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Modelling the Nonlinear Oscillations Due to Vertical Bouncing Using a Multi-Scale Restoring Force System Identification Method

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Abstract

Human vertical bouncing motion is studied using a system identification method. A multi-scale mathematical model is identified directly from real experimental data to characterise the nonlinear oscillation associated with the vertical bouncing. A new method which combines the restoring force surface method and the iterative orthogonal forward regression algorithm is proposed to determine the model structure and estimate the associated parameters. Two types of sub-models are used to study the nonlinear oscillations in different scales. Results show that the model predicted outputs provide excellent predictions of the experimental data and the models are capable of reproducing the nonlinear oscillations in both time and frequency domain.

Key words: iterative orthogonal forward regression, iOFR, restoring force surface method, multi-scale, radial basis function, hybrid model

1. Introduction

Studies of the induced dynamic load that arises from people walking and bouncing is an important subject in many fields including biomechanics, medical science, sports science, robotics, control systems, and also civil engineering. Many authors have studied the motion of the human body in walking, jumping, and bouncing from different aspects (Blickhan, 1989; Ernesto & Tianjian, 2009; Harman, Rosenstein, Frykman, Rosenstein, & Kraemer, 1991; Hof, Van Zandwijk, & Bobbert, 2002;

Vitomir Racic & Chen, 2015; V. Racic & Pavic, 2010a, 2010b; V. Racic, Pavic, & Brownjohn, 2009; Spägele, Kistner, & Gollhofer, 1999a, 1999b; van Werkhoven & Piazza, 2013).

A complete representation of body motion introduced by bouncing includes the modelling of vertical, lateral, and longitudinal motions (Garcia, 1999). The vertical component is often studied as part of the crowd-structure interaction in civil engineering while the lateral and longitudinal components are often studied for the lateral stability of the human body during walking and bouncing. In this paper, only the vertical motion due to human bouncing will be investigated.

In this paper, the motion of a marked point on the chest of a test subject during bouncing is recorded and investigated using a system identification method. The modelling of the motion of human body can be very complicated because of the following difficulties. Firstly, the human body is composed of several connected segments: head, trunk, arms, legs, feet and so on. These segments are connected by joints and interact with each other in motion. Each of these segments may have a complex effect on the motion of a specific point and the effect is unknown. For instance, the motion of the head depends on the movements of trunk, legs, ankles, and so on. In the robotics, especially in the investigation of the stability of robots, the motion of a robot is often simplified as a multi-link inverted pendulum. The movement of the top-end could be very complex because of the effects from the lower segments of the system. Another source of complexity is that the mass of the human body is neither lumped in a mass centre nor distributed uniformly. Therefore, modelling the motion of the marked point using a first principles method can be very difficult. In this paper a system identification method is used to study the motion of a marked point of human body which is on a relatively high position of the human body and the motion of this point is of rich dynamics. In the investigation of complex systems, a system identification method often has significant advantages. The system to be identified is considered as a black box, which avoids the complex underlying mechanism in the system. The data of interest are collected through experimental methods and the relationships between these observations are studied.

In this paper, a continuous time model will be identified for the body motion in vertical bouncing by studying the relationships between the displacement, velocity and accelerations. This method is known as the restoring force surface method (RFS). Restoring force surface method as an ideal method for the study of nonlinear dynamics has been widely used since the first introduction (Masri, Bekey, Sassi, & Caughey, 1982; Masri & Caughey, 1979). Restoring force surface method which converts the problem of modelling nonlinear dynamics into the surface fitting in the state space significantly simplified the modelling process. The restoring force surface is usually reconstructed using the Chebyshev polynomials and nonparametric methods. The nonparametric restoring force surface often yields insight into the physical system. This allows one to qualitatively study the primary difficulties encountered in nonlinear system identification: what nonlinearities involve in the system and how these nonlinearities affect the dynamics behaviours of the system. For some more complex cases where the restoring force surface cannot be represented using a uniform nonlinear function, a piecewise models (Allen, Sumali, & Epp, 2008) and local restoring force surface method have been studied (S. W. R. Duym, Schoukens, & Guillaume, 1996). For a complete discussion of the restoring force surface method, readers are referred to Worden and Tomlinson's book (Worden & Tomlinson, 2001) and the related papers (Worden, 1990a, 1990b). Some quantitative methods have also been intensively studied especially the direct parameter estimation methods (Worden & Tomlinson, 2001). However, the problems that which set of nonlinearities are involved in the system dynamics and how to get a minimum set of nonlinearities which is sufficient to represent the systems seems not to be perfectly answered.

In this paper, a new method which combines the restoring force surface method with the iterative orthogonal forward regression (iOFR) algorithm (Yuzhu Guo, L.Z. Guo, S. A. Billings, & H. L. Wei, 2015c) will be introduced to try to give a satisfying answer to these problems. The orthogonal forward regression algorithm (is also known as forward orthogonal least squares regression algorithm) and the associated error reduction ratio (ERR) have been proved to be powerful tools for determination of nonlinear model structures in various ranges of applications (S. A. Billings, 2013). The OFR algorithm has recently been used to identify nonlinear continuous time models (Yuzhu Guo, Guo, Billings, & Lang, 2015; Yuzhu Guo, Guo, Billings, & Wei, 2016). The iOFR algorithm is an improvement to the classic OFR algorithm. The iOFR has been proved to work better under a non-persistent excited condition (Yuzhu Guo, L. Z. Guo, S. A. Billings, & H. L. Wei, 2015b). In the application of the orthogonal forward regression algorithm, a very wide range of terms can be used according the needs of the practical systems, such as, polynomials, rational functions, spline functions, radial basis functions (RBFs), wavelet functions, and so on (Stephen A. Billings, Wei, & Balikhin, 2007; Wei, Zhu, Billings, & Balikhin, 2007). Because of the complexity of the system under consideration, three different types of regressors will be used in this paper to model the body motion due to vertical bouncing: the polynomials, multi-scale radial basis functions (or wavelets) and the hybrid regressors which combine the first two kinds of functions.

