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Answers to the reviewers comments.

Following are the answers to the comments of reviewers. The answers are in blue text. All queries have been addressed and author(s) have tried their best to make modifications as suggested.

Reviewer #1:

The work provides an innovative mathematical methodology regarding the selection process and association process for IEEE 802.22 networks. The approach is largely evaluated for several cases and is robust enough to serve as a knowledge base for the Base Station.

Since the discussed environment is related to TV white spaces I would suggest to add to the introduction a well established review work on the topic, namely,

* Athina Bourdena, Prodromos Makris, Dimitrios N. Skoutas, Charalabos Skianis and George Kormentzas, "Joint Radio Resource Management in Cognitive Networks: TV White Spaces Exploitation Paradigm", in Evolution of Cognitive Networks and Self-Adaptive Communication Systems." IGI Global, 2013. 1-438, doi:10.4018/978-1-4666-4189-1

To make the work more informative and interesting, the reference [16], i.e.,

> "Bourdena A, Makris P, Skoutas D N, Skianis C, Kormentzas G, Pallis E, Mastorakis G. Joint radio resource management in cognitive networks: TV white spaces exploitation paradigm. Evolution of Cognitive Networks and Self-Adaptive Communication Systems, IGI Global, 2013, 50-80."

is appended in the first paragraph of Section 1: Introduction part, page no. 1 as suggested.

Moreover, some indication/discussion on the associated complexity for implementing such a knowledge mechanism could benefit the readership.

The reviewer's suggestion is appreciable. We intend to work on implementation of our proposed algorithms for cognitive radio network. For this we are constructing a testbed and also, we have established a research group in the department in which a number of researchers are working on modelling and spectrum sensing issues. We are at the initial phase and hopefully the next work will include the research with some implementations.

Reviewer #3:

The analyzed mathematical model shows interesting results especially considering the average ranging success delay experienced by the majority of the CPEs. The topic is in line with the actual international research trend in order to find solutions for improving the network capacity. The description of the problem is well presented in a mathematical and physical perspective. The results are clearly exposed and discussed adding a critical point of view. In my opinion the paper can be accepted for publication in this Journal.

A Framework for Dynamic Selection of Backoff Stages during Initial Ranging Process in Wireless Networks

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Keywords: Customer premises equipment, contention window, ranging request collision probability, average ranging success delay

Abstract: The only available solution in the IEEE 802.22 standard for avoiding collision amongst various contending customer premises equipment (CPEs) attempting to associate with a base station (BS) is binary exponential random backoff process in which the contending CPEs retransmit their association requests. The number of attempts the CPEs send their requests to the BS are fixed in an IEEE 802.22 network. This paper presents a mathematical framework that helps the BS in determining at which attempt the majority of the CPEs become part of the wireless regional area network from a particular number of contending CPEs. Based on a particular attempt, the ranging request collision probability for any number of contending CPEs with respect to contention window size is approximated. The numerical results validate the effectiveness of the approximation. Moreover, the average ranging success delay experienced by the majority of the CPEs is also determined.

1. Introduction

Due to the rapid growth of wireless devices and ever increasing bandwidth demands from users, more and more spectrum resources are required. Within the conventional spectrum framework, most of the spectrum bands have been exclusively allocated to specific licensed services. However, reports on spectrum usage measurements show that a lot of licensed bands, such as those for TV broadcasting, are underutilized, which results in spectrum wastage [1], [2], [3], [4]. The emerging cognitive radio (CR) technology has been proposed as a new solution to increase the efficiency of spectrum utilization. It is different from conventional radio devices in the sense that it can equip users with cognitive capability and reconfigurability [5], [6], [7], [8]. Exploiting these capabilities, a CR can sense and collect data about transmission frequency, bandwidth, power, modulation, etc. from its surrounding environment. After incorporating this information, it can identify the best available spectrum to meet communication requirements, and then can reconfigure its operational parameters according to the collected information in order to attain the optimal performance. The researchers use this technology in various wireless networks to understand how to improve the network capacity [9], [10], [11]. Due to this technology, the Federal Communications Commission (FCC) has allocated the licensed bands to unlicensed users [12], [13]. The IEEE 802.22 working group has been formed to develop the first international standard for wireless regional area networks (WRAN). Its basic purpose is to provide broadband access in remote and rural areas by effectively utilizing the unused TV band provided that no harmful interference is caused to the licensed users which may be digital TV, analog TV or wireless microphone [14], [15]. For more details about TV band and their related important issues, readers are directed to [16].

Unlike IEEE 802.16, IEEE 802.22 operations are mostly targeted at rural and remote areas, and its geographic area is significantly bigger. Moreover, no incumbent protection scheme is included in IEEE 802.16. Each WRAN cell contains one base station (BS) and a number of CPEs. If more than one CPE attempts to associate with the BS at the same time, a collision occurs. In this case, the binary exponential random backoff (BERB) process is carried out. According to this process, the CPE adjusts its internal backoff window and re-attempts to associate with the BS. If collision occurs again, the CPE is involved in a second backoff phase and does the same procedure before the maximum number of attempts expires as specified by the IEEE 802.22 standard [17]. This paper is the extension of our previous work [18]. It focuses on a mathematical model that helps the BS: 1) in finding at which attempt the majority of the CPEs become part of a WRAN cell from a particular number of CPEs; 2) based on a particular attempt, the collision probability is approximated; and, 3) the average ranging success delay experienced by the majority of the CPEs is determined. The paper is organized as follows. In the next section, the main features and challenges of an IEEE 802.22 network is presented. In Section 3, the collision probability and the average ranging success delay are briefly explained. In Section 4, numerical results are discussed and a mathematical framework is developed based on a number of contending CPEs. Finally, the concluding remarks are discussed in Section 5.

2. Overview of IEEE 802.22

Before this research proceeds further to analyse the collision probability and associated backoff delay, the main features of IEEE 802.22 networks are presented.

2.1 *Architecture*

The main components of IEEE 802.22 networks are the BS and the CPEs as shown in Figure 1. A BS, in general manages all the activities within the cell, including medium access, allocations to achieve quality of service (QoS) and association with the network based on spectrum etiquette. In order to ensure primary protection, the IEEE 802.22 system follows a strict master/slave architecture, wherein the BS performs the role of master and the CPEs acts as the slave. No CPE is allowed to transmit without receiving authorization from a BS. Normally, the main function of BS/CPEs can be classified into two categories: sensing and transmitting/receiving data. Spectrum sensing plays an important role in detecting the incumbent and discovering spectrum opportunities. If any of the channels used by an IEEE 802.22 network are occupied by an incumbent, the primary task of 802.22 devices is to vacate the channel within the channel specified time (2 seconds) and switch to some other vacant channel. In order to acquire the knowledge about the presence of incumbent, both BS and CPEs perform channel sensing periodically. The general spectrum sensing process is carried out by fast sensing and fine sensing. Fast sensing is performed more quickly at the same time as carrying out data transmission. Fine sensing is carried out on the basis of fast sensing results. Fine sensing takes as much time as the time required for fast sensing [17].

