UNIVERSITY of
BRADFORD

Library

The University of Bradford Institutional Repository

http://bradscholars.brad.ac.uk

Link to original published version: *https://doi.org/10.4218/etrij.17.0117.0246*

Citation: Salam AOA, Sheriff RE, Al-Araji SR, Mezher K and Nasir Q. (2017) Novel Approach for Modeling Wireless Fading Channels using a Finite State Markov Chain. ETRI Journal. 39(5): 718-728.

Novel Approach for Modeling Wireless Fading Channels Using a Finite State Markov Chain

Ahmed Abdul Salam, Ray Sheriff, Saleh Al-Araji, Kahtan Mezher, and Qassim Nasir

Empirical modeling of wireless fading channels using common schemes such as autoregression and the finite state Markov chain (FSMC) is investigated. The conceptual background of both channel structures and the establishment of their mutual dependence in a confined manner are presented. The novel contribution lies in the proposal of a new approach for deriving the state transition probabilities borrowed from economic disciplines, which has not been studied so far with respect to the modeling of FSMC wireless fading channels. The proposed approach is based on equal portioning of the received signal-to-noise ratio, realized by using an alternative probability construction that was initially highlighted by Tauchen. The associated statistical procedure shows that a first-order FSMC with a limited number of channel states can satisfactorily approximate fading. The computational overheads of the proposed technique are analyzed and proven to be less demanding compared to the conventional FSMC approach based on the level crossing rate. Simulations confirm the analytical results and promising performance of the new channel model based on the Tauchen approach without extra complexity costs.

I. Introduction

Wireless communication systems suffer problems originating from adverse multipath fading in transmission channels that is more prone to dynamic variations. The severity of such fading is highly dependent on terminal mobility and tall obstacles on the ground. An appropriate channel model and simulation approach need to be investigated to account for these channel effects. The recently envisioned software-defined radio (SDR) is considered an enabling technology that can flexibly adapt against stringent channel conditions to maintain an adequate quality of service. Vital details of channel state information can hence be utilized to allow SDR to reconfigure its internal parameters to combat channel variations. The SDR is commonly known to be essential in cognitive radio (CR), and it can employ the adaptive coded modulation (ACM) technique to cope with dynamic channel changes. It is therefore imperative in the assessment of any wireless communication system to retain the design and analysis of channel fading as accurately as possible.

Spurred by the aforementioned challenges, many recent studies were devoted to the investigation of such channel characterization and simulation in wireless communication systems. A good survey on the finite state Markov chain (FSMC) origin of developments can be found in [1] and the references provided therein. It has been indicated that such statistical modeling can be traced to the initial efforts by Gilbert for a two-state channel crossover in wireline telephone circuits with burst noise, which was then

improved by Elliot, in the early 1960s. This model is commonly known as the Gilbert-Elliot channel (GEC) in the literature, and it is intended to approximate the Rayleigh channel fading behavior by only two states of channel quality. The basic idea behind such a conceptual approximation is to find a tractable methodology for the formulation and calculation of channel information capacity and associated bit error rates (BER). Therefore, the FSMC approach can be instead called an information-theoretic channel approximation.

In the GEC model, each state represents a specific channel quality, which is either noisy or noiseless. Generally, a binary symmetric channel with a given crossover probability can be associated with each state so that the channel quality for each state can be identified. The Rayleigh fading effect in this case is assumed to be wide stationary, which means the crossover and transition probabilities have fixed values not altered by time, and the FSMC is of the first-order type. The GEC model has been widely espoused for performance studies in wireless fading channel environments. However, the GEC has severe limitations, especially in cases where channel characteristics are highly likely to change dramatically [2].

Several studies on FSMC channel modeling can be identified in the literature, but only a few are described here. An attempt to establish a connection between Rayleigh fading channels and their FSMC counterparts can be found in [2]. In this study, the signal-to-noise ratio (SNR) was partitioned into a finite number of intervals corresponding to an FSMC model. The zone between any two levels represents the fading channel state, and hence the transition and crossover probabilities from one state to another can be interrogated using such an analytical-approximation approach. A methodology to partition the received SNR into a finite number of states according to the time duration of each state was developed and analyzed in [3].

Alternatively and instead of the SNR range, the dynamic range of channel fade amplitude was considered for partitioning in [4]. It expanded the FSMC by introducing intermediate channel states between adjacent symbol epochs of the actual de-interleaver output. Such an expansion claims that the FSMC is usable in real situations of non-interleaved channels with fast fading (fast Doppler frequency to symbol-transmission-rate ratio) or interleaved (correlated) channels such as in diversity combining. The validity of adopting the first-order FSMC was examined using an alternate approach of an autocorrelation function over consecutive data samples [5]. A different approach using adjacent transition (AT)

was proposed to construct an FSMC model to represent the Rayleigh fading channel [6]. The AT method generally differs from the equal probability (EP) in [2] and equal duration (ED) in [3].

From a wider perspective and out of many recent special-purpose applications, two particular examples are brought forward in regard to FSMC viability. The first involves FSMC modeling for wireless transmission losses that can be implemented to discriminate between wireless and congestion-related losses in data networks using real channel traffic [7], while the second involves FSMC-based spectrum sensing policies in a cognitive radio (CR) system. This system leverages past sensing outcomes of several cooperating secondary users (SUs) to decide which channel of primary users (PUs) should be sensed by each SU at a given time [8]. In recognition of the distinctive features of FSMC practices, such cornerstone applications represent new research trends in wireless communication systems.

Recent studies claimed that FSMC channel approximation can also be tangible in other dynamic applications. For example, FSMC channel modeling based on Nakagami statistics in massive multiple-input multiple-output (MIMO) schemes was reported for vehicle-to-infrastructure [9] and high-speed railways [10]. The Gamma shadowing effect of the people movement approach using FSMC modeling in the ultra-wide band broadcast was attempted in [11].