The iterative orthogonal forward regression restoring force surface method is capable to model a complex restoring force surface. For example the non-uniform restoring force surface can be identified using piecewise spline functions or radial basis functions as regressors to obtain a parsimony model structure. In this paper, a hybrid model which combines the polynomial and radial basis functions (S. A. Billings & Wei, 2005) will be combined with the restoring force surface method to give a coarse to fine representation of the nonlinear oscillation in body motion.

Since the body motion in this study is the source of the human-introduced dynamic forces the study of the nonlinear oscillation is of a significant meaning in study of the ground reaction force. The introduce method can also be directly used to study the relations between the body motion and the ground reaction forces by replacing the measured restoring force with the ground reaction force(Y. Guo et al., 2017). The nonlinear oscillation of the marked point on body trunk instead of the ground reaction force is studied because experimental data show that the motion of body trunk is of a richer dynamics compared with the ground reaction force. A direct recovering of the ground reaction forces using the same method will be studied in a separate paper.

This paper is organised as follows: Section 2 briefly explained the experiment from which data were collected and the preliminary analysis of these data. Section 3 introduced the new iterative orthogonal forward regression restoring force surface method. Three different kinds of sub-models are used to identify the body motion in section 4. The advantages and disadvantages of each model are discussed. The conclusions are finally drawn in section 5.

2. Experiment and data analysis

2.1 Experiment

The experiment was conducted in the Department of Civil and Structural Engineering, the University of Sheffield, Sheffield, UK. This study has been approved by the Research Ethics Committee of the University of Sheffield and conforms to the ethical guidelines.

A test subject bounced on an AMTI BP-400600 force plate following a 1.2 Hz metronome beat. The body motion was measured using optical motion capture technology. Two markers were attached on the chest and pelvis of the test subject, respectively. Cameras recorder the movements (displacements, velocities and accelerations) of the target markers in real time. The test last 35s and the recorded signals were sampled at 200Hz. Figure 1 shows the recorded displacement, velocity, and the acceleration signals of the chest marker.

In this paper the vertical motion of the marker on chest is studied. This point is studied because this point is at a relatively high position of the human body. The effects from the lower parts of human body are involved in the motion. This makes the motion of this point possesses rich dynamics. At the same time, since this point was chosen on the line of symmetry of the body the influent from the lateral movement is small and negligible.

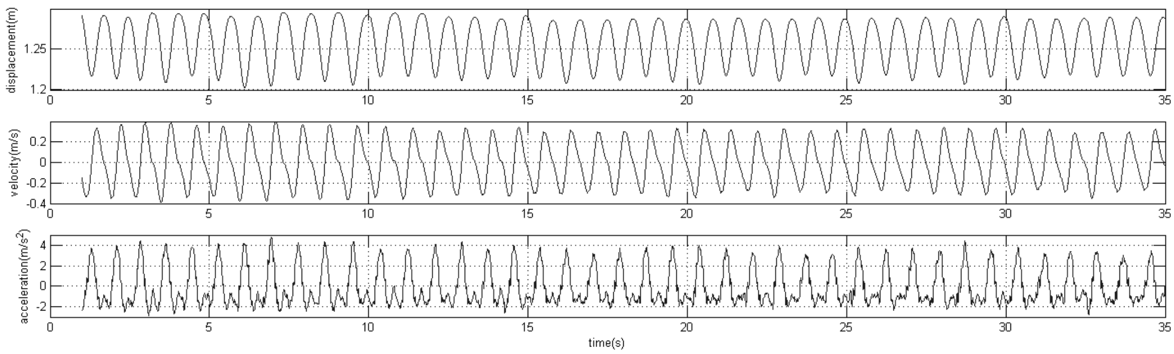


Figure 1 Recorded signals

2.2 Analysis of the experimental data

Fast Fourier transforms of the recorded displacement, velocity, acceleration series show that the main spectral component of these signals is at 1.2 Hz and the second order harmonic at 2.4 Hz. The higher-order harmonic components are small. The spectrum of the displacement at the frequencies over 3 Hz is very smooth and the higher-order harmonics can hardly be observed. This is because the displacement signal is integratal of the velocity and acceleration and the integrands have typical low pass property. Since there is no external excitation in this autonomous system the harmonics in these signals are introduced by nonlinearities.

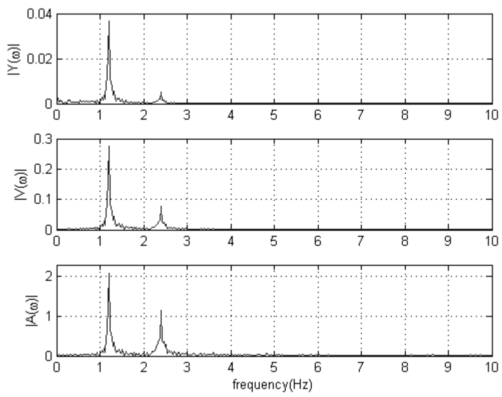


Figure 2 The frequency spectra of the displacement, velocity, and acceleration signals

