Analysis of R/C frames considering cracking effect and plastic hinge formation

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Abstract. The design of reinforced concrete buildings must satisfy the serviceability stiffness criteria in terms of maximum lateral deflections and inter story drift in order to prevent both structural and non-structural damages. Consideration of plastic hinge formation is also important to obtain accurate failure mechanism and ultimate strength of reinforced concrete frames. In the present study, an iterative procedure has been developed for the analysis of reinforced concrete frames with cracked elements and consideration of plastic hinge formation. The ACI and probability-based effective stiffness models are used for the effective moment of inertia of cracked members. Shear deformation effect is also considered, and the variation of shear stiffness due to cracking is evaluated by reduced shear stiffness models available in the literature. The analytical procedure has been demonstrated through the application to three reinforced concrete frame examples available in the literature. It has been shown that the iterative analytical procedure can provide accurate and efficient predictions of deflections and ultimate strength of the frames studied under lateral and vertical loads. The proposed procedure is also efficient from the viewpoint of computational time and convergence rate. The developed technique was able to accurately predict the locations and sequential development of plastic hinges in frames. The results also show that shear deformation can contribute significantly to frame deflections.

Keywords: reinforced concrete (R/C) structure, nonlinear analysis, analytical method, static analysis, iteration method

1. Introduction

In recent years, the importance of serviceability limit state for design becomes more apparent due to the availability of high strength steel and concrete coupled with availability of more accurate and efficient analytical procedures. In the design of reinforced concrete buildings, the control of lateral drift is a critical design criterion that must be satisfied to prevent large second-order P-delta effects. Due to the low tensile strength of concrete, cracking, which is primarily load dependent, is an inevitable phenomenon occurring at service loads, generally leading to reduction in flexural and shear stiffnesses of reinforced concrete members and, consequently, deflection increase. The formation of plastic hinges also decreases the structure stiffness and, thus, causing the deflections to increase significantly. It is therefore important to consider the plastic hinge formation together with concrete cracking in the structural analysis and design of reinforced concrete frames in order to obtain more accurate prediction of deflections and capacity of reinforced concrete frames.

Two basic types of algorithm, namely the finite element and direct stiffness methods, have been extensively developed to analyze reinforced concrete structures. The finite element method directly uses the stress and strain as the variant, while the direct stiffness method is based on internal forces. Over the past three decades, significant advancements to the finite element method for the analysis of reinforced concrete frames have been proposed, including the type of finite elements used, the constitutive models adopted and the methods of stiffness evaluation and mesh discretization. Several researchers (Ingraffea and Grestle 1985, Polak 1995, Chan et. al. 2000, Wang and Hsu 2001, Kwak and Song 2002, Kwak and Kim 2004, Spiliopoulos and Lykidis 2006, Stramandinoli and Rovere 2008) proposed constitutive models for reinforced concrete following the rules of nonlinear elasticity and plasticity. These include the softening effects in compression, the effect of cracking in tension and shear retention. On the other hand, the procedures concerning the type of finite elements and mesh discretization can be classified as microelement and macroelement approaches. The microelement is defined in such way that the structure is divided into many small finite elements including the two dimensional and three dimensional elements modeling concrete, bar elements modeling steel, and discrete crack representation modeling cracks. However, the macroelement approach incorporates such factors as the cracking effect, mechanical aggregate interlock and bond-slip behavior into a comprehensive constitutive model of reinforced concrete. In this approach, each finite element represents both concrete and steel, and the local phenomena are incorporated into a constitutive model used to obtain stiffness matrix of a macroelement. However, most of these analyses have restrictions in their application to large scale reinforced concrete buildings (Chen 1982, Berzegar 1989, Nilson 1967, Vecchio and Emara 1992, 1993, Bratina et. al. 2004, Karthiga et al. 2014).

A simplified method for the analysis of reinforced concrete frames with cracked beam and column elements was

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developed by Dundar and Kara (2007), in which the probability-based effective stiffness models, ACI (1995) and CEB (1985) were used to evaluate the effective moment of inertia of the cracked members. Shear deformations were also taken into account in the analysis and reduced shear stiffness was considered by using effective shear modulus models. However, the effect of plastic hinge formation on the behavior of reinforced concrete frames was not considered in the analysis. The formation of plastic hinges causes reduction in the structure stiffness and, thus, an increase in deflections of frames. Also consideration of plastic hinge formation in the analysis is important to obtain the failure mechanism and ultimate capacity of frames.

Several experimental studies have been conducted on the determination of the plastic hinge length of reinforced concrete members (Tang et al. 2016, Barrera et al. 2011, Elmenshawi et al. 2012, Yang et al. 2013, Bae and Bayrak 2008). On the other hand, a large number of techniques and models have been proposed in the past decades by numerous researchers to investigate the inelastic behaviour of plastic hinge length for accurate evaluation of the reinforced concrete structural ductility (Panagiotakos and Fardis 2001; Lu et al. 2005; Berry et al. 2008; Elmenshawi et al. 2012, Ning and Li 2016, Babazadeh et al. 2016, Yang et al. 2013, Dadi and Agarwal 2015). However, due to the complexity and nonlinearity involved in the plastic hinge mechanism, large differences are encountered when investigating these models.

Following a comprehensive finite element analysis of plastic hinge region of reinforced concrete beams, Zhao et al. (2014) concluded that none of the existing methods for the prediction of plastic hinge length is adequate and correctly included all major factors affecting the load carrying and deformation capacities of reinforced concrete members. In contrast, Lopez et al. (2016) showed that the plastic hinge modelled with few empirical expressions available in the literature would be adequate for simulating the behavior of reinforced concrete buildings located in seismic areas. Lopez et al. (2016) also suggested other equations for the yield moment, chord rotation and ultimate chord rotation of rectangular RC sections based on genetic algorithm. They concluded that proposed equations provide more accurate results than other expressions and reduce computational time needed to simulate the yield and ultimate behaviour of RC rectangular sections.

In practice, the analysis of reinforced concrete frames are usually carried out by using linear elastic models which either neglect the cracking effect or consider it by reducing the stiffness of members arbitrarily. If cracking occurs in some members due to loads, the flexural and shear stiffnesses of these cracked members will decrease, resulting in additional deflection and a redistribution of internal forces. The formation of plastic hinges at the ends of members has also a significant effect on the behavior and strength of frames. The predictions of plastic hinges and progressive collapse of reinforced concrete structures are essential to prevent catastrophic collapse of structures. Therefore, an analytical model which can include the effects of concrete cracking on the flexural and shear stiffnesses of members and consider the plastic hinge formation in the analysis would be very useful.

In the present study, a computer program based on the iterative procedure has been developed for the analysis of reinforced concrete (R/C) frames with cracked beam and column elements. The formation of plastic hinges at the ends of members is also taken into account in the analysis. The analytical procedure is based on the stiffness matrix method and involved the plastic hinge formations under increasing load until a failure condition is achieved. This method accounts for the zero-length hinge formation without considering the finite-length hinges. In obtaining the flexibility influence coefficients, a cantilever beam model is used. The effective flexural stiffness of a cracked member is evaluated by the ACI and probability-based effective stiffness model. In the analysis, shear deformation effect is also taken into account and the variation of shear stiffness due to cracking is considered by employing reduced shear stiffness models available in the literature. The analytical procedure has been compared with experimental results of three reinforced concrete frames tested in the literature.