Given the conceptual facts exposed above, the main contributions of this paper can be summarized as follows:

- Establish the analytical tools to design and test Rayleigh fading channels using autoregressive (AR) models.
- Show that the first-order AR(1) model is sufficient to achieve adequate results without extra overhead.
- Illustrate that the strategy of approximating fading statistics by first-order FSMC performs broadly well.
- Debut the approximation notion settled by Tauchen for FSMC modeling of wireless fading channels.
- Validate the supremacy of the new FSMC modeling using the Tauchen method compared with classical methods.

The rest of this paper is organized as follows. Section II provides a brief background on channel fading. Section III presents the AR channel, while the FSMC is given in Section IV. The new approach of statistical modeling based on the Tauchen procedure is outlined in Section V. The simulation results are illustrated in Section VI and are followed by concluding remarks.

II. Fading Channel Background

Time-varying multipath channels can be either time or frequency selective, or both together. As such, they are labeled as doubly selective. The speed of time and frequency variations determines whether a channel is of a slow fading or fast fading nature. In general, commercial land wireless communication systems are assigned predefined channels with small frequency bandwidths. Hence, our approach will consider a narrowband baseband signal over a flat-fading Rayleigh channel only.

Consider the linear dynamic model for a received baseband signal at the output of a matched filter given by

$$y(k) = h(k)x(k) + n(k), \qquad (1)$$

where $h(k)$ is the fading channel envelope, which is a recursive complex-valued random process, and $n(k)$ is the complex zero-mean additive white Gaussian noise (AWGN) $\sim \mathcal{CN}(0, \sigma_n^2)$. The amplitude a and phase θ components of the fading channel are governed by $h(k) = a(k)e^{j\theta(k)}$. The fading channel can be decomposed into its real and quadrature components $h(k) = h_r(k) + jh_q(k)$, and each is given by $\sim \mathcal{N}(\mu, \sigma_h^2)$. The mean value μ represents the line-of-sight propagation component (LOS). If it is zero, the fading process is called Rayleigh fading; otherwise, it is called Rician fading, as recommended by the ITU [12]. In multipath channels, the fading amplitude a has a Rayleigh probability distribution function (PDF) [1], [13], [14]

$$f_h(a) = \frac{a}{\sigma_h^2} \exp\left(-\frac{a^2}{2\sigma_h^2}\right), a \in [0, \infty). \qquad (2)$$

and the fading channel phase has a uniform PDF as below:

$$f_h(\theta) = \frac{1}{2\pi}, \theta \in [0, 2\pi). \qquad (3)$$

In the same manner, but with a slightly different portrayal, such a Rayleigh PDF can also be implied for the fading SNR random process [3], [6], [7]. Even though multipath fading gains and their real and imaginary components are driven by uncorrelated wide-sense stationary (WSS) AWGN, it is known that the time variations of such channels interact in a correlated manner. Such intercorrelation will determine the channels' statistical, time, and frequency characteristics. It is hence essential to seek a tractable mathematical model to accurately describe the dynamic time variations of fading channels. The most widely accepted statistical model in such cases was developed by Jake and Clark (JC) [1], [5], [13], [14]. In the JC model, the autocorrelation function

(ACF) of the real and imaginary components of a fading channel gain is given by

$$\begin{aligned} R_h(T_s) &= E\{h_r(k)h_r^*(k - T_s)\} \\ &= E\{h_q(k)h_q^*(k - T_s)\} \\ &= \sigma_h^2 J_o(2\pi f_D T_s) \end{aligned} \qquad (4)$$

where $J_o(.)$ is the zeroth-order Bessel function of the first kind, $f_D = (v/c)f_o$ is the Doppler spread frequency, v is the terminal travelling speed, f_o is the carrier frequency, c is the speed of light, and T_s is the channel symbol duration. The power spectral density (PSD) of the above ACF is denoted by

$$S(f) = \begin{cases} \dfrac{\sigma_h^2}{2\pi f_D} \dfrac{1}{\sqrt{1 - \left(f/f_D\right)^2}}, & \text{for } |f| < f_D \\ 0, & \text{otherwise.} \end{cases} \qquad (5)$$

The channel fading characteristics are determined by the Doppler frequency owing to the motion of a mobile terminal. In slow or flat-fading channels, the channel coherence time $T_c \approx 1/f_D$ is larger than the symbol period T_s, alternatively $f_D T_s \ll 1$. For extremely fast-fading channels, $f_D T_s \gg 1$. However, such channels are not commonly encountered in most real-world wireless communication systems.

III. AR Channel Modeling

The AR filters are finite impulse response (FIR) structures commonly utilized to approximate fading channels of particular time and frequency responses. This is a result of the traceable computation of their parameters and correlation properties. Let us assume we have a frequency-nonselective fading channel with L resolvable paths. This channel is anticipated to be slowly varying and constant during the observation interval. By using a conventional tapped delay line model with tap spacing equal to T_s, the fading formula for real and imaginary components is given by [1], [14], [15]

$$h(k) = -\sum_{l=1}^{L} a_l h(k - l) + w(k), \qquad (6)$$

where $\{a_1, a_2, \ldots, a_L\}$ are the AR filter coefficients of order L and $|a_l| < 1$, denoted as AR(L), and $w(.)$ is a complex AWGN process with uncorrelated real and imaginary components and is denoted by $\sim \mathcal{CN}(0, \sigma_w^2)$.

The ACF and PSD approximates generated by the above AR model need to be evaluated. This can be done by either comparing against empirical channel measurements taken from the field, or from a specified analytical

approach as defined in equations (4) and (5) above. Since the plausibility of the first option is highly dubious owing to practical and economical constraints, the second option is employed herein instead.

