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A Universal Two Way Approach for Estimating Unknown Frequencies for Unknown Number of Sinusoids in a Signal Based on Eigenspace Analysis of Hankel Matrix

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Abstract We develop a novel approach to estimate the n unknown constituent frequencies of a noiseless signal that comprises of unknown number, *n*, of sinusoids of unknown phases and unknown amplitudes. The new two way approach uses two constraints to accurately estimate the unknown frequencies of the sinusoidal components in a signal. The new approach serves as a verification test for the estimated unknown frequencies through the estimated count of the unknown number of frequencies. The Hankel matrix, of the time domain samples of the signal, is used as a basis for further analysis in the Pisarenko harmonic decomposition. The new constraints, the Existence Factor (*EF*) and the Component Factor (*CF*), have been introduced in the methodology based on the relationships between the components of the sinusoidal signal and the eigenspace of the Hankel matrix. The performance of the developed approach has been tested to correctly estimate any number of frequencies within a signal with or without a fixed unknown bias. The method has also been tested to accurately estimate the very closely spaced low frequencies.

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1. INTRODUCTION

Consider a signal *y(t)* consisting of multiple (*n*) sinusoids such that each sinusoid can have unknown amplitude (*a*) and unknown phase (θ). Equation (1) represents such a signal with a fixed bias of C_0.

( ) Σ (( ) (1)

Where ≠ 0 is the associated amplitude of the individual sinusoid

> 0 is the frequency of individual sinusoid such that ≠ for 0<a<n+1 and 0<b< n+1

is the associated phase of the individual sinusoid.

n is of an unknown value.

Estimating the unknown frequencies of the individual sinusoids present in *y(t)* becomes relatively easier if the number of sinusoids present in the *y(t)* are known. Several methods have been developed to estimate the unknown frequencies of multiple sinusoids. For an unbiased estimation of a single sinusoid, a globally convergent scheme was presented in [1] using the lattice-based adaptive infinite impulse response (IIR) notch filter in the discrete time domain. This approach was extended in [2] for continuous time domain. In [3] the concept of nonlinear time scaling is applied to the approach presented in [1] to derive the global asymptotic convergence of the estimated frequency. However the methods presented in [1], [2] and [3] were limited to the frequency estimation of only a single sinusoid. A global estimation of multiple unknown frequencies was presented in [4] and [5], using the concept of adaptive filters presented in [6]. However the functionality of the approaches presented in [4] and [5] are limited in their need for prior knowledge of the number of sinusoids, *n*, present in the signal. This paper presents a new approach to accurately estimate all the unknown frequencies without any prior knowledge of the number of sinusoids or their associated amplitudes and phases. The proposed method employs eigenspace analysis for the square symmetric matrix of the Hankel matrix [7], building upon the Pisarenko algorithm [8] for the decomposition of noiseless signal. Equation (2) and (3) represent the Hankel matrix **H** and the associated square symmetric matrix **S**. The matrix **H** represents the m samples of the signal *y(t)* taken at regular interval of time and has the order of L × (m-L+1).

$$\mathbf{H} = [\ (\ )\quad (\ )\quad (\quad)\ ;\ (\ )\quad (\ )\ ;\ (\ )\quad (\quad)\quad (\ )\ ] \qquad (2)$$

$$\mathbf{S} = \mathbf{H} \times \mathbf{H}` = [\quad] \qquad (3)$$

Where

H is the Hankel matrix of order L × (m-L+1)

H` is the transpose of matrix **H**

= such that 0<u<L+1 and 0<v<L+1

The matrix **H** forms the basis for frequency estimation of constituent sinusoids as its order determines the order of matrix **S** which is used for the eigenspace analysis and ultimately leads to frequency estimation. The eigenvalues and the corresponding eigenvectors of **S** reflect upon the correct frequency estimation. Let matrix **E** represents the matrix of eigenvalues of **S** and matrix **V** represents the matrix of eigenvectors of **S** such that first vector of **V** corresponds to first eigenvalue in **E.**

$$\mathbf{E} = \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & \lambda_n \end{bmatrix} \qquad (4)$$

$$\mathbf{V} = [\quad] \qquad (5)$$

Where each eigenvalue, corresponds to an eigenvector .

