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Practical approach to predict the shear strength of fibre-reinforced clay

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Abstract

1   Carpet waste fibres have a higher volume to weight ratios and once discarded into landfills, these fibres occupy a

2   larger volume than other materials of similar weight. This research evaluates the efficiency of two types of carpet

3   waste fibre as sustainable soil reinforcing materials to improve the shear strength of clay. A series of consolidated

4   undrained (CU) triaxial compression tests were carried out to study the shear strength of reinforced clays with 1%,

5   to 5% carpet waste fibres. The results indicated that carpet waste fibres improve the effective shear stress ratio and

6   deviator stress of the host soil significantly. Addition of 1%, 3% and 5% carpet fibres could improve the effective

7   stress ratio of the unreinforced soil by 17.6%, 53.5% and 70.6%, respectively at an initial effective consolidation

8   stress of 200 kPa. In this study, a nonlinear regression model was developed based on a modified form of the

9   hyperbolic model to predict the relationship between effective shear stress ratio, deviator stress and axial strain of

10   fibre-reinforced soil samples with various fibre contents when subjected to various initial effective consolidation

11   stresses. The proposed model was validated using the published experimental data, with predictions using this model

12   found to be in excellent agreement.

13

14   **Key words**: Geosynthetics, Shear strength, Carpet waste fibre, Reinforced soil, Clays, Modified hyperbolic model

Notations:

Basic SI units are shown in parentheses.

15	q_{ratio}	deviator stress ratio (dimensionless)
16	p'_{ratio}	average mean effective stress ratio (dimensionless)
17	q	deviator stress (Pa)
18	p'	mean effective stress (Pa)
19	q_{re}	deviator stress of fibre-reinforced soil (Pa)
20	q_{un}	deviator stress of unreinforced soil (Pa)
21	$(\frac{q}{p'})_{re}$	effective stress ratio of fibre-reinforced soil (dimensionless)
22	$(\frac{q}{p'})_{un}$	effective stress ratio of unreinforced soil (dimensionless)
23	p'_{re}	mean effective stress of fibre-reinforced soil (Pa)
24	p'_{un}	mean effective stress of unreinforced soil (Pa)
25	σ'_1	major principal effective stress (Pa)
26	σ'_3	minor principal effective stress (Pa)
27	z, w	coefficients of the linear relationship between q_{ratio} and p'_{ratio} (dimensionless)
28	ε	axial strain in triaxial shear test (dimensionless)
29	a,b,n, g, h, t	modified hyperbolic model parameters (dimensionless)
30	a*,b*,n*, g*, h*, t*	normalised modified hyperbolic model parameters (dimensionless)
31	σ'_c	initial effective consolidation stress (Pa)
32	f	fibre content (dimensionless)
33	$k_{1a,g}, k_{2a,g}, k_{1b,h}, k_{2b,h}, k_{1n,t}, k_{2n,t}$	coefficients of the linear equations for normalised model parameters
34	(dimensionlesss)	

1. Introduction

35 The carpet manufacturing industry produces a large quantity of fibre wastes due to aspects such as the end of line
36 leftovers, stop-start wastage, yarn breakages, faults and quality control. In addition, as the demand for new carpets
37 increases, substantial amounts of old and post-consumer carpet wastes have to be disposed to landfills. Synthetic
38 fibres used in carpet manufacturing are particularly problematic as they do not degrade with time and once dumped
39 into landfills, may release colour pigments in the surrounding soil. This may pollute the underground water
40 reservoirs, which are potential sources for domestic use. Local communities, businesses and governmental agencies
41 are increasingly encouraged to re-use and recycle carpet waste fibres, so as to minimise the need for landfilling.
42 Virgin discrete synthetic/natural fibres have been used in geotechnical engineering practice to improve the stress-
43 strain response of fine and coarse-grained soils. Upon mixing with soil particles, short discrete fibres behave like
44 tension resisting elements interlocking soil particles to partially withstand shear stresses within the soil. This results
45 in the formation of a composite coherent soil matrix that possesses superior strength properties with reduced and/or
46 eliminated continuous planes of weakness at failure (Ahmad et al., 2010). The reinforcing role of short virgin
47 synthetic fibres in coarse-grained soils with loose to dense states has been studied previously (Maher and Gray 1990;
48 Nataraj 1997; Consoli et al. 2007; Diambra et al. 2007, 2010; Chen and Loehr 2009; Hamidi and Hooresfand 2013;
49 Miranda Pino et al. 2015; and Jamsawang et al. 2015). The behaviour of fibre-reinforced fine-grained soils has also
50 been studied previously (Falorca et al. 2006; Casagrande et al. 2006; Özkul and Baykal 2007; Al-Akhras et al. 2010;
51 Babu and Chouksey 2010; Ekinci and Ferreira 2012; Maliakal and Thiyyakkandi 2013; Sadeghi and Beigi 2014 and
52 Botero et al. 2015; Khatri et al. 2015; and Anggraini et al. 2016). The reported research findings are in agreement in
53 that fibre reinforcement improves the stress-strain behaviour, unconfined compression strength, shear strength and
54 ductility of the soil and reduces the consolidation settlement of clay. Currently, the research theme within the area of
55 soil fibre reinforcement is strongly favoured towards the use of virgin short fibres, mostly with granular soils. There
56 is presently a research gap on the utilisation of recycled carpet waste fibres for the reinforcement of clays, with only
57 limited current publications on this aspect (Murray et al. 2000, Ghiassian et al. 2004, Fatahi et al. 2012, 2013 a & b
58 and Mirzababaei et al. 2013 a & b, 2017 b).

59 Murray et al. (2000) conducted a consolidated undrained compression triaxial testing program to investigate the
60 shear strength of a reinforced sandy silt soil with nylon waste fibres and virgin fibrillated polypropylene fibres. The
61 results indicated that adding up to 3% waste nylon fibre or a maximum of 1% polypropylene virgin fibre can
62 significantly increase the peak shear strength of the soil and can reduce its post-peak reduction in a ductile manner.
63 Based on a series of drained compression triaxial tests on sand samples reinforced with carpet waste strips,
64 Ghiassian et al. (2004) reported a satisfactory degree of improvement in the peak shear strength of sand that could
65 be achieved by either increasing the strip content at a constant aspect ratio or by increasing the aspect ratio at a
66 constant strip content.

67 Recently, Fatahi et al. (2012, 2013 b) reported the improving effect of carpet waste fibre addition to cement
68 stabilised soft kaolin and stated that the fibre type and content could significantly influence the peak shear strength,

69 stiffness and brittleness of the soft clays. The initial Young's modulus of the cement-stabilised clay increased with

70 the addition of polypropylene fibres, but reduced with the addition of carpet and steel fibres.

71 Mirzababaei et al. (2013 a and b) carried out an extensive testing programme to investigate the unconfined

72 compression strength and swelling pressure of expansive clays reinforced with carpet waste fibres and concluded

73 that inclusion of up to 5% carpet waste fibres could result in reduction of the swelling pressure and increasing the

74 unconfined compression strength of the clay. However, the relative gain in unconfined compression strength and

75 reduction in swelling pressure is highly dependent on the initial dry unit weight and the moisture content of the clay.

76 Mirzababaei et al. (2017 b) also reported significant enhancement in the bearing capacity of a model footing on a

77 clay slope reinforced with carpet waste fibre. In this study, the stress-strain behaviour of a clay reinforced with two

78 types of carpet waste fibre was analysed by conducting a number of consolidated undrained triaxial compression

79 tests. Thus the experimental programme in this research seeks to evaluate the feasibility to sustainably reuse carpet

80 waste fibres with non-uniform lengths and thicknesses to enhance the shear strength of clay.

81 Several analytical models are available to estimate the stress-strain behaviour of fibre-reinforced granular soils

82 based on the force-equilibrium approach (Waldron 1977; Gray and Ohashi 1983; Maher and Gray 1990; Ranjan et

83 al. 1996; Michalowski 2008; Shukla et al. 2010), energy-based model (Michalowski and Zhao 1996) and discrete

84 framework model (Zornberg 2002). There are also several statistical models to predict the shear strength of fibre-

85 reinforced granular soils (Ranjan et al 1996; Babu and Vasudevan 2008) with input parameters including some or

86 any of the following: fibre content, fibre aspect ratio, fibre surface friction, confining pressure, fibre length, cohesion

87 and internal friction angle of the fibre-reinforced soil. Cam Clay constitutive model has also been reformulated to

88 account for the estimation of the deviator stress of fibre-reinforced clays (Chen 2007; Babu and Chouksey 2010;

89 Diambra and Ibraim 2014 and Nguyen and Fatahi 2016).

90 Although all proposed analytical and constitutive models agree well with the experimental data, the parameters

91 required to execute these models are complex and require a number of triaxial, direct shear and consolidation tests.

92 Therefore, due to the variability of soil types, fibre types and the complex interfacial behaviour of the soil at the

93 interface with fibres, it is required to estimate the shear strength properties of the fibre-reinforced soil with less

94 effort, so as to obtain the soil/fibre input parameters to ease its application in practice. In this study, a simple

95 regression model is developed to predict the relationship between the effective shear strength ratio (q/p'), deviator

96 stress (q) and the axial strain of a fibre-reinforced clay subjected to axial deviator stress at any fibre content and

97 confining pressure regardless the fibre type and clay type.

2. Materials

98   The soil sample used in this study was collected from a site in the Northwest region of the United Kingdom. The soil

99   was classified as low plastic clay according to the Unified Soil Classification System (USCS) with a plasticity index

100 of 17% and specific gravity of 2.68. The grain size analysis of the soil indicated that it contained 55.78% fine grains

101 and 44.22% coarse grains. **Table 1** shows the properties of the soil. Two different types of carpet waste fibre (i.e.,

102 herein called GBF and ABF) were utilised in this investigation and supplied by Carpet Recycling UK and Milliken

103 Carpet Europe as waste by-products from the production line (i.e. from shearing and/or edge trimming of the

104   carpet). The GBF consisted predominantly of propylene fibres whereas the ABF included short nylon fibres. **Table**

105   **2** presents typical specifications of the carpet waste fibres used in this study. Due to the waste origin of the fibres

106   used in this study, the length of the fibres ranged from 2 to 20 mm with diverse thicknesses from 80 μm to 1,500

107   μm.

3. Experimental Programme

108   In this study, because carpet fibre is classified as waste material, it was decided to investigate the addition of a high

109   proportion of waste fibres on the shear strength of the clay. Therefore, fibre contents of 1%, 3%, and 5% of the dry

110   mass of soil were selected to reinforce the clay. Because of reduced workability and practical difficulties to mix

111   higher fibre contents evenly with the soil, the maximum fibre content used in this study was limited to 5%. Standard

112   Proctor compaction tests were carried out on the unreinforced and fibre-reinforced clay. The compaction test results

113   revealed that the maximum dry unit weight of the soil decreases with fibre inclusion. **Figure 1** presents the

114   compaction curves of the unreinforced and carpet waste fibre-reinforced soil with both fibre types. To achieve a

115   uniform distribution of fibres in the clay sample, various approaches were undertaken including mixing water

116   soaked fibres with dry clay, addition of dry fibres to the wet clay followed by mixing and mixing dry clay and fibres

117   followed by spraying water on the mixture. It was concluded that spraying water on a dry mixture of fibre and clay

118   to achieve the desired moisture content results in producing relatively uniform samples with less fibre tangling.