The nonlinearities can obviously be observed in the phase portrait of these signals which are shown in Fig 3, especially in the acceleration- displacement phase plane in Fig 3 (c). The system is nonlinear because the phase portrait of a linear second order autonomous system should be on a three dimensional plane. Namely, the displacement $y(t)$, velocity $\dot{y}(t)$ and the acceleration $\ddot{y}(t)$ satisfy a linear equation

$$m\ddot{y}+c\dot{y}+ky=0 \tag{1}$$

Rearrangeing equation (1) yields

$$m\ddot{y}=-c\dot{y}-ky \tag{2}$$

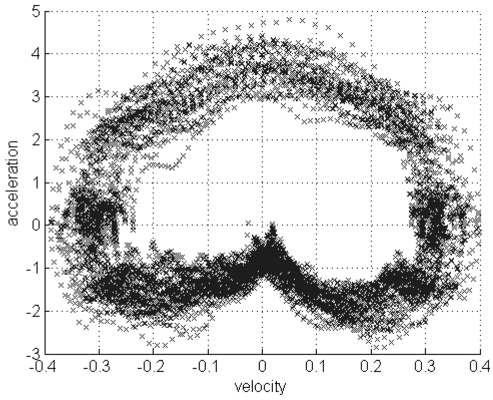
The right hand side of the equation represents the restoring force of a linear system composed of the elastic force $-ky$ and the damping force $-c\dot{y}$, where c and k are the dumping coefficient and stiffness, respectively. Fig 3 (b) shows that the scattering of data in the three-dimensional state space is flat in the acceleration direction which forms a surface in the state space. However the scattering of the data is far from a plane. That is the data does not satisfy the linear relationship (1) but a nonlinear one. A general form for nonlinear second order systems can be given as

$$m\ddot{y}=f\left(y,\dot{y}\right) \tag{3}$$

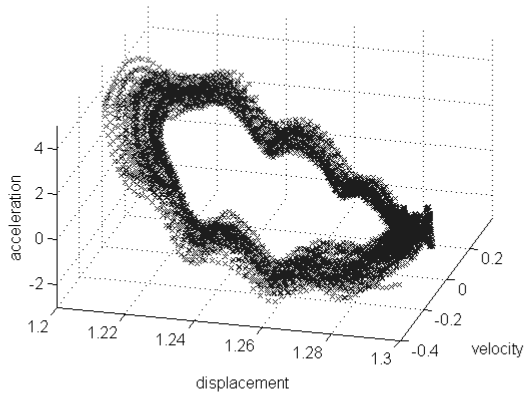
where the internal restoring force is a nonlinear function of the displacement and the velocity. Equation (3) shows the basic idea of the restoring force surface method (Masri & Caughey, 1979) which will be used to identify the model of the system in the next section.

A further observation of Fig 3 (a), (d), and (b) shows that the phase portrait of the system is almost symmetrical about the plane $\dot{y}=0$. That is the marked point moves at certain acceleration when the displacement and the amplitude of velocity are specified disregarding the direction of the velocity. Moreover, the change of acceleration along the velocity direction is insignificant. That is, the directional derivative of the restoring force along the velocity is small. This means that the influence of the velocity on the restoring force is insignificant and the restoring force mainly comes from a nonlinear elastic force which is a function of displacement. As a result it is reasonable to assume that the restoring force (or acceleration) does not depends on the value of the velocity and can be described as a univariate function of displacement. This assumption simplifies the system identification procedure in the next section. Under the above assumption the study of the restoring force surface in a three dimensional state space reduces to a study of the nonlinear elastic restoring force in the acceleration-displacement plane.

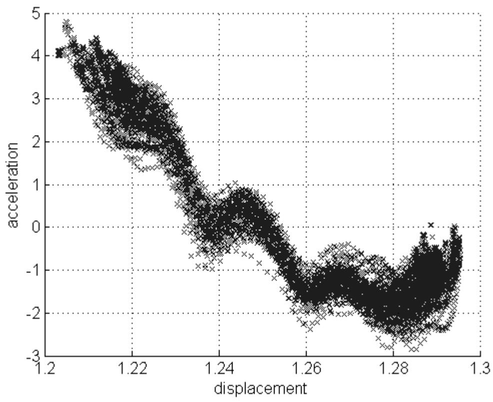
The nonlinear relationship between acceleration and displacement is shown in the projection of the phase portrait on the acceleration-displacement plane. Fig 3 (c) shows that acceleration and displacement are of a complex nonlinear relationship which is hardly to be represented using a uniform nonlinear function in the whole phase plane. This nonlinear relationship will be studied in next section using three different types of nonlinear terms: polynomial terms, multi-scale radial basis functions, and the hybrid term set which combines the previous two kinds of terms.



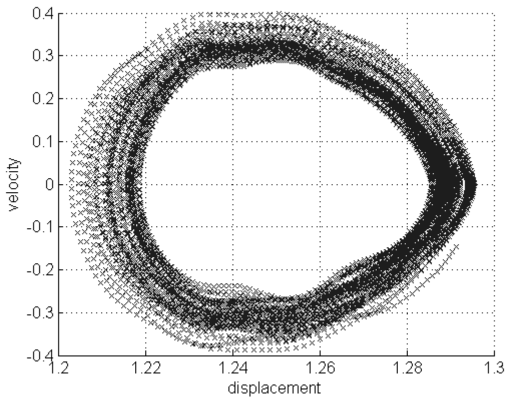
(a)



(b)



(c)



(d)

Figure 3 Data in the phase space

3. Orthogonal forward regression restoring force surface method

The restoring force method is based on Newton's second law

$$m\ddot{y} + f(y, \dot{y}) = u(t) \tag{4}$$

where $f(y, \dot{y})$ is the restoring force which is generally a non-linear function of the displacement and velocity. A trivial re-arrangement of equation (4) gives

$$\ddot{y} = \frac{1}{m} u(t) - \frac{1}{m} f(y, \dot{y}) \tag{5}$$

For a certain time instance t_i , a triplet $\langle y(t_i), \dot{y}(t_i), \ddot{y}(t_i) \rangle$ is specified where the first two values indicate a point in the phase plane and the third gives the height of the restoring force above that point. The main idea of the restoring force surface method is to reconstruct the restoring force as a function of velocity and displacement using measured force (or acceleration) and kinematic data at discrete time instants.

In this paper, the orthogonal forward regression algorithm is employed to determine the structure of the nonlinear restoring force surface. It is well known that the orthogonal forward regression algorithm and the ERR criterion are very effective nonlinear system identification method and have been successfully used to identify nonlinear systems in various applications. In this algorithm, a large enough term dictionary is firstly constructed. Any type of linear and nonlinear terms and their combination can be included in the candidate term set. The significant terms in the dictionary will be selected into the final model one by one based on the ERR criterion until a stop criterion is satisfied. The candidate terms are orthogonalised in each step to minimise the information redundancy in the final model. The obtained models which include the least number of terms and possess a strong descriptive power are often near to optimal.

