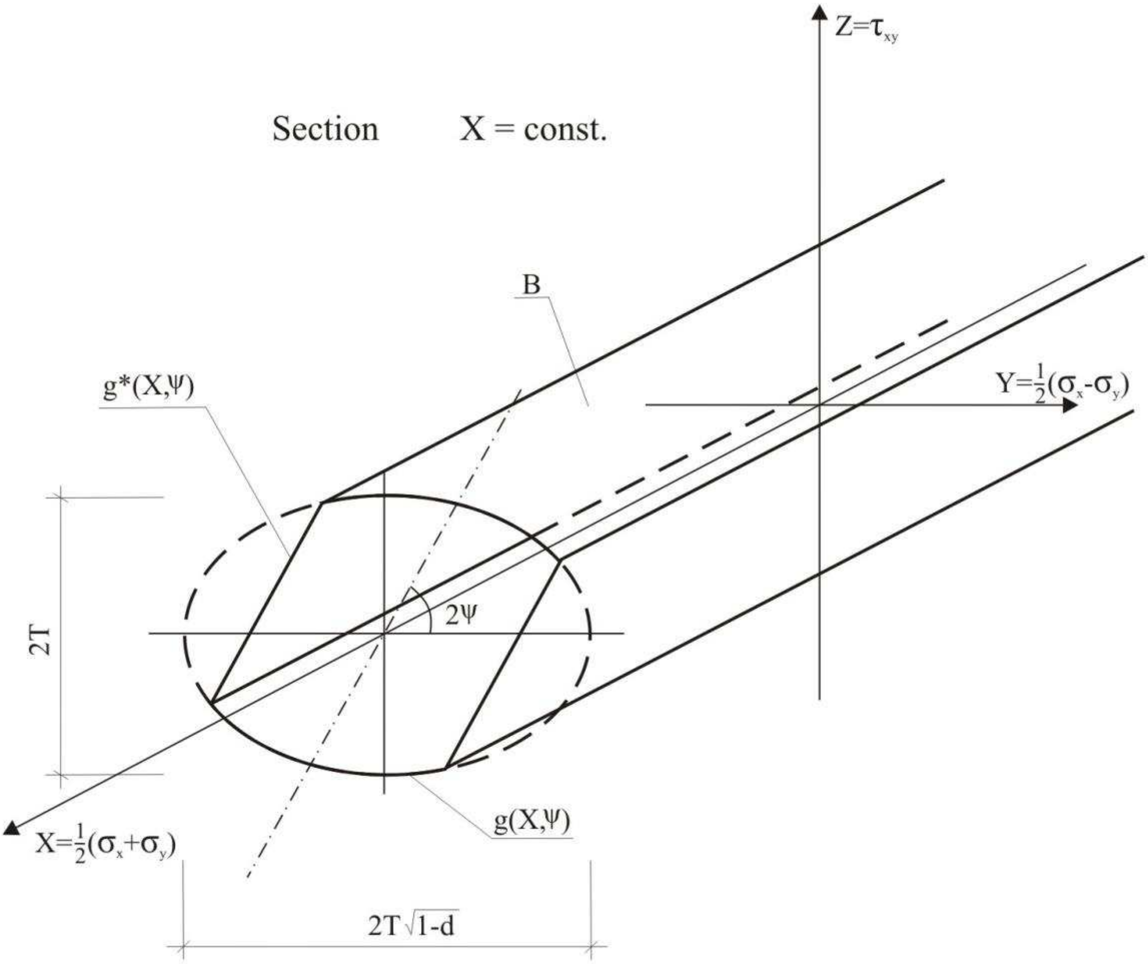
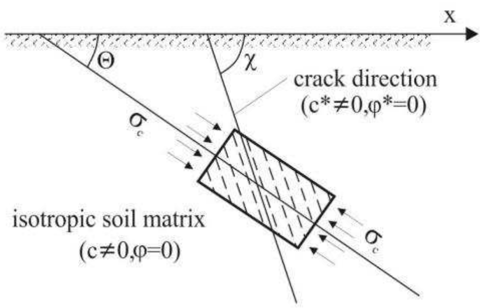
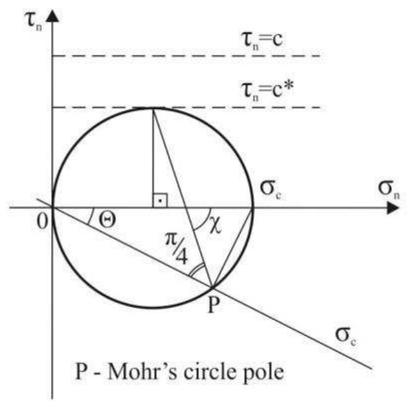

$Z=\tau_{xy}$
Section X = const.
B
$g^*(X,\Psi)$
$Y=\frac{1}{2}(\sigma_x-\sigma_y)$
2T
2Ψ
$g(X,\Psi)$
$X=\frac{1}{2}(\sigma_x+\sigma_y)$
$2T\sqrt{1-d}$