119   However, other examined methods led to the formation of fibre lumps or balling during preparation that could affect

120   the homogeneity of the sample. Therefore, the dry soil and fibres were mixed thoroughly in a sealed container by

121   shaking. Subsequently, the calculated amount of water to reach the target moisture content was sprayed over the dry

122   mixture of soil and fibre in several stages, followed by hand mixing to prepare a uniform mixture. The prepared

123   mixture was kept in a sealed container for 3 to 4 days before the test day. This procedure, could possibly increase the

124   mixing efficiency and therefore, ensure a relative uniformity of fibre distribution in the fibre-reinforced soil sample.

125   A series of consolidated undrained triaxial tests were carried out on unreinforced and fibre-reinforced clay samples

126   following BS 1377-8:1990 standard. Unreinforced and fibre-reinforced cylindrical clay samples with a diameter of

127   38 mm and height of 76 mm were prepared at the same dry unit weight of 17.8 kN/m^3 and their equivalent moisture

128   content on the dry side of the compaction curve (refer to **Figure 1**; see solid points). Soil samples were saturated

129   with a B Skempton ratio of at least 0.97 using steps of back pressure and cell pressure and subsequently

130   consolidated at initial effective consolidation stresses of 50 kPa, 100 kPa and 200 kPa, respectively to simulate the

131   medium to relatively high in-situ effective stress ranges and consequently sheared at axial strain rate of 0.13%/min.

132   There are a few studies investigating the effect of sample size on the unconfined compression strength (UCS) and

133   shear strength of fibre-reinforced clays. Xiao et al., (2014) experienced a significant increase in the UCS of 7-day

134   cured fibre-reinforced clay when the sample size increased from 50 mm to 100 mm. However, the change was minor

135   when the sample size was changed from 100 mm to 150 mm. They also reported insignificant size effect on UCS of

136   14-day and 28-day cured fibre-reinforced samples. They also investigated the consolidated undrained and

137   consolidated drained shear strength of fibre-reinforced clay and found that up to the consolidation stress of 110 kPa,

138   the sample size does not affect the deviator stress of the fibre-reinforced clay significantly.

139   Nataraj and McManis (1997) also investigated the mechanical behaviour of fibre-reinforced clay using UCS test,

140   direct shear test and CBR test. They reported an increase in UCS with the increase in sample size from 33 to 70 mm.

141   However, they found the strength of 100 mm fibre-reinforced sample was slightly lower than that for 70 mm

142   sample. Ang and Loehr (2003) also investigated the effect of sample size on the UCS of clays with diameter from 38

143   mm to 152 mm. They observed the highest UCS for 70 mm samples. They also concluded that sample size

144   influences the UCS of the clay compacted at dry side of the optimum value significantly. Therefore, although there

145   are no definite published results confirming the effect of sample size on the shear strength of clays in consolidated

146   triaxial tests, a sample diameter to fibre length ratio of 8 is desirable as recommended by Xiao et al. (2014). In this

147   study, due to the waste origin of the carpet fibres and their random size including fibre lengths from 2 mm to 20 mm

148   and fibre diameters from 80 μm to 1,500 μm, the above criteria may have been met partially as there was no control

149   on picking appropriate fibre lengths for sample preparation. On the other hand, smaller samples may not allow full

150   mobilisation of the reinforcing role of fibres in fibre-reinforced soil resulting in underestimating the shear strength

151   of fibre-reinforced clay (Ang and Loehr 2003).

4.   Results and Discussions

152   The contributing effect of fibre reinforcement on the shear strength behaviour of clay samples in terms of deviator

153   stress ratio-mean effective stress ratio (q_{ratio}/p'_{ratio}), effective stress ratio (q/p') and deviator stress (q) versus

154   axial strain was analysed and discussed in the following sections.

4.1.   Effective stress ratio and deviator stress of fibre-reinforced clay

155   The stress-strain relationships during the consolidated undrained triaxial test were normalised and plotted for

156   effective stress ratio (q/p') and deviator stress versus axial strain for both unreinforced and fibre-reinforced soil

157   samples at different initial effective consolidation stresses of 50 kPa, 100 kPa and 200 kPa as shown in **Figures 2**

158   and **3**. **Figure 2** shows that inclusion of GBF fibre enhanced the effective stress ratio and deviator stress of the host

159   soil significantly. Based on the graphs shown in **Figure 2a**, fibre-reinforced soil with 1%, 3% and 5% GBF fibre

160   content improved the effective stress ratio of the unreinforced soil at initial effective consolidation stress of 200 kPa

161   by 17.6%, 53.5% and 70.6%, respectively (i.e., measured at an axial strain of 20%). Although the effective stress

162   ratios of the fibre-reinforced soil with 3% and 5% GBF fibre contents were almost similar at all effective

163   consolidation stresses, their deviator stress behaviour was markedly different. This is mainly due to a simultaneous

164   increase in both deviator stress and mean effective stress with the increase in fibre content resulting in less growth in

165   effective stress ratio with the increase in fibre content. The ratio of deviator stress of 5% fibre-reinforced soil to that

166   of 3% fibre-reinforced soil at an axial strain of 20% measured for initial effective consolidation stress of 200 kPa

167   was determined to be 2.43 for GBF fibre-reinforced soil. The inclusion of 1%, 3% and 5% GBF fibre improved the

168   deviator stress of the soil by 4.0%, 35.3% and 229.4% for the test with initial effective consolidation stress of 200

169   kPa (i.e., measured at an axial strain of 20%).

170   ABF fibre-reinforced soils also demonstrated significant improvement in effective stress ratio and deviator stress

171   over those of unreinforced soil. At an axial strain of 20%, ABF fibre-reinforced soil samples subjected to initial

172   effective consolidation stress of 200 kPa, produced 20.0%, 32.4% and 37.5% effective stress ratio improvement with

173   inclusion of 1%, 3% and 5% fibre, respectively (see **Figure 3a**). The inclusion of 1%, 3% and 5% ABF fibre, also

174   improved the deviator stress of the soil by 16.7%, 38.4% and 129.3% at initial effective consolidation stress of 200

175   kPa (i.e., measured at an axial strain of 20%, **Figure 3b**). Fibre-reinforced soil samples with 3% fibre showed

176   slightly higher effective stress ratio at initial effective consolidation stress of 50 kPa than that of 5% fibre-reinforced

177   soil samples (See **Figures 2a and 3a**). However, with an increase in initial effective consolidation stress to 100 kPa

178   and 200 kPa, the observed order was reversed. This observation for the behaviour of fibre-reinforced soil is

179   consistent with results previously reported by Freilich and Zornberg (2010) as they observed an increase in

180   effectiveness of fibres on the shear strength of the soil with the increase in the effective confining pressure. At

181   higher initial effective consolidation stresses prior to the shearing stage, due to increase in confining pressure and

182   hence improved interaction between soil particles and fibres, the fibres were increasingly stretched. Therefore, upon

183   initiating the shear stage, the fibres contribute better to distributing the applied shear stresses into a wider area.

184   However, at lower initial effective consolidation stresses, the fibres may not interact effectively with soil grains and

185   hence may slip in shear. Hence at relatively low confining stresses, the fibres may not effectively confine soil grains

186   nor add surplus strength to the reinforced soil. Therefore, an increase in the fibre content at a low initial effective

187   consolidation stress may have an adverse effect on the strength of the reinforced soil without necessary soil grain

188   harnessing effect.

189   **Figures 2** and **3** also demonstrate that inclusion of fibres resulted in turning the plastic stress-strain behaviour of the

190   unreinforced soil to a strain hardening behaviour. The shear strength behaviour of a soil is partly dependent on its

191   initial moisture content and dry unit weight (Lamb and Whitman 1979). Reduction in the dry unit weight and

192   increase in the moisture content of a soil mass subjected to shear stresses results in a rapid shear failure (Newcomb

193   and Birgisson 1999). Although the moisture contents of the prepared soil samples were different and increased with

194   fibre content, the shear strength ratio of the fibre-reinforced soil was superior to that of the unreinforced soil. The

195   observed stress-strain response indicated that the contribution of the fibres to improve the shear strength of the soil

196   samples prepared at an identical dry unit weight and different moisture contents, compensated for the loss of shear

197   strength due to an increase in moisture content of the fibre-reinforced soil samples.

198   Repeatability and reliability of the shear strength test results of the fibre-reinforced soil are paramount for acquiring

199   an accurate mechanical behaviour, deeper understanding and successful modelling of the fibre-reinforced clay

200   behaviour. Although, a consistent and careful control was undertaken during the sample preparation phase to ensure

201   uniform distribution of fibres within the clay sample, to confirm the reliability of results three tests repeated on the

202   fibre-reinforced clay samples with 5% ABF fibre content at initial consolidation stresses of 50, 100 and 200 kPa,

203   respectively. **Figure 4** compares the original and the repeat test results and shows a good repeatability of the results.

204   In this study, the deviator stress ratio and mean effective stress ratio are defined as the ratio of the deviator and mean

205   effective stress of the fibre-reinforced soil to that of the unreinforced soil, respectively:

206   Deviator stress ratio $(q_{ratio}) = \frac{q_{re}}{q_{un}}$ (Eq.1)

207   Mean effective stress ratio $(p'_{ratio}) = \frac{p'_{re}}{p'_{un}}$ (Eq.2)

208   where; q is the deviator stress $(\sigma'_1 - \sigma'_3)$, p' is the mean effective stress $((1/3)(\sigma'_1 + 2\sigma'_3))$ and $\sigma'_{1,3}$ are the major and

7

209   minor principal effective stresses

210   Note: re: reinforced, un: unreinforced

211   **Figure 5** presents the relationships between q_{ratio} and p'_{ratio} of the fibre-reinforced soil samples. Illustrated data in

212   this figure are related to the data of q_{ratio}, and p'_{ratio} of the fibre-reinforced soil samples subjected to initial

213   effective consolidation stresses of 50 kPa, 100 kPa and 200 kPa and calculated at axial strains of 5%, 10%, 15% and

214   20%, respectively. **Figure 5** also shows that q_{ratio} changed rather linearly with p'_{ratio} at all fibre contents for both

215   fibre types and the deviator stress ratio and mean effective stress ratio of the fibre-reinforced soil increased with

216   fibre content. Assuming q_{ratio} a linear function of p'_{ratio}, we can write:

217    $q_{ratio} = z \times p'_{ratio} + w$ (Eq.3)

218   **Equation 3** shows that a linear relationship existed between the deviator stress ratio and mean effective stress ratio

219   of the fibre-reinforced soil. However, this equation did not specifically elaborate on the above relationship as a

220   function of axial strain during triaxial compression test, fibre content and initial consolidation stress of the soil

221   sample. Therefore, it is required to develop models to correlate the effective stress ratio and deviator stress of the

222   fibre-reinforced soil to the axial strain, fibre content and initial consolidation stress of the clay sample during the

223   triaxial compression test.

4.2. Predicting the effective stress ratio and deviator stress of fibre-reinforced soil

224   Duncan-Chang model (i.e., hyperbolic stress-strain theory, 1970) is an incremental nonlinear stress-dependent

225   model that explains the nonlinearity, stress-dependent and inelastic behavioural feature of cohesive and cohesionless

226   soils with a simple form. Horpibulsuk and Miura (2001) and Horpibulsuk and Rachan (2004) used a modified form

227   of the Duncan-Chang hyperbolic model to capture the undrained and drained behaviour of uncemented and cement

228   stabilised clays. The modified hyperbolic model can exhibit strain softening behaviour following the destruction of

229   the sample at peak and also the strain hardening behaviour. Therefore, in this study, the modified hyperbolic model

230   proposed by Horpibulsuk and Miura (2001) was used to characterise the nonlinear stress-strain behaviour of the

231   fibre-reinforced clay.