2. Stiffness Models & Analysis Method

2.1 Effective flexural and shear stiffness models for cracked members

The flexural stiffness of a concrete member varies along its length due to the presence of cracks that can occur from the applied loading. At crack locations, concrete carries essentially zero tension. Between cracks, however, concrete participates in resisting tensile stresses because of bond between reinforcement and concrete. This effect is often referred to as tension stiffening and is taken into account with the effective moment of inertia I_{eff} (Cosenza 1990). Fig. 1 presents a typical moment curvature relationship taking into account tension stiffening. The effective moment I_{eff} of inertia in a region subjected to a moment M greater than the section cracking moment M_{cr} lies between the limit values of I_1 and I_2, which are the moments of inertia of the gross uncracked and cracked transformed sections, respectively.

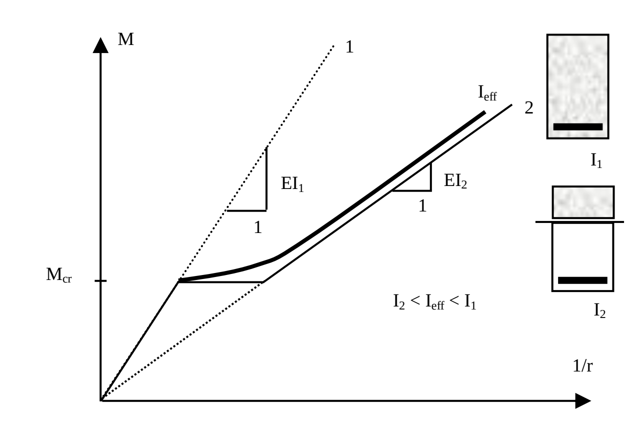

Fig. 1 Moment-curvature relationships of reinforced concrete section

In the present study, the ACI model (ACI 318-14), which includes the effect of cracking and participation of tensile concrete to flexural rigidities, is adopted as given below:

$$I_{eff} = \left(\frac{M_{cr}}{M}\right)^m I_1 + \left[1 - \left(\frac{M_{cr}}{M}\right)^m\right] I_2 \quad \text{for } M \geq M_{cr} \tag{1a}$$

$$I_{eff} = I_1 \quad \text{for } M < M_{cr} \tag{1b}$$

where m=3. This equation was first proposed by Branson (1963) with m=4 when I_{eff} is required for the calculation of curvature of an individual section. The cracking moment M_{cr} is calculated using the following equation:

$$M_{cr} = \frac{(f_r + \sigma_v) I_1}{y_t} \tag{2}$$

where σ_v is the axial compressive stress, f_r is the flexural tensile strength of concrete, and y_t is the distance from the centroid of gross section to extreme fiber in tension.

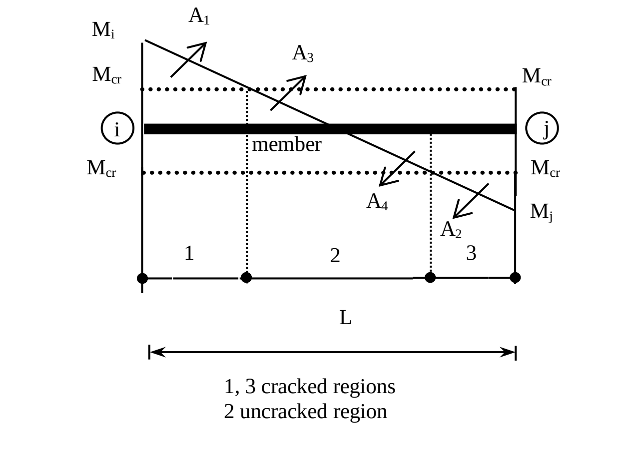

Fig. 2 Cracked and uncracked regions of the member

In the literature (Sakai and Kakuta 1980, Cosenza 1990), the effective moment of inertia given by the ACI (2014) is perhaps the most widely used and accurate among the commonly accepted simplified methods for the estimation of instantaneous deflection. Although ACI model is usually considered for beams, in the present study this model is also used for columns but the axial force is taken into account in the determination of the cracking moment.

In addition to the ACI model (2014), the probability-based effective stiffness model (Chan et. al. 2000) has been considered for the calculation of the effective flexural stiffness of cracked members. In the probability-based effective stiffness model, the effective moment of inertia is also obtained as the ratio of the area of moment diagram segment over which the working moment exceeds the cracking moment M_{cr} to the total area of moment diagram for each cracked element by the following form (see Fig.2):

$$A_{cr} = A_1 + A_2 = \int_{M(x) \geq M_{cr}} M(x) \tag{3a}$$

$$A_{uncr} = A_3 + A_4 = \int_{M(x) < M_{cr}} M(x) \tag{3b}$$

$$A_t = A_{cr} + A_{uncr} \tag{3c}$$

$$P_{uncr}[M(x) < M_{cr}] = \frac{A_{uncr}}{A_t} \tag{3d}$$

$$P_{cr}[M(x) \geq M_{cr}] = \frac{A_{cr}}{A_t} \tag{3e}$$

$$I_{eff} = P_{uncr} I_1 + P_{cr} I_2 \tag{3f}$$

where A_{cr} is the area of moment diagram segment over which the calculated moment due to the applied load exceeds the cracking moment M_{cr} and A_t is the total area of moment diagram. In the same equation, P_{cr} and P_{uncr} are the probability of occurrence of cracked and uncracked sections, respectively.

Both the ACI and Probability-based effective stiffness models represent a weighted average of the uncracked (E_cI_1) and cracked (E_cI_2) flexural stiffness of concrete members. However, they adopt different approach in determining the weight of uncracked and cracked moment of inertia of concrete members to evaluate the effective stiffness of the cracked member. Hence the probability-based effective stiffness model can be considered as an alternative model over ACI model for the effective flexural stiffness of cracked members in the analysis of concrete frames. Comparisons of both techniques against experimental results will be presented later in this paper.

Shear deformation in frame structures can be significant and thus can be of practical importance in their design and behavior. The effective shear modulus of concrete due to cracking is accounted for by employing the models developed by Al-Mahaidi (1978) and Yuzugullu and Schnobrich (1973). Al-Mahaidi (1978) suggested the following hyperbolic expression for the reduced shear stiffness G_c to be employed in the constitutive relation of cracked concrete

$$\bar{G}_c = \frac{0.4 G_c}{\varepsilon_1 / \varepsilon_{cr}} \tag{4}$$

where G_c is the elastic shear modulus of uncracked concrete, ε_1 is the principal tensile strain normal to the crack and ε_{cr} is the cracking tensile strain. Details of these models can be found in Dundar and Kara (2007).

In the computer program developed in the present study, the aforementioned models are adopted for the effective flexural and shear stiffnesses of cracked concrete.