There are few methods proposed in the literature to adjust the AR model parameters per the desirable fading covariance statistics. Chief among the various methods is one that employs Yule–Walker (YW) equations [1], [14], [15]. The YW approach is considered further here because the viability of other methods is susceptible to terms of extra complexity and computational demand. First, we need to define the PSD of the AR(L) fading model [14], [15]:

$$S_{hh}(f) = \frac{\sigma_w^2}{\left|1 + \sum_{l=1}^{L} a_l e^{-j2\pi fl}\right|^2}. \tag{7}$$

The AR(L) parameters are given in termsf the desired model ACF. $R_{hh}(l)$ is hence described recursively below:

$$R_{hh}(l) = \begin{cases} -\sum_{m=1}^{L} a_m R_{hh}(l-m), & l \geq 1 \\ -\sum_{m=1}^{L} a_m R_{hh}(-m) + \sigma_w^2, & l = 0 \end{cases} \tag{8}$$

and in matrix form given by

$$\mathbf{R}_{hh}\mathbf{a} = -\mathbf{v}, \tag{9}$$

where

$$\mathbf{R}_{hh} = \begin{bmatrix} R_{hh}(0) & R_{hh}(-1) & \dots & R_{hh}(-L+1) \\ R_{hh}(1) & R_{hh}(0) & \dots & R_{hh}(-L+2) \\ \vdots & \vdots & \ddots & \vdots \\ R_{hh}(L-1) & R_{hh}(L-2) & \dots & R_{hh}(0) \end{bmatrix}, \tag{10a}$$

$$\mathbf{a} = [a_1\, a_2 \cdots a_L]^{\mathrm{T}}, \tag{10b}$$

$$\mathbf{v} = [R_{hh}(1)\ R_{hh}(2)\ \cdots R_{hh}(L)]^{\mathrm{T}}, \tag{10c}$$

$$\sigma_w^2 = R_{hh}(0) + \sum_{l=1}^{L} a_l R_{hh}(-l). \tag{10d}$$

Solving the L set of YW equations for the desired ACF, the generated AR(L) process yields the following ACF estimate:

$$\hat{R}_{hh}(l) = \begin{cases} R_{hh}(l), & 0 \leq l \leq L \\ -\sum_{m=1}^{L} a_m \hat{R}_{hh}(l-m), & 1 > L. \end{cases} \tag{11}$$

It is important to note that $\mathbf{R}_{hh}$ is a positive definite Toeplitz type to permit the application of the above equations.

IV. FSMC Channel Modeling

A conventional first-order FSMC model is explored here, relying on common aspects in [1]–[8] and [13]. Despite the fact that some of these references have challenged such a scheme, many studies have shown that the first-order system is sufficient for adequate and tractable analysis and results. The methodology proposed in [2], which constituted the basis of other studies [3]–[8] and [13], will be adopted herein to form an FSMC channel to reflect the Rayleigh fading statistics.

An FSMC channel model is a discrete stochastic process in which the current state depends on the complete history of past states through the most recent state only. Among several different approaches, it can be built by partitioning γ (the received SNR) into a fixed number of states or intervals. Let us consider an N channel state space $S = \{s_1, s_2, \dots, s_N\}$ and the corresponding BER, or crossover (also called transition) probability P_{en} where the subscript "e" stands for the error, and $n \in \{1, 2, \dots, N\}$. If we are using M-ary constellation symbols, which is the common case, then these crossover probabilities represent the symbol error rate (SER). Let $P_{n,j}$ be the state transition probability and π_n be the steady state probability such that $\sum_{n=1}^{N} \pi_n = 1$ for the simplest equiprobable SNR quantization method. The PDF given in (2) is revised to consider the AWGN and instantaneous γ, recalling that the fade amplitude is characterized by Rayleigh statistics, while the power of which has an exponential probability distribution given by

$$f(\gamma) = \frac{1}{\bar{\gamma}} e^{-\gamma/\bar{\gamma}}, \gamma \in [0, \infty), \tag{12}$$

where $\bar{\gamma}$ is the average SNR. The first-order FSMC constitutes the transitions that occur between adjacent states, and hence the probability $P_{n,j} = P[s_n|s_j] = 0$ if $|n - j| > 1$. Let the boundary separation between different channel states represented by $\Gamma = \{\Gamma_1, \Gamma_2, \dots, \Gamma_N\}$, that is, when the received SNR is in the interval $[\Gamma_n, \Gamma_{n+1}]$ at time k, the channel state is defined to be s_n. Then, the forward and backward transition probabilities and the level crossing rate (LCR) are given by [1]–[4], [6], [13], respectively:

$$P_{n,n+1} \approx \frac{\Phi(\Gamma_{n+1})T_s}{\pi_n}, n = 0, 1, \dots, N-1, \tag{13}$$

$$P_{n,n-1} \approx \frac{\Phi(\Gamma_n)T_s}{\pi_n}, n = 1, \ldots, N, \qquad (14)$$

$$\Phi(\Gamma_n) = \sqrt{\frac{2\pi\Gamma_n}{\bar{\gamma}}} f_D e^{-\Gamma_{n/\bar{\gamma}}}. \qquad (15)$$

These expressions assume the time axis is divided into "slots" of identical size equivalent to the symbol period T_s.

V. FSMC Approximation Method

There are many varieties of statistical approximation methods in the literature. The method that will be explored herein has solid foundations in economic, finance, and econometric modeling systems. Tauchen pioneered this field, and his work is well known and adopted in many studies. He contributed to solving functional equations where the state variables have autoregressive patterns [16], [17]. Computational simplicity and generating almost accurate results under uncorrelated error terms are recognized features attributed to this approximation scheme. To the best knowledge of the authors, this approach has never been addressed or attempted properly in the signal processing field. Hence, this paper contributes the first lead of deploying the Tauchen procedure in the FSMC modeling of wireless fading channels.