232   In order to consider the nonlinear behaviour (i.e., strain hardening/softening) of the effective stress ratio and deviator

233   stress of the fibre-reinforced clay, the variation of the effective stress ratio and deviator stress of the fibre-reinforced

234   soil with the axial strain in terms of modified hyperbolic relation takes the following forms:

235    $\frac{q}{p'} = \frac{\varepsilon}{a+b\varepsilon^n}$ a, b and n are the modified hyperbolic model parameters for effective stress ratio (Eq.4)

236    $q = \frac{\varepsilon}{g+h\varepsilon^t}$ g, h and t are the modified hyperbolic model parameters for deviator stress   (Eq.5)

237   The parametric study of the influence of model parameters on the evolution of effective stress ratio ($\frac{q}{p'}$) and deviator

238   stress (q) are shown in **Figures 6** and **7**, respectively. **Figure 6** shows that *a* controlled the value of the initial

239   tangent to the $q/p' - \varepsilon$ behaviour while *b* accounted for the residual value and *n* was responsible for the slope of the

240   post peak drop of the $q/p' - \varepsilon$ curve.

8

241 The effective stress ratio and deviator stress data of the consolidated undrained triaxial tests on both unreinforced
242 and fibre-reinforced soil samples were used to determine the coefficients of the modified hyperbolic model using a
243 nonlinear regression analysis. **Figures 8** and **9** show the calculated coefficients for both unreinforced and fibre-
244 reinforced soil samples tested at different initial effective consolidation stresses. The calculated hyperbolic model
245 parameters for the stress-strain behaviour of the fibre-reinforced samples with different fibre contents and subjected
246 to different initial effective consolidation stresses showed a nonlinear trend for both ABF and GBF fibres.
247 Therefore, to develop a set of meaningful correlations between calculated coefficients and the fibre content, the
248 parameters were further normalised to the values of initial effective consolidation stress and fibre content. The
249 following equations show the relationships between the normalised coefficients and the initially non-normalised
250 coefficients used in the modified hyperbolic model. **Figures 10** and **11** depict the relationships between the
251 normalised parameters and the fibre content.

252 $a^* = \log(\frac{a}{100\sigma_c'})(1 + 100f) = k_{1a} \times f + k_{2a}$ (Eq.6)

253 $b^* = \log\left(\frac{b}{100}\right)(1 + 100f) = k_{1b} \times f + k_{2b}$ (Eq.7)

254 $n^* = Exp(-n)(1 + 100f) = k_{1n} \times f + k_{2n}$ (Eq.8)

255 $g^* = \log(\frac{\sigma_c'g^2}{100})(1 + 100f) = k_{1g} \times f + k_{2g}$ (Eq.9)

256 $h^* = \log\left(\frac{\sigma_c'h^2}{100}\right)(1 + 100f) = k_{1h} \times f + k_{2h}$ (Eq.10)

257 $t^* = Log(100t)(1 + 100f) = k_{1t} \times f + k_{2t}$ (Eq.11)

258 where; σ_c' stands for the initial effective consolidation stress, f stands for the fibre content and
259 $k_{1a,g}, k_{2a,g}, k_{1b,h}, k_{2b,h}, k_{1n,t}, k_{2n,t}$ are the coefficients of the linear equations (see **Figures 10 and 11**).

260 Therefore, the original coefficients of the modified hyperbolic model can be expressed based on the fibre content,
261 initial effective consolidation stress and the linear equations presented in **Figures 10** and **11** using **equations 12** to
262 **17**.

263 $a = \sigma_c'10^{(2+\frac{k_{1a} \times f + k_{2a}}{1+100f})}$ (Eq.12)

264 $b = 10^{(2+\frac{k_{1b} \times f + k_{2b}}{1+100f})}$ (Eq.13)

265 $n = -\ln(\frac{k_{1n} \times f + k_{2n}}{1+100f})$ (Eq.14)

266 $g = \sqrt{\frac{1}{\sigma_c'}10^{(2+\frac{k_{1g} \times f + k_{2g}}{1+100f})}}$ (Eq.15)

267 $h = \sqrt{\frac{1}{\sigma_c'}10^{(2+\frac{k_{1h} \times f + k_{2h}}{1+100f})}}$ (Eq.16)

268 $t = 10^{(-2+\frac{k_{1t} \times f + k_{2t}}{1+100f})}$ (Eq.17)

269 Combining **equations 4** and **5** with **equations 12** to **17** results in the following equations for predicting the effective
270 shear stress ratio and deviator stress of the fibre-reinforced clay:

9

271 $$\left(\frac{q}{p'}\right)_{re} = \frac{0.01 \times \varepsilon}{\sigma'_C \times 10^{\frac{k_{1a} \times f + k_{2a}}{1+100f}} + 10^{\left(\frac{k_{1b} \times f + k_{2b}}{1+100f}\right)} \varepsilon^{-\ln\left(\frac{k_{1n} \times f + k_{2n}}{1+100f}\right)}}$$ (Eq.18)

272 $$q_{re} = \frac{0.01 \times \varepsilon}{\sqrt{\frac{1}{\sigma'_C}10^{\frac{k_{1g} \times f + k_{2g}}{1+100f}} + \varepsilon 10^{\left(-2+\frac{k_{1t} \times f + k_{2t}}{1+100f}\right)} \sqrt{\frac{1}{\sigma'_C}10^{\frac{k_{1h} \times f + k_{2h}}{1+100f}}}}}$$ (Eq.19)

273 **Equations 18** and **19** can also be rewritten for unreinforced soil (i.e., $f = 0$):

274 $$\left(\frac{q}{p'}\right)_{un} = \frac{0.01 \times \varepsilon}{\sigma'_C \times 10^{k_{2a}} + 10^{k_{2b}} \times \varepsilon^{-\ln(k_{2n})}}$$ (Eq.20)

275 $$q_{re} = \frac{0.01 \times \varepsilon}{\sqrt{\frac{1}{\sigma'_C}10^{k_{2g}} + \varepsilon 10^{(k_{2t}-2)} \sqrt{\frac{1}{\sigma'_C}10^{k_{2h}}}}}$$ (Eq.21)

276 **Figure12** compares the predicted data using the developed regression model for the relationships between effective
277 stress ratio and axial strain of the unreinforced and fibre-reinforced soil samples studied in this research.

4.3. Proposed method to predict the stress ratio of fibre-reinforced clays

278 In this study, the modified hyperbolic model was used to predict the effective stress ratio and deviator stress of the
279 fibre-reinforced clay for practical application. The model parameters can be calculated using the following steps:

1) Use **equations 4** and **5** to fit the test data of unreinforced and fibre-reinforced soil samples using nonlinear regression analysis and determine a, b and n or g, h and t.

2) Use **equations 6** to **11** to calculate the normalised parameters a*, b* and n* or g*, h* and t*.

3) Draw the best fit line through the graphs of normalised parameters (calculated in step 2) versus fibre content to calculate $k_{1a,g}$, $k_{2a,g}$, $k_{1b,h}$, $k_{2b,h}$, $k_{1n,t}$ and $k_{2n,t}$.

4) Use **equations 18** and **19** to predict the effective stress ratio and deviator stress of the fibre-reinforced clay, respectively.

287 The model calibration phase as explained above aims to capture the inherent properties of the soil and fibres into the
288 equation in form of model parameters. Therefore, once the model is calibrated based on a particular clay type and a
289 particular fibre type, it can predict the effective shear stress ratio and deviator stress of the clay at different axial
290 strains during the triaxial compression test. The model parameters can be adequately estimated using a minimum of
291 2 tests on an unreinforced soil (at two different initial effective consolidation stresses) and two tests on fibre-
292 reinforced clay (with a fibre content at two different initial effective consolidation stresses). A medium range fibre
293 content is recommended for calibration stage. To increase the accuracy of the model it is recommended to use the
294 triaxial test data of three unreinforced soil samples tested at different effective consolidation stresses.

4.4. Validation of the model

295 The proposed model was validated by comparing its predictions with some of the published experimental data
296 available in the literature. The stress-strain data of the chosen research works were extracted by digitising the
297 reported graphs.

10

298   Wu et al. (2014) carried out a series of undrained triaxial tests on fibre-reinforced silty clay samples with 0.5%, 1%

299   and 1.5% sisal fibres. They reported a gradual increase in deviator stress of fibre-reinforced soil with an increase in

300   fibre content. However, the rate of deviator stress improvement was declined beyond 1% fibre content. **Figure 13**

301   shows the predicted data using the model introduced in this study. The model parameters were calculated using

302   experimental data of the unreinforced and 1% fibre-reinforced samples tested at consolidation stresses of 100 kPa

303   and 400 kPa, respectively. The developed model in this study could predict the experimental results at 200 kPa

304   consolidation stress for 0.5%, 1% and 1.5% fibre-reinforced clay with 0.5% to 5% underestimation.

305   Babu and Chouksey (2010) also carried out a series of undrained tests on randomly distributed fibre-reinforced clay

306   samples with 0.5%, 1% and 2% coir fibres. The results indicated that the fibre-reinforced soil gained a higher

307   strength than unreinforced soil with the increase in fibre content to the studied limit of 2% fibre content. They also

308   developed an analytical model based on Cam Clay model to predict the shear strength of fibre-reinforced soil.

309   Although their model fitted well with the experimental results, they did not validate their model with published data.

310   **Figure 14** shows the experimental results reported in their article and the predicted data using the proposed model in

311   this study. The model parameters were acquired from triaxial test data carried out on unreinforced and 1% fibre-

312   reinforced soil at consolidation stresses of 50 and 150 kPa, respectively. According to **Figure 14**, the ratios of the

313   effective stress ratio of 0.5%, 1% and 2% coir fibre-reinforced clay predicted by the developed model in this study

314   to that of the experimental results at consolidations stress of 100 kPa at 12% axial strain were 1.03, 0.99 and 0.95,

315   respectively.

316   Ozkul and Baykal (2007) reported an increase in drained and undrained deviator stress of saturated silty clay

317   reinforced with 10% short tyre buffing fibres. They also determined a limiting confining pressure of 200-300 kPa

318   beyond which the presence of rubber fibres reduced the shear strength of the soil. The undrained experimental data

319   reported by Ozkul and Baykal (2007) was used to validate the model developed in this study. Therefore, the model

320   parameters were calculated using the test data of unreinforced soil at consolidation stresses of 100 kPa and 300 kPa

321   in addition to the single test data of fibre-reinforced soil at consolidation stress of 300 kPa. **Figure 15** shows the

322   experimental data and predicted deviator stress curves. The ratios of the deviator stresses of 10% rubber fibre-

323   reinforced clay predicted by the developed model in this study to that of the experimental results at consolidation

324   stresses of 100 kPa, 200 kPa and 300 kPa at 12% axial strain were 1.04, 0.97 and 0.99, respectively.