Assuming that the eigenvalues are arranged in the ascending order, represent the minimum eigenvalue of **S** and represents the corresponding eigenvector. According to the Pisarenko algorithm, polynomial roots of exist in the form of complex conjugate pairs and reflect the presence of sinusoids as according to Euler's relationship equations [9], the sinusoids can also be expressed in the form of complex exponential pairs shown in equation (6), (7) and (8).

$$= \quad +i \qquad (6)$$

$$= \quad + \qquad (7)$$

$$= \quad - \qquad (8)$$

If there is a prior knowledge of the number of sinusoids in a signal, the order of Hankel matrix can be determined such that the eigenspace analysis will give a correct estimation of frequencies of the known number of sinusoids; otherwise the right order of the Hankel matrix cannot be determined and subsequently the correct eigenspace analysis using the Pisarenko algorithm cannot be performed, resulting in either too many or too few complex conjugate pairs in the polynomial roots of the eigenvector . Even if the number of sinusoids is known, for closely spaced multiple frequencies, the correct estimates of frequencies become difficult. The new approach overcomes this limitation and can estimate the frequencies for an unknown number of sinusoids present in a signal.

2. UNIVERSAL ESTIMATION APPROACH

The new method uses a two way constraint approach to correctly estimate the frequency of the unknown sinusoids. Both the constraints are derived from the eigenspace analysis of the matrix S which is obtained from matrix **H** using the equation (3). The two constraints, developed in the new methodology, are the Existence Factor (*EF*) constraint and the Component Factor *(CF)* constraint. Both these constraints must be satisfied for correct estimation of the unknown frequencies and for correct detection of the number of sinusoids present in the signal. Both constraints are also linked together. The combination of the two constraints leads to successful estimation of unknown multiple frequencies.

2.1. First Constraint :: Existence Factor (*EF*)

The problem arises at the point when the number of sinusoids, present in the signal *y(t),* is not known. If the exact number of sinusoids is known then the order of matrix **H** can be selected such that of **S** will only have exactly same number of complex conjugate pairs as the number of sinusoids in the signal. But if the number of sinusoids present in the signal is not known then the right order of matrix **H** cannot be selected which will result in either too many or less number of complex conjugate pairs for of **S**, which will result in inaccurate estimation or false estimation of the frequencies of constituent sinusoids of the signal.

The new method overcomes this problem by considering the fact that does provide the estimate of the frequencies of constituent sinusoids but the new method also exploits the fact that there exists a relationship between the actual frequencies of the unknown number of sinusoids and the order of the matrix **S**. The order of matrix **S** determines the number of eigenvectors of **S** as shown in equation 5 that the matrix **S** of order $M \times M$ has M eigenvectors. The polynomial roots of all these M eigenvectors have some complex conjugate pairs common for all M eigenvectors. These common complex conjugate pairs in M eigenvectors of **S** are the complex exponential pairs of the constituent sinusoids. There is one pure constant term which may also be common to polynomial roots of all M eigenvectors and this common term represents the existence of a fixed bias term along with the sinusoids. First constraint factor is developed based on this relationship. For n number of sinusoids present in a signal and for matrix **S** of order $M \times M$, the relationship between the unknown frequencies of n sinusoids with M can be represented with new existence factor (*EF*) as follows:

Existence Factor *EF*: $\{\ 0 > EF \geq M - (2n + 1)\ \}$ (9)

This new term is developed based on the fact that each sinusoid can be represented as pair of complex exponential and each complex exponential corresponds to a principal vector in the eigenspace. The addition of one is to consider the fixed bias in *y(t)*, if present. It shows that out of M eigenvectors of **S**, *EF* number of eigenvectors will have the some polynomial roots in the form of conjugate pair that will be common to all *EF* number of eigenvectors, hence resulting in the true frequency estimation from these common conjugate pair roots. It is also consistent with the concept that out of M eigenvalues, *EF* numbers of eigenvalues are of the same order of the minimum eigenvalue and the vectors corresponding to all these eigenvalues will have the polynomial roots that facilitate the frequency estimation through the common conjugate pairs of polynomial roots.

2.2. Second Constraint :: Component Factor (*CF*)

The second constraint is called Component Factor (*CF*) and is also derived from the eigenspace of **S.** For the square matrix **S**, the eigenvalues of **S** and the eigenvectors of **S** are related as

(10)

➔ (11)

➔ ( ) (12)

Where

V is the matrix representing the eigenvectors of **S**; and

is the matrix representing the eigenvalues of **S** and **I** is the identity matrix. We are interested in the non-trivial solution, i.e. for $\mathbf{V} \neq \mathbf{0}$.