By applying the orthogonal forward regression algorithm, various ranges of terms can be used to recover the restoring force surface such as: polynomial terms, rational functions, radial basis functions, wavelets functions, and so on, and also the hybrid models which combining different type of terms. This will greatly extend the application of the restoring force surface method to the dynamic systems with complex nonlinearities.

The new orthogonal forward regression restoring force surface method can then be summarised as:

(i) Design the experiment and prepare data. For example, select appropriate input signals to produce good data for the next identification. Record all the displacement, velocity, and acceleration data or obtain the data using a numerical integral or differentiation. This step is exactly same as the classical restoring force surface method.

(ii) Construct a candidate term dictionary which should be large enough to cover all the correct nonlinearities involved in the system. The term dictionary can be composed of any type of terms, polynomials, wavelets, trigonometric functions, and so on. All these terms are function of either displacement or velocity or both.

(iii) Apply the iOFR algorithm to select the significant terms from the dictionary and construct a parsimony model.

(iv) Verify the obtained models using model validation methods or model perdition. A commonly used model validation method is the high order correlation test method (S. A. Billings & Zhu, 1994; BlLlings & Voon, 1986). Two kinds of predictions are often used for model validation: one-step-ahead prediction and model predicted output. The model predicted output predicts the long term behaviours of a system. A good model predicted output often means that the identify model can represents the original system.

A very important issue encountered when applying the restoring force surface method is to design an appropriate excitation signals. One of the important criteria is that the phase trajectory covers as much of the phase plane as possible thus allowing one to construct a connected and continuous force surface (S. Duym & Schoukens, 1995; Worden, 1990a). Therefore the restoring force surface method is often used for the modelling of the nonlinear dynamical systems with external inputs. Directly applying the restoring force surface method to an autonomous nonlinear system could be difficult(Y. Guo, L. Z. Guo, S. A. Billings, & H.-L. Wei, 2015a). This is because the scattering points are sparse in a three-dimensional state space for a nonlinear oscillation.

The nonlinear dynamics of the body motion which considered in this paper is autonomous and the external force $u(t)$ is zero. However, according to the assumption in subsection 2.2 that the restoring force is a univariate function of displacement, equation (5) can then be written as

$$\ddot{y} = \frac{1}{m} f(y) \tag{6}$$

This assumption makes the identification process enforceable since the data is sufficient for the recovering of a restoring force curve in the two-dimensional displacement-acceleration plane (see fig 3 (c)) although the data scattering is sparse in a three dimensional state space.

It is worthy mentioning that this assumption seems not to be sufficiently supported because of the sparseness of the data in the phase space. However, the identification results prove the correctness of the assumption. That is a univariante restoring surface is adequate to represent the nonlinear body motion.

4. Identification of the nonlinear oscillations

As what has been observed, the motion of the marked point in the vertical bouncing behaves as a nonlinear oscillation. In this section, this nonlinear oscillation will be modelled using the new introduced orthogonal forward regression restoring force surface method. Three different types of candidate terms are used to represent the nonlinear system, including polynomial model, multi-scale radial basis function model, and a hybrid model. The advantages and disadvantages of each model are discussed in detail.

4.1 Identification of a polynomial model

Polynomial nonlinearities are widely used for the identification of nonlinear system because of the inherent advantages of this kind of model. The nonlinear relationship between acceleration and the displacement is firstly identified using this kind of model. Following the identification programme given in section 3, a third order polynomial model is obtained. The identified model is given as follows and the results are shown in Table 1. A total number of four terms are selected in the final model.

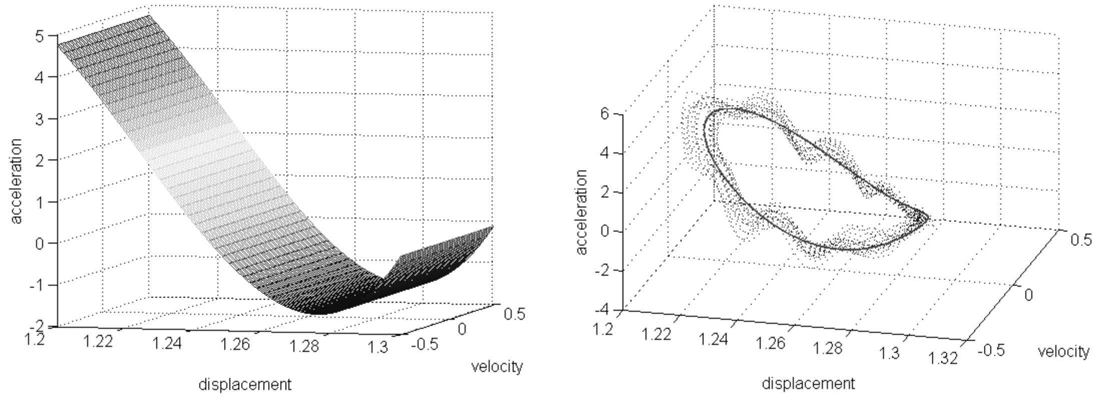
$$\ddot{y} = -24485.68 + 60276.76\,y - 49365.42\,y^2 + 13451.68\,y^3 \tag{7}$$

It can be observed that, using the iOFR algorithm, the first select term does not have a very large ERR, which is different from the classic OFR algorithm. A less dominant first term may make the term selection less greedy and a better model structure can be obtained (Yuzhu Guo, Guo, et al., 2015c). In the obtained model the constant term produced the greatest ERR because the restoring force curve does not go through the origin of the acceleration-displacement plane.

Table 1 The results of the forward regression

Terms	Err (%)	Coefficient	Standard Deviations
y^3	0.3714	13451.68	526.7102
1	79.6868	-24485.68	1033.9431
y^2	13.1757	-49365.42	1978.9077
y	0.6663	60276.76	2477.8056

Based on the simple polynomial model, the restoring force surface can be reconstructed as Fig 4 (a).