325   In another study, Nguyen and Fatahi (2016) reported the mechanical behaviour of fibre-reinforced cemented clay

326   with polypropylene fibres using a series of consolidated undrained triaxial tests. They also developed a constitutive

327   model called C3F based on Modified Cam Clay (MCC) model to predict the shear strength of fibre-reinforced clays

328   with and without cementation. Their model could predict the shear strength of the fibre-reinforced cemented clay in

329   a good agreement with the experimental results. **Figure 16** shows the undrained experimental results on fibre-

330   reinforced clay samples stabilised with 15% cemented, predicted results using C3F model and also the predicted test

331   data using the method introduced in this study. The model parameters were acquired using data of unreinforced and

332   0.5% fibre-reinforced soil tested at effective consolidation stresses of 400 kPa and 800 kPa, respectively. According

333   to **Figure 16**, at consolidation stress of 200 kPa and 12% axial strain the developed model in this study

334 overestimated the deviator stress of 0.3% and 0.5% fibre-reinforced soil by 4% and 15% respectively and the C3F

335 model underestimated both by 4%. However, at 800 kPa consolidation stress and 12% axial strain, the developed

336 model in this study and C3F model overestimated the deviator stress of 0.3% and 0.5% fibre-reinforced soil both by

337 the same values of 16% and 10%, respectively.

338 Khatri et al. (2016) also carried out a series of undrained triaxial tests on fibre-reinforced clay with 0.4% to 1.6%

339 coir fibre. They reported that the shear strength of fibre-reinforced clay was improved with the increase in fibre

340 content and developed a successful hyperbolic model to predict their own data. However, they did not correlate the

341 model parameters with fibre content or consolidation stress and their model was not validated against available data

342 in the literature. **Figure 17** compares the experimental data and those predicted by the developed model in this

343 study. The model parameters were calculated using experimental data of unreinforced and 0.8% fibre-reinforced soil

344 at consolidation stresses of 77.48 kPa and 313.92 kPa, respectively. The developed model in this study

345 underestimated the effective stress ratio of the 0.4%, 0.8% and 1.6% fibre-reinforced soil at consolidation stress of

346 77.48 kPa and 12% axial strain by 10%, 10% and 3%, respectively. However, at consolidation stress of 313.92 kPa

347 and 12% axial strain, the effective stress ratio of 0.4%, 0.8% and 1.6% fibre-reinforced soil was overestimated by

348 18%, 1% and 5%, respectively.

349 **Table 3** demonstrates a comparison of experimental and predicted results of above-studied research works. The

350 developed nonlinear regression model based on the modified hyperbolic model and the proposed technique to

351 acquire the model parameters in this study could capture the experimental stress-strain results reasonably well

352 considering the less effort to calculate the model parameters compared to using constitutive models.

353 The **Supplemental Material** for this article explains the technique to calculate the model parameters for the

354 modified hyperbolic model based on the triaxial test results reported by **Wu et al. (2014)**.

5. Conclusions

355 In this research, the shear strength behaviour of a clay reinforced with two types of carpet waste fibre was

356 investigated by conducting a set of consolidated undrained triaxial shear tests on fibre-reinforced samples with 1%,

357 3% and 5% fibre contents. The fibre-reinforced soils exhibited significant variation in the Proctor compaction curve

358 to that of the unreinforced soil. Therefore, to eliminate the effect of initial unit weight on the stress-strain results, all

359 samples were prepared at an identical dry unit weight (i.e., 17.8 kN/m^3) and their respective moisture contents based

360 on the standard Proctor compaction curves for the unreinforced and fibre-reinforced soils.

361 Fibre reinforcement using waste carpet fibres improved the stress-strain behaviour of the clay significantly and the

362 contribution of the fibres to increase the effective stress ratio and deviator stress of the soil increased with fibre

363 content. At higher initial effective consolidation stresses prior to the shearing stage, fibres stretch increasingly and

364 therefore, contribute better to distribute the disturbing shear stresses into a wider area upon initiating the shear stage.

365 However, at lower initial effective consolidation stresses, fibres cannot stretch effectively and may slip during the

366 shear. Hence, fibres may not confine soil grains and add surplus strength to the reinforced soil. Therefore, increase

367 in fibre content at a low initial effective consolidation stress may have an insignificant effect on the shear strength of

368   the fibre-reinforced soil without necessary soil grain harnessing effect. Increase in fibre content results in changing

369   the stress-strain behaviour of the unreinforced soil sample to a strain hardening behaviour. This research indicated

370   that up to 5% carpet waste fibre could be optimally used for improving the shear strength behaviour of clay and

371   hence introduces carpet waste fibres as sustainable soil reinforcing materials to improve the shear strength of weak

372   soils.

373   Two nonlinear regression models were developed based on the modified hyperbolic model to predict the effective

374   stress ratio and deviator stress of the fibre-reinforced clay. The model parameters can be determined from the

375   triaxial test results on unreinforced and a fibre-reinforced soil sample at two different consolidation stresses. The

376   model can predict the stress-strain curve of the fibre-reinforced clay with knowing the initial effective consolidation

377   stress and the fibre content. The model was verified with the available data in the literature and predictions by the

378   model agreed reasonably well with the published experimental data. The proposed model is relatively simple

379   compared to other developed regression models and constitutive models reported in the literature in that it eliminates

380   the requirement for advanced soil testing and knowledge of fibre properties. This model can be used as a tool to

381   predict the shear strength of fibre-reinforced clay in geotechnical engineering practice with limited knowledge of the

382   soil properties and less number of testing.

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388

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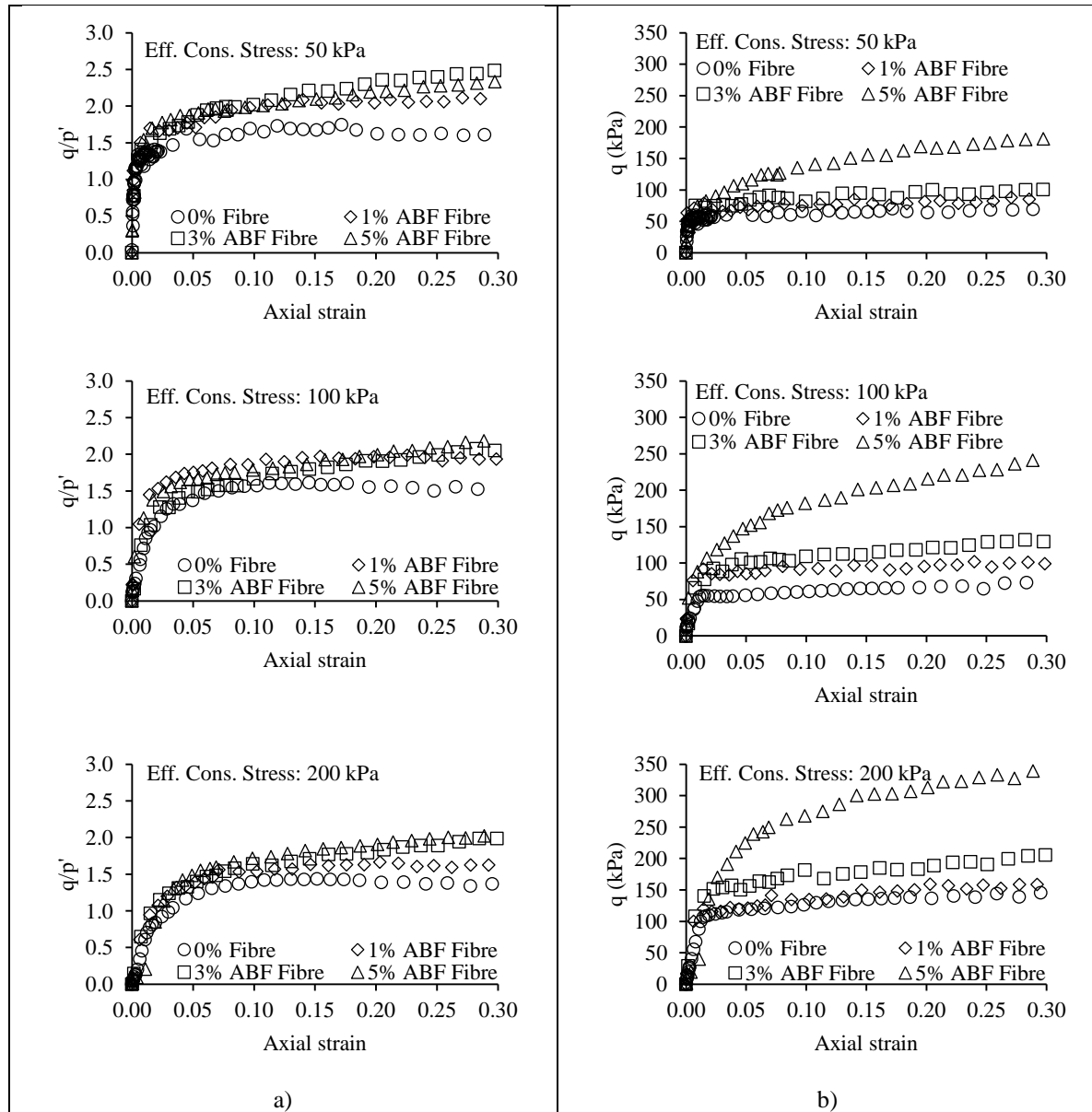496   52(8), 593-604.

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531   Figure 2. Effective stress ratio a) and deviatoric stress b) behaviours of GBF fibre-reinforced clay

Figure 3. Effective stress ratio a) and deviatoric stress b) behaviours of ABF fibre-reinforced clay

| a) Eff. Cons. Stress: 50 kPa | b)) Eff. Cons. Stress: 100 kPa | c) Eff. Cons. Stress: 200 kPa |
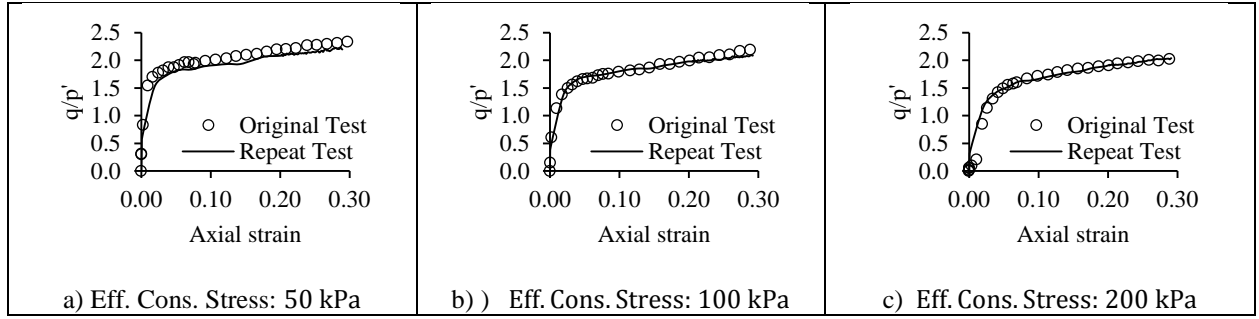
533   Figure 4. Results of repeated tests on 5% ABF fibre-reinforced clay