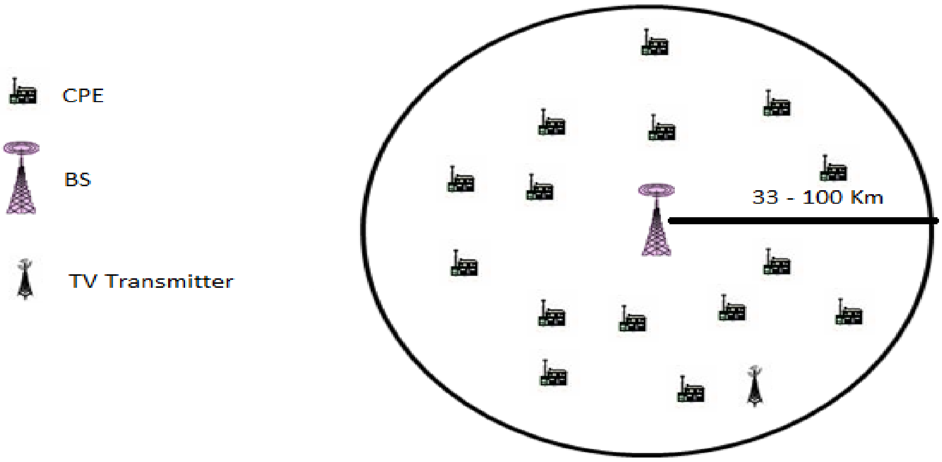

Figure 1: An example of IEEE 802.22 network.

2.2 *Mac layer*

Most of the characteristics of existing MAC of IEEE 802.22 are similar to the MAC of 802.11 and 802.16. However, some characteristics of IEEE 802.22 differentiate it from other standards.

2.2.1 Initial connection establishment

The initial connection establishment mechanism in IEEE 802.22 is different from that of 802.11 and 802.16 standards [19] in that this mechanism is completely centralized. Since IEEE 802.22 systems share the spectrum band with the incumbents, there is no predefined channel for the CPE to associate with the BS. A CPE does not have any information about what the channel is used for when establishing its initial connection with a BS. Furthermore, a CPE is not allowed to select any channel for association; rather the BS provides the required permission.

When a CPE is powered on, it follows the procedure of listen before talking by scanning the TV band to determine the vacant and occupied channels. On the basis of its observation, it establishes a spectrum usage report. The same procedure of spectrum sensing is also carried out by the BS and it periodically broadcasts the necessary information on the operating channel. The broadcast propagated from the BS is differentiated from the TV broadcasts by the preamble transmitted at the beginning of each frame. If a CPE captures a particular frame, it then tunes to that frequency and transmits a unique identifier in the uplink direction. The BS, on the other hand, becomes aware of the presence of a CPE. The authentication and registration process are then to be carried out. After establishing the association, the CPE sends the spectrum usage report to the BS in the form of feedback. On the basis of this feedback, the BS takes a necessary decision on spectrum utilization. In the case where more than one CPE attempts to establish an initial connection with the BS, a contention-based connection setup is performed.

2.2.2 Frequency of operation and service coverage

Normally, the 802.22 system operates on 6 MHz, 7 MHz and 8 MHz channels depending upon the regulatory domain and specifies spectral efficiencies in the range of 0.5 bit/sec/Hz up to 5 bit/sec/Hz, thus resulting in an average data rate of 18 Mbps and maximum data rate of up to 30 Mbps in 6 MHz TV channel (particularly in United States) [17]. The system incorporates orthogonal frequency division multiplexing (OFDM) for downlink and uplink, and orthogonal frequency division multiple access (OFDMA) for uplink when more than one CPE transmits to the BS at the same time [standard]. The 802.22 system architecture consisting of one or more PHY/MAC air interface modules has the capability to take benefit from the availability of multiple vacant TV channels. Moreover, the channel carriers can be fragmented so that IEEE 802.22 devices can share the spectrum with incumbents such as wireless microphones that use only 1 or 2 MHz of the entire channel bandwidth. This functionality of aggregating/fragmenting carriers also avoids interference and cross-talking by tuning out of partial channels, leading to improved spectrum utilization.

An important feature that makes IEEE 802.22 WRAN superior to other IEEE 802 standards is its coverage area, which can be extended up to an 100 Km radius if power is not an issue. As shown in Table 1, an IEEE 802.22 network has a large coverage area as compared with other networks. This is primarily due to the lower frequencies of TV bands leading to lower propagation path loss and thus resulting in reducing power spectral density. However, this enhanced coverage range offers technical challenges as well as opportunities.

Table 1: 802.22 WRAN classifications with respect to other wireless standards.

Standard	Network type	Range	Data rate	Frequency
IEEE 802.22	WRAN	30 - 100 Km	18 Mbps -24 Mbps	54-862 MHz
IEEE 802.16	MAN (Yi-Max)	1- 2 Km	54 Mbps	< 60 GHz
IEEE 802.11b	LAN (Yi-Fi)	33 m	11 Mbps	2.4 GHz
IEEE 802.11a	LAN (Yi-Fi)	20 m	54 Mbps	5 GHz
IEEE 802.15	PAN (Bluetooth)	10 m	1 Mbps	2.4 GHz

2.3 Major Challenges in IEEE 802.22 Networks

Since, in IEEE 802.22 networks, the devices share the spectrum bands with the incumbents, they have no prior knowledge of what frequency bands the other devices are working on. Therefore, there is a need for dynamic spectrum access among IEEE 802.22 networks, so that the interference among the networks can be minimized [20], [21]. In other earlier standards, self-coexistence and security issues are taken into account after the specification has been finalized, however IEEE 802.22 considers a proactive approach and includes self-coexistence protocols and procedures for enhanced MAC as a revision of the initial CR standard conception and definition [22]. This gives rise to the following MAC layer challenges:

- How will BS in one IEEE 802.22 WRAN cell select a channel from the set of channel(s) that can be employed for communication across the entire cell so that the interference to/from other WRAN cells is minimized?
- How can these channels be selected such that interference to/from other primary networks is minimized?

2.3.1 Self-coexistence

Since, multiple IEEE 802.22 WRAN cells can operate on the same or overlapping regions and the number of vacant TV channels is limited, the issue of self-coexistence among multiple BSs/operators in an overlapping area is significant. In such regions with incumbents, which may be digital TV, analog TV or wireless microphone, vacant channels are already commodities of demand. In the situation when more than one IEEE 802.22 network overlaps, it is highly possible that the operators will make an effort to act greedily and use the entire available bandwidth. As, all the operators may react in the same way, this may lead to interference among IEEE 802.22 networks themselves. Therefore, an efficient channel scheme needs to be incorporated in order to avoid interference amongst the networks [23].

