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Haar Wavelet Collocation Method for Thermal Analysis of Porous Fin with Temperature-dependent Thermal Conductivity and Internal Heat Generation

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Abstract

In this study, the thermal performance analysis of porous fin with temperature-dependent thermal conductivity and internal heat generation is carried out using Haar wavelet collocation method. The effects of various parameters on the thermal characteristics of the porous fin are investigated. It is found that as the porosity increases, the rate of heat transfer from the fin increases and the thermal performance of the porous fin increases. The numerical solutions by the Haar wavelet collocation method are in good agreement with the standard numerical solutions.

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1. Introduction

In recent times, the study into thermal analysis in porous fin has experienced significant research, following the pioneer work on heat transfer enhancement through porous fin by (Kiwan and Al-Nimr [1]). In solving for the nonlinearity arising from heat transfer analysis in porous fin, Kiwan [1-4], and Golar and Baker [5] applied Runge-Kutta for the thermal analysis in porous fin. Kundu [6-9] and Taklifi *et.al* [10] applied the Adomian decomposition method (ADM) on the performance and optimum design analysis of the fins while Saedodin and Sadeghi [11], Darvishi *et al.* [12] and Moradi *et al.* [13] and Ha *et al.* [14] adopted the homotopy analysis method (HAM) to provide solution to the natural convection and radiation in porous and porous moving fins. Hatami and Ganji [15] applied the least square method (LSM) to study the thermal behaviour of convective-radiative in porous fin with different sections and ceramic materials. Hoshyar *et al.* [16] used Homotopy perturbation method and collocation method for the thermal performance analysis of porous fins with temperature-dependent heat generation. Also, Rostamiyaan *et al.* [17] applied variational iterative method (VIM) to provide analytical solution for heat transfer in porous fin. Ghasemi *et al.* [18] used the differential transformation method (DTM) for analysing heat transfer in porous and solid fin. Most of these approximate methods give accurate predictions only when the nonlinearities are weak or for small values of the fin thermo-geometric parameter. However, these methods often fail to predict accurate solutions for strong nonlinear models. Moreover, these approximate methods often involve complex mathematical analysis leading to analytic expression involving a large number terms. In addition, in practice, approximate analytical solutions with large number of terms are not convenient for use by designers and engineers, since they require high computational cost and time in analysing the nonlinear problem. It is in view of these issues that led to the adoption of different wavelet collocation methods for solving different nonlinear equations. The ease of application, simplicity and fast rate of convergence have in recent times made these methods gained popularity in nonlinear analysis of systems and they have been applied to nonlinear problems in heat transfer analysis of fins [19-24]. In the class of the wavelets, the relative simplicity, high accuracy and conveniences, have made Haar wavelet method an interesting and effective tool for solving linear and nonlinear differential equations.

Thus, in this paper, we applied the Haar wavelet collocation method (HWCM) to numerically study the thermal analysis of porous fin with temperature-dependent thermal conductivity and internal heat generation. The numerical solutions are used to investigate the effects of various parameters on the thermal behaviour of the fin.

2. Problem Formulation

Fig. 1 shows a straight porous fin of length L and thickness t exposed on both faces to a convective environment at temperature T_∞. Assuming the porous medium is homogeneous, isotropic and saturated with a single-phase fluid, the physical properties of solid as well as fluid are considered as constant except density variation of liquid, which may affect the buoyancy term where Boussinesq approximation is employed. The fluid and porous mediums are in locally thermodynamic equilibrium in the domain, surface radiative transfers and non-Darcian effects are negligible, the temperature variation inside the fin is one-dimensional i.e. temperature varies along the length only a nd remain constant with time and there is no thermal contact resistance at the fins base and the fin tip is adiabatic type.