a) GBF fibre reinforced clay   b) ABF fibre-reinforced clay
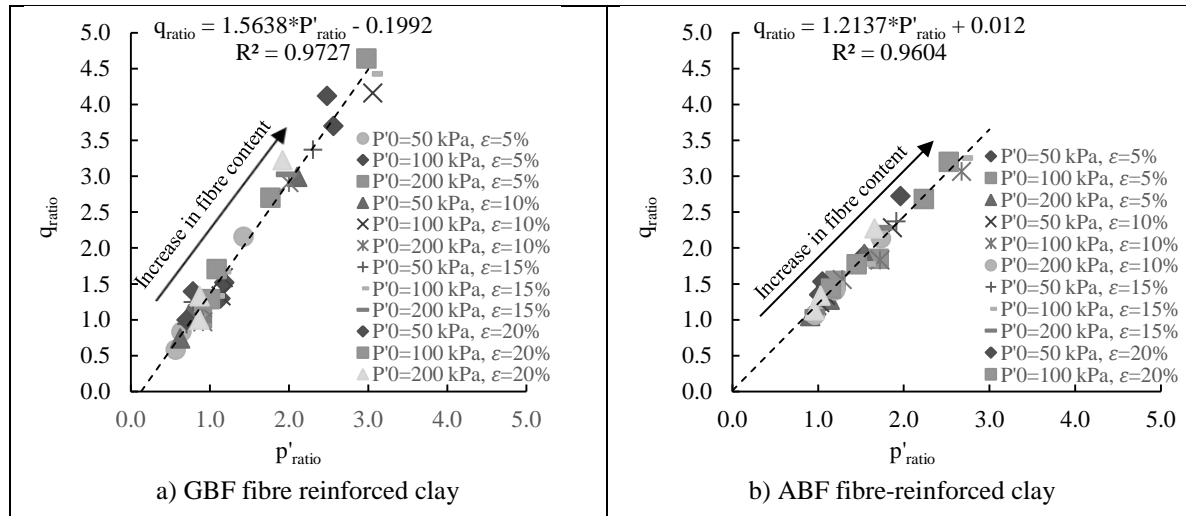
534   Figure 5. The relationship between deviator stress change ratio and mean effective stress change ratio of fibre-

535   reinforced clays

a)   b)   c)
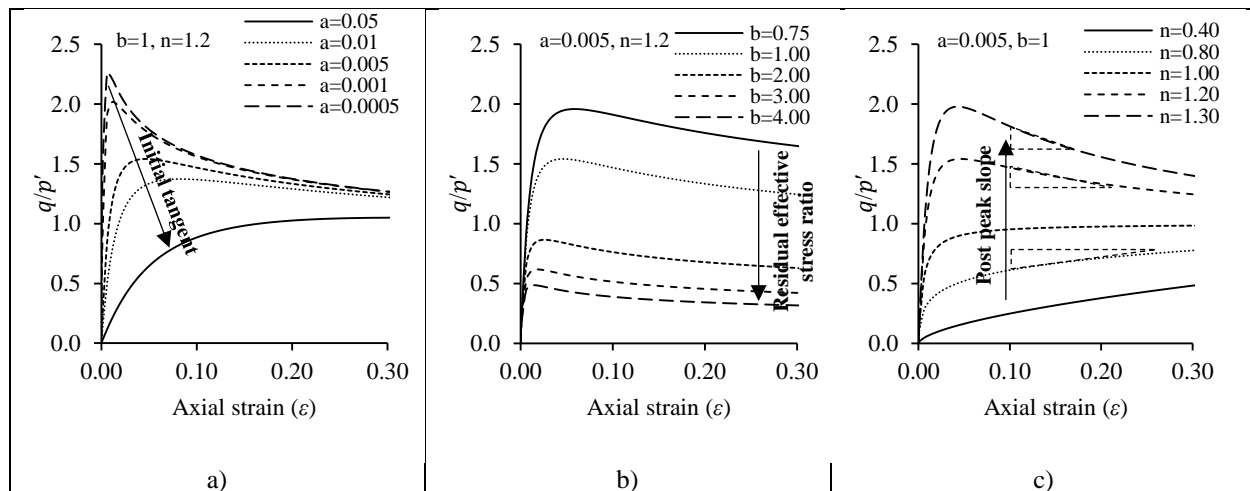
536   Figure 6. Parametric study of the modified hyperbolic model parameters on the effective stress ratio of the soil: the

537   effect of a) parameter 'a' b) parameter 'b' c) parameter 'n' on the modified hyperbolic function

23
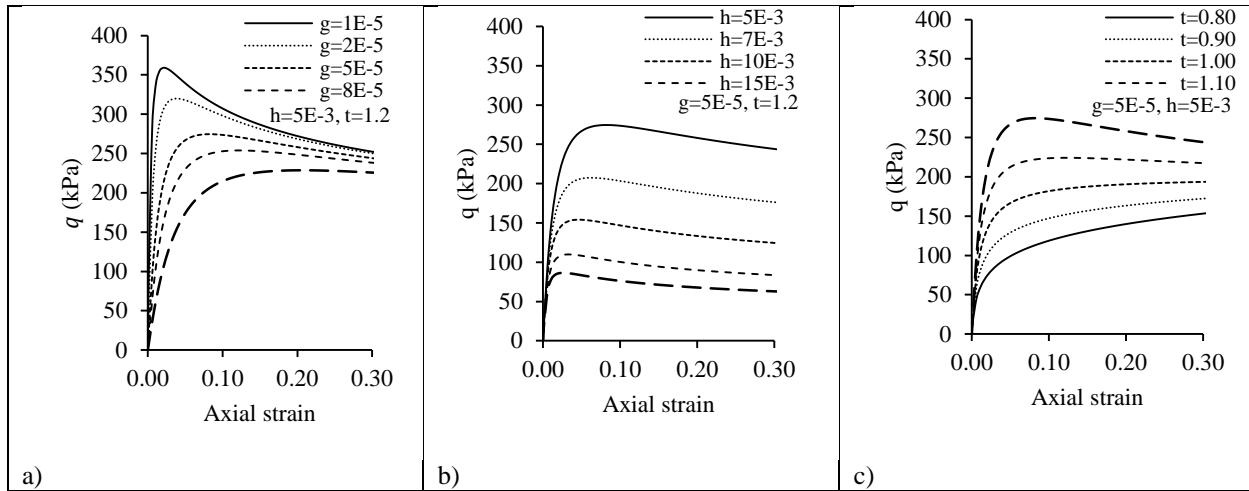
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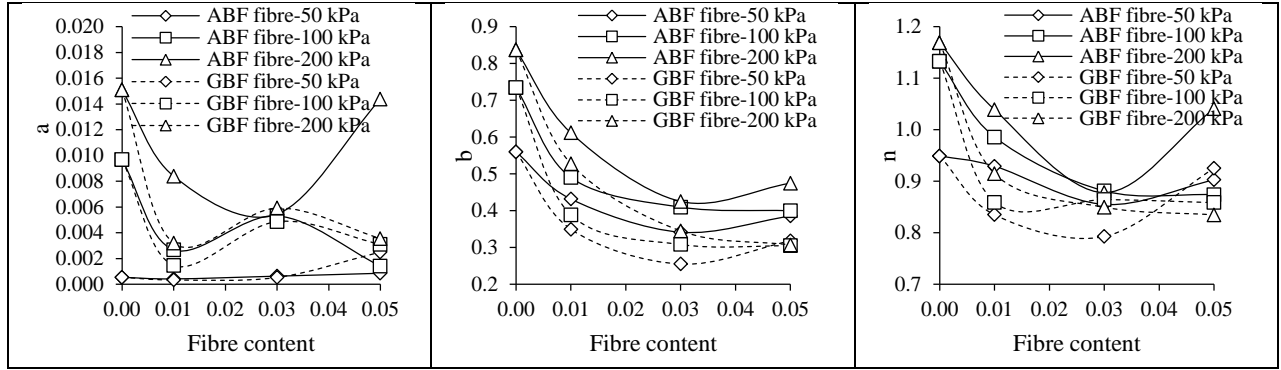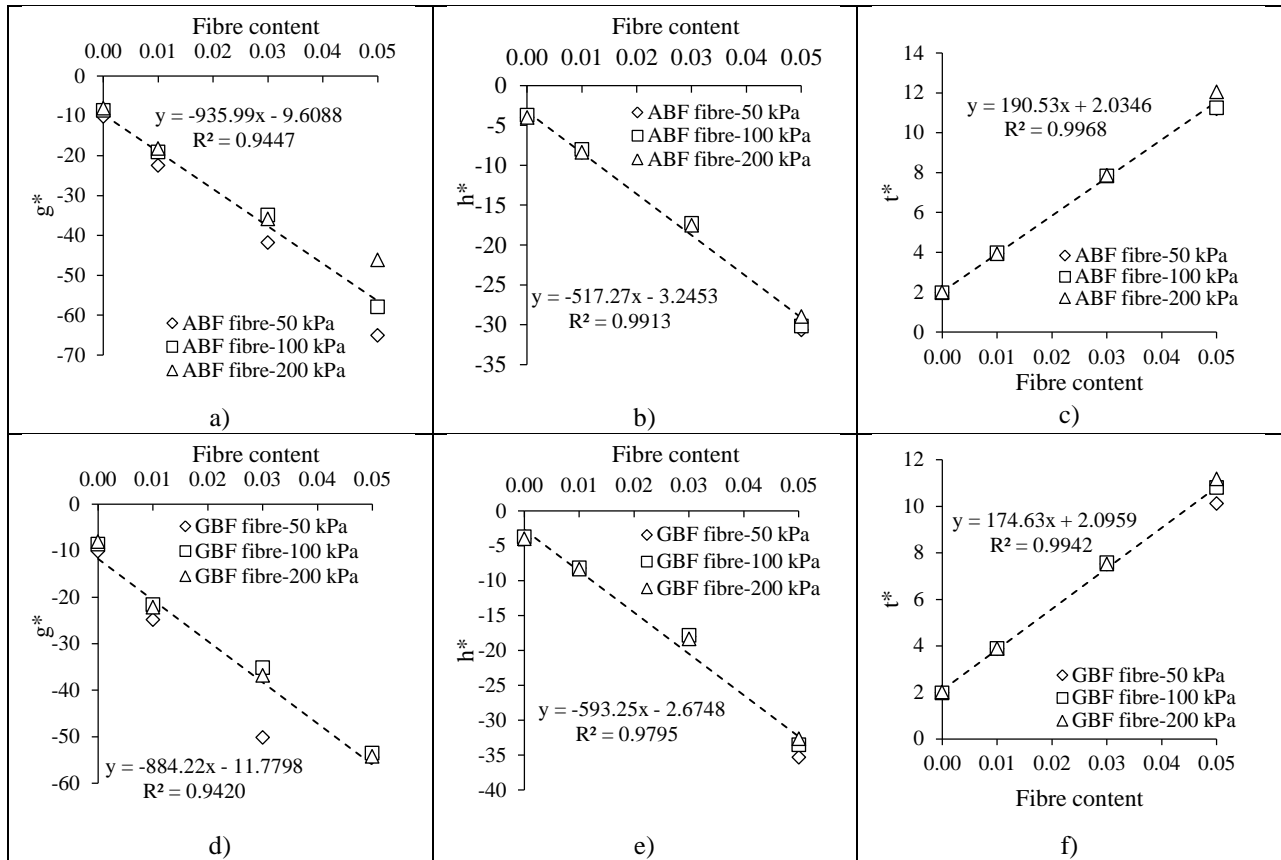
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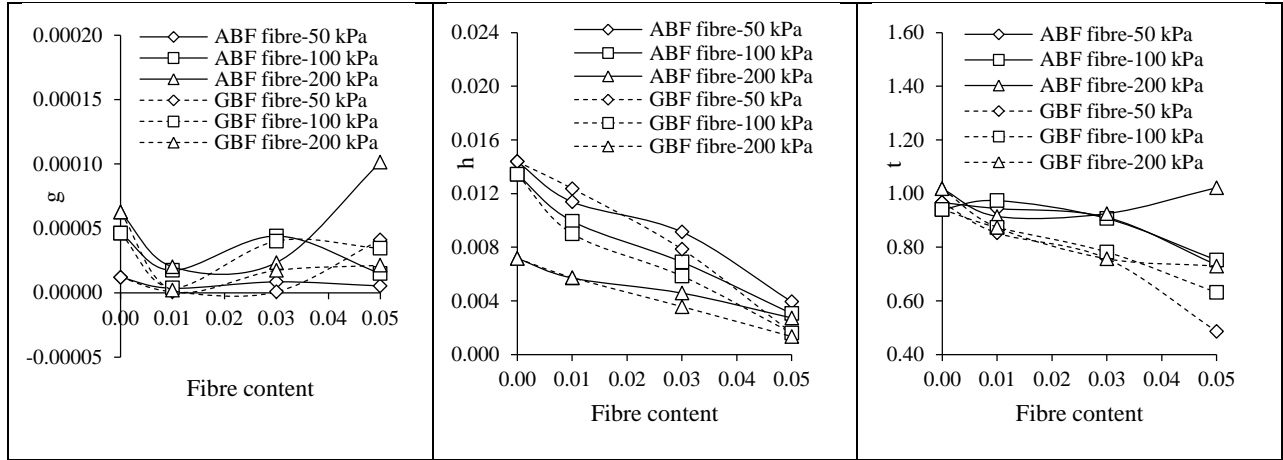
541   Figure 9. Modified hyperbolic model parameters for $q - \varepsilon$ behaviour of the fibre-reinforced soil sample
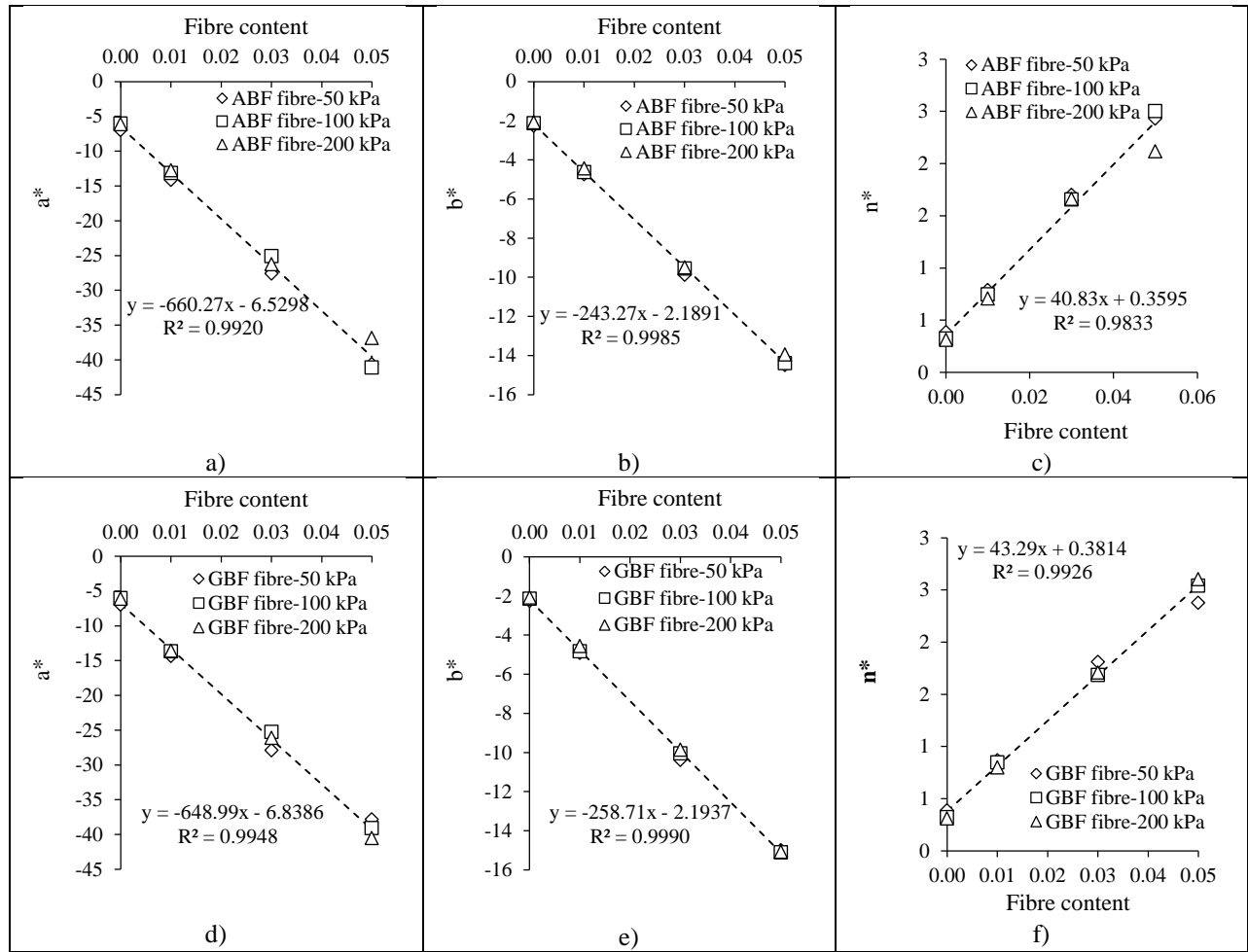
542   Figure 10. Normalised modified hyperbolic model parameters for $q/p' - \varepsilon$ behaviour of the fibre-reinforced soil