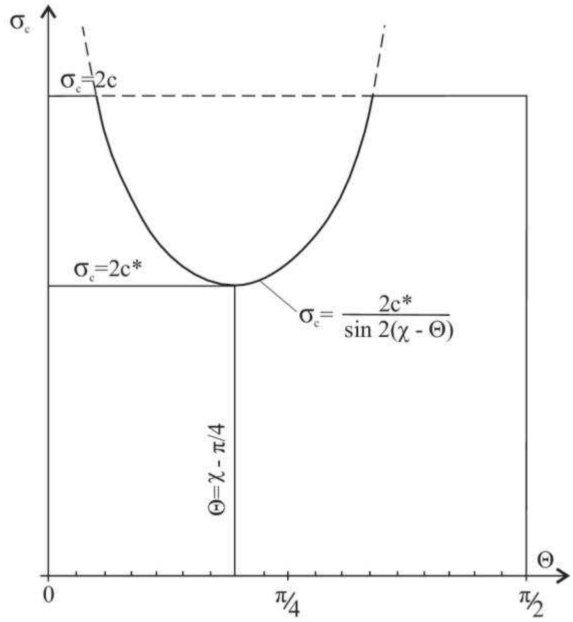
Fig. 8


x
Θ
χ
crack direction
(c*≠0,φ*=0)
σc
isotropic soil matrix
(c≠0,φ=0)
σc

(a)

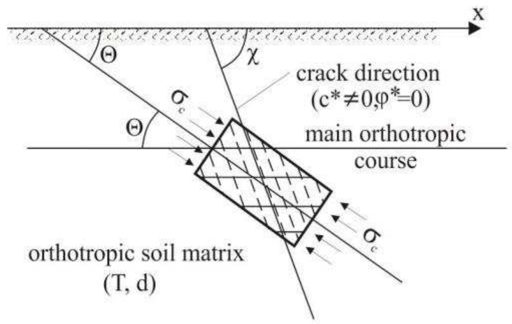

τn
τn=c
τn=c*
σc
σn
0
Θ
χ
π/4
P
σc
P - Mohr's circle pole

(b)

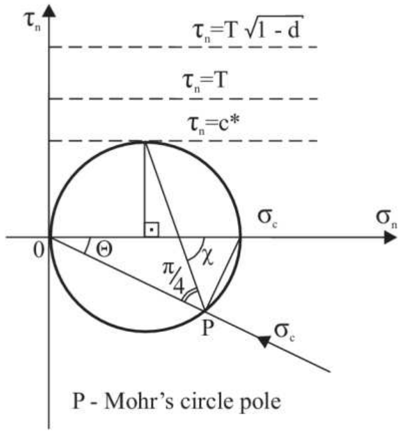

σc
σc=2c
σc=2c*
σc = 2c* / sin 2(χ - Θ)
Θ=χ - π/4
Θ
0
π/4
π/2

(c)

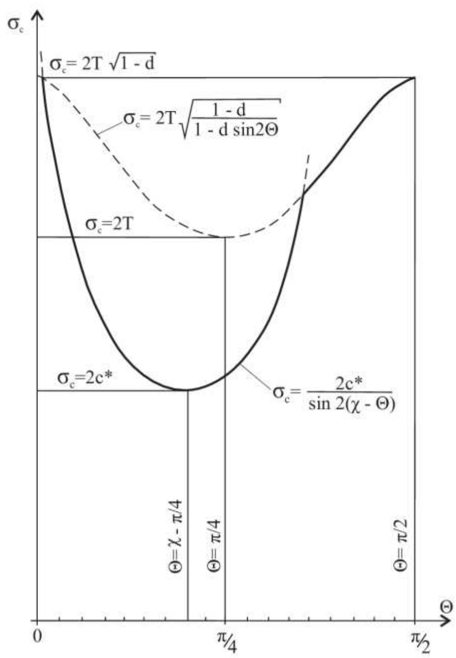
Fig. 9


x
Θ
χ
crack direction
(c*≠0,φ*=0)
σc
Θ
main orthotropic
course
σc
orthotropic soil matrix
(T, d)

(a)


τn
τn=T √1 - d
τn=T
τn=c*
σc
σn
0
Θ
π/4
χ
P
σc
P - Mohr's circle pole

(b)


σc
σc= 2T √1 - d
σc= 2T √((1 - d)/(1 - d sin2Θ))
σc=2T
σc=2c*
σc= 2c*/sin 2(χ - Θ)
Θ=χ - π/4
Θ= π/4
Θ= π/2
Θ
0
π/4
π/2

(c)