2.1 Basic equations and formulation of analytical procedure

In the present study, the analytical process is based on the stiffness matrix method. Due to the nonlinear behavior of materials, the load is incrementally increased and, when a certain loading renders the determinant of the system stiffness matrix negative, the analysis is terminated. The analytical procedure deals with the plastic hinge formation together with concrete cracking in reinforced concrete members. A plastic hinge is assumed to be developed at a

certain section when the moment exceeds the plastic moment capacity of such section obtained by calculating the sectional moment carrying capacity of individual members at yielding of longitudinal steel reinforcement. This procedure also accounts for the zero-length hinge formation without considering finite-length hinges.
In the current analysis, the member is assumed to consist of two zones as shown in Fig 3. The first zone is the plastic hinge zone assumed to have zero length. The second zone is also cracked and uncracked regions of the member.
In the evaluation of the member stiffness equation, the flexibility influence coefficients of a member are first obtained, and then using compatibility conditions and equilibrium equations, the stiffness matrix of a member with some regions in cracked state are evaluated. The basic formulations of the flexibility influence coefficients of a member are given below and full details of the formulations can be found in Dundar and Kara (2007). When the plastic hinges are active at the ends of the member, the development of the member stiffness matrix is also described in the following section.
Fig. 3 shows a typical member, and positive end forces with corresponding displacements. A cantilever model is used for computing the relations between nodal actions and basic deformation parameters of a general planar element (Fig. 4). The basic deformation parameters of a general planar element may be established by applying unit loads in turn in the directions of 1, 2 and 3. Then, the compatibility conditions give the following equation in matrix form:

$$\begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} = \begin{bmatrix} f_{11} & 0 & 0 \\ 0 & f_{22} & f_{23} \\ 0 & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} \quad (5)$$

in which, f_{ij} is the displacement in the i-th direction due to the application of a unit load in the j-th direction, and can be obtained by using the principle of virtual work as follows:

$$f_{ij} = \int_0^L \left(\frac{M_i M_j}{E_c I_{eff}} + \frac{V_i V_j}{G_c A} s + \frac{N_i N_j}{E_c A} \right) dx \quad (6)$$

In Eq. (6), M_i, M_j, V_i, V_j, N_i and N_j are the bending moments, shear forces and axial forces due to the application of a unit load in i-th and j-th directions, respectively, E_c denotes the modulus of elasticity of concrete, s and A are the shape factor and cross sectional area of element section, respectively.
The stiffness influence coefficients are simply obtained by inverting the flexibility matrix in Eq. (4) using the equilibrium conditions as follows

$$\begin{gathered} k_{11} = 1 / f_{11} = E_c A / L \\ k_{22} = f_{33} / (f_{22} f_{33} - f_{32} f_{23}) \\ k_{23} = k_{32} = - f_{23} / (f_{22} f_{33} - f_{32} f_{23}) \\ k_{33} = \ f_{22} / (f_{22} f_{33} - f_{32} f_{23}) \\ k_{12} = k_{13} = k_{31} = 0. \\ k_{52} = - k_{55} = -k_{22} \\ k_{53} = -k_{23} \\ k_{62} = -k_{65} = k_{22} L - k_{32} \\ k_{66} = k_{33} + k_{22} L^2 - k_{32} L \end{gathered} \quad (7)$$

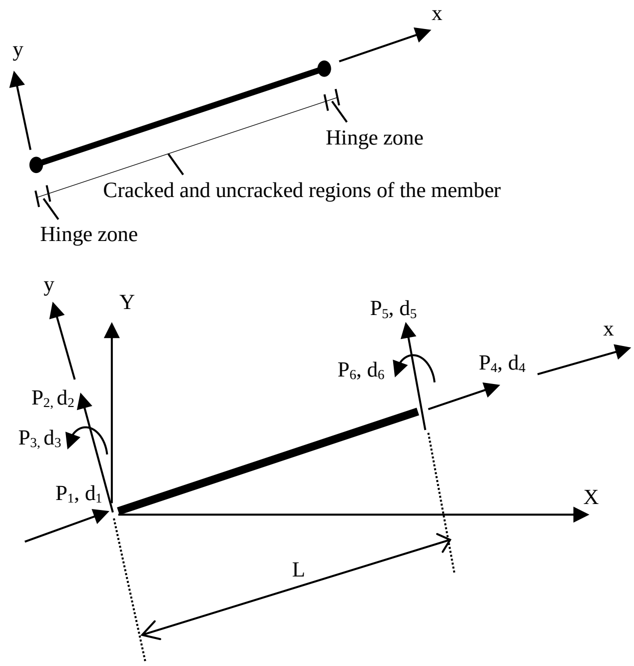

Fig. 3 A typical two dimensional member with positive end forces and displacements

Fig. 4 A cantilever model for computing the relations between the nodal actions and basic deformation parameters

The reinforced concrete members crack at relatively low load levels and have varying degrees of cracking ranging from uncracked to fully cracked regions due to the lateral and vertical loads. It should be noted that, since the member has cracked and uncracked regions, the integral operations used for obtaining the flexibility influence coefficients are carried out in each region individually.
When only the cracking of structural concrete is considered in the analysis, the proposed procedure can be applied to reinforced concrete frames without subdividing the member under distributed load into many elements (Kara and Dundar 2009, Tanrikulu et. al. 2000). In such case, the member fixed-end forces for the case of point and uniformly distributed loads are evaluated by means of the compatibility and equilibrium conditions. If the plastic hinge formation is taken into account together with the concrete cracking in the analysis, the reinforced concrete frames are analyzed by dividing the member under distributed loads into many elements and applying the distributed loads as equivalent concentrated loads at nodes

distributed along the length of the member. The program also automatically divides the member under distributed loads without spending extra effort. The cracked member stiffness equation can be obtained as

$$\underline{P} = \underline{k}\,\underline{d} \tag{8}$$

where $\underline{k}$ (6x6) is the stiffness matrix, $\underline{d}$ (6x6) is the displacement vector and $\underline{P}$ (6x1) is the end force vector of the member. Eq. (8) is given in the member coordinate system (x, y). Hence, it should be transformed to the structure coordinate system (X, Y).

moment of the section), a plastic hinge is formed at the same section location. The plastic moment of each member of the frame structure is obtained by calculating the sectional moment carrying capacity of the individual members at the yielding of the longitudinal reinforcement. In obtaining the plastic moment capacity of members, the equivalent rectangular stress block of concrete in compression proposed by ACI (2014) is adopted and the reinforcing steel bars are assumed to be elasto-plastic ($\sigma_s = E_s{*}\varepsilon_s \le f_y$ and σ_s and ε_s are the stress and strain in reinforcing steel bars, respectively; E_s and f_y are the