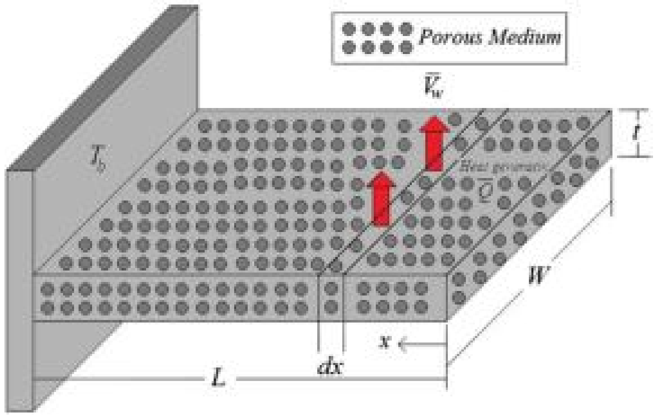


Fig. 1. Schematic of the longitudinal porous fin geometry with the internal heat generation

Using Darcy's model and its assumptions, the governing equation for the heat transfer in the fin is given as

$$\frac{d}{dx}\left[k_{eff}(\mathrm{T})\frac{dT}{dx}\right]-\frac{h(T-T_\infty)}{t}-\frac{\rho c_p g\beta' K(T-T_\infty)^2}{t\nu_f}+q_a(T)=0 \tag{1}$$

The boundary conditions are:

$$x=L,\quad T=T_b$$
$$x=0,\quad \frac{dT}{dx}=0 \tag{2}$$

The temperature-dependent thermal conductivity and internal heat generation are given by

$$k_{eff}\left(T\right)=k_a[1+\lambda(T-T_\infty)] \tag{3}$$

$$q_{\mathrm{int}}\left(T\right)=q_a[1+\psi(T-T_\infty)] \tag{4}$$

Upon substituting eqns. (3) and (4) into eqn. (1), we arrived

$$\frac{d}{dx}\left[[1+\lambda(T-T_\infty)]\frac{dT}{dx}\right]-\frac{h(T-T_\infty)}{k_a t}-\frac{\rho c_p\,\mathrm{g}K\beta'(T-T_\infty)^2}{k_a t\nu_f}+\frac{q_a}{k_a}[1+\psi(T-T_\infty)]=0 \tag{5}$$

The dimensionless form of eqn. (5) can be developed if we use the following dimensionless parameters in eqn. (6)

$$X=\frac{x}{L},\quad \theta=\frac{T-T_\infty}{T_b-T_\infty},\ Ra=\left(\frac{\beta' gT_b t^3}{\nu_f^2}\right)\left(\frac{\rho c_p\nu_f}{k_{eff,a}}\right),\quad Q=\frac{q\nu_f t}{\rho\mathrm{c}_p\beta'\,\mathrm{g}K(T_b-T_\infty)^2},\quad M^2=\frac{hL^2}{k_{eff,a}t},Da=\frac{K}{t^2},$$
$$S_h=\frac{RaDa(L/t)^2}{k_{eff,a}},\quad \gamma=\psi(T_b-T_\infty),\quad \beta=\lambda(T_b-T_\infty) \tag{6}$$

The dimensionless governing differential Eq. (7) and the boundary conditions

$$\frac{d^2\theta}{dX^2}+\beta\theta\frac{d^2\theta}{dX^2}+\beta\left(\frac{d\theta}{dX}\right)^2-M^2\theta-S_H\theta^2+S_HQ(1+\gamma\theta)=0$$

The boundary conditions are: (7)

$$X = 1, \quad \theta = 1$$

$$X = 0, \quad \frac{d\theta}{dX} = 0 \tag{8}$$

3. Solution Procedure

It is very difficult to develop a closed-form solution for the above non-linear eq. (7). Therefore, recourse should be made to either approximation analytical method, semi-numerical method or numerical method of solution. In this work, Haar wavelet collocation is used. The procedures are given below.