To use the approach laid down by Tauchen, one must assume the process values stay within bounded intervals to solve the problem at hand. As stated earlier in this paper, these intervals are curbed by N different channel states, which are generated by the AR(1) channel model as given in (6). These intervals and states are also assumed to be equally spaced to make the Tauchen approach valid. Let the probability of $w(k)$ be such that $P[w(k)] \leq u = F(u/\sigma_w)$, where u is any value and F is the cumulative distribution function (CDF) with unit variance. The following assumptions were made by Tauchen:

$$\begin{aligned} s_1 &= -s_N, \\ \zeta &= s_n - s_{n-1}, \\ s_N &= m\sigma_h = m(\sigma_w^2/(1-a^2))^{1/2}, \\ s_n &= s_1 + \frac{(n-1)(s_N - s_1)}{N-1}, \end{aligned} \qquad (16)$$

where m is any multiplicity number. There is no particular rule established to set the value for this multiplication parameter; however, [16], [17] stated that $1.2 \times \ln(N)$ and 3 could be proposed. From this, one can calculate the transition probabilities for $j \in [2, N-1]$ as below:

$$P_{n,j} = F\left(\frac{s_j - as_n + \zeta/2}{\sigma_w}\right) - F\left(\frac{s_j - as_n - \zeta/2}{\sigma_w}\right). \qquad (17)$$

This expression can be thought of as the probability that the event $as_n + w \in [s_j - \zeta/2, s_j + \zeta/2]$ takes place. The transition probability from state n to state 1 is given by

$$P_{n,1} = F\left(\frac{s_1 - as_n + \zeta/2}{\sigma_w}\right) \qquad (18)$$

and the transition probability of leaving state n to state N is

$$P_{n,N} = 1 - F\left(\frac{s_N - as_n - \zeta/2}{\sigma_w}\right). \qquad (19)$$

The above discrete probabilities converge in a weak sense to their continuous terms in the stochastic recursive model.

VI. Complexity Analysis

The computational complexity of the proposed Tauchen approximation for FSMC channel modeling is assessed with reference to the conventional LCR approach in this section. The computational complexity usually includes the overall operation of mathematical addition, subtraction, multiplication, and division procedures. The complexity analysis of FSMC channel models is a challenging threefold task. While assuming the observed samples are statistically independent, the complexity involves the following three steps that need to be efficiently computed: 1) probability of the observation sequence for a given model, 2) selection of the corresponding state sequence, and 3) adjustment of the model parameters. These steps are common in the general context of most hidden Markov models encountered in various applications [18], [19].

As stated earlier, the widely adopted procedure for the iterative estimation of LCR-FSMC parameters is the forward–backward (FB) algorithm. When given a finite data sequence of $k \in [1, K]$ samples for training, the FB algorithm smoothly evaluates the likelihood of such data and coordinates the sufficient statistics for the FSMC updated parameters according to the Baum-Welch (BW) algorithm [18], [19]. Some approximations need to be considered in order to proceed with the computational analysis. The linear expressions given in (13) and (14) can be applied directly, while the Taylor series expansion is the best candidate to approximate the exponential and square root functions given in (15). The CDF of the Tauchen-FSMC model expressed in (17) can be attended by using the famous error function $F(z) = erf(z)$. For the purpose of presentation clarity and consistency, and by referring to the relevant functions in [20], these

approximations are provided as shown below:

$$e^{-z} = \sum_{i=0}^{\infty} \frac{z^i}{i!} = 1 - z + \frac{z^2}{2!} - \cdots, \qquad (20)$$

$$\sqrt{z} = f(z_o) + (z - z_o)\dot{f}(z_o) + (z - z_o)^2 \ddot{f}(z_o) + \cdots, \quad (21)$$

$$\operatorname{erf}(z) = \frac{2}{\sqrt{\pi}} \left\{ z - \frac{z^3}{3 \times 1!} + \frac{z^5}{5 \times 2!} + \cdots \right\}, \qquad (22)$$

where $f(z_o)$ is $\sqrt{\cdot}$ at arbitrary constant z_o, $\dot{f}(z_o)$ is the first derivative of $\sqrt{\cdot}$ at z_o, and so on to the end of series.

The steady-state probabilities π_n, where each denotes that the FSMC attributes originate in state n, are assumed to be fixed indicating that they remain in their initial conditions. This is valid in the context of WSS processes where the channel parameters are expected to have trivial variations. Furthermore, we address the common regime of weak channels by having the value of z be small enough to make such an assumption admissible. This also is a common practice in most studies since previous results showed that most partitioning occurs in regions of low SNR levels where the error probability is significant [3], [7]. Therefore, the first two terms of (20) and the second term only of (21) are used to approximate function (13) to (15), while the first term of (22) is accounted for by the approximation of (17).

By successively performing the above procedures over the entire K training population, the time operational requirements $\mathcal{O}(.)$ were developed for both the LCR-FSMC and Tauchen-FSMC modeling schemes, as shown in Table 1. The parameters that are calculated just once in the Tauchen-FSMC model are discarded from the operational complexity assessment.

Using the partition policy of 10 states, the above total computation figures would be 26×10^6 and 10×10^6 for the LCR and Tauchen FSMC models, respectively, for a 10^5-long training sequence. This is a modest example that is applicable to Rayleigh faded channels; however, the difference in computational figures gets exponentially larger as the number of states increases. The situation of extensively large number of states is expected in diverse applications, such as speech recognition where the acceptable minimum number of states lies between 32 and 256 for modest performance [18]. Irrespective of the situation, the difference in the time computational loads, and also with regard to memory, is undoubtedly apparent in favor of the Tauchen-FSMC model compared with LCR-FSMC.

VII. Simulation Results

Simulation results are provided to verify the performance of the proposed Tauchen-FSMC model approximation.

An AR(1) model generator based on the JC fading channel is developed first. The simulation scenario assumes a slowly fading channel with one resolvable path and without an LOS component. The channel fading parameters are governed by the values $f_D T_s = 0.01$ and $T_s = 0.1$ ms. This is equivalent to a vehicular mobility of 60 km/h, which is typically expected as the average speed in urban areas. The SNR is assumed to be 0 dB, and the time stream of the generated Rayleigh channel envelope is depicted in Fig. 1. An AR(1) model is simulated using $a = 0.8$, as shown in Fig. 1, while the associated ACF and the bell-shape-like PSD trends of which are shown in Figs. 2 and 3, respectively.