2.3.2 The hidden incumbent problem

If an incumbent like a TV transmitter operates with the same frequency in the vicinity of the CPE but outside the BS sensing contour, the BS has no way of detecting this TV transmission. Therefore, the BS is hidden from this incumbent. Conversely, one or more CPEs associated to it are not hidden from the incumbent as shown in Figure 2. This problem is said to be the hidden incumbent problem [24], [25], [26]. In this case, the CPE can identify the incumbent transmission by performing periodic in-band channel sensing, however the BS cannot. The BS, on the other hand, will keep on transmitting leading to interference of the devices in the hidden incumbent area. The CPE has no way of reporting this incumbent to the BS. If it transmits on the operating channel, it will cause harmful interference to the incumbent, due to the centralized nature of an IEEE 802.22 WRAN cell. In addition, the CPE is not allowed to select any other channel except the operating channel to inform the BS about the presence of an incumbent. Similarly, if a CPE is powered on, it first attempts to associate with the BS. To achieve this, it scans the TV band for periodic broadcasts. If there is an incumbent near to the CPE transmitting with the same frequency as that of the periodic broadcast frequency and is outside the BS sensing area, the CPE will not be in a position to decode the BS broadcasting frequency due to the interference from the incumbent. This results in the possibility of three cases:

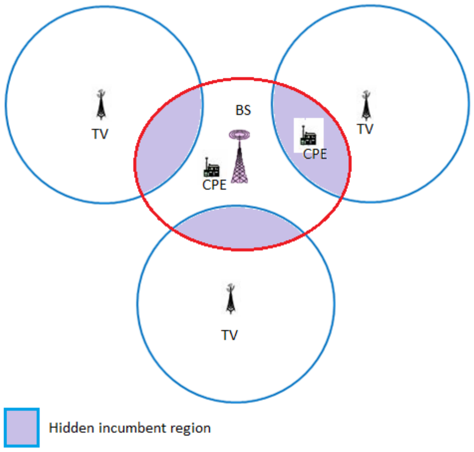

Figure 2: IEEE 802.22 hidden incumbent scenario.

- The CPE may think that there is no BS transmitting at that time leading to a switch off.
- In the same way, if the BS does not obtain any feedback from the CPE, it may assume that no CPE is live and may stop broadcasting after a certain number of broadcasting periods.
- During the broadcasting period, the BS may cause harmful interference to the incumbents.

3. Collision Probability and Backoff Delay

In this section, the ranging request collision probability and average ranging success backoff delay will be discussed.

3.1 *Collision probability*

A collision takes place when more than one CPE attempts to associate with the BS in the same transmission opportunity. In this case, the corresponding ranging request collision probability $\left(P_c(j)\right)$ is defined as [19]:

$$P_c(j) = \begin{cases} 0 \quad if\ n < 2 \\ \sum_{\phi=2}^{n} \binom{n}{\phi} p_j^{\phi} (1-p_j)^{n-\phi} \quad if\ n \geq 2 \\ where\ \ p_j = \frac{1}{W_j}, W_j = 2^j W_0, for\ j = 0,1,2, \dots m-1 \end{cases} \tag{1}$$

where n is the number of contending CPEs, which is equal to the sum of previously contended CPEs and current attempted CPEs. Whilst m is the maximum number of attempts for each attempted CPE and j is the number of backoff stages such that $j \leq m-1$. Each stage has a backoff window corresponding to the states at that stage. W_0 is the initial contention window size defined by the BS, W_j is the internal backoff window incorporated by the CPE for the jth backoff stage, p_j is the probability of selecting K_j from W_j. The CPE selects a random number K_j from the set $\{0, 1, \ldots W_j - 1\}$ such that $0 \leq K_j < W_j$.

The expression given in (1) at the j^{th} backoff stage is valid only under the following assumptions [15]:

- Each CPE has geolocation information acquiring capability and sensing capability.
- Each CPE updates the required parameters correctly after the BS recommendations.
- When BS successfully receives the association request without requiring any modification, it will associate the CPE with the network.
- During association process if an incumbent is detected, the CPE will not proceed further on the operating channel according to IEEE 802.22 spectrum etiquette [17]. Assume that no incumbent will occur on the operating channel during this process.
- The BS has the information about how many CPEs are attempting to join the WRAN cell.

3.2 *Backoff delay*

Average ranging success delay (D) is defined as [19],

$$\begin{aligned} D = {} & \big(1 - P_c(0)\big)[D_0 + t] + P_c(0)\big(1 - P_c(1)\big)\left[\sum_{j=0}^{1} D_j + 2t\right] + P_c(0)^2\big(1 - P_c(2)\big)\left[\sum_{j=0}^{2} D_j + 3t\right] \\ & + P_c(0)^3\big(1 - P_c(3)\big)\left[\sum_{j=0}^{3} D_j + 4t\right] + \cdots \\ & + P_c(0)^{m-1}\big(1 - P_c(m-1)\big)\left[\sum_{j=0}^{m-1} D_j + mt\right] \qquad (2) \end{aligned}$$

where m is the total number of backoff stages allowed in an IEEE 802.22 network, D_j is the delay at j^{th} backoff stage and t = BS response time = 10 ms, defined in the IEEE 802.22 standard.

4. Numerical Results and Discussion

The maximum number of attempts for association with the BS in IEEE 802.22 is 5, i.e. $m = 5$, therefore, $j = 0, 1, 2, 3, 4$. Using equation (1) with $W_0 = 8$, the collision probability for number of contending CPEs, $n = 5, 10, 20, 40, 80$ is illustrated in Figure 3 with respect to number of attempts. At the first attempt, i.e. $m = 1$, $j = 0$, the collision probability for $n = 5$ is 0.1207. When n is changed from 5 to 10, the collision probability is changed from 0.1207 to 0.3611. This means that when the number of contending CPEs is doubled, the collision probability is more than double at that initial contention window size. Due to this, more CPEs are involved in the BERB process. When n is increased either from 10 to 20 or from 20 to 40, i.e. 100%, the collision probability is increased from 0.3611 to 0.7330, i.e. 103%, and from 0.7330 to 0.9678, i.e. 32% respectively. Only a small number of CPEs can establish their connections with the BS at the first attempt on this particular window size, W_0. Again, when n is further increased by 100%, i.e. from 40 to 80, the collision probability is further increased to 0.9997. At this stage, it is very difficult for the CPEs to become part of the WRAN cell as shown in Figure 4. From this analysis, it is observed that, at the given initial window size, the collision probability increases when increasing the number of contending CPEs.

Now, at the second attempt, i.e. m = 2, the collision probability for $n = 5$ is 0.0344. There is a significant reduction of 71% in collision probability when m changes from 1 to 2. This reduction is primarily due to the increased window size because of first backoff. The collision probability for $n = 10$ and $n = 20$ reduces from 0.3611 to 0.1259 and from 0.7330 to 0.3582 respectively as shown in Figure 3. Due to this decrement in collision probability, more CPEs join the IEEE 802.22 network. At $n = 40$, the collision probability reduces to 25%, which implies that there are more chances for the CPEs to associate with the BS. Similarly, for $n = 80$, collision probability reduces to just 3%. Note that, because of the large value of n, this reduction in the collision probability is low.