Let $\mathrm{M} = 2^j$ (J is the maximum level of resolution); the i^{th} wavelet is defined as;

$$h_i(x) = \begin{cases} 1, & for\ x \in [\alpha, \xi), \\ -1, & for\ x \in [\xi, \gamma) \quad i = 2, ..., 2M \\ 0, & elsewhere \end{cases} \tag{9}$$

where $\alpha = \frac{k}{2^j}$, $\xi = \frac{k+0.5}{2^j}$, and $\gamma = \frac{k+1}{2^j}$. Here $k = 0, 1, ..., 2^j - 1$ is the translation parameter and $j = 0, 1, ..., J$ is the dilatation parameter. The number of wavelets is given by $i = 2^j + k + 1$ and the maximum value is $i = 2M$.

For $i = 1$, it is assumed that;

$$h_1(x) = \begin{cases} 1, & for\ x \in [0, 1). \\ 0, & elsewhere \end{cases} \tag{10}$$

Introducing the following notations;

$$p_{i,1}(x) = \int_0^x h_i(x')\,dx', \tag{11}$$

$$p_{i,v+1}(x) = \int_0^x p_{i,v}(x')\,dx', \qquad i = 2, 3, \ldots \tag{12}$$

The above integrals can be evaluated using Eq. (9) and are given by;

$$p_{i,1}(x) = \begin{cases} x - \alpha & for\ x \in [\alpha, \xi), \\ \gamma - x & for\ x \in [\xi, \gamma), \\ 0 & elsewhere \end{cases} \tag{13}$$

$$p_{i,2}(x)=\begin{cases}\frac{1}{2}(x-\alpha)^2 & for\ x\in[\alpha,\xi),\\ \frac{1}{2^{2j+2}}-\frac{1}{2}(\gamma-x)^2 & for\ x\in[\xi,\gamma),\\ \frac{1}{2^{2j+2}} & for\ x\in[\gamma,1),\\ 0 & elsewhere\end{cases}$$

(14)

Also, it is assumed that;

$$C_{i,1}=\int_0^1 p_{i,1}(x')dx'.$$

(15)

The collocation points are defined as:

$$X_j=\frac{j-0.5}{2M},\qquad j=1,2,\ldots,2M$$

(16)

Re-writing eqn. (7) as:

$$\theta''=f(X,\theta,\theta').$$

(17)

The Haar wavelet collocation method for the above nonlinear problem subject to the conditions is given as follows:

$$\theta(0)=\lambda_1,\qquad \theta'(1)=\lambda_2$$

(18)

Also, it is assumed that;

$$\theta''(X)=\sum_{i=1}^{2M}a_i h_i(X).$$

(19)

Integrating Eq. (19) from 0 to X, the derivative $\theta'(X)$ can be expressed as:

$$\theta'(X)=\theta'(0)+\sum_{i=1}^{2M}a_i p_{i,1}(X).$$

(20)

Putting $X=1$ in eqn. (20) and using the second boundary condition, the value of $\theta'(0)=\lambda_2-a_1$. Thus, eqn. (20) can then be re-written as:

$$\theta'(X)=\lambda_2-a_1+\sum_{i=1}^{2M}a_i p_{i,1}(X).$$

(21)

Now, again integrating eqn. (21) from 0 to X and using the first boundary condition;

$$\theta(X)=\lambda_1+(\lambda_2-a_1)X+\sum_{i=1}^{2M}a_i p_{i,2}(X)$$

(22)

Eqn. (22) is obtained. Substituting the values of $\theta(X)$, $\theta'(X)$ and $\theta''(X)$ in Eq. (17) and applying discretization using collocation points given in Eq. (16), a nonlinear system is obtained;