This is known as the characteristic determinant of **S** and $|\mathbf{S} - \mathbf{EI}| = \mathbf{0}$ is the characteristic equation [10]. Solution of the characteristic equation gives the eigenvalues, , of **S** and these eigenvalues satisfy following two relationships [10]:

i. The trace of matrix **S** is basically the sum of the diagonal values of the matrix **S** and is equal to the sum of the eigenvalues.
ii. The determinant of matrix **S** is equal to the product of the eigenvalues.

The first relationship is used to develop the second constraint, *CF*, that accurately determines the number of principal components presents in the signal's eigenspace. The trace of the **S** is related to its eigenvalues . Starting from the maximum eigenvalue of **S**, the number of eigenvalues that sum up to the trace of **S** represent the number of significant components present in the signal *y(t).* Here we have performed the equality test between the trace of **S** and the sum of eigenvalues of **S** up to thirteen decimal places. Let $T_r(\mathbf{S})$ represent the trace of **S**, then the *CF* can be represented as:

$CF = [Max\ (k)\ \{\sum\qquad (\ )\ \}$ (13)

Each and every sinusoid will have two significant complex exponential components present in the signal whereas the fixed bias, if present, will have a single principal component present in the signal. Hence, the odd value of *CF* will show the presence of a fixed bias whereas the even value of *CF* will reflect the absence of a fixed bias.

2.3. Relationship Between Two Constraints

As both the constraints are developed from the eigenspace of **S**, so there exists a relationship between the two constraints. The *EF* value is related to the eigenvector in the eigenspace of **S** and *CF* is related to the eigenvalues in the eigenspace of **S**. Each eigenvalue in the eigenspace corresponds to an eigenvector and as the *CF* gives the number of eigenvalues that satisfy the defined criteria, this *CF* also gives the number of eigenvectors that will not satisfy the criteria defined under *EF*, Hence, the

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relationship between the *EF* and *CF* can be written as shown in equation (14) by modifying the equation (9).

Existence Factor *EF*: $\{ 0 > EF \geq M - (CF+1) \}$ (14)

The comparison between the equation (9) and equation (14) gives an impression that value of *CF* is same as 2*n* but it is only with the case when there is no fixed bias present in the signal. For the signal that has fixed bias present in the signal, the value of *CF* will be 2*n* plus one for the fixed bias presence.

2.4. Frequency Estimation Scheme

The unknown frequencies of sinusoids within a signal are estimated using the constraints *EF* and *CF* through following steps:

i. Formation of matrix **H** from the signal's samples.
ii. Calculation of **S**.
iii. Finding the matrix **E** and **V** of **S.**
iv. Calculating the polynomial roots of all the vectors in **V**
v. Estimating the frequencies **,** from the polynomial roots of **V** by taking the imaginary part of the natural logarithm of the polynomial roots**.**
vi. Taking the frequencies calculated, with unit magnitude of the polynomial roots, for $\mathbf{V_1}$ as the reference frequencies .
vii. The presence of is counted in **,** and if the count satisfies the first constraint *EF* then the frequencies in that have satisfied the *EF* constraint are the estimated frequencies of the constituent sinusoids of signal *y(t).*
viii. The constraint condition for *CF* is evaluated and the count of eigenvalues that have satisfied the constraint condition is the *CF* value.
ix. The relationship equation (14) is tested for evaluated values of *EF* and *CF*. If the equation (14) is satisfied then the frequencies found under step vi are the true frequencies of the sinusoids present in the signal *y(t).*
x. If any of the constraints is not satisfied or test for equation (14) is not satisfied then the procedure is repeated with larger number of samples of signal *y(t)* to from higher order matrix **H**.

3. SIMULATION AND RESULTS :: PERFORMANCE EVALUATION

Multiple simulations have been carried out to check the performance of the new method for estimating the unknown frequencies of unknown constituent sinusoids for multiple signals. The simulations have been carried out with multiple signals and each signal has been represented with constituent sinusoids of highly varying features like high to low values of frequencies and amplitudes, very closely spaced low frequencies, presence and absence of fixed bias and varying phases of constituent sinusoids.