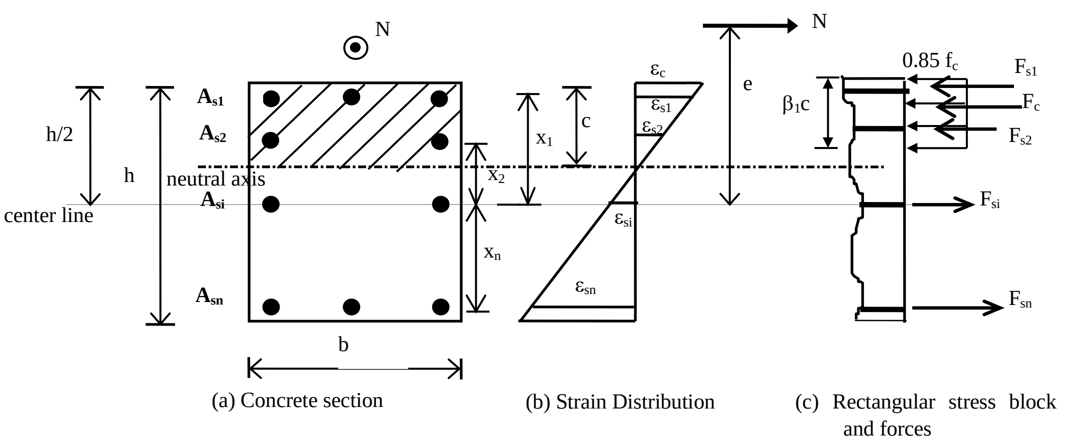

Fig. 5 Strains, stresses and forces on the reinforced concrete section

In the cracked regions where the applied moment is larger than or equal to the cracking moment, the flexural tensile cracks develop in the tensile fibers of the flexural members, and, consequently, I_{eff} and $\overline{G}_c$ vary with M along the region. Therefore, the integral values in these regions should be determined by a numerical integration technique. The flexural and shear stiffnesses of a cracked member vary according to the amount of crack formation along the member. Changes in the stiffness of the cracked members lead to a certain transfer of the internal forces of these members to other uncracked members, resulting in cracking of some of otherwise uncracked members. The variation of the effective moment of inertia and effective shear modulus of concrete in the cracked regions necessitates the redistribution of the internal moments and forces in the structure. The formation of plastic hinges at the ends of the member also influences the redistribution of the internal forces. Hence, iterative procedure should be implemented to obtain the final deflections and internal forces of the structure.

2.3 Analysis of the effect of plastic hinge formation on member stiffness

In a reinforced concrete member, a plastic hinge takes place in the critical section, where tensile reinforcing bars reach yielding due to external load. The bending moment at which a plastic hinge is formed at a section of a structure is called the section plastic moment M_p. When the moment at any section exceeds its plastic moment, M_p, (i.e. the yield

modulus of elasticity and yield strength of steel, respectively). The interaction between bending and axial force is also considered in the analysis. Details of the equations used for obtaining the plastic moment capacity of the member are given in Ersoy and Ozcebe (2014) (See Fig. 5).

The plastic hinge formation leads to a reduction in the stiffness and thus an increase in the deflection of reinforced concrete frames. The formation of plastic hinges has also a significant effect on the behavior and failure mechanism of frames. The shear capacity of members is assumed to be sufficient to carry shear forces that develop in the reinforced concrete members. However, the modifications in the member stiffness matrix in case of formation of plastic hinges at member ends are explained below.

If a member includes a plastic hinge at one end, say end i, and if the rotation of this hinge is treated as an extra degree of freedom, the rotation at end i of the member is $d_3 + d_{3h}$ where d_3 is the rotation of the node at end i and d_{3h} is the additional rotation due to the plastic hinge formation. The stiffness equations for a typical member are given in Eq. (9) below.

The first six rows and columns of the member stiffness matrix contain the usual terms for concrete cracking analysis. The hinge terms in the seventh row and column repeat the terms in the third row and column. A similar additional eighth row and column arise when the plastic hinge is formed at end j of the member (See Fig. 6).

$$\begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ M_{pi} \\ M_{pj} \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} & k_{15} & k_{16} & k_{13} & k_{16} \\ k_{21} & k_{22} & k_{23} & k_{24} & k_{25} & k_{26} & k_{23} & k_{26} \\ k_{31} & k_{32} & k_{33} & k_{34} & k_{35} & k_{36} & k_{33} & k_{36} \\ k_{41} & k_{42} & k_{43} & k_{44} & k_{45} & k_{46} & k_{43} & k_{46} \\ k_{51} & k_{52} & k_{53} & k_{54} & k_{55} & k_{56} & k_{53} & k_{56} \\ k_{61} & k_{62} & k_{63} & k_{64} & k_{65} & k_{66} & k_{63} & k_{66} \\ k_{31} & k_{32} & k_{33} & k_{34} & k_{35} & k_{36} & k_{33} & k_{36} \\ k_{61} & k_{62} & k_{63} & k_{64} & k_{65} & k_{66} & k_{63} & k_{66} \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \\ d_6 \\ d_{3h} \\ d_{6h} \end{bmatrix} \qquad (9)$$

In the present study, a step-by-step incremental load is applied. In each loading step, iterative procedure is employed where the formation of plastic hinges and cracks occurring in the members are examined. When plastic hinges are formed, they have to be considered in the following steps of analysis. The iterative procedure is applied until the convergence criterion, which will be explained in the next section, is less than a predefined tolerance. In the iterative analytical procedure, member equations are first obtained and then the system stiffness matrix and system load vector are assembled. Finally, the system displacements and member end forces are obtained by solving the system equation. The system stiffness matrix and load vector are obtained by using code number technique. Additional rotation due to plastic hinge at the end of the member is also inserted to the system stiffness matrix by using this technique. The procedure is repeated step by step in all iterations. The analysis is terminated when sufficient plastic hinges have formed for the structure to have lost its stiffness, i.e. the determinant of the structure stiffness matrix equals to zero or negative.

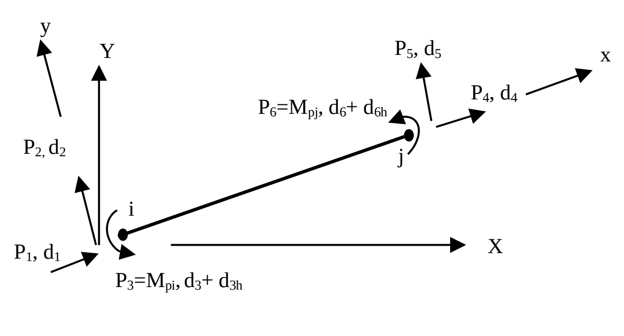

Fig. 6 The member end forces and displacements with plastic hinges at each end

3. Applications, Results and Discussions

In the present study, a general purpose computer program developed for the analysis of reinforced concrete frames based on the iterative procedure is coded in FORTRAN 77 language. The flow chart of the solution procedure of the program is presented in Fig. 7. The proposed analytical procedure provides the history of nonlinear behavior of reinforced concrete frames due to external loads in an incremental manner. In the analysis, over reduction of stiffness in some members at one iteration may cause smaller redistributions of internal forces for these members and, therefore, result in excessive increase in the stiffness of these cracked members in the subsequent iteration. The relative increase of flexural stiffness of some members attracts transfer of more internal forces to these members, thus leading to further stiffness reduction of these members. The alternate increase and decrease in stiffness of members is likely to generally cause a non-convergent procedure. The formation of plastic hinges also effects the redistribution of internal forces. Therefore, in the procedure, the member end forces used at each iteration are taken as the mean value of the end forces of all previous iterations. In the program, the following,

$$\left| \frac{P_i^n - P_i^{n-1}}{P_i^n} \right| \le \varepsilon \qquad (10)$$

is used as a convergence criterion in each loading step. Here, n is the iteration number, ε is the convergence factor and P^n_i (i=1,6) is the end forces of each member of the structure for the n-th iteration. As seen in the flow chart of the program, the analytical procedure initially takes the members into account to be uncracked and performs the linear elastic analysis of the structure. The flexural and shear stiffnesses of cracked members are then decreased by the effective stiffness models available in the literature. In the meantime, the formation of plastic hinges is also considered. The computer program developed based on the proposed procedure generates the plastic hinges automatically, and performs incremental step by step analysis. The iterative procedure is applied until the convergence criterion is less than the predefined tolerance in each loading step. Due to space limitation, the listing of the computer program is not given in the paper.