(a) (b)

Figure 4 Reconstruction of the restoring force surface using the polynomial model

Simulating the polynomial model, the comparison of the model predicted outputs with the experimental data is shown in Fig 5. Details of the predicted acceleration signal are shown in Fig 6 by zooming in Fig 5. Figure 4 (b) shows the predicated trajectory in the phase space. Figure 4, 5 and 6 show that the obtained polynomial model captures the main morphology of the experimental data but misses some details.

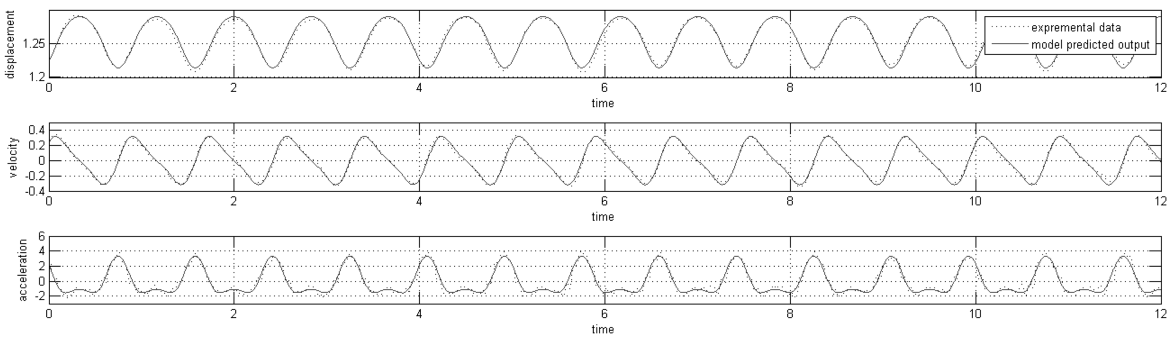


Figure 5 The model predicted output of the polynomial model

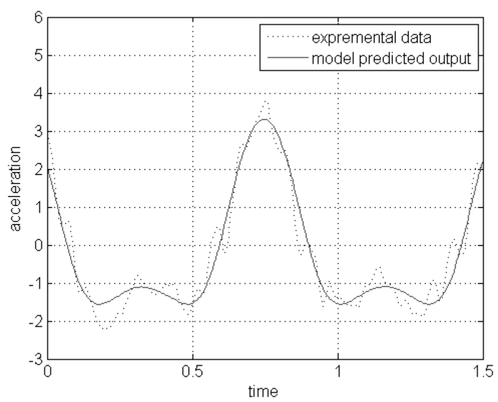


Fig 6 Enlarged plot of the acceleration signal

Fig 7 shows the comparison of the frequency spectra of the model predicted output and the experimental data. It is shown that the simple polynomial model is capable to reproduce the basic frequency component and the harmonics.

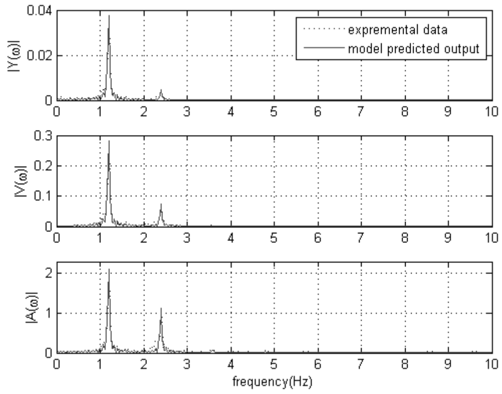


Fig 7 Frequency components of the model prediction output

The nonlinear terms in model (7) represent the nonlinear stiffness in the system, which plays a crucial role in the generation of the harmonics. The polynomial models inherently possess many significant advantages. (i) The polynomial model is of a very simple structure and easy to simulate and to realise as a real physical system. (ii) Owing to the polynomial structure, the model can be easily used for further analysis in both time and frequency domains. There are many mature techniques for the analysis of systems with polynomial nonlinearities. For example, the perturbation method and bifurcation analysis (Khalil, 2002), the describing function method (Kochenburger, 1950), and the Volterra series theory and the associated generalised frequency response functions (Chua & Yaw-Shing, 1982; Rugh, 1981). (iii) The output of the system is strictly harmonic signals. That is all the higher frequencies are strict integer times of a basic frequency component.

However, problems start to occur when the restoring force surfaces are complex. This is simply because the discontinuities and nonuniformities are very difficult to model using inherently smooth polynomial terms. What make the condition worse is that the higher order polynomial model may easily sink into the difficulty of instability. This is easy to explain. In order to satisfy the requirement of a best fit some of the higher-order stiffness with negative coefficients may emerge in the final model with a large coefficient. These terms are physically sensible and yield instability results.

4.2 Radial basis function model

In order to characterise the system more accurately, radial basis functions are used as the term dictionary of the iterative orthogonal forward regression restoring force surface method in this section. Radial basis functions (RBF) whose value depends on the distance from the centre points can efficiently characterise the position related local information. The radial basis functions should be a good choice for the nonlinearity in this case.

The nonlinear restoring force surface is then represented as the linear combination of a series radial basis functions $\phi_i(y)$, (i =*1, 2, …, n*), with the weights w_1, w_2, …, w_n, respectively .

$$f(y) = \sum_{i=1}^{n} w_i \phi_i(y) \tag{8}$$

The Gaussian functions are chosen as the “mother” function of the multi-scale radial basis functions.

$$\phi_i(x) = e^{-\frac{\|x - c_i\|^2}{2\sigma_i^2}} \tag{9}$$

where $\|\bullet\|$ is the norm to define the distance from x to the *i*th centre c_i .