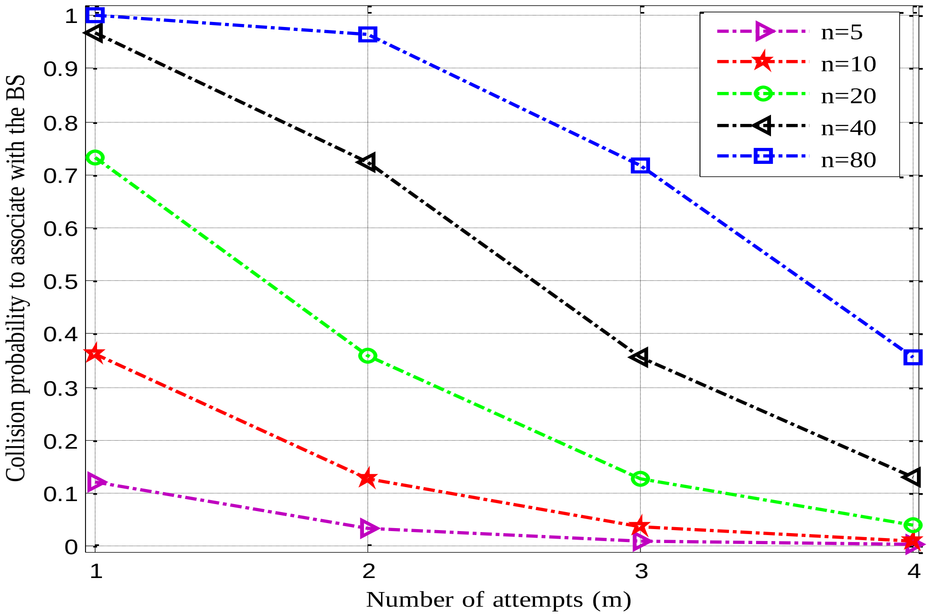

Figure 3. Collision probability verses number of attempts to associate with the BS.

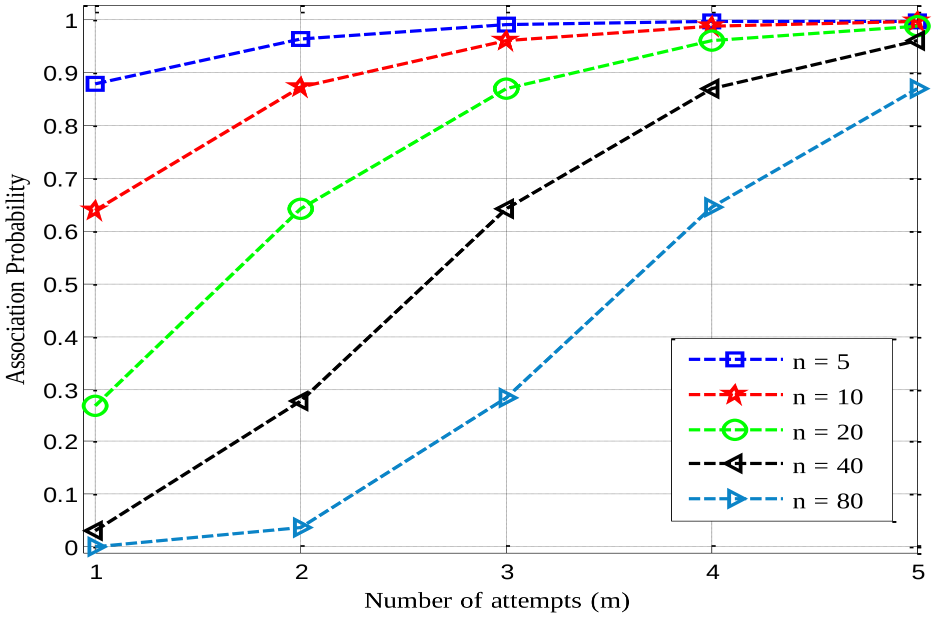

Figure 4. Association probability verses number of attempts to associate with the BS.

From this analysis, it is important to see that the collision probability for all contending CPEs at $m = 2$ reduces significantly when compared with $m = 1$. Again, at the third attempt, i.e. $m = 3$, there is little change in the collision probability for $n = 5$. This means, the collision probability becomes saturated with the current backoff window size. There is no for benefit of going towards the 2nd backoff stage for this particular number of contending CPEs. For $n = 10$ and $n = 20$, the decrement in the collision probability is 70% and 64% respectively, which shows that the majority of the contending CPEs establish their connections with the BS as can be observed from Figure 4. Similarly, for $n = 40$ and $n = 80$, the reduction in collision probability is increased significantly. There is a similar case for $m = 4$. Now finally, at the last attempt, $m = 5, j = 4$, the collision probability is saturated for $n = 5$, $n = 10$ and $n = 20$ at the 4th backoff window. The collision probability for $n = 40$ approaches zero and almost all contending CPEs establish their association with the BS. For $n = 80$, the collision probability is just 0.1297, which implies that majority of the contending CPEs join the IEEE 802.22 network as can be seen from Figure 4. From this discussion, it is clear that the collision probability not only depends upon the number of contending CPEs but also on the size of the initial contention window.

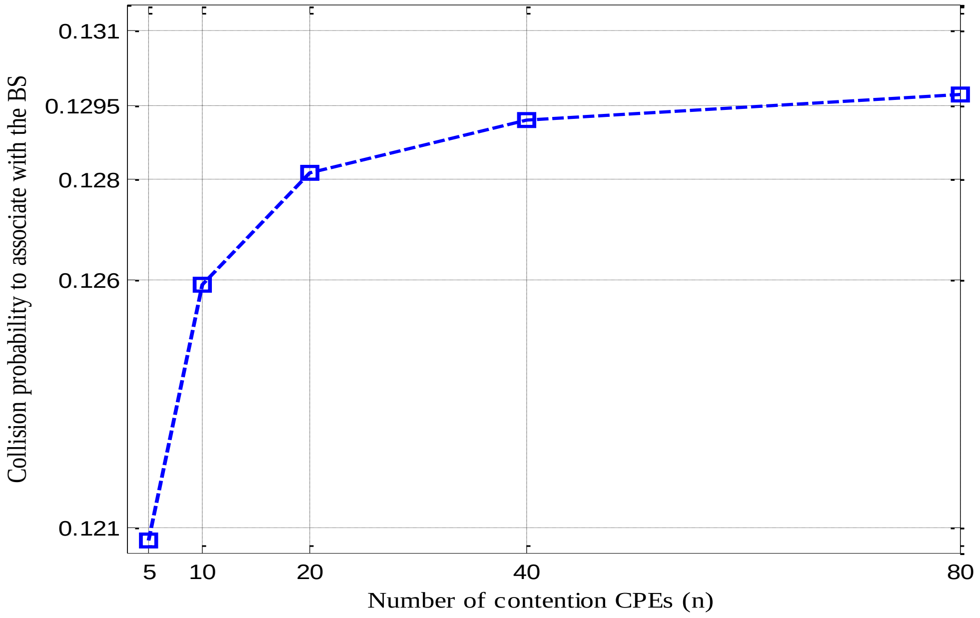

Figure 5. Collision probability verses number of contending CPEs.