The ACF figure is generated using the YW algorithm. Applying the fast Fourier transform (FFT) yields the PSD. The JC channel model is also depicted in Figs. 2 and 3 for comparison with the AR(1) results. It is evident that the JC model has a greater resemblance to the Bessel function in the time domain, while it accurately identifies the Doppler frequency f_D in the frequency domain. On the other hand, AR(1) is considered an approximation to such JC model behavior. AR(1) follows the envelope of the Bessel-like
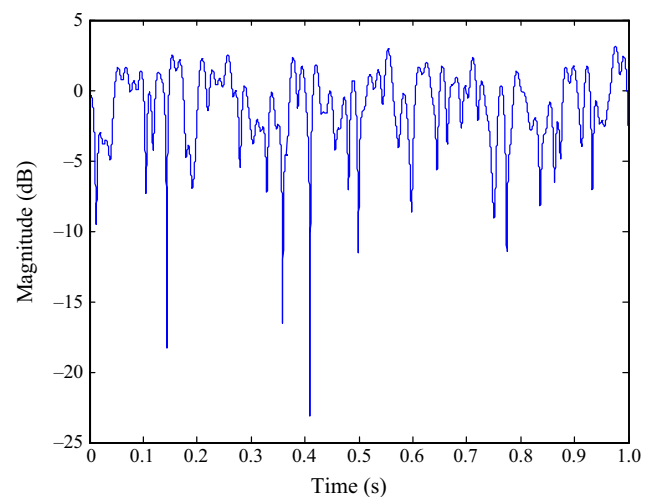
Table 1. FSMC complexity operations.

Model	Mul/div	Add/sub	Total
LCR-FSMC	$\mathcal{O}(2(N^2+N)K)$	$\mathcal{O}(4NK)$	$\mathcal{O}(2(N^2+3N)K)$
Tauchen-FSMC	$\mathcal{O}(2NK)$	$\mathcal{O}(8NK)$	$\mathcal{O}(10NK)$

Fig. 1. Envelope of AR(1) slow fading Rayleigh channel for $a = 0.8$.

function of the JC model in the time domain, while the PSD of which starts to pick up almost nearby f_D in the frequency domain. To some extent, such a simple approximation could be acceptable in general applications. When more accurate channel results are required, the order of the autoregressive model needs to be increased. This agrees with suggestions in some studies to increase the order and make it tens, or a few hundred, to fit particular applications. However, this will be at the cost of extra computational loads.

Next, the simulation of first-order FSMC channel models is considered. The channel fading envelope, or its associated SNR, is partitioned into 10 equal intervals. First, the LCR method and the transition probabilities are analyzed. Their cumulative trend and LCR curve are calculated as given earlier, the results of which
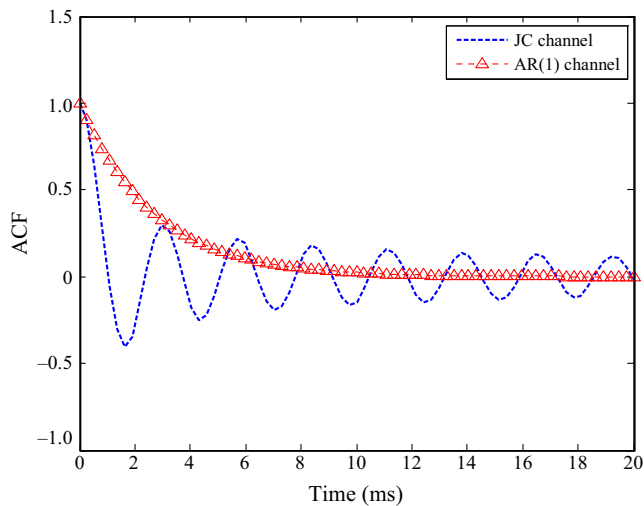

Fig. 2. ACF of AR(1) slow fading Rayleigh channel for $a = 0.8$.
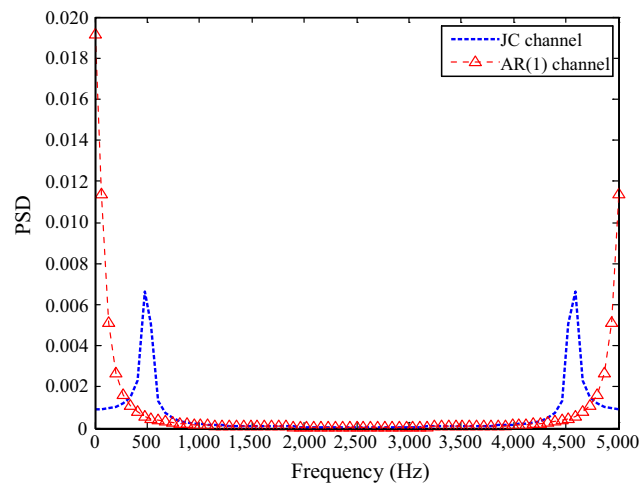

Fig. 3. PSD of AR(1) slow fading Rayleigh channel for $a = 0.8$.

are depicted in Figs. 4–6, respectively. As expected, the transition probabilities and their cumulative trends show sharp crossovers between adjacent states and with very small probabilities to transit to other far states. The footprint contours exhibit heavy concentrations of states transiting to themselves or to their neighboring states. This confirms the applicability of this statistical model as claimed by most prior studies.

Figure 6 shows a persistence rate for a state to remain or to cross to the next neighboring states only. This rapidly declining exponential curve obviously depicts the state's trend for short-time traversing rather than long-time traveling to distant states. The more state intervals that are considered, the sharper this exponential behavior, and hence shorter transition paths are consolidated. This suggests having a larger pool of state partitions. However, there is no feasible tool to examine the influence of the envelope fading parameter a on the LCR-FSMC model. Hence, this model can be identified as being insensitive to fading strength variations, which is one of the main findings of this paper.

