Octupole correlations in the structure of 0_2^+ bands in the $N = 88$ nuclei ^{150}Sm and ^{152}Gd

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Knowledge of the exact microscopic structure of the 0_1^+ ground state and first excited 0_2^+ state in ^{150}Sm is required to understand the branching of double β decay to these states from ^{150}Nd. The detailed spectroscopy of ^{150}Sm and ^{152}Gd has been studied using (α,xn) reactions and the γ-ray arrays AFRODITE and JUROGAM II. Consistently strong $E1$ transitions are observed between the excited $K^\pi = 0_2^+$ bands and the lowest negative parity bands in both nuclei. These results are discussed in terms of the possible permanent octupole deformation in the first excited $K^\pi = 0_2^+$ band and also in terms of the "tidal wave" model of Frauendorf.

I. INTRODUCTION

Recent measurements of the double β decay to the first excited 0_2^+ states in ^{150}Sm [1] and ^{100}Ru [2] demand a full understanding of the exact microstructure and wave functions of the final states as well as the parent ground 0_1^+ states of ^{150}Nd and ^{100}Mo. Accurate calculations of the nuclear matrix elements, involving the initial and final states, are necessary if fundamental questions of the properties of neutrinos are to be answered [3–5]. A determination of the partial lifetime of the $2\beta0\nu$ neutrinoless decay could establish the Majorana/Dirac nature of neutrinos and the ordering of the neutrino masses. Experimentally the 2β decay to the 0_2^+ states is important as the excited states emit two characteristic γ rays giving a vital extra signature of the decay [1,2]. In principle having these two extra γ rays, besides the two decay electrons, could lengthen measurable 2β decay partial lifetimes from the current ∼10^{20} years to the ∼10^{24} years estimated for Majorana $2\beta0\nu$ decay [5].

Evidence indicating that the 0_2^+ states in $N = 88$ and 90 nuclei are not β vibrations [6] but 2p-2h neutron states lowered

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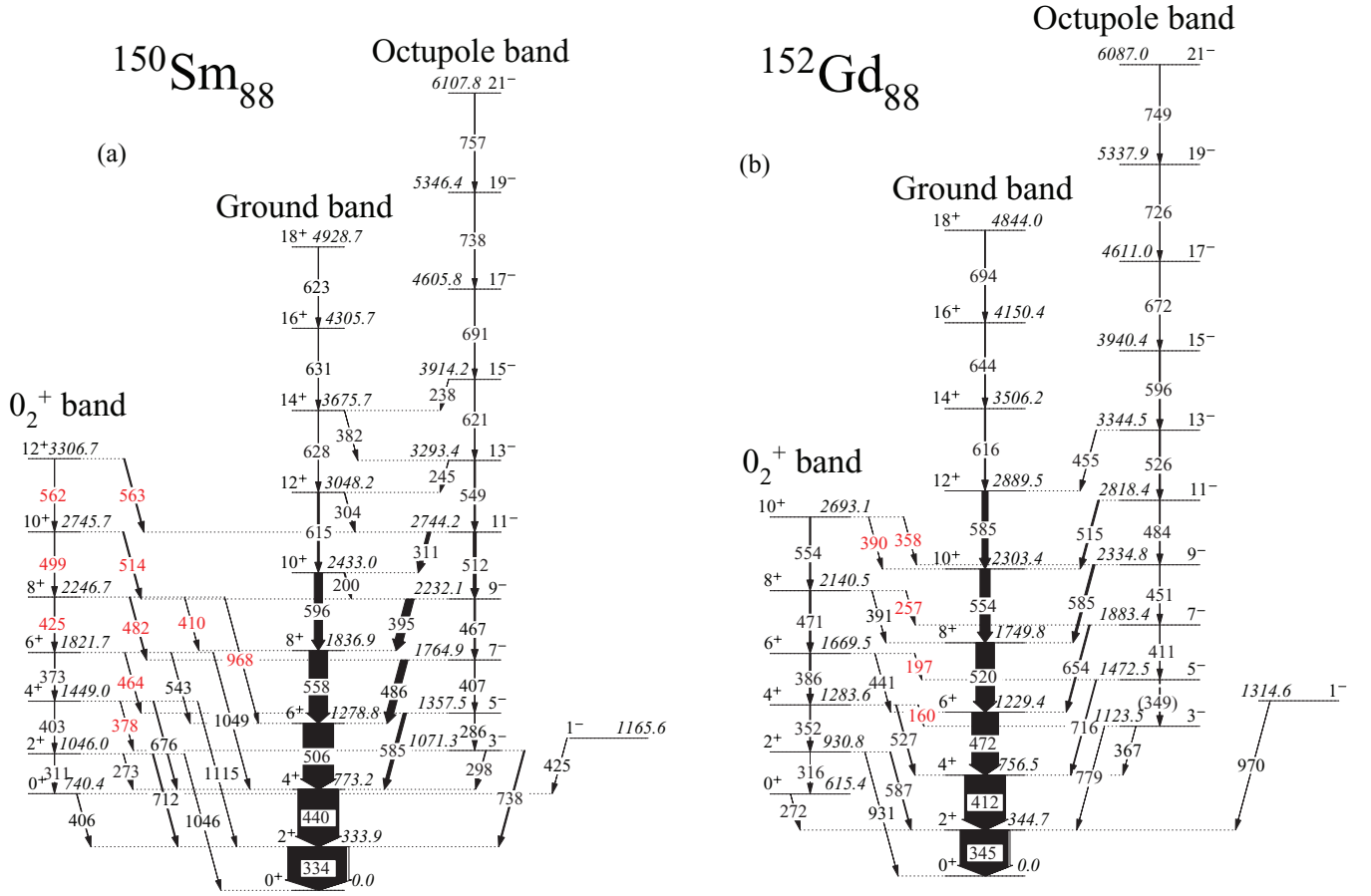‖Present address: TRIUMF, 4004 Wesbrook Mall, Vancouver, British Columbia V6T 2A3, Canada.