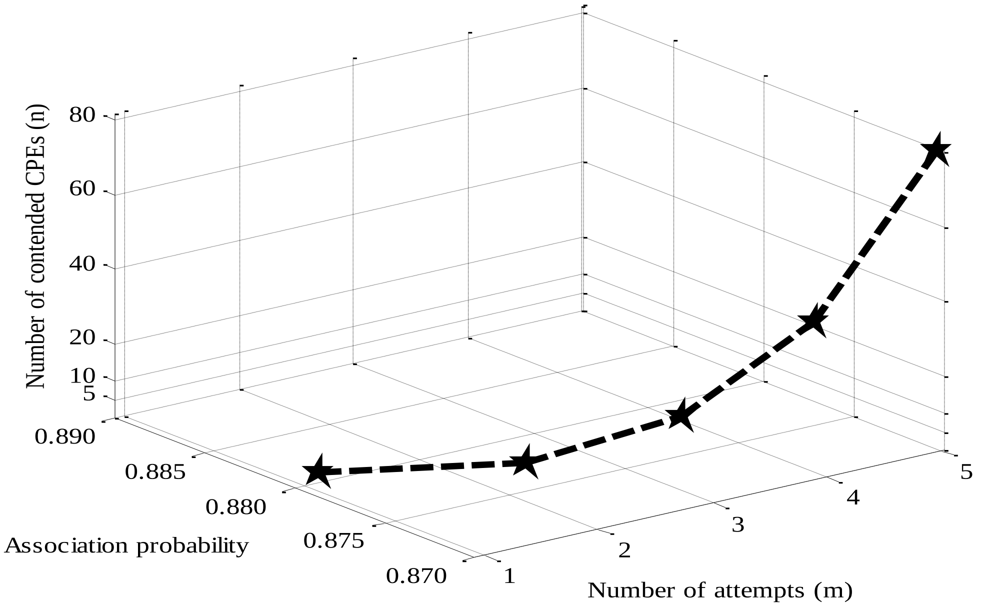

Figure 6. Association probability as a function of maximum number of attempts and the number of contending CPEs. It is clear that more than 80% CPEs are associated with the BS.

If we consider n = 80 as a base reference, then by using Figure 3, we see that there is a shift of -1 between $n = 40$ and the base reference. Similarly, there is a shift of -2 between $n = 20$ and the base reference, and so on. Finally, there is shift of -4 between $n = 5$ and $n = 80$. Therefore, this can be modeled as:

$$n = 80 \times 2^{\alpha}$$

$$\alpha = log_2\left(\frac{n}{80}\right) = g(n) \tag{3}$$

where α is the shift and n is the number of contending CPEs. Let Attempt (m, n) be a function of maximum attempts allowed by the IEEE 802.22 network and the number of CPEs attempting to associate with the base station, defines the attempt number at which the majority of the contending CPEs can associate with the BS. Then,

$$\text{Attempt }(m,n) = m + g(n)$$

$$\text{Attempt }(m,n) = m + log_2\left(\frac{n}{80}\right) \quad (4)$$

Generally, it can be written as:

$$\text{Attempt }(m,n) = m + log_2\left(\frac{n}{n_1}\right)$$

where n_1 is base reference and $n_1 \geq n$, $log_2\left(\frac{n}{n_1}\right) \leq m$, Attempt $(m,n) > 0$

After exploiting equation (4), for $n = 5, 10, 20, 40, 80$, the specific attempt number can be computed. For example, $n = 40$, then, in this case, $\text{Attempt }(5{,}40) = 5 + log_2\left(\frac{40}{80}\right) = 5 + log_2\left(\frac{1}{2}\right) = 5 - 1 = 4$, which implies that at the 4th attempt, the maximum number of CPEs out of 40 contending CPEs can join the system at the given initial window size. From the obtained attempt number, the particular backoff stage (j),

$$j = \text{Attempt }(m,n) - 1 \quad (5)$$

is calculated and then put into equation (1) in order to achieve the corresponding collision probability. The number of contending CPEs with the desired attempt number along with the respective association probability is illustrated in Figure 6, whereas Figure 5 depicts the respective collision probability. It is clear that more than 80% of CPEs establish their association with the BS in the IEEE 802.22 network.

The expression given in (1) is dependent upon n and W_j. In order to reduce the number of dependencies and to make it simple, we procede as follows. Taking $W_0 = 8, 16, 32, 64$, the collision probability is dynamically calculated depending upon n and j, where j is also computed dynamically using equation (5). Now, for $n = 5$ at $W_0 = 8$, the value of j comes out to be 0 and the corresponding collision probability is 0.120728. This indicates that at this initial contention window size, the majority of the CPEs can associate with the BS at first backoff stage or at first attempt. At $W_0 = 16$, the collision probability is further reduced to 0.034405 while keeping the value of n and j the same. The reason of this reduction is due to the higher window size as the number of CPEs is less than the window size. Therefore, the given CPEs may choose different numbers from the range of internal backoff windows. It is also noted that when initial contention window size is doubled, the collision probability is decreased by 76%. Similarly when window size is increased from 16 to 32 and 32 to 64 for the same number of contending CPEs, the collision probability is decreased by 73% and 74% respectively, i.e. the maximum number of CPEs will associate with the BS with limited attempts as shown in Figure 7.

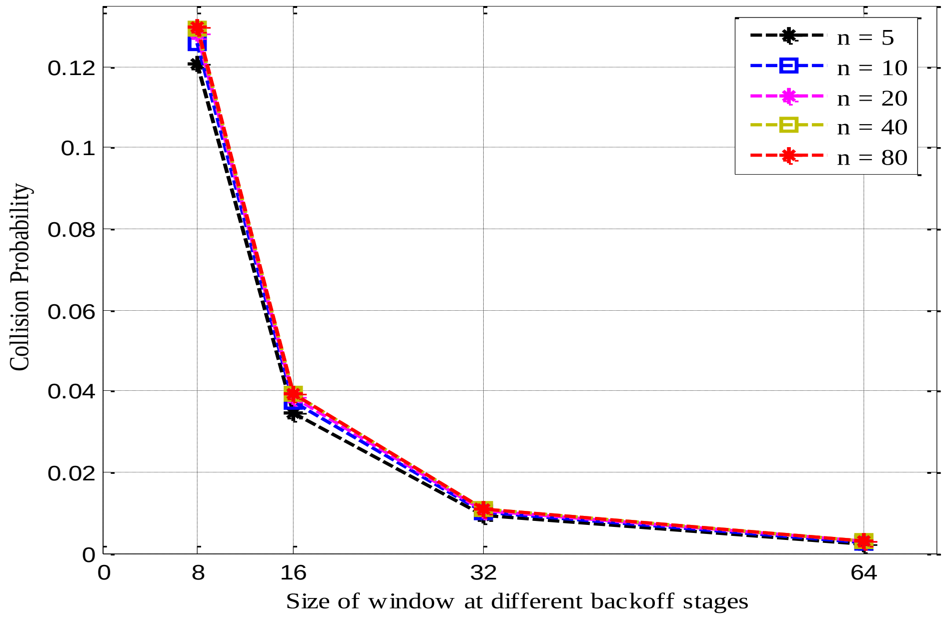

Figure 7: Collision probability at different values of n vs. contention window size at different backoff stages.