543   samples: a,b,c) ABF fibre-reinforced soils d,e,f) GBF fibre-reinforced soils

544   Figure 11. Normalised modified hyperbolic model parameters for $q - \varepsilon$ behaviour of the fibre-reinforced soil

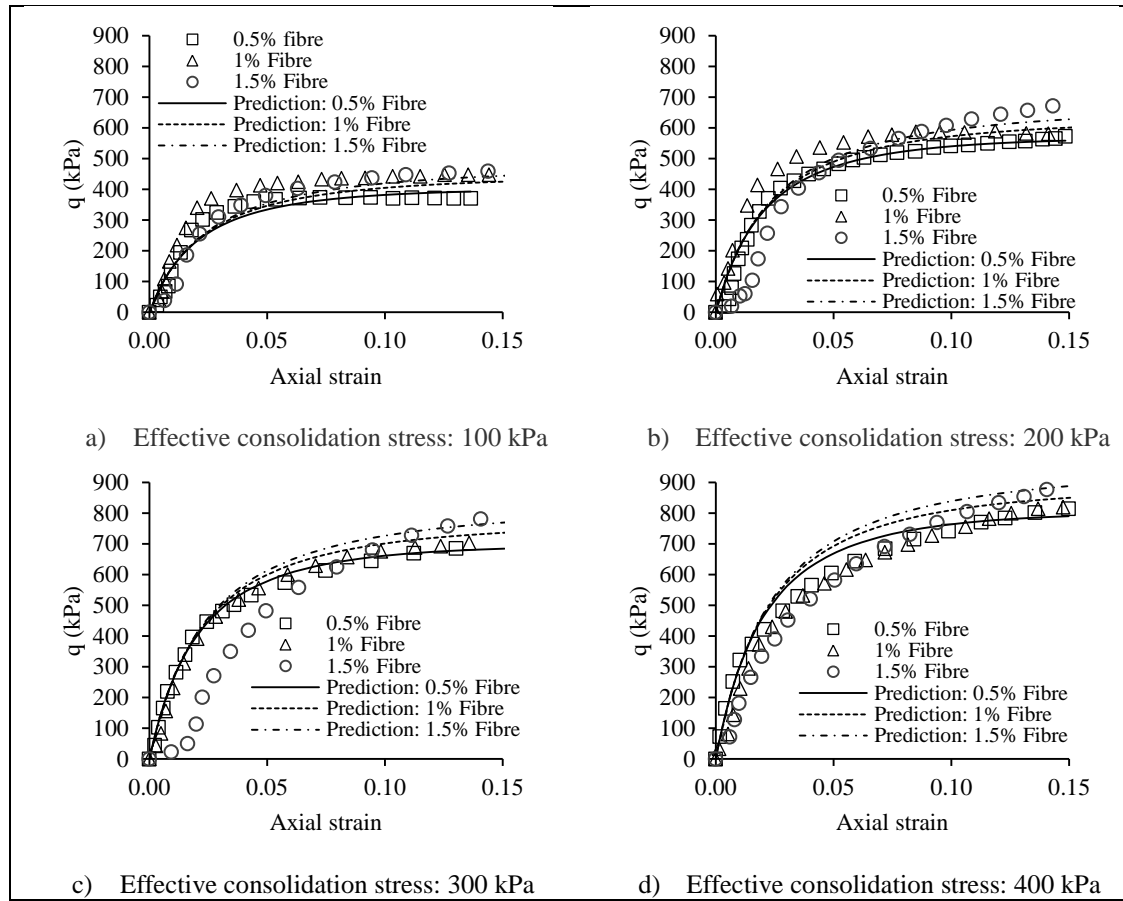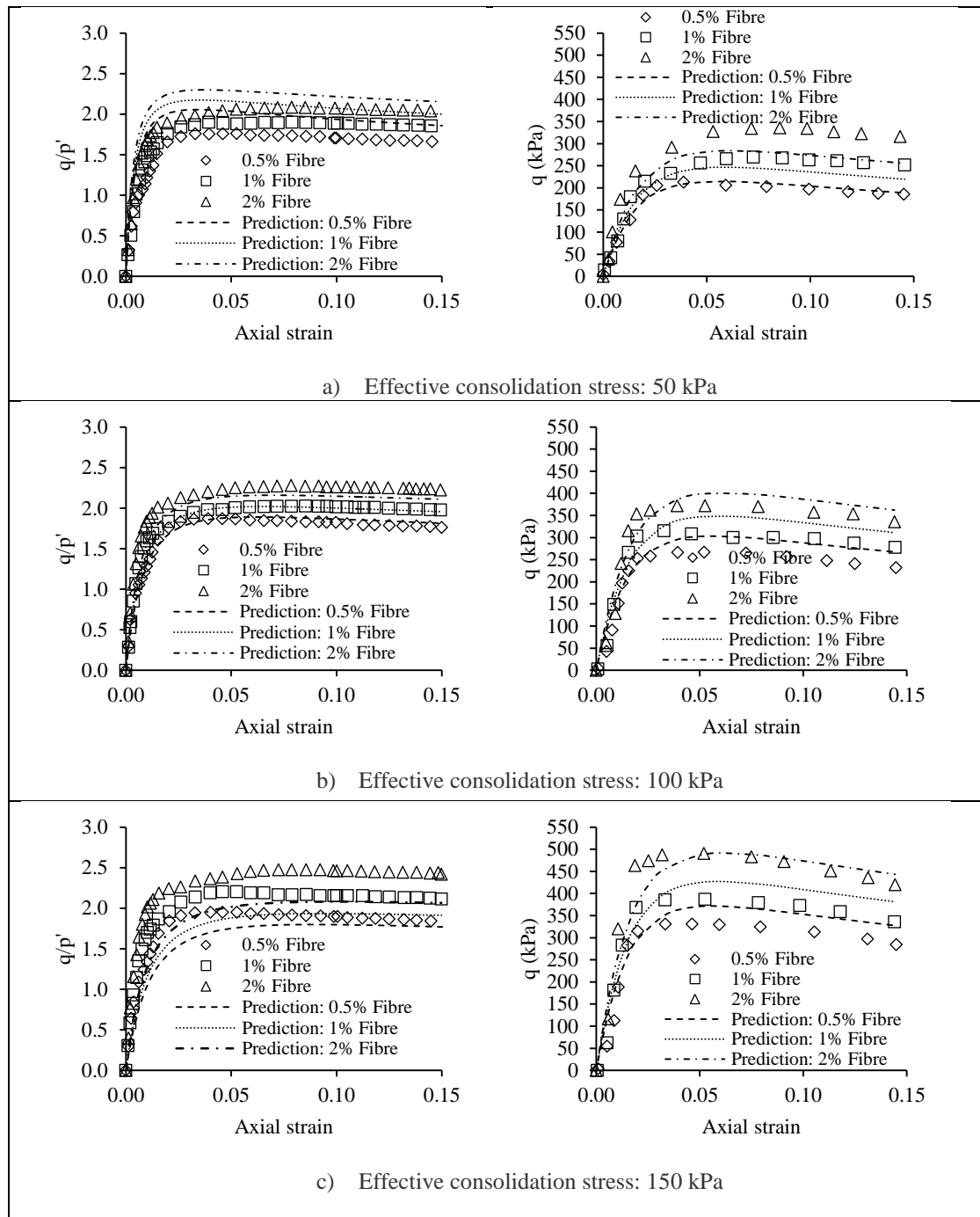545   samples: a,b,c) ABF fibre-reinforced soils d,e,f) GBF fibre-reinforced soils
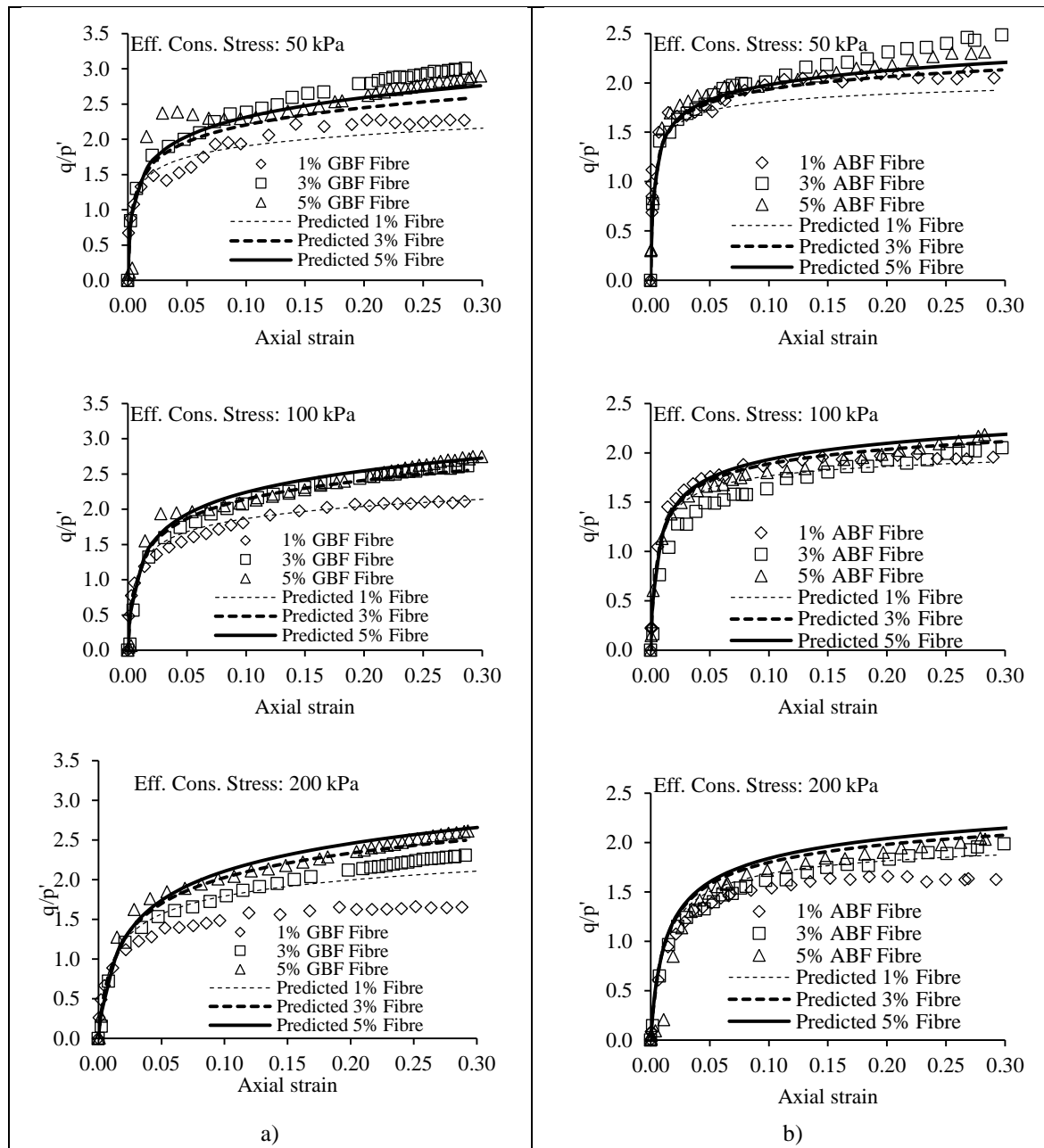
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a) Effective consolidation stress: 100 kPa   b) Effective consolidation stress: 200 kPa

c) Effective consolidation stress: 300 kPa   d) Effective consolidation stress: 400 kPa