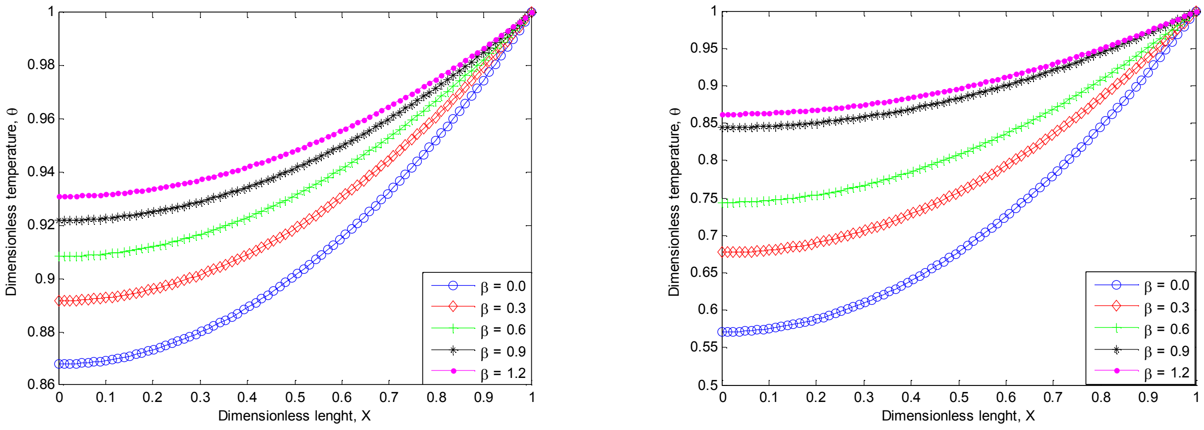
$$\sum_{i=1}^{2M}a_i h_i(X_j)=\left[f(X_j,\lambda_1)+(\lambda_2-a_1)X_j+\sum_{i=1}^{2M}a_i p_{i,2}(X_j),(\lambda_2-a_1)+\sum_{i=1}^{2M}a_i p_{i,1}(X_j)\right]$$

(23)

Solving the above $2M\times 2M$ system using Newton's method, the unknown Haar coefficient $a_i's,\ i=1,\ 2,\ldots,\ 2M$ are obtained which are eventually used to find the approximate solution.

4. Result and Discussion

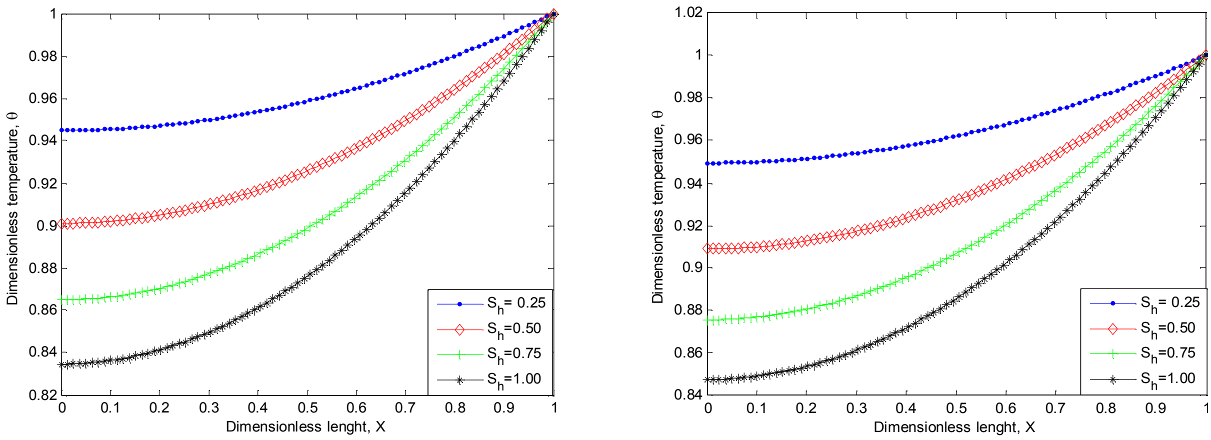
Figs. 2a and b shows the effects of thermal conductivity parameters on the temperature distribution. The figures indicate that as thermal conductivity parameter increases, the dimensionless temperature distribution in the fin decreases.