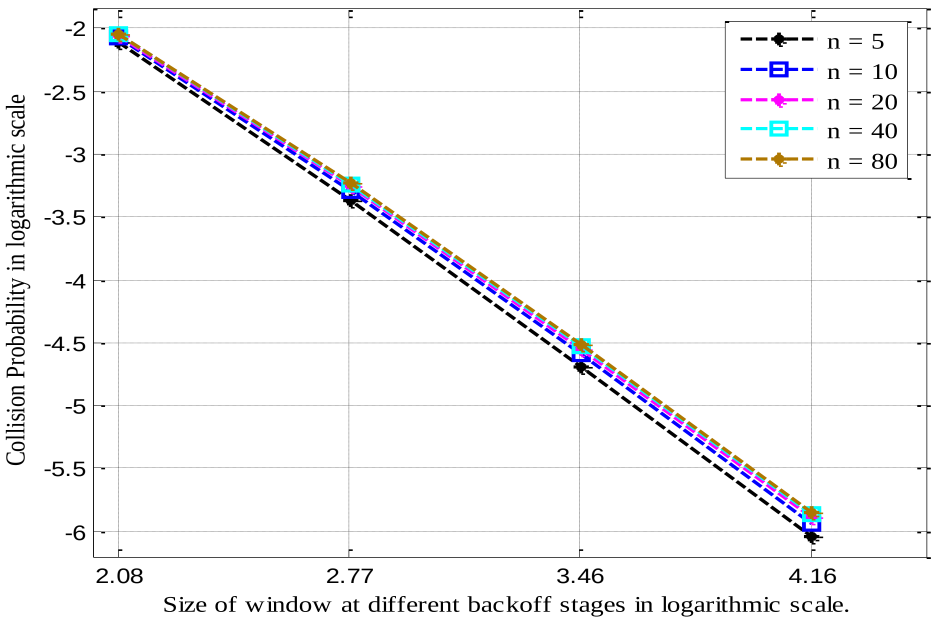

Figure 8: Collision probability vs. contention window size in logarithmic scale.

If n is doubled from 5 to 10, then the maximum backoff stage for this number is calculated dynamically as: Attempt $(m,n) = 5 + log_2\left(\frac{10}{80}\right) = 2,\ j = 2 - 1 = 1$. This means that when there are 10 CPEs in the sytem, the majority of the CPEs will contend upto the second backoff stage. Now at $W_0 = 8$, the collision probability is 0.125899, which is further reduced to 0.037193 when W_0 is doubled. This means that when the window size is increased by 100%, the collision probability is decreased by 70% for these numbers of contending CPEs. In this case there will be an increased number of chances for CPEs to become the part of WRAN cell. Again, when contention window size is gradually changed from 16 to 32 and then from 32 to 64, the collision probability is significantly reduced by 72% and 73% respectively.

Now for $n = 20$ and $n = 40$, the collision probabilities at $W_0 = 16, 32, 64$ reduce to 70%, 73%, 74% respectively from the original values at $W_0 = 8$. Similarly, for $n = 80$, the reduction in collision probabilities is 70%, 72%, 74% with respect to the original value at $W_0 = 8$. It is noted that when window size increases for the same number of contending CPEs, the collision probability reduces significantly. The reason behind this is, for a bigger window size, CPEs may choose bigger random numbers and hence there will be fewer chances of selecting the same transmission opportunity to transmit the ranging request, which leads to a reduced chance of collision.

In order to obtain a straight forward relationship between collision probability and initial contention window size, Figure 7 demonstrates the use of a logarithmic scale. Now, using Figure 8 and to obtain a mathematical expression, we apply least square approximation on γ data points, where $\{(x,y) \in \gamma, where\ x \in log\ P_c(j), y \in log\ W_j\}$. The estimated collision probability is achieved as follows:

$$\hat{P}_c(j) = e^a\ {W_j}^b \tag{6}$$

where a and b are parameters given by

$$b = \frac{\sum_{i=1}^{\gamma}\left(log\ W_j\right)_i \sum_{i=1}^{\gamma}(log\ P_c(j))_i - \gamma \sum_{i=1}^{\gamma}(log\ P_c(j))_i\left(log\ W_j\right)_i}{\left(\sum_{i=1}^{\gamma}\left(log\ W_j\right)_i\right)^2 - \gamma \sum_{i=1}^{\gamma}\left(log\ {W_j}^2\right)_i} \tag{7}$$

$$a = \frac{\sum_{i=1}^{\gamma}(log\ P_c(j))_i}{\gamma} - b\ \frac{\sum_{i=1}^{\gamma}\left(log\ W_j\right)_i}{\gamma} \tag{8}$$

The procedure for the derivation of these parameters is discussed in Appendix A. Now, using (7) and (8), we come up with

$$\hat{P}_c(j) = \begin{cases} e^{1.83}\, W_j^{-1.88} & for\; n = 5 \\ e^{1.85}\, W_j^{-1.87} & for\; n = 10 \\ e^{1.81}\, W_j^{-1.84} & for\; n = 20 \\ e^{1.83}\, W_j^{-1.84} & for\; n = 40 \\ e^{1.80}\, W_j^{-1.83} & for\; n = 80 \end{cases} \tag{9}$$

Taking averages and finally, we get

$$\hat{P}_c(j) = 6.17 \times \frac{1}{W_j^{1.85}} \qquad \text{for } n > 0 \tag{10}$$

where $W_j = 2^j\, W_0, \qquad W_0 \geq 2$

and $\quad j = \text{Attempt}(m,n) - 1, \quad \text{Attempt}\,(m,n) > 0$

The expression given in (10) gives the estimated collision probability at which the maximum number of contending CPEs can join the IEEE 802.22 network. In order to test the accuracy of the approximation, we compare estimated collision probability $\left(\hat{P}_c(j)\right)$ with numerically calculated collision probability $\left(P_c(j)\right)$ at different numbers of contending CPEs. As shown in Figure 9, for small values of n, i.e., $n = 5$, $\hat{P}_c(j)$ is 0.121443, then the respective error is 0.001006 which is close to 0 as can be seen from Figure 10. When n changes from 5 to 10, the approximation error reduces to 0.000354. Also, when n increases from 10 to 20, the approximation error further reduces to 0.000147. Similarly, when n moves from 20 to 40 and then from 40 to 80, the approximation error first reduces to 0.000052 and then reduces to 0.00013. It is clear that the approximation error goes on decreasing as we increase the number of contending CPEs. Hence, it is established that the approximation presented is good enough for all values of n.

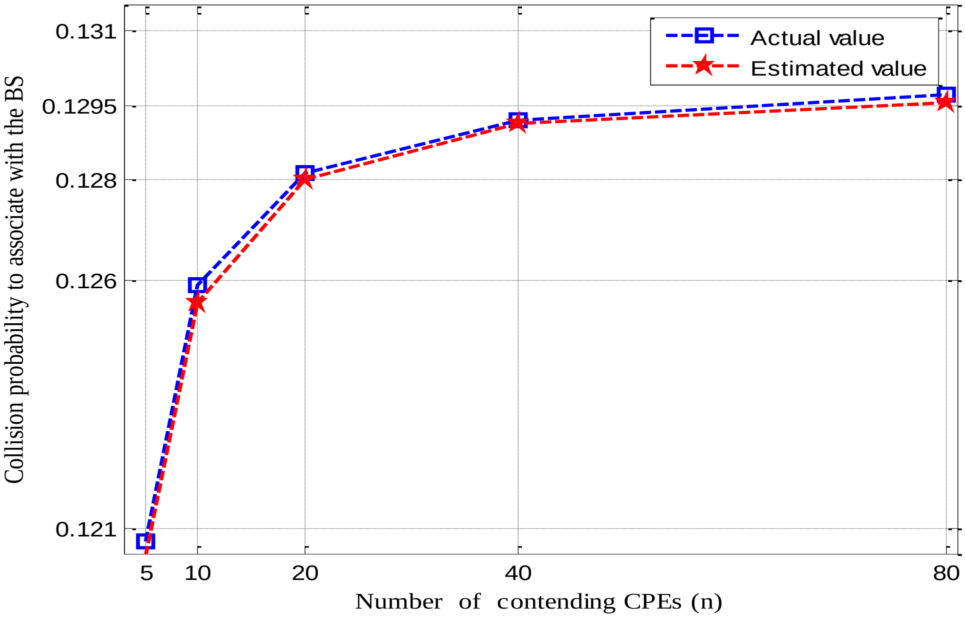

Figure 9: Actual and estimated collision probability at different number of contending CPEs.