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549   Figure 14. Experimental and predicted data for shear strength response of fibre-reinforced soil
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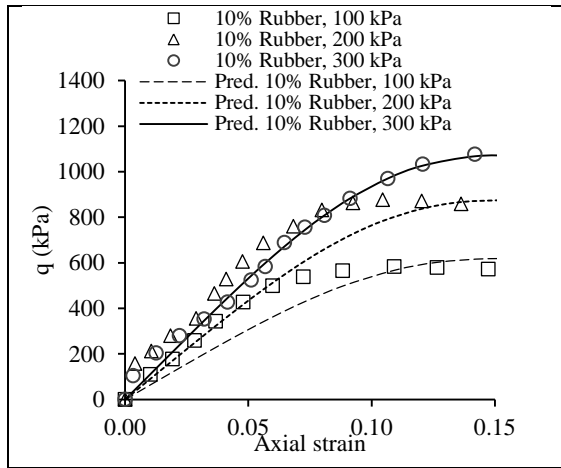
551   Figure 15. Experimental and predicted data for shear strength response of fibre-reinforced soil
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c) Effective consolidation stress of 600 kPa

d) Effective consolidation stress of 800 kPa

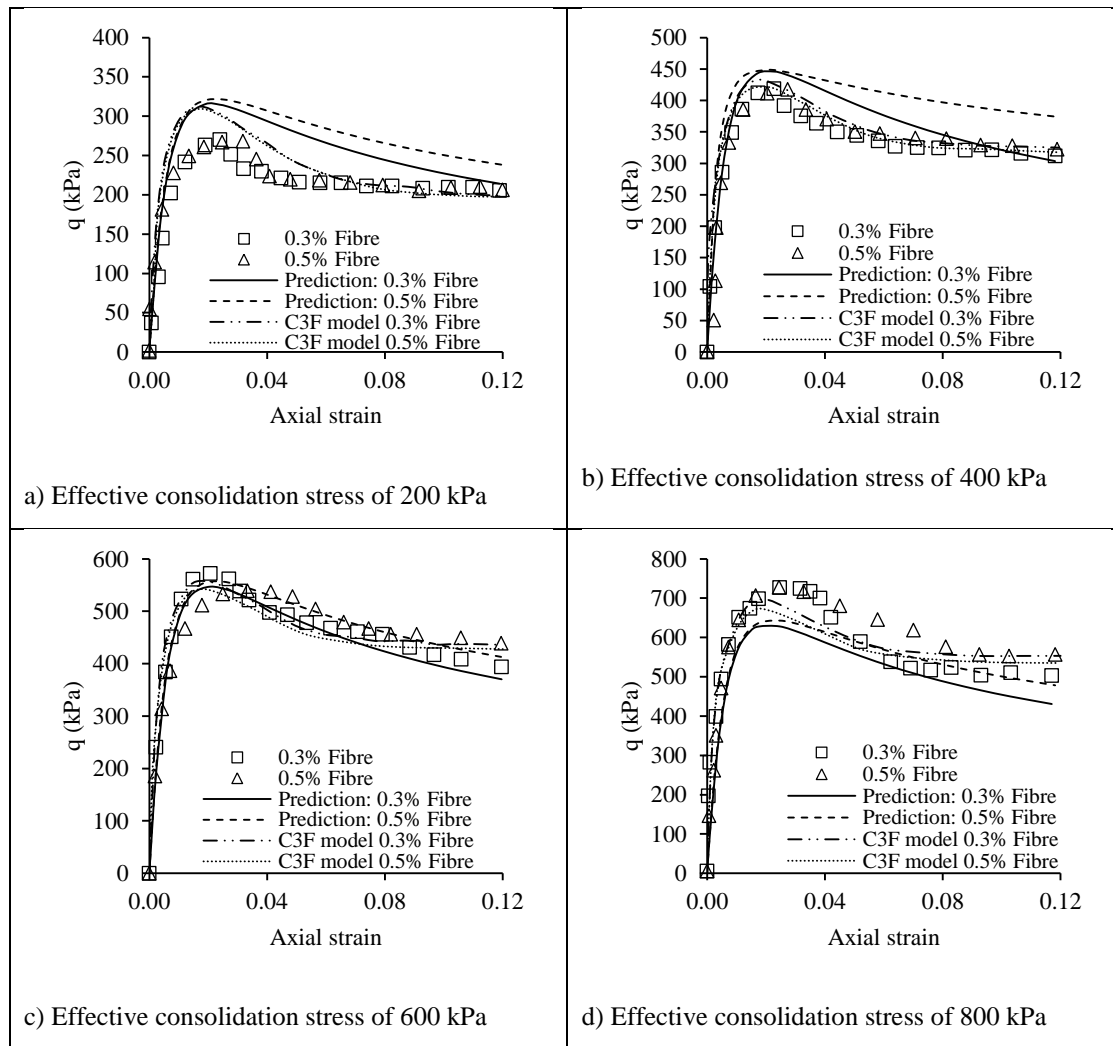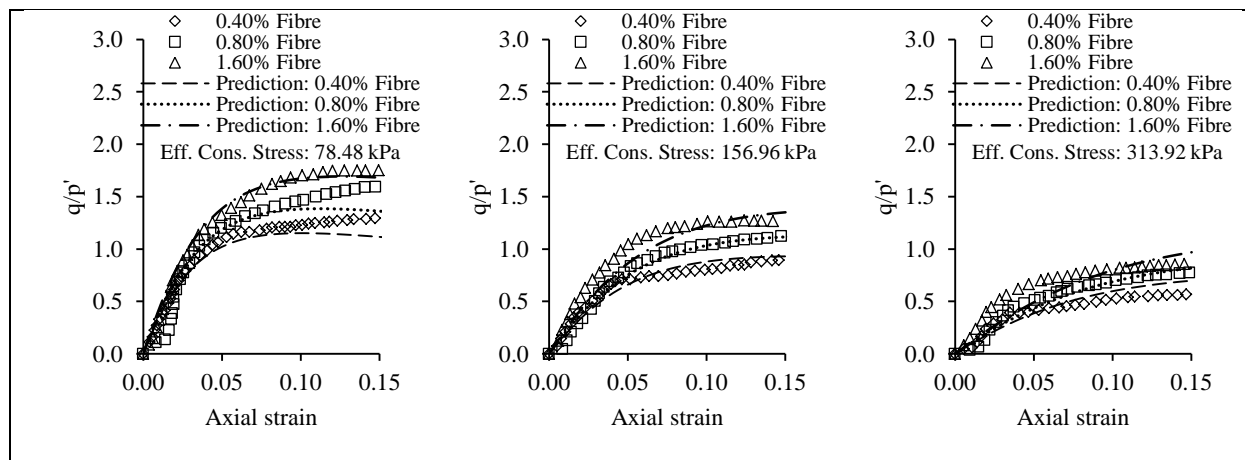553   Figure 16. Experimental and predicted data for shear strength response of fibre-reinforced soil
554   (Nguyen and Fatahi 2016)