According to the information show in the phase portrait the candidate term dictionary is constructed as follows. The centres c_i choose values between 1.2 and 1.3 for every 0.001 unit where the limits of the centres are determined by the phase plane trajectory. The scales σ_i change from 0.001 to 0.020 for every 0.001 unit to produce a sufficient cover to the range of the displacement, that is, the interval [1.2, 1.3]. An appropriate choice of candidate term set is crucial for the identification process. A large enough term set is needed to make sure that the underlying dynamics of the system can be well approximated using the candidate model building blocks. However, a very large term set will make the identification computationally intensive. The range of scales for the radial basis functions can be efficiently determined through an iterative process. Initially, select a relatively small range of scales in the dictionary and apply the orthogonal forward regression algorithm. Examine the obtained model to check whether more than one RBF functions with the same centre but different scales are selected. This often means that the dictionary is often not large enough because these RBF functions may be approached by less RBFs of a larger scale. Enlarge the range of scales and repeat the process until this condition does not happen in the final model.

The iOFR algorithm is then employed to select the significant terms from the term dictionary. The results are shown in Table 2. A total number of 9 terms are selected in the final model. These RBFs are shown in Fig 8. Adding these weighted terms together yields the final model.

Table 2 Sub-models of the system

Terms		Err (%)	Coefficient	Standard Deviations
c_i	σ_i			
1.211	0.0138	63.0126	3.3423	0.0149
1.276	0.0152	30.9499	-1.4356	0.0126
1.260	0.0026	0.4368	-0.7464	0.0278
1.227	0.0028	0.3922	0.6278	0.0262
1.248	0.0030	0.3286	0.4774	0.0252
1.237	0.0022	0.1971	-0.4468	0.0291
1.201	0.0044	0.1517	2.0520	0.1485
1.281	0.0028	0.1328	-0.3726	0.0244
1.291	0.0014	0.0923	-0.3560	0.0331

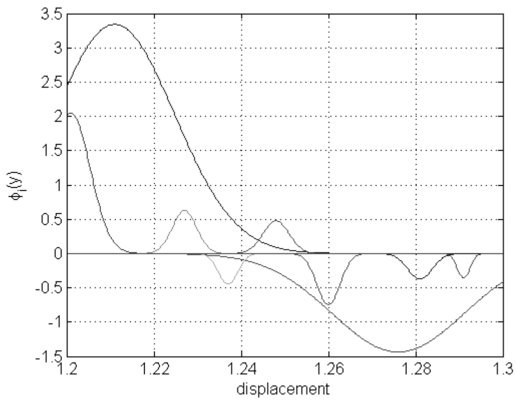


Figure 8 The terms in the final RBF model

The model predicted outputs are shown in Fig 9. The restoring force surface and the phase trajectory are show in Fig 10 (a) and (b), respectively. It is easy to observe that the RBF model represents the nonlinear restoring force better than the polynomial model did.

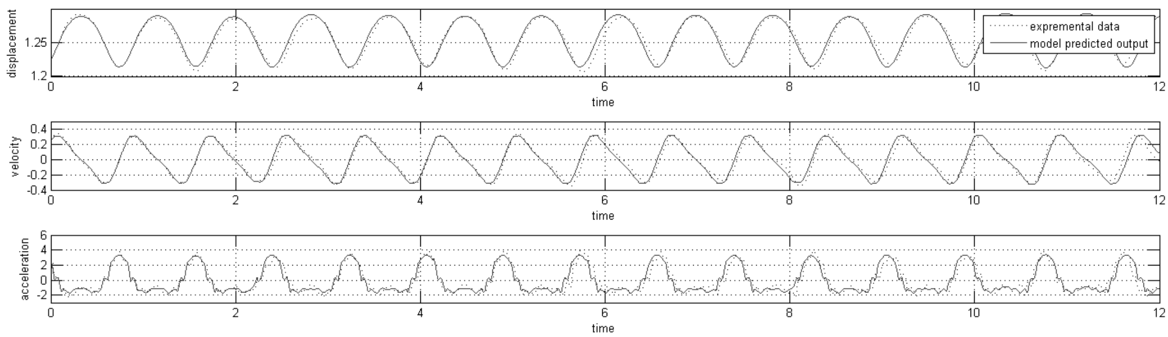
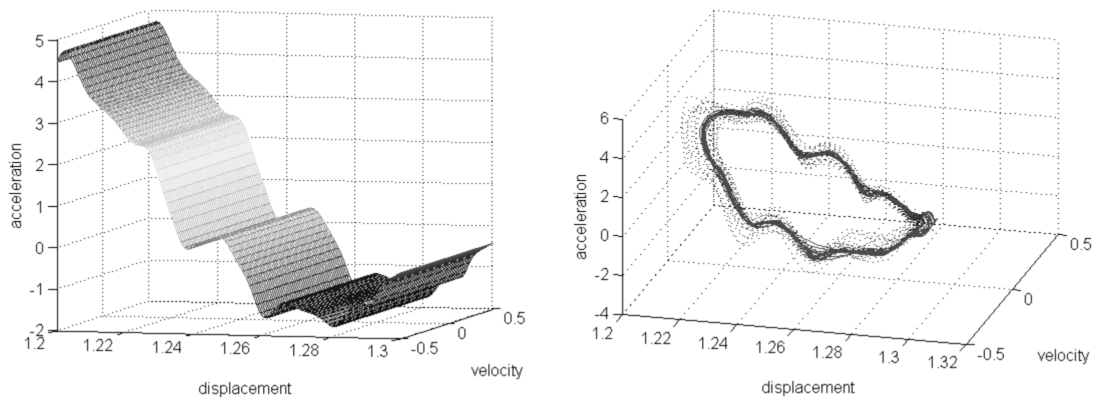


Figure 9 The model predicted outputs of the RBF model



(a) (b)

Figure 10 Reconstruction of the restoring force surface using the RBF model

The spectral analysis of the model predictions is shown in Fig 11. Although the RBF model prediction fits the experimental data better in the time domain and in the state space, the polynomial model prediction fits the data better in the frequency domain. There are more energy leaks around the harmonic frequencies.