In order to verify the applicability and to determine the limitation of the proposed analytical procedure, three examples are taken from the literature and solved using the proposed method as explained below.

3.1 Example 1

In this example, a two story reinforced concrete frame tested by Vecchio and Emara (1992) has been analyzed by the developed computer program. This reinforced concrete frame was designed with a center to center span of 3500 mm and a story height of 2000 mm as shown in Fig. 8. The concrete had a compressive strength of 30 MPa, and the reinforcing steel had a modulus of elasticity of 192500 MPa and a yield strength of 418 MPa. The testing setup involved first applying a total axial load of 700 kN to each column and maintaining this load throughout the test. The lateral load (Q) was, then, monotonically applied until the ultimate capacity of the frame was achieved. The reinforced concrete frame is modeled by four columns and two beam elements as shown in Fig. 8. The reinforcing steel in the beams and columns, the span and the loads are also shown in the figure.

In the analysis, I_{eff} is estimated using ACI (2014) and probability-based effective stiffness models and $\overline{G}_c$ is evaluated using Al-Mahaidi's model. The flexural stiffness reductions of beams and columns with increasing lateral loads are also obtained by using probability-based effective stiffness model. The flexural tensile strength f_r and modulus

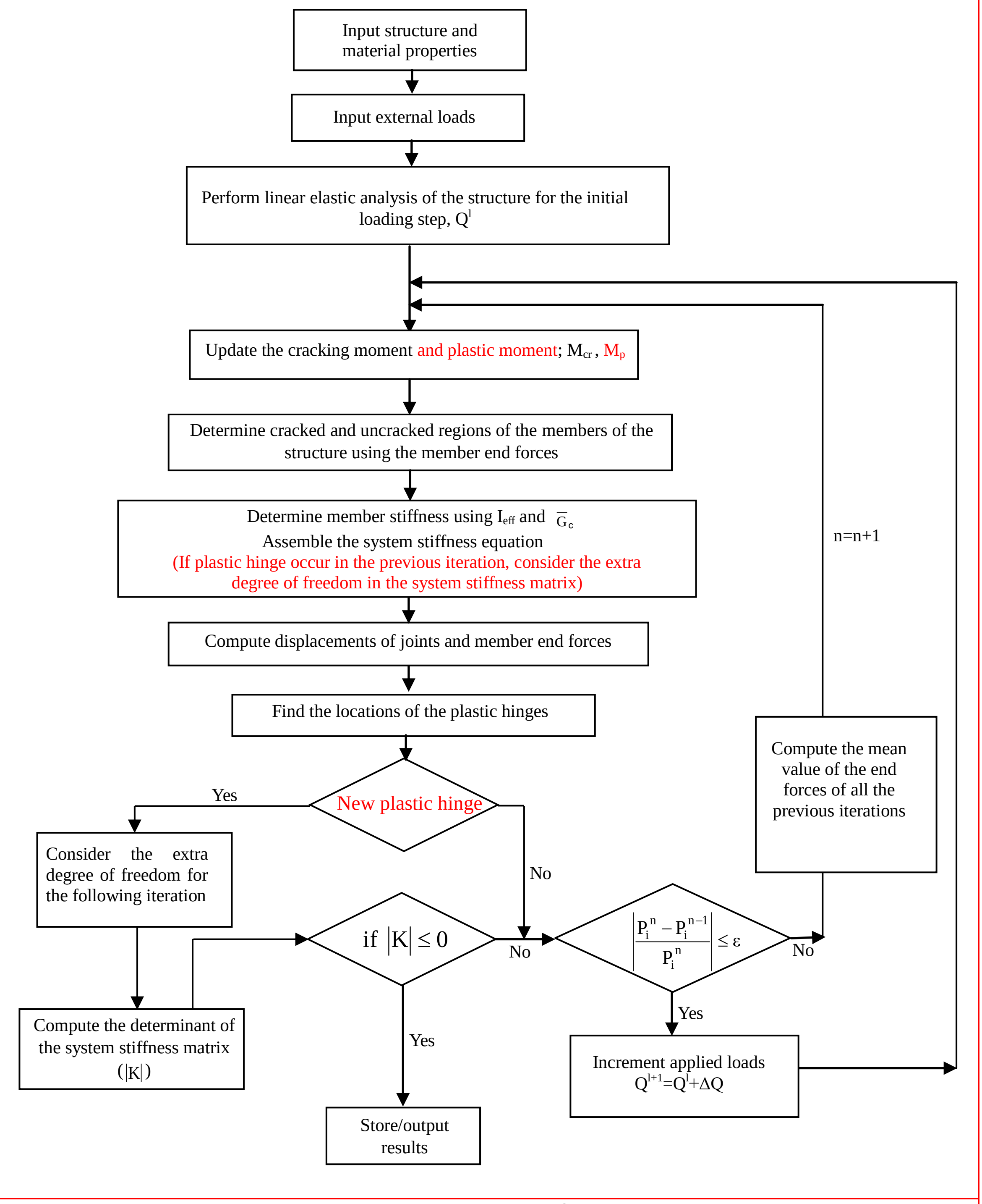

Fig. 7 Solution procedure of the program

of elasticity E_c of concrete were not measured in the experiment and, therefore, the following equations recommended by ACI 318-14 (2014) are used:

$f_r = 0.62\sqrt{f_c'}$ (N/mm^2)	(11a)
$E_c = 4730\sqrt{f_c'}$ (N/mm^2)	(11b)

where f_c' is the compressive strength of concrete. The plastic moment capacity of the members is also obtained as expressed in subsection 2.3.

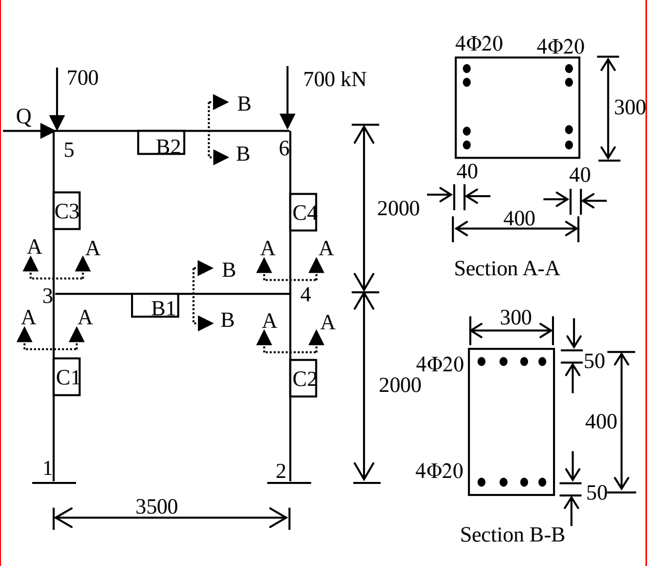

Fig. 8 Dimensions of reinforced concrete frame model (dimensions in mm), (Vecchio and Emara (1992)).

Fig. 9 presents a comparison of the lateral top deflections using the proposed technique predictions and actual test results. It is observed that the analytical procedure predicts the lateral deflection with good accuracy up to a load equal to 93% of test collapse load. The linear elastic method, not considering the cracking effect and plastic hinge formation in the analysis, significantly underestimates the lateral deflection even at service loads. The reason is that the first flexural stiffness reductions caused by initial cracking of structural members usually occurs at a very low load level.