Now, we find the average ranging success delay (D) experienced by the majority of the CPEs before associating with the BS. By exploiting equation (5), backoff stage (j) is first calculated depending upon the number of contending CPEs (n) and then using this value in the calculation of W_j based on the size of the initial contention window (W_0). After using this information in (10) and (2), $P_c(j)$ and D can be computed. Assume that $W_0 = 8$. At $n = 5$ then $D = 13.54\, ms$. This indicates that the majority of the CPEs will associate in less than 20 ms as shown in Figure 11. When n is doubled, i.e. $n = 10$ then $D = 27.57\, ms$. Majority of the CPEs will experience sufficient delay due to 2nd backoff stage. When the number of contending CPEs increases from 10 to 20, then the majority have to wait about 78 ms for association. This is due to approaching the 3rd backoff stage, where each CPE has to select an opportunity slot from a larger contention window than the initial contention window specified. The same is true for $n = 40$. Finally, when n changes from 40 to 80, then D increases from 224 ms to 457 ms. The majority of the CPEs will take an increased amount of time for association with the BS due to availing maximum backoff stages. It is clear that the average ranging success delay of the majority of the CPEs is highly dependent upon the number of contending CPEs as well as the initial contention window size.

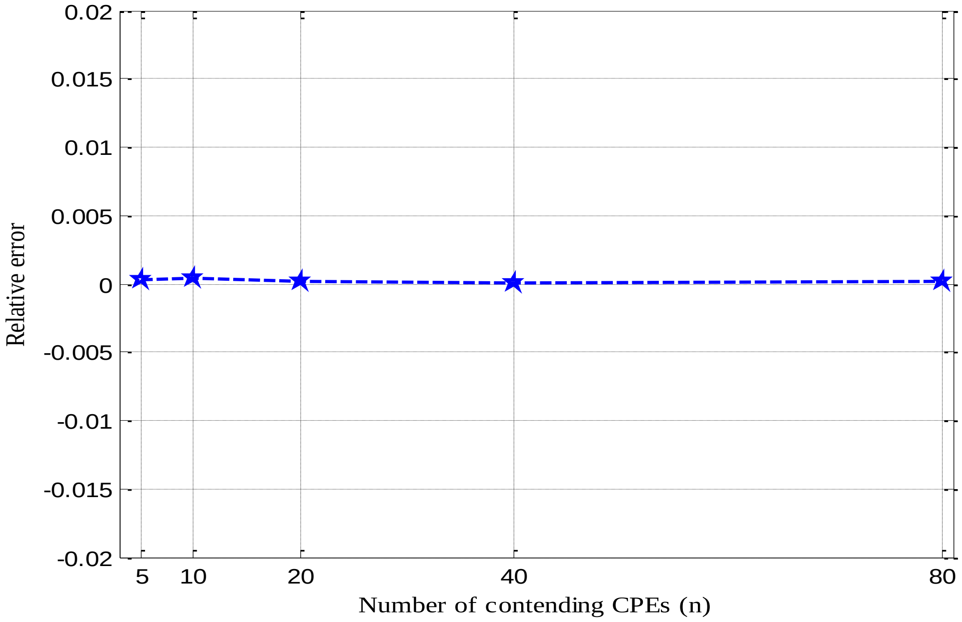

Figure 10: The relative error of the approximation.

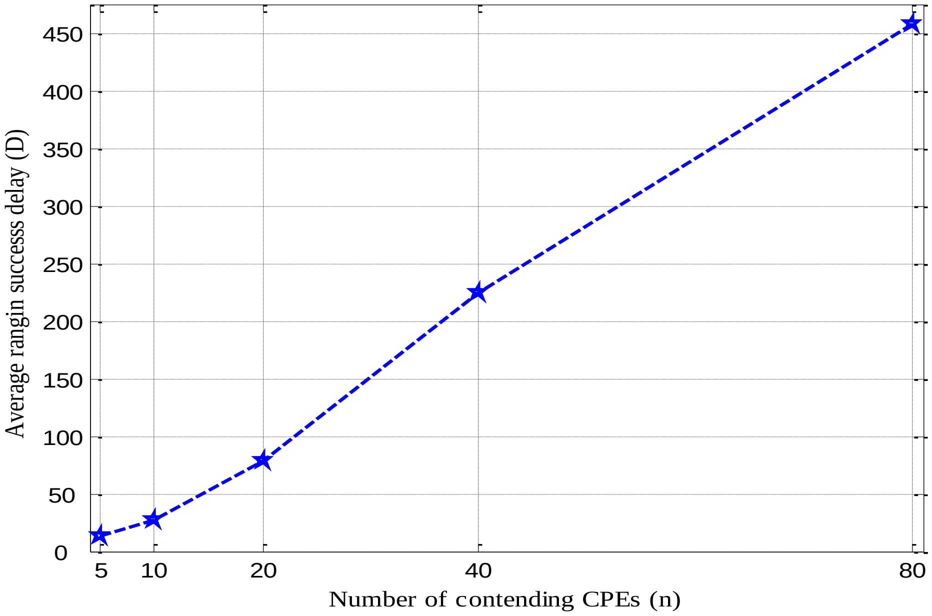

Figure 11: Average ranging success delay for n number of contending CPEs at $W_0 = 8$.

5. CONCLUSION

After turning on the CPE, its first and most important job is to associate with the BS. The association probability is highly dependent upon the number of contending CPEs as well as the attempt number. The number of attempts which a CPE can avail at association process is fixed in the IEEE 802.22 network. This work presents a mathematical framework to help the BS in finding the particular attempt at which the maximum number of CPEs from the contending CPEs has been reached in terms of becoming the part of WRAN cell. Depending upon particular attempts, the collision probability with respect to contention window size is approximated. In addition, the average ranging success delay experienced by the majority of the CPEs is also computed.