(a) (b)

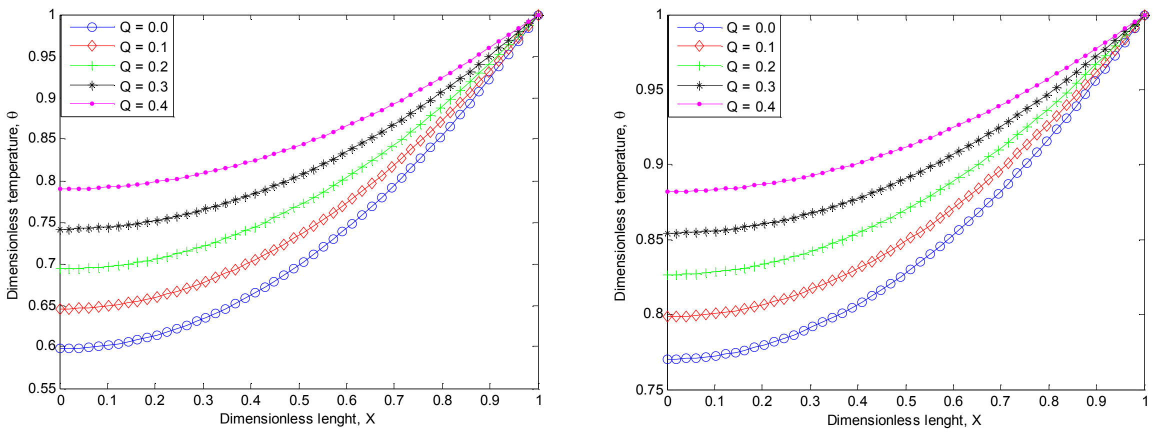
Fig. 2. Dimensionless temperature distribution in the fin parameters for varying thermo-geometric parameter when: (a) S_h,=0.5, M=0.3, Q=0.4, γ=0.2 (c)) M=0.8, S_h,=0.1, Q=0.4, γ=0.2

Also, Figs. 3a-b show the effects of porosity on the temperature distribution. As the porosity parameter increases, the temperature decreases rapidly and the rate of heat transfer through the fin increases. The increase in the porous parameter, S_h implies increase in the Darcy and Raleigh number of the porous fin and the figures depict that, increase in the porosity of the fin improves fin efficiency due to increase in convection heat transfer.



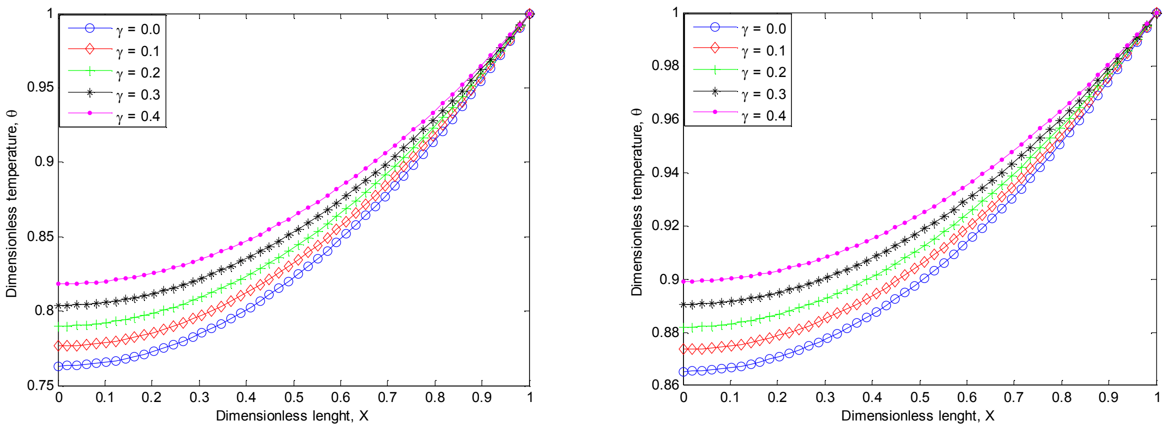
(a) (b)