into the pairing gap by configuration dependent pairing has been presented in Refs. [7,8]. It was argued that they are classic examples of "pairing isomers" [9] forming a "second vacuum" [7] on which a complete set of excited deformed states are built that are congruent to those built on the 0_1^+ ground state. Evidence for this [10] has also been found in ^{152}Sm. The importance of neutron pair correlations in the structure of 0^+ states involved in 2β decay has been highlighted in Ref. [4]. There have been other descriptions of these 0_2^+ states including them being candidates for $s = 1$ states arising from X(5) symmetry [11].

Nuclei with 88 neutrons are at the very start of the deformed region past the magic number 82. Static quadrupole moments, measured by Coulomb reorientation of the 2^+ states for the stable $N = 86$–90 nuclei $^{146,148,150}_{60}$Nd and $^{148,150,152}_{62}$Sm are reported [12,13] as $-0.79(9)$, $-1.46(13)$, $-2.0(5)$ and $-1.0(3)$, $-1.3(2)$, $-1.68(2)$ eb respectively. These data demonstrate that the $N = 88$ nuclei, ^{148}Nd and ^{150}Sm, are weakly deformed. This is confirmed by measurements of total neutron cross sections for the Nd and Sm isotopes [14] and by Coulomb excitation experiments [15]. The $N = 88$ nuclei with less than 60 protons, that have been investigated using fission fragment spectroscopy, have been suggested as candidates for having a static octupole deformation at medium spins [16–18]. The basis for this suggestion was that strong $E1$ transitions were observed from the positive parity yrast, ground state rotational band to the lowest negative parity band as well as the usual transitions from the first negative parity band to the ground state band. These $E1$ transitions have to compete

FIG. 1. (Color online) Partial level schemes of ^{150}Sm (a) and ^{152}Gd (b) showing the ground state, second vacuum 0_2^+, and octupole bands. New transitions and $E1$ transitions from the second vacuum 0_2^+ bands to the octupole bands are shown in red.

with the strong $E2$ in-band transitions. Interleaving of positive and negative parity states is a property of nuclei with a static octupole deformation [19–22].

The $N = 88$ nuclei have the additional remarkable features; they are at a peak in the $|M(E3)|^2$ transition strength of $0_1^+ \rightarrow 3_1^-$ transitions for even-even nuclei as a function of neutron number (see Fig. 1 in Ref. [23]); they also have very strong $E0$ transitions from the band built on the 0_2^+ states to the ground state bands [24,25]. Generally, strong $E3$ transitions have been accounted for [22] by the proximity of $\Delta j^\pi = 3^-$ shell model orbits near the Fermi surface. For $N = 88$ nuclei these are $i_{13/2}$–$f_{7/2}$ for neutrons and $h_{11/2}$–$d_{5/2}$ for protons. Recent reflection-asymmetric relativistic mean-field calculations [26] and folded Yukawa-Strutinsky calculations with particle number projection [27] indicate that ^{150,152}Sm and ^{152}Gd could have a permanent octupole $Y_{3,0}$ deformation.