Second, the Tauchen approximation method is examined for the first-order FSMC channel model. Figures 7 and 8 illustrate the results implemented for the same envelope parameter $a = 0.8$ as before. Despite the results showing statistical channel features almost comparable to those of the LCR method, a more resilient behavior can now be detected. This is mainly attributed to the direct influence of the a parameter on the autoregressive filter bandwidth. In other words, such an effect is explicitly interpreted in terms of the channel memory and its profound reliance on old or current states. The smaller the bandwidth, the stronger the link to current states. The opposite is also true, and hence there would be
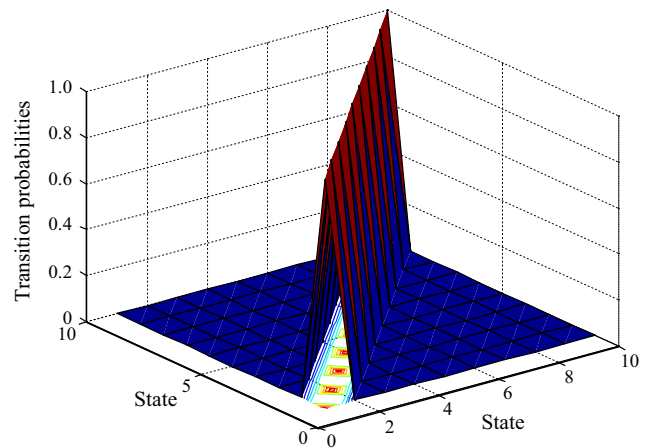

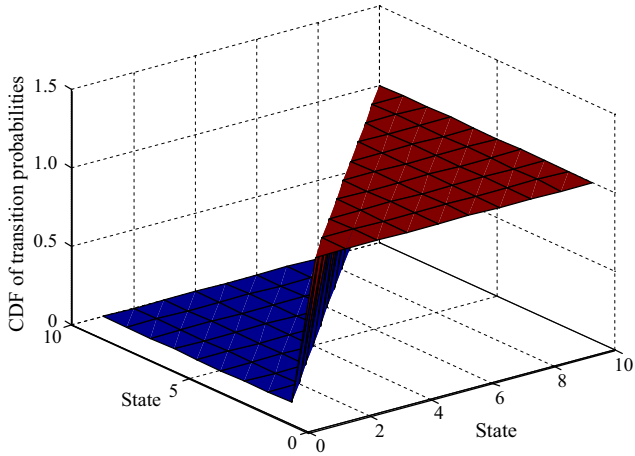Fig. 4. Transition probabilities of LCR-FSMC channel.

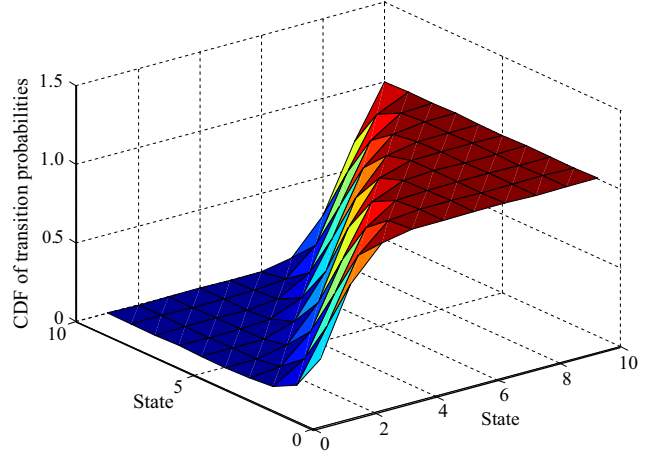Fig. 5. Cumulative transition probabilities of LCR-FSMC channel.

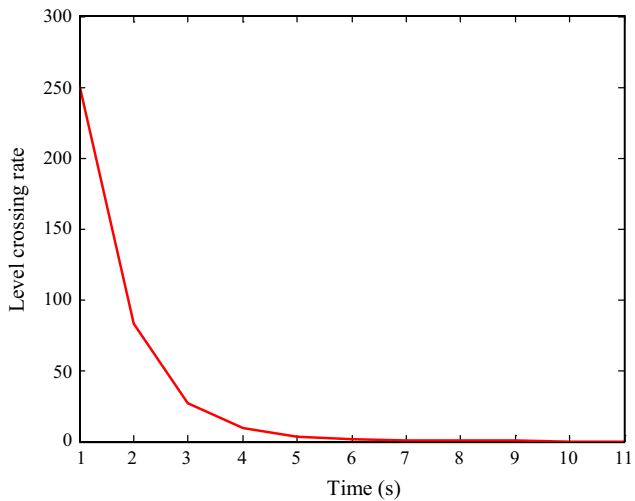Fig. 8. Cumulative transition probabilities of Tauchen-FSMC channel for $a = 0.8$.
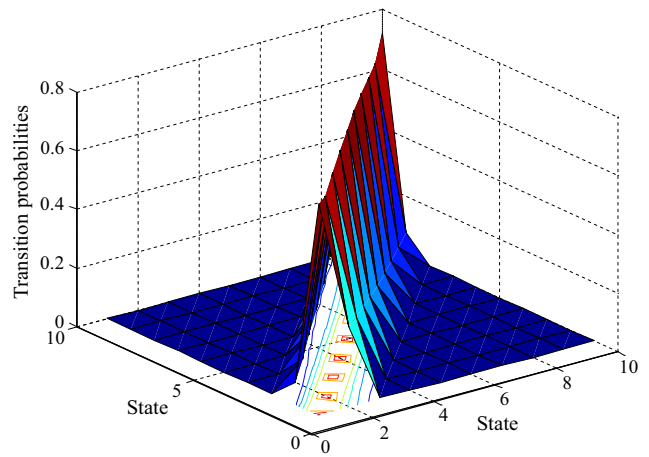

Fig. 6. LCR trend of LCR-FSMC channel.