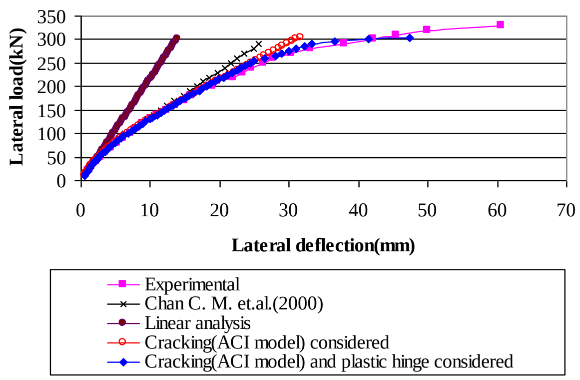

Fig. 9 Comparison among linear elastic method and analytical and experimental results for the lateral deflection of the second story

The results of the proposed analytical procedure considering the variation of shear stiffness due to cracking and formation of plastic hinges at the ends of the members give a better prediction of deflections than the technique presented in Chan et. al. (2000). The effect of plastic hinge formation on the lateral deflection of the frame is also shown in Fig. 9. It can be seen that the formation of plastic hinges at ends of members decreases the structure lateral stiffness, thus, causing the deflection to increase significantly. The predicted deflections of the second story are higher with the development of hinges compared with values obtained when the plastic hinge formation is ignored in the analysis.

Fig. 10 presents a comparison of the deflections at the second level of the frame using different models for the effective moment of inertia of the cracked members. It should be noted that different models provide quite similar results.

The comparison between the experimental and analytical results for the rotation at joint 5 is shown in Fig. 11(a). The moment-rotation diagram at end of the second story beam (joint 5) of the frame is also shown in Fig. 11(b). It can be seen from Fig. 11(a) that the proposed analytical procedure predicts the joint rotation with reasonable accuracy.

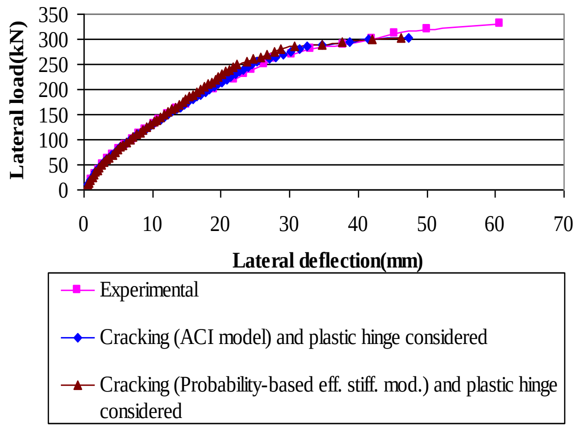

Fig. 10 Numerical comparison of lateral deflection obtained by various models for the effective flexural stiffness

Fig. 12 shows the theoretical influence of shear deformation on the total lateral deflection of the reinforced concrete frame. As seen from the figure, the influence of shear deformation results in an increase in the lateral deflection of the structure, particularly near the ultimate loads where deflections are approximately increased by as much as 14%. Consideration of the shear deformation effect in the analysis leads to prediction of a slight decrease in loading capacity and large lateral displacements compared to values obtained when shear deformation is not considered in the analysis of the frame. These results agree well with that found by Vechio and Emara (1992).

The sequence of the formation of plastic hinges in the reinforced concrete frame obtained by the proposed procedure is also shown in Fig. 13. It is seen that the first two hinges are formed at the ends of the first story beam. Then, two more hinges develop at the ends of the second story beam. Shortly after, the last two hinges are formed at the base of the columns, converting the frame into a mechanism. These analytical results are almost consistent

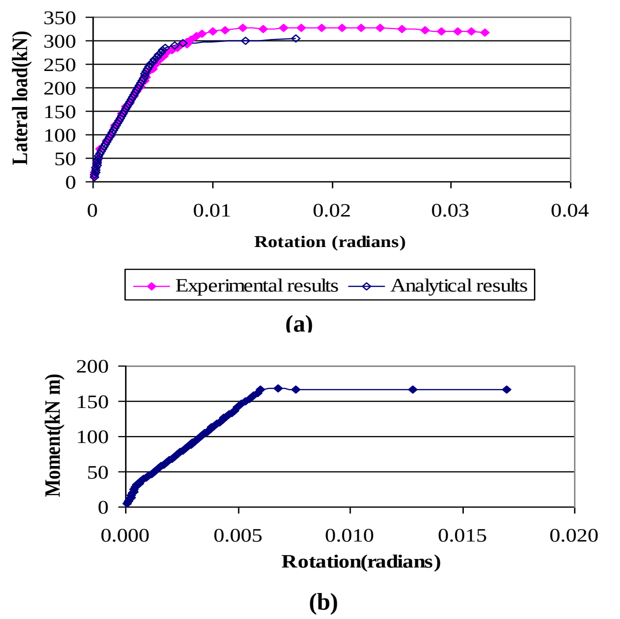

Fig. 11 **(a)** Comparison between experimental and analytical results of the rotation at joint 5
(b) Moment-rotation diagram at the beam of the second story (near joint 5)

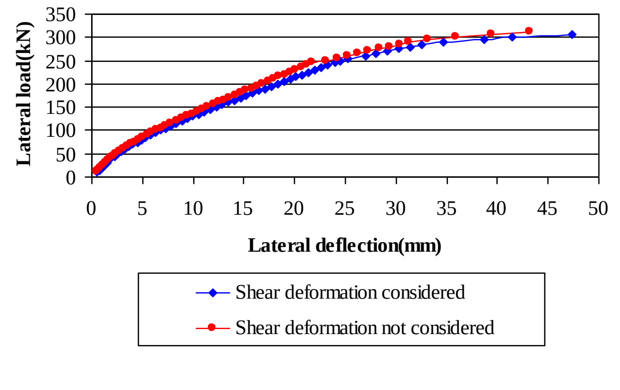

Fig. 12 Theoretical influence of shear deformation on lateral deflection of frame

with the test results (Vecchio and Emara 1992).The load carrying capacity of the frame is also underestimated by the proposed procedure with a difference of 7%.

The theoretical results of cracking sequence and flexural stiffness reductions of beams and columns with respect to the lateral applied load are also shown in Fig. 14. As seen from the figure, the beams of the first and second stories are initially cracked, followed by cracking of the two columns at the first story, C1 and C2, and, then, both columns at the second story, C3 and C4 cracked. From Fig. 14, it is shown that, at 50% of the ultimate lateral load, the two columns at the first story have 67 and 72%, respectively, of their gross moment of inertia, and the two beams at the first and second stories have 48 and 49% of their uncracked values. The columns of the second story have also 89 and 93% of the gross moment of inertia. However, when the

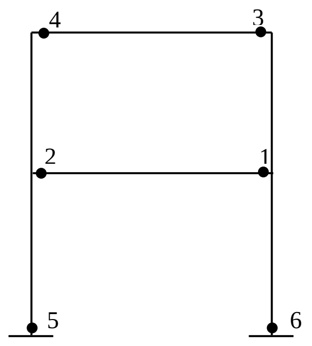

Fig. 13 The sequential formation of the plastic hinges at the ends of the member

lateral load reaches 80% of the ultimate load, the stiffness of the four columns further decreases to the 58, 61, 64 and 69%, respectively, whereas the two beams maintain 45-46% of the uncracked values. These results provide design engineers with significant information on the sequence of member cracking.