The nucleus ^{150}Sm has its first negative parity band at an unusually low excitation energy. Indeed, this negative parity band is actually yrast at spin 11^- [28]. $E1$ transitions have been observed [29,30] both ways between the positive parity yrast states, at 10^+ and above, and the negative parity band. It was conjectured that the yrast states were associated with a static octupole deformation beyond 10^+.

We have made extensive spectroscopic measurements in the nuclei $^{150}_{62}$Sm$_{88}$ and $^{152}_{64}$Gd$_{88}$ using modern spectrometers.

We report here on the first observation of consistent $E1$ transitions in deformed nuclei from levels in the first excited 0_2^+ bands to the lowest negative parity bands. Decays of this kind have not been observed in any of the deformed nuclei with $N \geqslant 90$ [31]. We will refer to the lowest negative parity band in any nucleus as the *octupole band* for brevity. These bands in ^{150}Sm and ^{152}Gd have traditionally been assumed to be $K^\pi = 0^-$ vibrational bands with only natural parity states 1^-, 3^-, 5^-, ... and no even-spin signature partners.

II. EXPERIMENTAL PROCEEDURE AND RESULTS

The ^{152}Gd$_{88}$ nucleus was studied using the ^{152}Sm$(\alpha,4n)^{152}$Gd reaction at 45 MeV, a self-supporting target of 4 mg cm^{-2}, and the iThemba LABS escape-suppressed γ-ray spectrometer array AFRODITE [32] consisting of 8 HPGe (high-purity germanium) clover detectors in BGO (Bismuth Germinate) shields. About 5.10^8 $\gamma\gamma$ coincidences were obtained and DCO (Directional Correlations from Oriented nuclei) ratios and linear polarizations at $90°$ were measured to establish spins and parities [33]. The nucleus ^{150}Sm was studied with the ^{148}Nd$(\alpha,2n)^{150}$Sm reaction at 25 MeV, a self-supporting target of 5 mg cm^{-2}, and the Jyväskylä JUROGAM II escape-suppressed γ-ray spectrometer array [34] consisting of 24 clover and 15 tapered
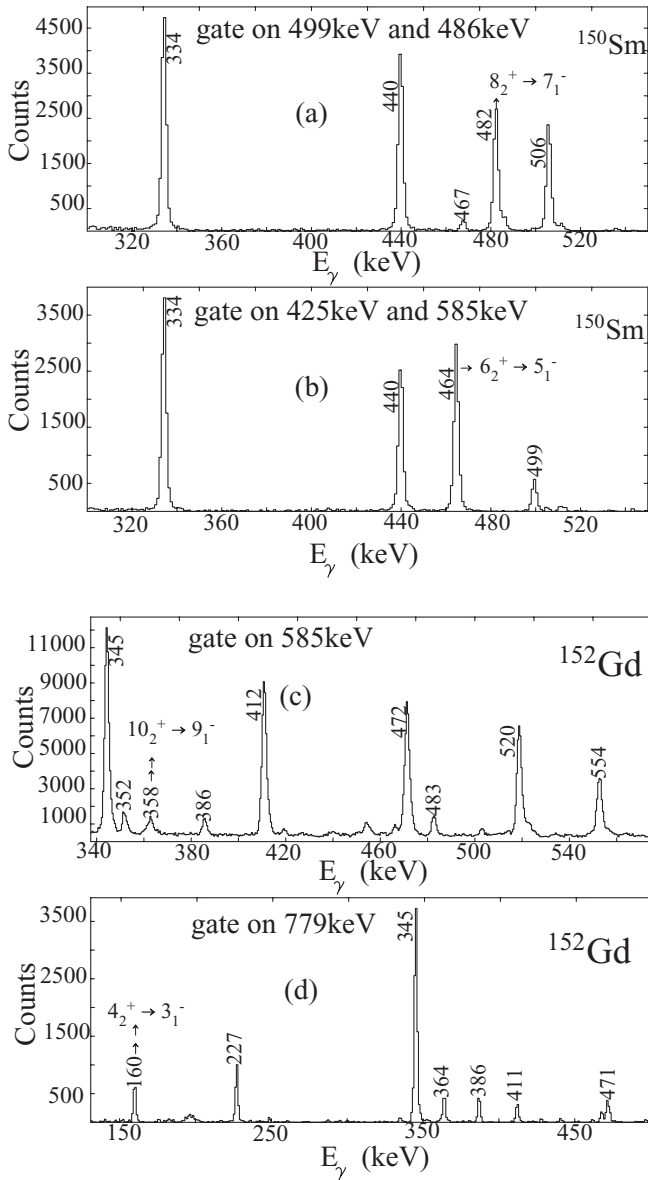
FIG. 2. Coincidence spectra illustrating the positioning of the 0_2^+ to octupole band $E1$ transitions. (a) ^{150}Sm $\gamma\gamma\gamma$ data for the 482 keV $8_2^+ \to 7_1^-$ γ ray. (b) ^{150}Sm $\gamma\gamma\gamma$ data for the 464 keV $6_2^+ \to 5_1^-$ γ ray. (c) ^{152}Gd $\gamma\gamma$ data for the 361 keV $10_2^+ \to 9_1^-$ γ ray. (d) ^{152}Gd $\gamma\gamma$ data for the 159 keV $4_2^+ \to 3_1^-$ γ ray.