NB: All the estimated frequencies have been rounded off up to six decimal places. Hence the error, , for estimated frequencies is less than 10^{-6} radians.

3.1. Estimating Single Frequency

Consider a signal comprising of single sinusoid as shown below

( ) (( )

Table 1 presents the parameters such as amplitude (*a*), frequency (*ω*) and phase (*θ*) of the test signal 1 shown in **Fig.1**,

Table 1 Parameters of Test Signal 1

Total Number of sinusoids in the signal n = 1			
Constituent Sinusoid	Radian Frequency ω	Amplitude a	Phase θ
1^{st}: sin	2.5	2	0

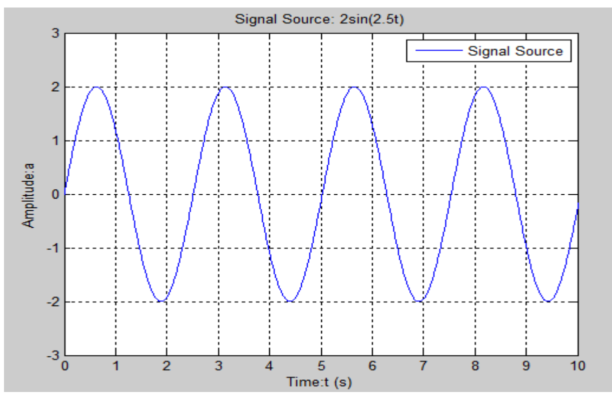

Figure 1 Test Signal 1

Table 2 below shows the performance of the new method to estimate the constituent frequencies of test signal 1 with 10 samples taken from the test signal 1 with sampling time of 10ms.

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Table 2 : Frequency Estimation for Test Signal 1

Estimated Number of sinusoids in the signal $n = 1$								
Frequency Pair	**Estimated Radian Frequency ($\dot{\omega}$)**	**Order of S (M × M)**	**Existence Factor (EF)**	**EF≥M – (2n+1)**	**CF**	**EF≥M – (CF+1)**	**Actual Radian Frequency (ω)**	**Error (ε) Radian (ω - $\dot{\omega}$)**
1^{st}	2.5	5 × 5	2	TRUE	2	TRUE	2.5	$<10^{-6}$

3.2. Estimating Two Frequencies

Consider a signal comprising of two sinusoids as shown below

( ) (( ) + ( )

Table 3 presents the parameters such as amplitude (a), frequency (ω) and phase (θ**)** of the test signal 2 shown in **Fig.2**,

Table 3: Parameters of Test Signal 2

Total Number of sinusoids in the signal n = 2			
Constituent Sinusoid	**Radian Frequency ω**	**Amplitude a**	**Phase θ**
1^{st}: sin	2.5	2	0
2^{nd}: sin	6.1	3.6	-π /2

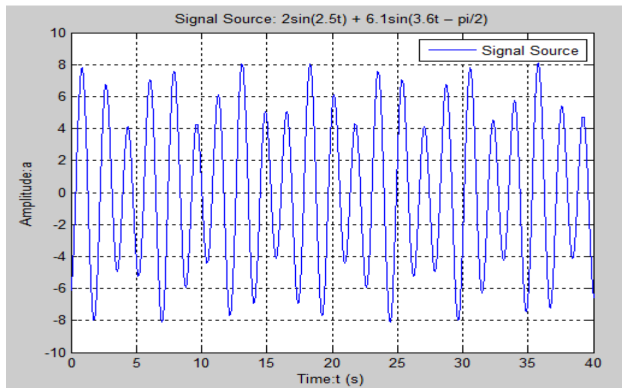

Figure 2 Test Signal 2

Table 4 below shows the performance of the new method to estimate the constituent frequencies of test signal 2 with 40 samples taken from the test signal 1 with sampling time of 10ms.