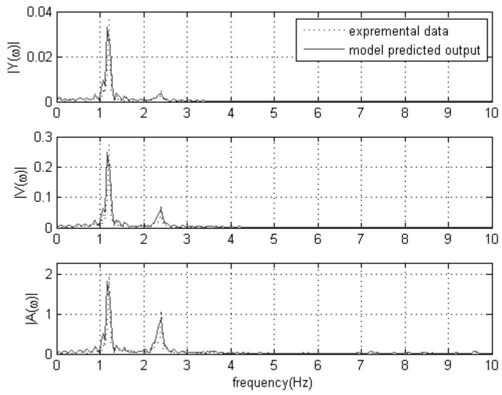


Figure 11 The spectra of the model prediction outputs of the RBF model

Fig 9, 10 and 11 show that the model predicted outputs agree with the experimental data. The results show that the radial basis functions successfully overcome the divergence problem which may happen in the high order polynomial model fitting. This shows the powerful fitting ability of the radial basis function model. Theoretically, the multi-scale radial basis functions are able to recover any complex restoring force surface and the multi-scale radial basis function approximations are asymptotically optimal, in the sense of convergence rate (Stephen A. Billings et al., 2007). This means a parsimonious model can be obtained.

However one obvious disadvantage of the method is that the model is unphysical and the coefficients obtained for the expansion cannot directly yield information about the physical quantities in the system, such as the damping and stiffness of the structure. Another disadvantage, which was shown in the spectral analysis of the reproduced signals, is that the reproduced signals may have much richer frequency components than a harmonic signal. This is not what expected because additional frequency components which are not in the experimental data may be introduced through the RBF model.

4.3 Hybrid model combining the polynomial and radial basis functions

According to the analysis in the previous sections, a model which possesses the advantages of both the polynomial model and the radial basis function model is expected. That is, the hybrid model can not only give an accurate description of the nonlinear dynamics but also accurately characterise the system in frequency domain. In this section a two-level hybrid model will be identified to describe the nonlinear oscillations in multi-scale from coarse to fine (Stephen A. Billings et al., 2007; Wei et al., 2007).

A hybrid model which includes both polynomial and RBF terms were used to describe the dynamics. The iOFR algorithm is again used to select the significant terms from the mixed term dictionary consists of polynomial and RBF candidate terms. This time, the centres of the radial basis functions keep same however the scales of the RBF terms are selected in a relatively smaller range from 0.0005 to 0.0030 to avoid the information overlap with the polynomial terms. This is because the hybrid model represents the behaviours of the nonlinear system in two different levels. The polynomials recover the main shape of the system and generate harmonic signals while the RBFs to characterise the local details. Therefore, the scale of the RBFs is chosen to focus on the local information and shield off the global information. While the polynomial terms characterise the large-scale information but neglect the local details. The determination of an appropriate scale range for the RBFs is easy to realise in the identification. Choose a relatively larger scale range, for example, the maximum scale in Table 2, and set the initial range as 0.0005 ~ 0.0152. Reduce the upper limit and apply the orthogonal forward regression algorithm until the polynomial terms start to appear in the model. In this example the scale range is chosen as 0.0005 ~ 0.0030.

Apply the orthogonal forward regression algorithm and stop the forward selection process as the sum of ERRs reaches 0.95 (Wei, Billings, & Liu, 2004). The obtained model structure and the associated coefficients are given as Table 3. The list of the ERRs shows that the polynomial terms play important roles in the final model. The comparison of the model predicted outputs and the real data is shown in Fig 12. The comparison of the frequency spectra of the predicted outputs and the real data is show in Fig 13. The results show that the model predicted outputs excellently agree with the experimental data both in the time domain and also in frequency domain.

Table 3 Sub-models of the hybrid model

Terms		Err (%)	Coefficients	Standard Deviations
c_i	σ_i			
1.216	0.0030	0.3818	-0.0053	0.0696
1.282	0.0030	0.1576	-0.3656	0.0271
1.225	0.0030	0.1417	0.4694	0.0517
1.208	0.0030	0.0925	-0.7889	0.0987
y^3		0.0751	761.67	18.0948
1		0.0535	784.30	18.1019
y^2		0.0402	-1454.4	34.2119
1.248	0.0030	0.0087	2.5037	0.3514

1.237	0.0023	0.0016	-0.4164	0.0466
1.260	0.0023	0.0017	-0.4009	0.0451
1.277	0.0030	0.0013	-0.2166	0.0305
1.291	0.0015	0.00064382	-0.3461	0.0477
1.229	0.0020	0.00049928	0.3819	0.0394
1.214	0.0010	0.00035982	-0.3333	0.0466
1.248	0.0026	0.00023084	-1.7875	0.3565
1.268	0.0016	7.2807e-05	0.1849	0.0455
1.287	0.0008	6.3289e-05	0.0878	0.0296
1.254	0.0016	4.8444e-05	0.1114	0.0447