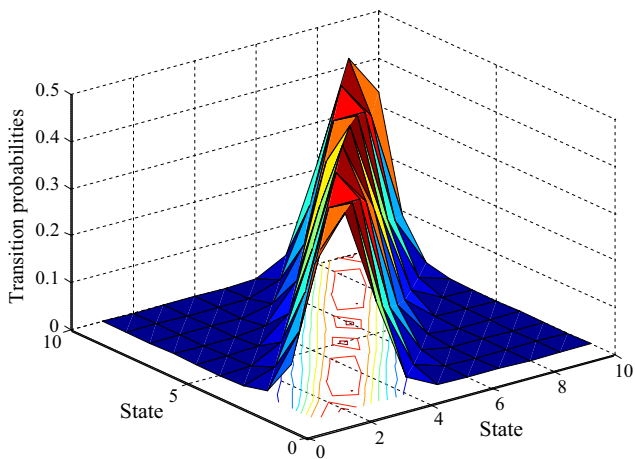Fig. 9. Transition probabilities of Tauchen-FSMC channel with $a = 0.95$.

more correlation governing remote jumps into past or future channel states.

Another envelope parameter value of $a = 0.95$ is examined to consolidate the above important finding, and the generated results are illustrated in Figs. 9 and 10. The transition probabilities and their cumulative trends show more resemblance to those obtained for the LCR method. Such statistical behavior reflects on the flexibility of the Tauchen model in determining the system autoregression order on the expressed statistical approximations. This is quite different than what was experienced [2], [3], [6] with respect to the static behavior of the LCR method, which hence can be considered as a merited feature counted for the Tauchen method.

It is worth mentioning that the selected values of the fading parameter a of 0.8 and 0.95 are in accordance with
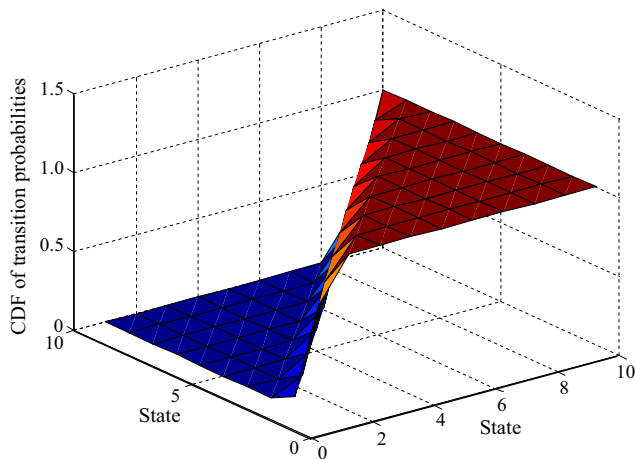

Fig. 7. Transition probabilities of Tauchen-FSMC channel for $a = 0.8$.

Fig. 10. Cumulative transition probabilities of Tauchen-FSMC channel with $a = 0.95$.

the ITU recommendations of expected channel coherence between 50% and 90% in reality [12].

VIII. Conclusions

Insightful guidelines for the essential requirements to build wireless fading channel simulators are provided. Two conceptual fading structures are discussed and investigated, namely, the AR and the FSMC models. The AR(1) models are examined and compared against the common simulation methods based on the JC channel design. The AR(1) models lead to results that could be acceptable to some extent, the decisive resolution of which is highly determined by the sensitivity and accuracy levels of the applications at hand. The higher the order of the AR models, the better statistical characteristics and time and frequency performances obtained for the channels being examined.

On the other hand, FSMC channel modeling is also investigated using the LCR and Tauchen approximation schemes. The latter scheme is proposed in this paper for the first time to deal with wireless fading channel applications. The LCR provided reasonable results, however, its static nature and insensitivity against the underlying channel autoregression model presents a limitation on the scheme that was assumed as prevailing in the literature for a considerable time. The remedy for such difficulty, and the retaining of the accessibility of the autoregression's direct influence on the statistical probability analytics of the FSMC, can be readily materialized, thanks to the flexibility gained by having the Tauchen modeling rendering such tasks easier than before.

The given simulation exercises confirmed the instrumental viability of the above claim, which could be considered as a cornerstone for the Tauchen-FSMC fading channel approximation paving the road for further future exploration. Whatever the FSMC model, a larger number of states produces better statistical performance at the expense of extra computations. Such computational demands were also quantified and assessed in this paper. In addition to the competent accuracy achieved by Tauchen-FSMC channel model, the complexity figures showed that this approximation methodology has favorably less computational overhead compared with the conventional LCR-FSMC approach.

Further challenges foreseen for the Tauchen FSMC approximation in future endeavors may constitute the calculation of performance probabilities in wireless communications, and channel state estimation involving ACM systems. Moreover, exploiting such a novel FSMC paradigm for spectrum access and management in CR systems might also be appealing for future attempts.

References

[1] P. Sadeghi et al., "Finite-State Markov Modeling of Fading Channels: A Survey of Principles and Applications," *IEEE Signal Process. mag.*, vol. 25, no. 5, Sept. 2008, pp. 57–80.

[2] H.S. Wang and N. Moayeri, "Finite-State Markov Channel-A Useful Model for Radio Communication Channels," *IEEE Trans. Veh. Technol.*, vol. 44, no. 1, Feb. 1995, pp. 163–171.

[3] Q. Zhang and S.A. Kassam, "Finite-State Markov Model for Rayleigh Fading Channels," *IEEE Trans. Commun.*, vol. 47, no. 11, Nov. 1999, pp. 1688–1692.

[4] F. Babich, O.E. Kelly, and G. Lombardi, "Generalised FSMC Model for Radio Channels with Correlated Fading," *IEEE Proc. Commun.*, vol. 48, no. 4, Apr. 2000, pp. 547–551.

[5] C.C. Tan and N.C. Beaulieu, "On First-Order Markov Modeling for the Rayleigh Fading Channel," *IEEE Trans. Commun.*, vol. 48, no. 12, Dec. 2000, pp. 2032–2040.