Appendix A

We derive closed form expressions for parameters a and b as follows:

Let e be the error between the actual collision probability and the estimated collision probability, then

$$e_i = log\left(P_c(j)\right)_i - a - b\,log\left(W_j\right)_i \tag{11}$$

Let E be the sum of the squared errors, then

$$E = \sum_{i=1}^{\gamma}(e_i)^2 = \sum_{i=1}^{\gamma}\left(log\left(P_c(j)\right)_i - a - b\,log\left(W_j\right)_i\right)^2 \tag{12}$$

For finding the values of a and b that minimize the sum of the squared errors, we put $\frac{\partial E}{\partial a} = 0$ and $\frac{\partial E}{\partial b} = 0$, therefore,

$$\gamma a + b\sum_{i=1}^{\gamma}\left(log\,W_j\right)_i = \sum_{i=1}^{\gamma}(log\,P_c(j))_i \tag{13}$$

$$a\sum_{i=1}^{\gamma}\left(log\,W_j\right)_i + b\sum_{i=1}^{\gamma}\left(log\,{W_j}^2\right)_i = \sum_{i=1}^{\gamma}(log\,P_c(j))_i\left(log\,W_j\right)_i \tag{14}$$

By doing, $(13) \times \sum_{i=1}^{\gamma}\left(log\,W_j\right)_i - (14) \times \gamma$, we get

$$b = \frac{\sum_{i=1}^{\gamma}\left(log\,W_j\right)_i \sum_{i=1}^{\gamma}(log\,P_c(j))_i - \gamma\sum_{i=1}^{\gamma}(log\,P_c(j))_i\left(log\,W_j\right)_i}{\left(\sum_{i=1}^{\gamma}\left(log\,W_j\right)_i\right)^2 - \gamma\sum_{i=1}^{\gamma}\left(log\,{W_j}^2\right)_i} \tag{15}$$

Now by using (14), we achieve a as:

$$a = \frac{\sum_{i=1}^{\gamma}(log\,P_c(j))_i}{\gamma} - b\,\frac{\sum_{i=1}^{\gamma}\left(log\,W_j\right)_i}{\gamma} \tag{16}$$

References

[1] Liang Y-C, Hoang AT, Chen H-H. Cognitive radio on TV bands: a new approach to provide wireless connectivity for rural areas. *IEEE Wireless Communication* 2008; 15(3): 16–22.

[2] Shellhammer SJ, Sadek AK, Zhang W. Technical challenges for cognitive radio in the TV white space spectrum. *IEEE Information Theory and Applications Workshop* 2009; 323-333.

[3] Aldabbagh G, Bakhsh ST, Akkari N, Tahir S, Khan S, Cioffi J. Distributed dynamic load balancing in a heterogeneous network using LTE and TV white spaces. *Wireless Networks* 2015; 21(7): 2413-2424.

[4] Saeed A, Ibrahim M, Harras KA, Youssef M. Toward dynamic real-time geo-location databases for TV white spaces. *IEEE Network* 2015; 29(5): 76-82.

[5] Haykin S. Cognitive radio: Brain-empowered wireless communications. *IEEE Journal on Selected Areas in Communications* 2005; 23: 210–220.

[6] Tawk Y, Costantine J , Christodoulou CG. Cognitive-radio and antenna functionalities: a tutorial. *IEEE Transaction on Antennas and Propagation Magazine* 2014; 56(1): 231 – 243.

[7] McLean RK et. al. An architecture for coexistence with multiple users in frequency hopping cognitive radio networks. *IEEE Transaction on Selected Areas in Communications* 2014; 32(3): 563 – 571.

[8] Hasegawa M, Hirai H, Nagano K, Harada H, Aihara K. Optimization for centralized and decentralized cognitive radio networks. *Proceedings of the IEEE* 2014; 102(4): 574-584.

[9] Qi J, Shimamoto S. A cognitive mobile sensor network for environment observation. *Telematics Informatics (Elsevier Journal)* 2012; 29(1): 26–32.

[10] Bicen AO, Sahin AB, Akan OB. Spectrum-aware underwater networks: cognitive acoustic communications. *IEEE Vehicle Technology Magazine* 2012; 7(2): 34–40.

[11] Delgado M, Rodríguez B. Opportunities for a more efficient use of the spectrum based in cognitive radio. *IEEE Latin America Transactions* 2016; 14(2): 610-616.

[12] Khan AA, Rehmani MH, Reisslein M. Cognitive radio for smart grids: survey of architectures, spectrum sensing mechanisms, and networking protocols. *IEEE Communications Surveys & Tutorials* 2016; 18(1): 860-898.

[13] Ahmed E, Gani A, Abolfazli S, Yao LJ, Khan SU. Channel assignment algorithms in cognitive radio networks: taxonomy, open issues, and challenges. *IEEE Communications Surveys & Tutorials* 2016; 18(1): 795-823.

[14] Afzal H, Awan I, Mufti MR, Sheriff RE. Modeling of initial contention window size for successful initial ranging process in IEEE 802.22 WRAN cell. *Simulation, Modelling, Practice and Theory (Elsevier Journal)* 2015; 51: 135-148.

[15] Afzal H, Awan I, Mufti MR, Sheriff RE. Modelling and analysis of customer premise equipments registration process in IEEE 802.22 WRAN cell. *Journal of Systems and Software (Elsevier Journal)* 2014; 98: 107-116.

[16] Bourdena A, Makris P, Skoutas D N, Skianis C, Kormentzas G, Pallis E, Mastorakis G. Joint radio resource management in cognitive networks: TV white spaces exploitation paradigm. Evolution of Cognitive Networks and Self-Adaptive Communication Systems, IGI Global, 2013, 50-80.

[17] 802.22-2011 - IEEE Standard for Information Technology--Telecommunications and information exchange between systems Wireless Regional Area Networks (WRAN)--Specific requirements Part 22: Cognitive Wireless RAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications: Policies and Procedures for Operation in the TV Bands 2011; 1-680.

[18] Afzal H, Awan I, Mufti MR. Mathematical modeling of association attempt with the base station for maximum number of customer premise equipments in the IEEE 802.22 network. *IEEE 3rd International Conference on Future Internet of Things and Cloud (FiCloud) Italy, Rome* 2015; 289–292.

[19] Afzal H, Awan I, Mufti MR, Sheriff RE. Performance analysis of contending customer equipment in wireless networks. *Journal of Systems and Software (Elsevier Journal)* 2016; 117: 357-365.

[20] Gardellin V, Das SK, Lenzini L. A fully distributed game theoretic approach to guarantee self-coexistence among WRANs. *IEEE INFOCOM-2010 Workshop on Cognitive Wireless Communications and Networking, San Diego, CA, USA* 2010; 1-6.

[21] Sengupta S, Chandramouli R, Brahma S, Chatterjee M. A game theoretic framework for distributed self-coexistence among IEEE 802.22 networks. *Proceeding of the IEEE GLOBECOM* 2008; 1–6.

[22] 802.22b-2015 - IEEE Standard for information technology--telecommunications and information exchange between systems - wireless regional area networks (WRAN)--specific requirements - Part 22: cognitive wireless RAN medium access control (MAC) and physical layer (PHY) specifications: policies and procedures for operation in the TV Bands - amendment 2: enhancement for broadband services and monitoring applications.

[23] Gardellin V, Das SK, Lenzini L. Self-coexistence in cellular cognitive radio networks based on the IEEE 802.22 standard. *IEEE Wireless Communications* 2013; 20(2): 52-59.

[24] Bian K, Park J-M, Chen L, Li, X. Addressing the hidden terminal problem for heterogeneous coexistence between TDM and CSMA networks in white space. *IEEE Transactions on Vehicular Technology* 2014; 63(9): 4450-4463.

[25] Popescu V, Fadda M, Morgade J, Angueira P. Co-channel and adjacent channel interference and protection issues for DVB-T2 and IEEE 802.22 WRAN operation. *IEEE Transactions on Broadcasting* 2014; 60(4): 693-700.

[26] Popescu V, Fadda M, Murroni M. Performance analysis of IEEE 802.22 wireless regional area network in the presence of digital video broadcasting – second generation terrestrial broadcasting services. *IET Communication* 2016; 10(8): 922–928.