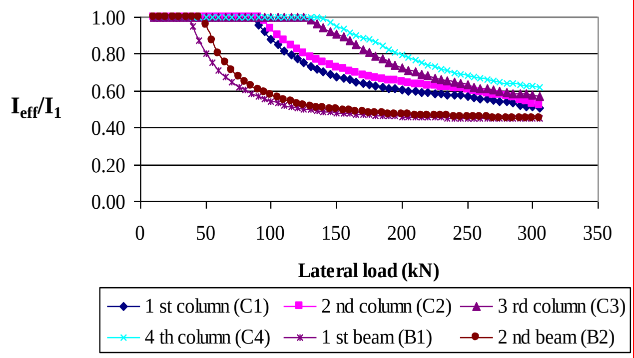

Fig. 14 Flexural stiffness reduction of beams and columns with respect to increasing lateral loads

3.2 Example 2

The applicability of the proposed procedure has been investigated using other frame test results (Chan et. al. 2000). As seen in Fig. 15, a two story reinforced concrete frame was designed with a center to center span of 3000 mm, a first story height of 1170 mm and a second story height of 2000 mm. The concrete had a compressive strength of 29 MPa, whereas the reinforcing steel had a modulus of elasticity of 200000 MPa and a yield stress of 460 MPa. The section dimensions of the columns were 250-mm wide and by 375 mm deep and the section dimensions of the beams were 250-mm wide and 350-mm deep. The testing sequence involved first applying a total axial load of 200 kN to each column and maintaining this load throughout the test. The lateral load (Q) was then monotonically applied until the ultimate capacity of the frame was achieved.

The reinforced concrete frame is modeled by four columns and two beam elements. The reinforcing steel in the beams and columns, the span and the loads are also shown in Fig. 15. In the analysis, ACI and probability-based effective stiffness models are used for the effective moment of inertia and Al-Mahaidi's model is used for the shear modulus of concrete in the cracked regions. The flexural stiffness reductions of beams and columns with increasing lateral loads are also obtained by using the probability-based effective stiffness model.

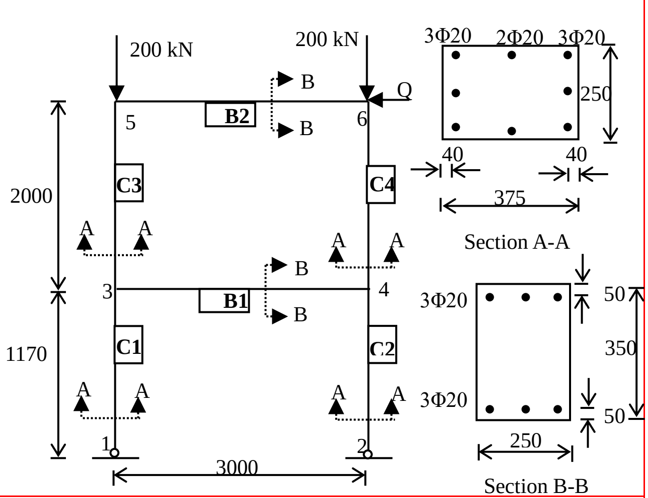

Fig. 15 Two story reinforced concrete frame model tested by Chan, C.M. et. al. (2000), (dimensions in mm).

The comparison between the test and theoretical results for the lateral deflection of the second story is presented in Fig.16. It can be seen from the figure that the analytical procedure predicts the lateral deflection of the second story with good accuracy to a load equal to 91% of the test collapse load. The effects of concrete cracking and plastic hinge formations on the lateral deflection of the frame can also be shown explicitly in Fig. 16. The cracking effect causes the reduction of the overall lateral stiffness which in turn results in an increase in the lateral deflection of the reinforced concrete frame. The formation of plastic hinges at the member ends also reduces the structure stiffness' and, thus, significantly increases the lateral deflection of the frame.

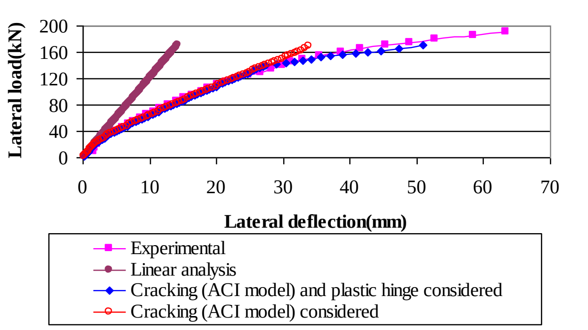

Fig. 16 Comparison between experimental and analytical results of the lateral deflection of the second story

Using the probability-based effective stiffness and ACI models for the effective flexural stiffness, the theoretical lateral deflections of second story are also obtained by the computer program as shown in Fig. 17. As seen from the figure, although different models have been used for the effective flexural stiffness, the results are very close to one another.

Fig. 18 also shows the theoretical influence of shear deformation on the total lateral deflection of the reinforced concrete frame. As depicted from this figure, the percentage of shear deformation in the total deflection increases with increasing lateral loads, particularly following crack developments and plastic hinge formations in the members. A slight decrease in the loading capacity is also predicted compared with the value obtained when the shear deformation effect is not considered in the analysis.

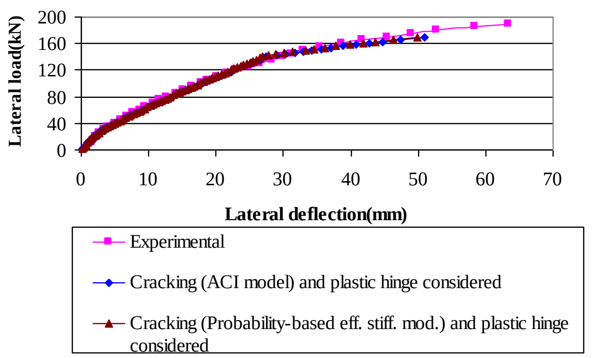

Fig. 17 Comparison of lateral deflections obtained, by various models for the effective moment of inertia

The sequence of development of plastic hinges in the reinforced concrete frame obtained by the proposed analytical procedure is also shown in Fig. 19. It is seen that the first two hinges are formed at the ends of the first story beam. Then, two more hinges are developed at the ends of the second story beam when the frame is converted to a mechanism and achieved its ultimate capacity. The program underestimates the ultimate load of the frame by a value of 9%. The prediction is reasonable, considering that there are many factors that may influence the results, particularly nonlinear behavior of the material properties.

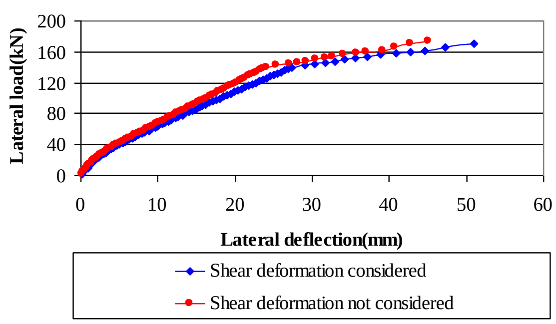

Fig. 18 Theoretical influence of shear deformation on the lateral deflection of frame

The variation of the flexural stiffness of beams and columns with respect to the lateral applied load is also shown in Fig. 20. It can be seen from the figure that the beams at the first and second stories, B1 and B2, crack first and, then, the two columns on the lateral loading side, C4 and C2, crack, respectively. Finally, the two columns on the opposite loading side, C3 and C1, start to crack, respectively. When the stiffness of the two beams are reduced to 50% of their uncracked stiffness, the stiffness of the two first-story columns has been reduced to 59-62% of their uncracked stiffness.