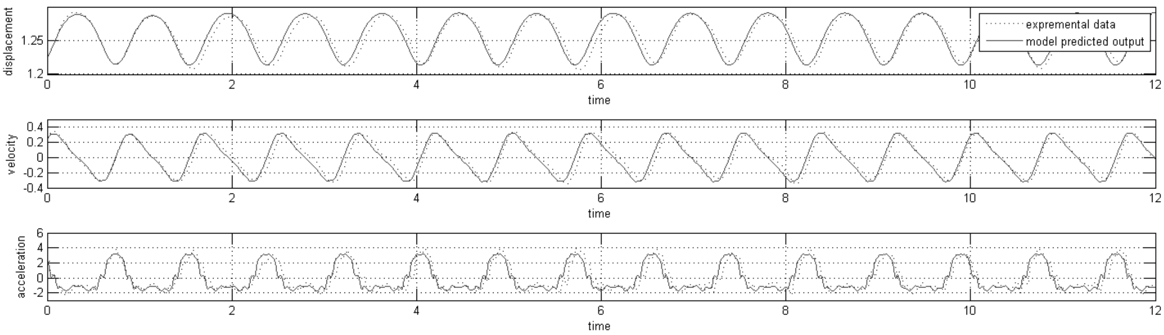


Figure 12 Model predicted output of the hybrid model

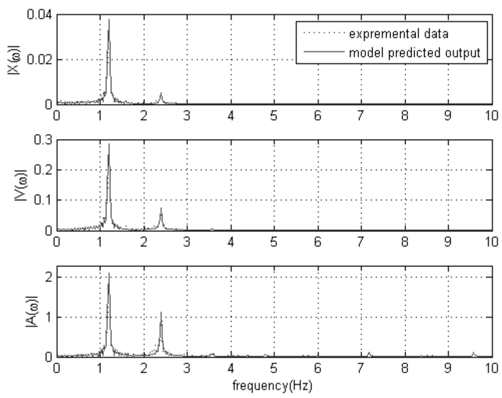


Figure 13 Spectra of the model predicted output of the hybrid model

However the hybrid model has not shown significant advantages over the RBF model so far. It is expected that the polynomial part of the hybrid model can also give an acceptable description of the system behaviours neglecting the details. Comparing Table 3 with Table 1, it can be observed that the three polynomial terms in the hybrid model is exactly same as the first three terms in the purely polynomial model.

Extracting the polynomial terms from the hybrid model yields a new polynomial model as

$$\ddot{y} = 784.30 - 1454.4\,y^2 + 761.67\,y^3 \tag{10}$$

Plot the hybrid model and the extracted polynomial model (10) in the displacement-acceleration phase plane. Fig 14 shows that model (10) fits the experimental data very well.

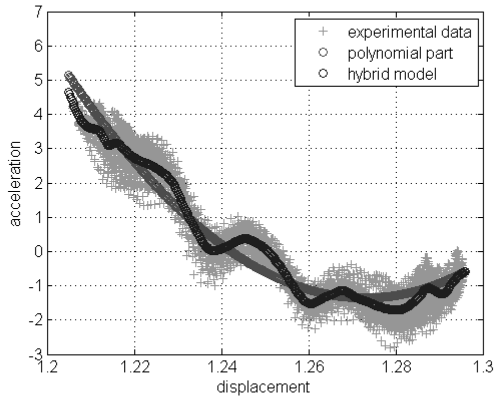


Fig 14 The fitness of the hybrid model in the displacement-acceleration phase plane

The model predicted output of the polynomial part (model(10)) and the spectra of the prediction are shown in Fig 15 and 16 respectively. Although the model predicted outputs do not fit the data very well after 6 seconds the model predictions perfectly reproduced the frequency spectra of the experimental data, which is even better than the pure polynomial model (7) did. This means that the polynomial part of the hybrid model is capable to represent the global behaviours neglecting some local details. This is because the Fourier transform always gives average information of the whole time series. Simulations show that the hybrid model works stably at an arbitrary long time.

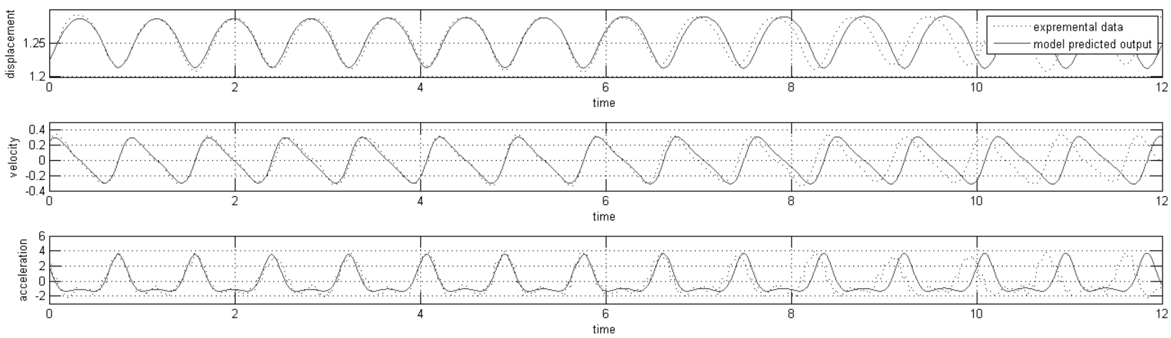


Fig 15 The model predicted output of the polynomial part

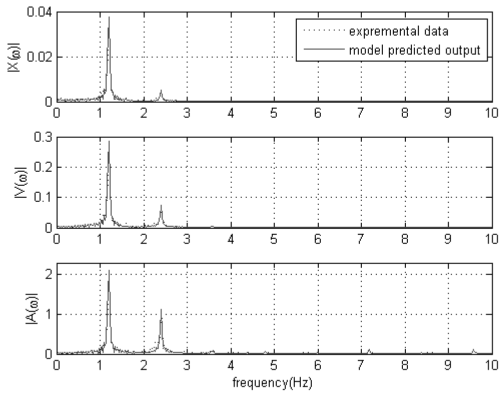


Fig 16 Spectra of the model predicted outputs by model (10)

To sum up, the hybrid model successfully provides a two-level coarse-to-fine, representation of the nonlinear systems. The polynomial terms give a coarse description which could characterise the main frequency components of the nonlinear oscillations whereas the complete model gives a more accurate description of both the local and global behaviours of the system.

5. Conclusions

The nonlinear oscillations existing in human bouncing has been investigated. A new system identification method which combines the restoring force method and the iterative orthogonal forward regression algorithm has been introduced. The system identification based method which avoids the difficulties in a first principle method is simple to carry out in the practical applications.

The obtained models have been shown to be able to reproduce the nonlinear oscillation both in the time and frequency domain. The new identification method itself extends the restoring force method to a more wide class of system with complex nonlinearities. Although a special example where the external input is zero and the effect of the velocity can be ignored has been studied in this paper this does not prevent the new orthogonal forward regression restoring force surface method to be a general choice for the investigation of the nonlinear dynamics.

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