Table 4 : Frequency Estimation for Test Signal 2

Estimated Number of sinusoids in the signal $n = 2$								
Frequency Pair	**Estimated Radian Frequency ($\dot{\omega}$)**	**Order of S (M × M)**	**Existence Factor (EF)**	**EF≥M – (2n+1)**	**CF**	**EF≥M – (CF+1)**	**Actual Radian Frequency (ω)**	**Error (ε) Radian (ω - $\dot{\omega}$)**
1^{st}	2.5	20 × 20	15	TRUE	4	TRUE	2.5	$<10^{-6}$
2^{nd}	6.1	20 × 20	15	TRUE	4	TRUE	6.1	$<10^{-6}$

3.3. Estimating Multiple Closely Spaced Low Frequency with High Variation in The Amplitude of the Constituent Sinusoids and In the Presence of DC Bias:

The final test has been carried out to test the performance of the new method for the estimation of low frequencies of multiple closely spaced sinusoids with high amplitude variation among the sinusoid along with the presence of fixed bias. Test signal 3 is shown below:

( ) (( ) (
) ( )
( )

( ) ( )

Table 5 presents the parameters such as amplitude (*a*), frequency (*ω*) and phase (*θ*) of the test signal 4 shown in **Fig.3**,

Table 5: Parameters of Test Signal 3

Total Number of sinusoids in the signal n = 6			
Constituent Sinusoid	**Radian Frequency ω**	**Amplitude a**	**Phase θ**
1st : sin	3.14	0.5	π
2nd: cos	0.62	-1	π/6
3rd: cos	5.65	0.1	-π/2
4th:cos	4.71	2	5π/3
5th: cos	1.57	-60	0
6th: cos	1.88	10	0
Fixed Bias	10		

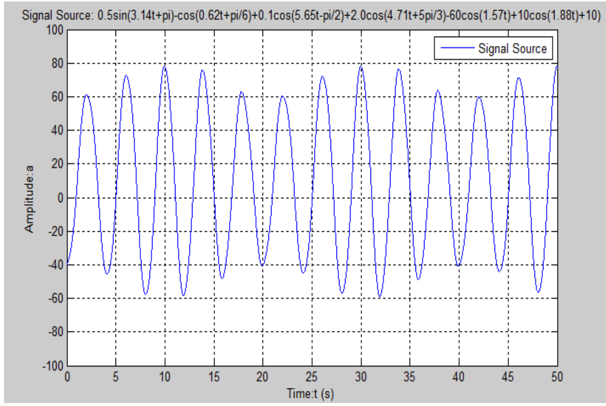

Figure 3 Test Signal 3

Table 6 below shows the performance of the new method to estimate the constituent frequencies of test signal 8 with 3000 samples taken from the test signal 3 with sampling time of 10ms. Table 6 also shows that the new approach has accurately estimated the closely spaced frequencies of test signal 8 in the presence of a fixed bias along with the high variations among the amplitudes of the constituent sinusoids.

The odd value of *CF* also reflects the presence of a fixed bias in the signal which is also reflected in the detected estimated frequencies with the frequency of zero.

Table 6: Frequency Estimation for Test Signal 3

Estimated Number of sinusoids in the signal $n = 6$								
Frequency Pair	**Estimated Radian Frequency ($\dot{\omega}$)**	**Order of S (M × M)**	**Existence Factor (EF)**	**EF≥M – (2n+1)**	**CF**	**EF≥M – (CF+1)**	**Actual Radian Frequency (ω)**	**Error (ε) Radian (ω - $\dot{\omega}$)**
1st	3.14	1500 × 1500	1487	TRUE	13	TRUE	3.14	$<10^{-6}$
2nd	0.62	1500 × 1500	1487	TRUE	13	TRUE	0.62	$<10^{-6}$
3rd	5.65	1500 × 1500	1487	TRUE	13	TRUE	5.65	$<10^{-6}$
4th	4.71	1500 × 1500	1487	TRUE	13	TRUE	4.71	$<10^{-6}$
5th	1.57	1500 × 1500	1487	TRUE	13	TRUE	1.57	$<10^{-6}$
6th	1.88	1500 × 1500	1487	TRUE	13	TRUE	1.88	$<10^{-6}$

The new method is applied to all the simulated test signals and the method has efficiently estimated the unknown frequencies of constituent sinusoids and has also correctly found the total number of sinusoids in the test signals. The technique is an efficient blind sinusoidal detection and estimation technique that accurately estimates the sinusoidal components without any prior knowledge regarding the sinusoidal components.

A qualitative and quantitative comparison of our proposed scheme has been performed with Multi Signal

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Classification (MUSIC) algorithm. On qualitative comparison with MUSIC algorithm, our proposed scheme has greater advantage as our proposed scheme does not need any prior knowledge of the number of frequency components present with in a signal whereas MUSIC algorithm requires the exact number of frequency components that need to be discovered. Another drawback of MUSIC algorithm is that it detects frequency components which are not present within the signal and this will occur if MUSIC algorithm is not provided with the true number of frequency components present with in the signal.