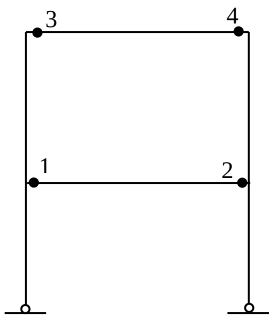

Fig. 19 The sequential formation of the plastic hinges at the ends of the member

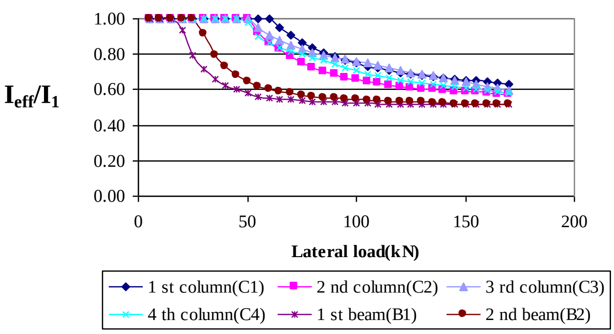

Fig. 20 Stiffness reduction of each member versus lateral loads

3.3 Example 3

In this last example, the single-story reinforced concrete frame tested by Ernst et. al. (1973) has been analyzed by the developed computer program (Fig. 21). The dimensions and reinforcement of the members and the loads are given in Table 1. This frame is analyzed under vertical loads that are applied gradually up to failure. In the analysis, I_{eff} is predicted using ACI model and $\overline{G}_c$ is evaluated using Al-Mahaidi's model.

The load-central deflection (joint 1) curves from the analytical predictions and the test results are shown in Fig. 22. The proposed analytical procedure predicts the vertical deflection with reasonable accuracy. From Fig. 22, it is also shown that the effects of concrete cracking and plastic hinge formations on the central deflection of the frame become significant.

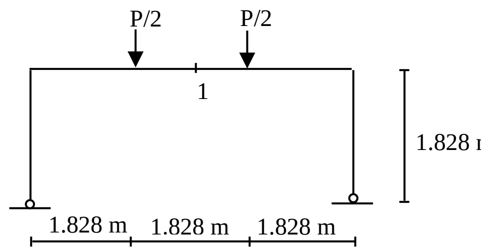

Fig. 21 One story reinforced concrete frame tested by Ernst et. al. (1973)

Table 1 Dimensions and reinforcement of the member

Member	The section dimensions(mm) (bxh)	Longitudinal bars (mm^2)	Concrete cover(mm)
Beam	152.4x228.6	387 (Outer) 258 (Inner)	38.1
Column	152.4x228.6	258 (Outer) 142 (Inner)	25.4

Fig. 23 also shows the sequence of the formation of plastic hinges in the reinforced concrete frame obtained by the proposed analytical procedure. The first two hinges are formed at the top ends of the columns. Then the other hinges develop at the locations of the point loads and the frame is achieved its ultimate capacity. The ultimate load capacity is predicted fairly accurate with computed values of 46 kN corresponding to 97% of the test value. These results provide engineers with significant information with respect to the behavior of the reinforced concrete frame.

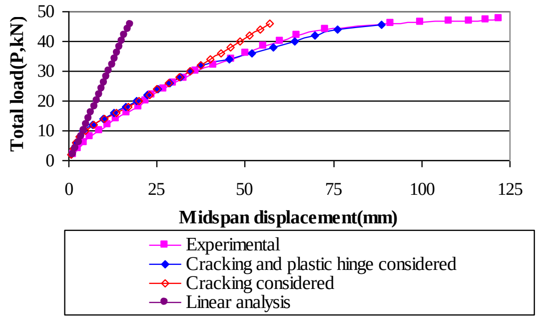

Fig. 22 Comparison between experimental and analytical results of the vertical deflection (joint 1)

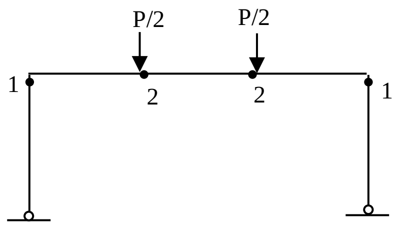

Fig. 23 The sequential formation of the plastic hinges

Fig. 24 also shows the influence of shear deformation on the total vertical deflection of the beam. It can be observed that the contribution of the shear deformation to the total vertical deflection of the beam increases with the lateral load increase.

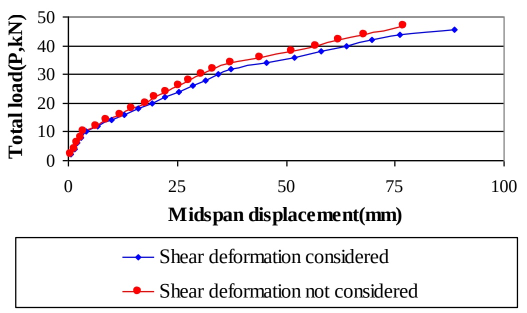

Fig. 24 Theoretical influence of shear deformation on the central deflection of the beam of the frame

4. Conclusions

An iterative procedure has been developed to analyze reinforced concrete frames. The effective flexural stiffness of a cracked member has been evaluated by the ACI and probability-based effective stiffness models. The variation of shear stiffness in the cracked regions of members has also been considered by employing reduced shear stiffness models available in the literature. In the analysis, the formation of plastic hinges at the ends of members is also taken into account. The proposed procedure is able to predict the locations and sequential formation of the plastic hinges in the frame members.

The stiffness matrix method has been employed to obtain the numerical solutions of the proposed analytical procedure. This iterative procedure is efficient from the viewpoints of computational effort and convergence rate to analyze reinforced concrete frames.

The validity of the proposed procedure has been tested by means of comparisons between the theoretical and experimental results of reinforced concrete frames available in the literature. The analytical procedure predicts the deflections and load carrying capacity of frames with reasonable accuracy.

Cracking sequence and flexural stiffness reductions of beams and columns with respect to the applied load can be obtained by the developed computer program. This feature can minimize the uncertainty of flexural stiffness of members and, therefore, provide design engineers with significant information on the consequence of cracking in members. The cracking effect on the reinforced concrete members has been found to cause reduction of overall lateral stiffness which, in turn, results in an increase in the lateral deflection of the reinforced concrete frames. The numerical results of the analytical procedure also indicate that the development of plastic hinges at the ends of members reduces the structure stiffness and, thus, causing the deflection to increase significantly.

Shear deformation is found to contribute the deflection of reinforced concrete frames. The analytical results indicate that the contribution of the shear deformation to the total defection of the reinforced concrete frames increases with increasing loads. It is therefore important to consider the reduction of shear stiffness for the cracked members in order to obtain more accurate results.

Although different models for the effective flexural stiffness have been used, the results are very close to one another.

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