HPGe detectors all in BGO shields. All events taken with JUROGAM II detectors are time stamped and merged into a time ordered stream [35]. The equivalent of $\mathbf{2.10^9}$ $\gamma\gamma\gamma$ triple coincidences were arranged in a cube and analyzed using RADWARE [36].

The partial decay schemes for ^{150}Sm and ^{152}Gd are shown in Fig. 1 for the ground, 0_2^+, and octupole bands. The 6^+, 8^+, 10^+, and 12^+ levels in the 0_2^+ band of ^{150}Sm are new. Figure 2 shows examples of coincident spectra which establish the $E1$ decays of the 0_2^+ band to octupole band levels. All of these $E1$ transitions are new except the 4^+ to 3^- transitions in both nuclei and the 6^+ to 5^- in ^{152}Gd. These transitions are listed in Table I together with their $B(E1)/B(E2)$ ratios.

TABLE I. $E1$ dipole transitions in ^{150}Sm and ^{152}Gd. The $B(E1)/B(E2)$ ratios for previously observed transitions are marked with an asterisk. Transitions from an excited 0_2^+ band are labeled 0_2^+, from a ground state band "gsb", and from an octupole band "oct".

Assignment	E_γ (keV)	$B(E1)/B(E2)$ (10^{-6} fm^{-2})
^{150}Sm		
$0_2^+ (4^+ \to 3^-)$	378	0.35(4)
$0_2^+ (6^+ \to 5^-)$	464	0.53(2)
$0_2^+ (8^+ \to 7^-)$	482	0.14(1)
$0_2^+ (10^+ \to 9^-)$	514	0.35(10)
gsb $(12^+ \to 11^-)$	304	0.27(2)*
gsb $(14^+ \to 13^-)$	382	0.12(2)*
^{152}Gd		
$0_2^+ (4^+ \to 3^-)$	159	0.041(1)
$0_2^+ (6^+ \to 5^-)$	198	0.15(1)
$0_2^+ (8^+ \to 7^-)$	259	0.12(1)
$0_2^+ (10^+ \to 9^-)$	361	0.13(1)
oct $(5^- \to 4^+)$	716	0.010(1)*
oct $(7^- \to 6^+)$	654	0.11(1)*
oct $(9^- \to 8^+)$	585	0.58(16)*
oct $(11^- \to 10^+)$	515	0.31(4)*
oct $(13^- \to 12^+)$	455	0.11(1)*

Table I also includes the previously measured $B(E1)/B(E2)$ ratios between the ground and octupole band in ^{150}Sm and the octupole to ground state band in ^{152}Gd. It can be seen that these ratios are very comparable for both nuclei and also with the same ratios in lighter $N = 88$ nuclei [16–18].