On quantitative comparison with MUSIC algorithm, our proposed scheme performs better than the MUSIC algorithm. Our proposed scheme detects the exact number of frequency components present with in the signal and error in frequency estimation is effectively lesser than the error of MUSIC algorithm. Table 7 presents the performance comparison of our proposed scheme with MUSIC algorithm for single frequency components. The Table 7 reflects the higher performance of our proposed scheme as the error of estimation for our proposed scheme is lesser than the error of estimation of MUSIC algorithm. Table 8 and Table 9 presents the performance comparison of our proposed scheme with MUSIC algorithm for multiple frequency components reflecting the higher performance of our proposed scheme as the error of estimation for our proposed scheme is lesser than the error of estimation of MUSIC algorithm.

Table 7. Performance Comparison with MUSIC for n = 1

Estimated Number of Sinusoids in the Signal **n =1** $\{10\sin(2\pi f_0 t)\}$						
Actual Frequency			Estimated Frequency		Estimation Error	
Test Case	Frequency Under Each Test		MUSIC	Proposed Scheme	MUSIC	Proposed Scheme
1	f_0	10	9.999999242	10	$\leq 10^{-6}$	$\leq 10^{-10}$
2	f_0	23	29.99999978	23	$\leq 10^{-6}$	$\leq 10^{-10}$
3	f_0	35	35.00000025	35	$\leq 10^{-65}$	$\leq 10^{-109}$

Table 8. Performance Comparison with MUSIC for n = 2

Estimated Number of Sinusoids in the Signal **n =2** $\{10\sin(2\pi f_0 t) + 10\sin(2\pi f_1 t)\}$						
Actual Frequency			Estimated Frequency		Estimation Error	
Test Case	Frequency Under Each Test		MUSIC	Proposed Scheme	MUSIC	Proposed Scheme
1	f_0	5	5.000000041	5	$\leq 10^{-7}$	$\leq 10^{-10}$
	f_1	15	14.99999991	15	$\leq 10^{-7}$	$\leq 10^{-10}$
2	f_0	37	36.999999913	37	$\leq 10^{-7}$	$\leq 10^{-10}$
	f_1	17	16.999999716	17	$\leq 10^{-7}$	$\leq 10^{-10}$
3	f_0	21	21.000000015	21	$\leq 10^{-7}$	$\leq 10^{-10}$
	f_1	49	48.999999825	49	$\leq 10^{-6}$	$\leq 10^{-10}$

Table 9. Performance Comparison with MUSIC for n = 4

Estimated Number of Sinusoids in the Signal **n =4** $\{1\sin(2\pi f_0 t) + 6\sin(2\pi f_1 t) + 5\sin(2\pi f_2 t) + 1\sin(2\pi f_3 t)\}$						
Actual Frequency			Estimated Frequency		Estimation Error	
Test Case	Frequency Under Each Test		MUSIC	Proposed Scheme	MUSIC	Proposed Scheme
1	f_0	10	10.0000000086	10	$\leq 10^{-8}$	$\leq 10^{-10}$
	f_1	20	20.0000000131	20	$\leq 10^{-7}$	$\leq 10^{-10}$
	f_2	30	29.9999999556	30	$\leq 10^{-7}$	$\leq 10^{-10}$
	f_3	40	39.9999999289	40	$\leq 10^{-7}$	$\leq 10^{-10}$
2	f_0	3	2.99999993663	3	$\leq 10^{-7}$	$\leq 10^{-10}$
	f_1	15	14.9999997807	15	$\leq 10^{-6}$	$\leq 10^{-10}$
	f_3	37	36.9999998448	37	$\leq 10^{-6}$	$\leq 10^{-10}$
	f_4	49	49.0000001909	49	$\leq 10^{-6}$	$\leq 10^{-10}$

4. CONCLUSION

The new method has been developed using the Hankel matrix and Pisarenko harmonic decomposition that accurately estimates the unknown frequencies of unknown number of sinusoids presents in a signal based on the new constraints *EF* and *CF*. The method also gives the exact number of sinusoid present in the signal. The method has also correctly estimated very closely spaced frequencies. The newly developed approach can be used to test the performance of noise filters by applying the method to the output of the filter.

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