[6] J.M. Park and G.U. Hwang, "Mathematical Modeling of Rayleigh Fading Channels Based on Finite State Markov Chains," *IEEE Commun. Lett.*, vol. 13, no. 10, Oct. 2009, pp. 764–766.

[7] D. Dalalah, L. Cheng, and G. Tonkay, "Modeling End-to-End Wireless Lossy Channels: A Finite-State Markov Approach," *IEEE Trans. Wireless Commun.*, vol. 7, no. 4, Apr. 2008, pp. 1236–1243.

[8] S. Fazeli-Dehkordy et al., "Markovian-Based Framework for Cooperative Channel Selection in Cognitive Radio Networks," *IET Commun.*, vol. 8, no. 14, 2014, pp. 2458–2468.

[9] R. Zhang et al., "Channel Measurement and Packet-Level Modeling for V2I Spatial Multiplexing Uplink using Massive MIMO," *IEEE Trans. Veh. Technol.*, vol. 65, no. 10, Oct. 2016, pp. 7831–7843.

[10] S. Lin et al., "Finite-State Markov Modeling for High-Speed Railway Fading Channels," *IEEE Anten. Wireless Propag. Lett.*, vol. 14, 2015, pp. 954–957.

[11] R. Zhang and L. Cai, "A Packet-Level Model for UWB Channel with People Shadowing Process Based on Angular Spectrum Analysis," *IEEE Trans. Wireless Commun.*, vol. 8, no. 8, Aug. 2009, pp. 4048–4055.

[12] Recommendation ITU-R P.1407-5, *Multipath Propagation and Parameterization of its Characteristics*, 09/2013.

[13] A. Goldsmith, *Wireless Communications*, Cambridge, UK: Cambridge University Press, 2005.

[14] K.E. Baddour and N.C. Beaulieu, "Autoregressive Modeling for Fading Channel Simulation," *IEEE Trans. Wireless Commun.*, vol. 4, no. 4, July 2005, pp. 1650–1662.

[15] H. Mehrpouyan and S.D. Blostein, "ARMA Synthesis of Fading Channels," *IEEE Trans. Wirel. Commun.*, vol. 7, no. 8, Aug. 2008, pp. 2846–2850.

[16] G. Tauchen, "Finite State Markov-Chain Approximations to Univariate and Vector Autoregressions," *Econ. Lett.*, vol. 20, no. 2, 1986, pp. 177–181.

[17] K.A. Kopecky and R.M.H. Suen, "Finite State Markov-Chain Approximations to Highly Persistent Processes," *Rev. Econ. Dynamics*, vol. 13, no. 3, July 2010, pp. 701–714.

[18] L.R. Rabiner, "A Tutorial on Hidden Markov Models and Selected Applications in Speech Recognition," *Proc. IEEE*, vol. 77, no. 2, Feb. 1989, pp. 257–286.

[19] W. Khreich et al., "On the Memory Complexity of the Forward–Backward Algorithm," *Pattern Recogn. Lett.*, vol. 31, no. 2, Jan. 2010, pp. 91–99.

[20] F.W.J. Olver et al., *NIST Handbook of Mathematical Functions*, Cambridge, UK; New York, USA: Cambridge University Press; NIST, 2010.

Ahmed Abdul Salam (IEEE M'00–SM'13) received the B.Sc. and M.Sc. degrees both in electrical engineering, and is currently pursuing his Ph.D. in wireless communications at the University of Bradford, UK. His current research interests include signal processing for adaptive coded modulation, automatic modulation classification, and spectrum sensing techniques with particular emphasis on cognitive radio applications.

Ray Sheriff is Professor of Electronic Engineering at University of Bradford, UK. He received the B.Eng. degree in electrical and electronic engineering from the University of Leeds, UK, in 1986. He then worked in the satellite communications industry before taking up a lectureship in 1991 at the University of Bradford, UK, where he received his Ph.D. in 1995 and MBA in 2010. His research interests are in the fields of mobile and satellite communication networks.

Saleh Al-Araji (IEEE M'96–SM'98) was a professor and department head at Khalifa University, UAE. Prior to that he was senior staff engineer with Scientific-Atlanta (now Cisco Systems, Inc.), Atlanta, Georgia, USA. During 1995/1996, he was a visiting professor at The Ohio State University, Columbus, Ohio, USA. He was a visiting professor at King's College, University of London, UK, during the summers of 1988 and 1989. He was professor and department head at the University of Baghdad, Iraq, and the University of Yarmouk, Jordan. He has published extensively and holds nine US patents. He received the B.Sc., M.Sc., and Ph.D. degrees from Swansea University, UK, all in electrical engineering.

Kahtan Mezher (IEEE M'99–SM'01) received the B.Sc. degree in electronic engineering in Iraq in 1982 and his M.Sc and Ph.D. degrees in electronic engineering from Bradford University, UK, in 1987 and 1992. He worked as postdoctoral RA on GAAS devices (1992–1994) and was a lecturer at Bradford University (1995–1999). He has worked at Khalifa University (UAE) since 1999 and currently is an associate professor in the ECE department. His research interests are in analogue electronics, analogue filters, CAD, and CR. He has published 44 international journal and conference papers. He is a member of IET and is a chartered engineer and member of the IEEE circuits and systems and electron devices societies.

Qassim Nasir (IEEE M'95) has been an associate professor and department chairman at the University Of Sharjah since 2016. Dr. Nasir received his B.Sc., M.Sc., and Ph.D. degrees from the University of Baghdad, Iraq, in 1977, 1984, and 1994, respectively. Prior to joining the University of Sharjah, UAE, Dr. Nasir worked with Nortel Networks, Canada, as a senior system designer for six years. He was an adjunct part-time assistant professor at Ottawa University, teaching telecommunication software engineering. Dr. Nasir has contributed to research in DPLL, telecommunication and network security, energy aware routing protocols, and haptic data transmission protocols.