III. DISCUSSION

The strong $E1$ transitions from the 0_2^+ band to the octupole band suggest that these bands should be related. Chasman [37] has used a schematic Hamiltonian with pairing forces and particle-hole octupole-octupole forces to calculate the properties of 0_2^+ levels in the light U nuclei. He finds that while the ground states remain only quadrupole deformed the first excited 0_2^+ states contain considerable octupole correlations. To test the conjecture that the 0_2^+ bands in ^{150}Sm and ^{152}Gd could have a permanent octupole deformation and form reflection asymmetric structures with their octupole bands, we use the criteria given in Ref. [22] that the ratio $\omega^-/\omega^+ \approx 1$ where ω^- is the rotational frequency observed in the negative parity octupole band and ω^+ is the rotational frequency observed in the 0_2^+ band. In Fig. 3 this ratio is plotted for ^{150}Sm and ^{152}Gd as a function of spin $\boldsymbol{I}$, pairing both the 0_2^+ and octupole bands and also pairing the ground state and octupole bands, and compared with the well established [22,38] octupole deformed nucleus ^{220}Ra. It is clear that for the comparison with the 0_2^+ band, the ratio for ^{150}Sm is 1.0 within 5% from $\boldsymbol{I} = 5$ onwards and is near 0.9 for ^{152}Gd. The ratio for both 0_2^+ bands is much nearer octupole deformed behavior than for the ground state bands and even for yrast ^{220}Ra. This would suggest that the 0_2^+ and octupole bands may form a reflection asymmetric structure with simplex quantum
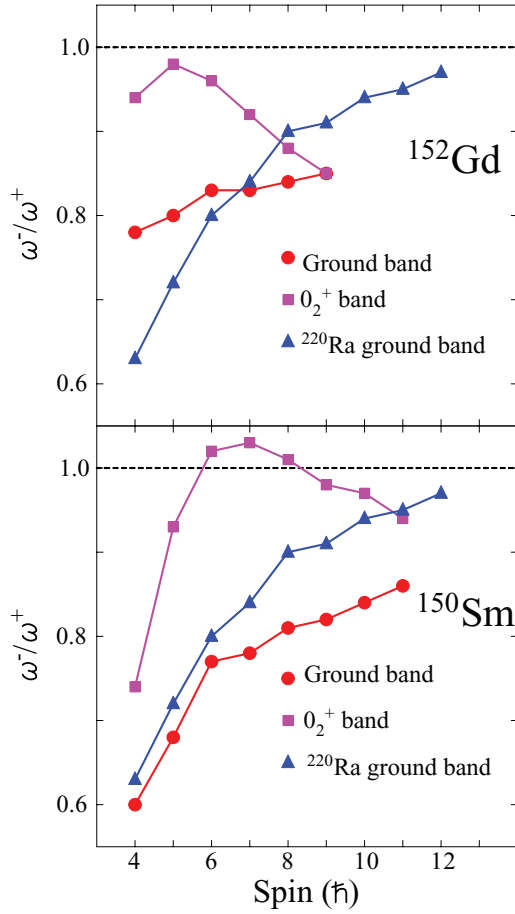
FIG. 3. (Color online) Frequency ratios ω^-/ω^+ as a function of spin I for the ground state band and the octupole band, and for the 0_2^+ band and octupole band, for ^{152}Gd and ^{150}Sm. They are compared with the established octupole deformed band in ^{220}Ra [22,38]. For structures with permanent octupole deformation, this ratio would be 1.0.

number $s = 1$, rather than the ground state and octupole bands as suggested in Ref. [30]. In $^{148}_{60}$Nd$_{88}$ Ibbotson *et al.* [18] find strong $E3$ strengths connecting the 0_2^+ band and octupole band.

In Fig. 1 it can be seen that there are pairs of levels in the 0_2^+ and octupole bands of ^{150}Sm that are nearly degenerate in energy. The pairs are $\{2^+,3^-\}$, $\{4^+,5^-\}$, $\{6^+,7^-\}$, $\{8^+,9^-\}$, $\{10^+,11^-\}$, and $\{12^+,13^-\}$. This is opposite to the more common degeneracy of $\{2^+,1^-\}$, $\{4^+,3^-\}$, etc. for a soft pear-shaped nucleus [21]. In order to increase the energy of the positive parity states with respect to the negative parity states, there would have to be strong mixing of the positive parity 0_2^+ band with the ground state band as suggested by Sheline [39] for similar phenomena in the actinide nuclei. In contrast, the energy sequence of the 0_2^+ and octupole bands in ^{152}Gd, apart from the 1^- state, is the same as that expected for a rigid pear shape [21]. In ^{148}Nd$_{88}$ [18] the 3^- octupole level is at 999 keV, lower than in both ^{150}Sm and ^{152}Gd, and the 0_2^+ state is at 917 keV, higher than in both ^{150}Sm and ^{152}Gd.