555   Figure 17. Experimental and predicted data for shear strength response of fibre-reinforced soil (Khatri et al. 2016)

556
Table 1. Soil Properties

Unified soil classification	CL
Specific gravity	2.68
Gravel (%)	1.42
Sand (%)	42.80
Fine content (%)	55.78
Liquid limit (%)	29.00
Plastic Limit (%)	12.00
Plasticity index (%)	17.00
Maximum dry unit weight (kN/m^3)	20.10
Optimum moisture content (%)	11.00
Swelling pressure (kPa)	76.20

557
Table 2. Properties of carpet waste fibres

	Composition	Specific Gravity*	Water Absorption* (%)	Composition (%)	Specific Tensile Modulus* (GPa/gram/cm^3)
GBF fibre	Polypropylene	0.90	0.0	60	0.27~0.44
	Styrene-Butadiene Rubber (SBR)	0.99	-	20	-
	Nylon	1.14	4.1-4.5	15	0.40~0.70
	Wool	1.32	13-15	5	0.27~0.40
ABF fibre	Nylon	1.14	4.1-4.5	100	0.40~0.70

558   *Recommended by the manufacturer

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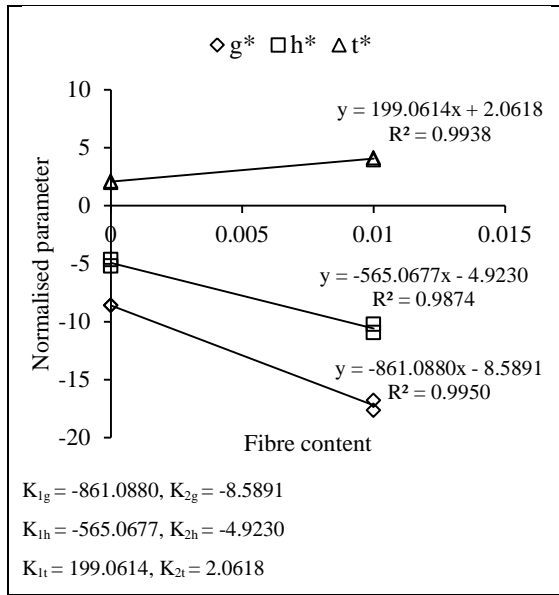Table 3. Comparison of experimental and predicted shear strength results

Test data	$\dfrac{(\frac{q}{p'})_{predicted}}{(\frac{q}{p'})_{experiment}}$	$\dfrac{q_{predicted}}{q_{experiment}}$	
$\varepsilon = 14.5\%, f = 0.5\%, \sigma'_c = 50\ kPa$	1.12	1.02	Babu and Chouksey
$\varepsilon = 14.3\%, f = 1\%, \sigma'_c = 50\ kPa$	1.08	0.87	(2010)
$\varepsilon = 14.3\%, f = 2\%, \sigma'_c = 50\ kPa$	1.06	0.81	
$\varepsilon = 14.4\%, f = 0.5\%, \sigma'_c = 100\ kPa$	1.03	1.16	
$\varepsilon = 14.4\%, f = 1\%, \sigma'_c = 100\ kPa$	0.99	1.12	
$\varepsilon = 14.4\%, f = 2\%, \sigma'_c = 100\ kPa$	0.94	1.08	
$\varepsilon = 14.5\%, f = 0.5\%, \sigma'_c = 150\ kPa$	0.96	1.15	
$\varepsilon = 14.5\%, f = 1\%, \sigma'_c = 150\ kPa$	0.90	1.14	
$\varepsilon = 14.4\%, f = 2\%, \sigma'_c = 150\ kPa$	0.85	1.05	
$\varepsilon = 13.6\%, f = 0.5\%, \sigma'_c = 100\ kPa$	-	1.06	Wu et al. (2014)
$\varepsilon = 14.4\%, f = 1\%, \sigma'_c = 100\ kPa$	-	0.94	
$\varepsilon = 14.4\%, f = 1.5\%, \sigma'_c = 100\ kPa$		0.96	
$\varepsilon = 14.1\%, f = 0.5\%, \sigma'_c = 200\ kPa$	-	0.98	
$\varepsilon = 14.1\%, f = 1\%, \sigma'_c = 200\ kPa$	-	1.03	
$\varepsilon = 14.3\%, f = 1.5\%, \sigma'_c = 200\ kPa$	-	0.93	
$\varepsilon = 14.1\%, f = 1.5\%, \sigma'_c = 300\ kPa$	-	0.98	
$\varepsilon = 14.1\%, f = 1.5\%, \sigma'_c = 400\ kPa$	-	1.01	
$\varepsilon = 14.7\%, f = 10\%, \sigma'_c = 100\ kPa$	-	1.08	Ozkul and Bykal (2007)
$\varepsilon = 13.6\%, f = 10\%, \sigma'_c = 200\ kPa$	-	1.01	
$\varepsilon = 14.2\%, f = 10\%, \sigma'_c = 300\ kPa$	-	0.99	
$\varepsilon = 10.5\%, f = 0.3\%, \sigma'_c = 600\ kPa$	-	0.94	Nguyen and Fatahi (2016)
$\varepsilon = 10.5\%, f = 0.5\%, \sigma'_c = 600\ kPa$	-	0.95	
$\varepsilon = 10.5\%, f = 0.3\%, \sigma'_c = 800\ kPa$	-	0.88	
$\varepsilon = 10.5\%, f = 0.5\%, \sigma'_c = 800\ kPa$	-	0.90	
$\varepsilon = 14.2\%, f = 0.4\%, \sigma'_c = 78.48\ kPa$	0.88	-	Khatri et al. (2016)
$\varepsilon = 14.1\%, f = 1.6\%, \sigma'_c = 78.48\ kPa$	0.97	-	
$\varepsilon = 14.1\%, f = 0.4\%, \sigma'_c = 156.96\ kPa$	1.04	-	
$\varepsilon = 14.2\%, f = 1.6\%, \sigma'_c = 156.96\ kPa$	1.06	-	
$\varepsilon = 14.1\%, f = 0.4\%, \sigma'_c = 313.92\ kPa$	1.22	-	
$\varepsilon = 14.5\%, f = 1.6\%, \sigma'_c = 313.92\ kPa$	1.12	-	

561

562   **SUPPLEMENTAL MATERIAL**
563

564   In this example, the model parameters of the introduced modified hyperbolic model are calculated for experimental

565   data reported by Wu et al. (2014). A nonlinear regression analysis using the modified hyperbolic function (Eq. 5)

566   was carried out on the stress-strain data of the unreinforced and 1% fibre-reinforced soil tested at effective

567   consolidation stresses of 100 kPa and 400 kPa, respectively. **Figure S1** shows the yielded modified hyperbolic and

568   normalised modified hyperbolic regression model parameters (**Equations 9** to **11**) for calibrating the model. In order

569   to determine the linear coefficients ($k_{1a,g}, k_{2a,g}, k_{1b,h}, k_{2b,h}, k_{1n,t}$ and $k_{2n,t}$), draw a graph of normalised modified

570   hyperbolic model parameters versus fibre content and find the best fit line (See **Figure S2**). Then use **equations 15**

571   to **17** to calculate the modified hyperbolic model parameters (g, h and t). Finally, use **equation 5** or **19** to predict the

572   deviator stress of the fibre-reinforced clay.

573
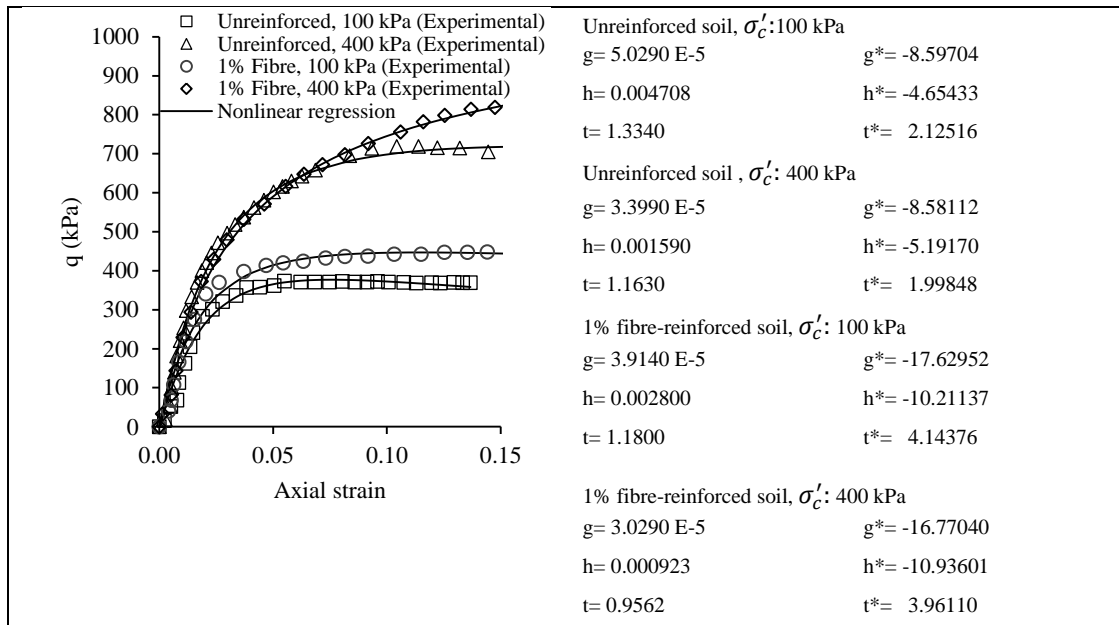
574   Figure S1. Experimental data and modified hyperbolic nonlinear regression analysis of deviator stress of
575   unreinforced and fibre-reinforced soil (Wu et al., 2014)
576

◇g* □h* △t*

K_{1g} = -861.0880, K_{2g} = -8.5891

K_{1h} = -565.0677, K_{2h} = -4.9230

K_{1t} = 199.0614, K_{2t} = 2.0618

577   Figure S2. Relationship between normalised modified hyperbolic parameters and fibre content