Fig. 3. Dimensionless temperature distribution in the fin parameters for varying thermo-geometric parameter when: (a) β=0.4, Q=0.3, γ=0, (b) β,=0.4, Q=0.3, γ=0.2



(a) (b)

Fig. 4. Dimensionless temperature distribution in the fin parameters for varying thermo-geometric parameter when: (a) S_h=1, β=-0.5, γ=0.2, (b) S_h=1, β,=0.5, γ=0.2



(a) (b)

Fig. 5. Dimensionless temperature distribution in the fin parameters for varying thermo-geometric parameter when: (a) S_h=1, β=-0.5, Q=0.4, (b) S_h=1, β=0.5, Q=0.4

Table 1: Comparison of results

X	NM	HWCRM
0.0	**0.9581**	**0.9571**
0.1	**0.9585**	**0.9585**
0.2	**0.9597**	**0.9597**
0.3	**0.9618**	**0.9618**
0.4	**0.9647**	**0.9647**
0.5	**0.9685**	**0.9685**
0.6	**0.9730**	**0.9730**
0.7	**0.9785**	**0.9785**
0.8	**0.9846**	**0.9846**
0.9	**0.9919**	**0.9919**
1.0	**1.0000**	**1.0000**

Table 1 shows comparison of results of the numerical solutions by the Haar wavelet collocation method (HWCRM) and with the standard numerical solutions (NM). From the Table, it is shown that excellent agreements are established between the numerical solutions by the Haar wavelet collocation and the standard numerical solutions.

5. Conclusion

In this work, thermal performance of a porous fin with temperature-dependent thermal properties and internal heat generation has been analysed using Haar's wavelet collocation method. The effects of various parameters on the thermal characteristics of the porous fin are investigated. It is found that as the porosity increases, the rate of heat transfer from the fin increases and the thermal performance of the porous fin increases. The numerical solutions by the Haar wavelet collocation method are in good agreement with the standard numerical solutions.

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Nomenclature

h_b	**Heat transfer coefficient at the base of the fin [$Wm^{-2}k^{-1}$]**	**Da**	**Darcy number**
c_p	**Specific heat of the fluid passing through porous fin**	g	Gravity constant [m/s^2]

	[J/kg-K]		
h	**Heat transfer coefficient over the fin surface [W/m^{2}K]**	L	Length of the fin, m
H	**Dimensionless heat transfer coefficient at the base of The fin [Wm^{-2}k^{-1}]**	Nu	Nusselt number
k	**Thermal conductivity of the fin material [Wm^{-1}k^{-1}]**	Ra	Rayleigh number
k_a	**Thermal conductivity of the fin material at the base of the fin [Wm^{-1}k^{-1}]**	S_h	Porosity parameter
k_{eff}	**Effective thermal conductivity ratio**	P	Perimeter of the fin
K	**Permeability of the porous fin [m^2]**	t	Thickness of the fin
M	**Dimensionless thermo-geometric parameter**	T	Fin temperature [K]
m	**Mass flow rate of fluid passing through porous fin kg/s]**	T_a	Ambient temperature K
Q	**Dimensionless heat transfer rate per unit area**	T_b	Temperature at the base of the fin [K]
Q_b	**Dimensionless heat transfer rate the base in porous fin**	w	Width of the fin
Q_s	**Dimensionless heat transfer rate the base in solid fin**	X	Dimensionless length of the fin
v	**Average velocity of fluid passing through porous fin [m/s]**	x	Axial length measured from fin tip (m)
q_b	**Heat transfer rate per unit area at the base [W/m^2]**	q	Internal heat generation

Greek Symbols

β	**Thermal conductivity parameter or non-linear parameter**	**δ**	**Thickness of the fin, m**
γ	**Dimensionless internal heat generation parameter**	δ_b	Fin thickness at its base
θ_b	**Dimensionless temperature at the base of the fin**	L	Length of the fin, m
β'	**coefficient of thermal expansion (K^{-1})**	θ	Dimensionless temperature
υ	**Kinematic viscosity (m^2/s)**	ε	Porosity or void ratio

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