Fig. 10

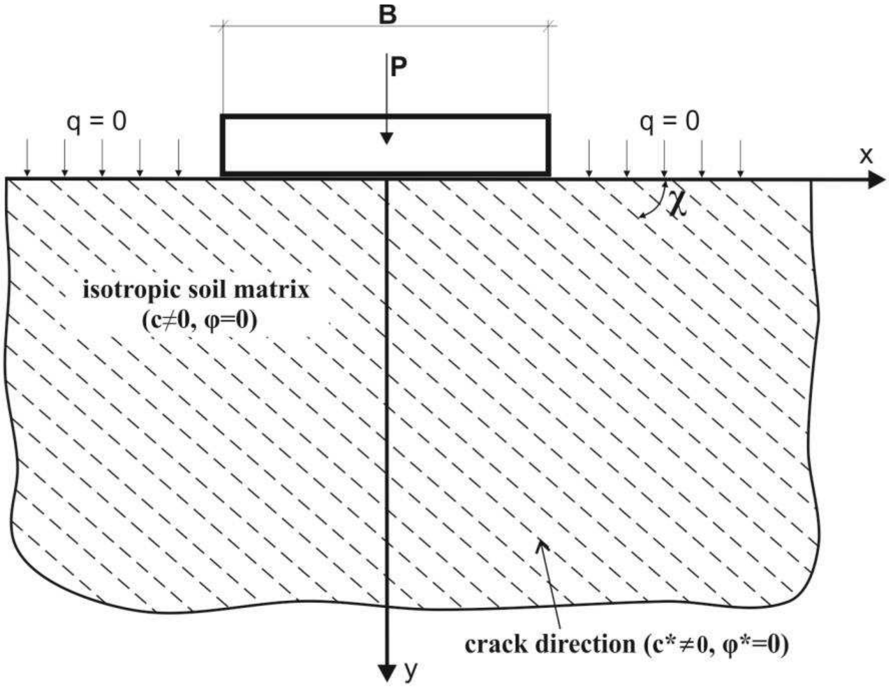
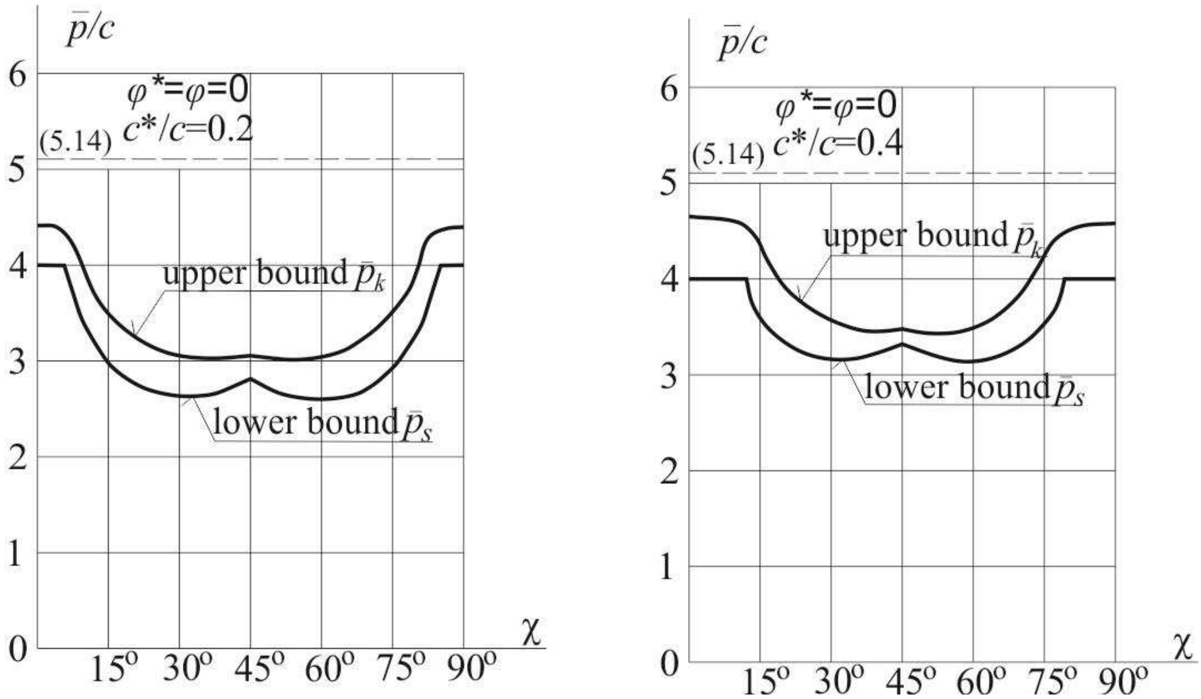

B
P
q = 0
q = 0
x
χ
isotropic soil matrix
(c≠0, φ=0)
crack direction (c*≠0, φ*=0)
y

Fig. 11



p̄/c
φ*=φ=0
(5.14) c*/c=0.2
upper bound p̄k
lower bound p̄s
χ
15° 30° 45° 60° 75° 90°
p̄/c
φ*=φ=0
(5.14) c*/c=0.4
upper bound p̄k
lower bound p̄s
χ
15° 30° 45° 60° 75° 90°

Fig. 12