A recent interpretation of low-lying negative parity bands has been given by Frauendorf [40,41] in terms of the

condensation of rotation-aligned octupole phonons or a surface "tidal wave" leading to heart shaped nuclei. In this model the rotation of the deformed nucleus aligns octupole phonons with the axis of rotation forming an yrast line of alternating parity states. Even numbers of phonons form the positive parity states and odd numbers the negative parity states. As the rotational frequency increases, the one phonon negative parity states
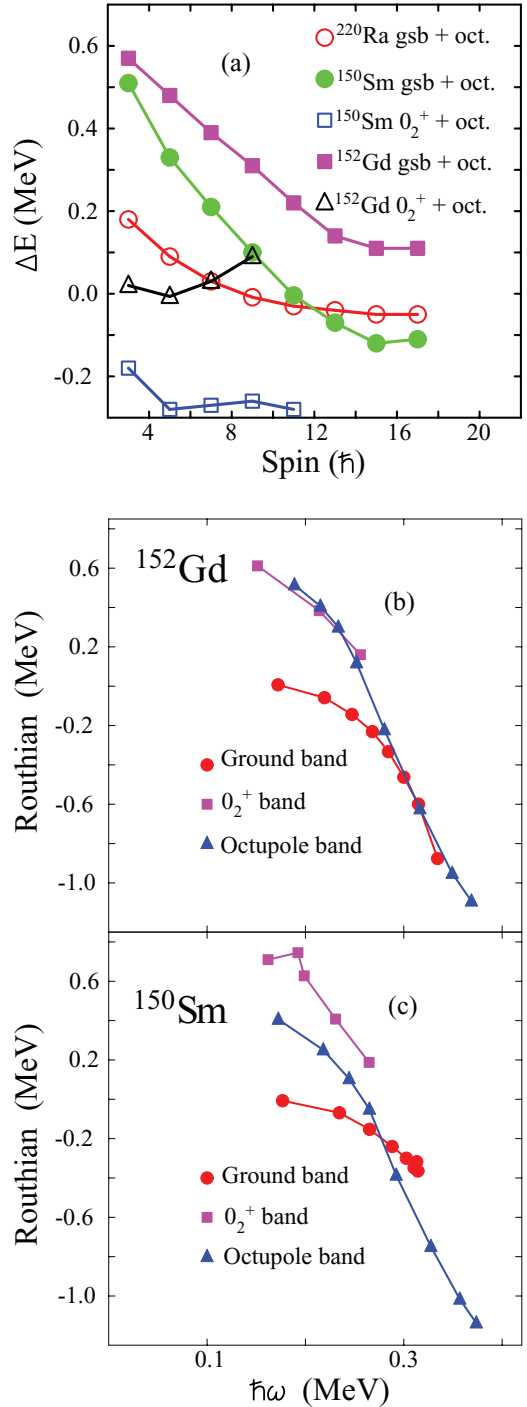

FIG. 4. (Color online) (a) Energy difference, Eq. (1), between negative and positive parity states in ^{150}Sm, ^{152}Gd, and ^{220}Ra. (b), (c) Energy in the rotating frame, Routhian, against rotational frequency $\hbar\omega$ for the bands in ^{152}Gd and ^{150}Sm.

become yrast, as they do in ^{220}Ra and ^{150}Sm but not in ^{152}Gd. As the second octupole phonon is aligned, the positive parity states again become yrast; again as in ^{220}Ra and ^{150}Sm. This is illustrated in Fig. 4(a) by taking the energy difference

$$\Delta E(I) = E_-(I) - \{E_+(I+1) + E_+(I-1)\}/2 \qquad (1)$$

between negative and positive parity states for ^{220}Ra and for both the ground and octupole bands and the $0_2{}^+$ and octupole bands in ^{150}Sm and ^{152}Gd. When $\Delta E(I)$ is positive the positive parity states are yrast and when $\Delta E(I)$ is negative the negative parity states are yrast.

For the octupole phonons to produce a tidal wave they have to have a critical frequency ω_c at which they coalesce. In Figs. 4(b) and 4(c) the Routhians, the energies e' in the rotating frame, are shown for ^{152}Gd and ^{150}Sm respectively. The Routhians for the bands in ^{150}Sm shown in Fig. 4(c) are not dissimilar to the idealized Routhians in Fig. 3(b) of Ref. [41]. The Routhians for ^{152}Gd, shown in Fig. 4(b) are also similar. The Routhian for the $0_2{}^+$ band in ^{150}Sm shown in Fig. 4(c) emphasizes that there is a marked change in slope between the 4^+ and 6^+ levels corresponding to the decrease in the in band γ-ray energy (Fig. 1). In principle a band based on a pair of noninteracting aligned 3^- octupole phonons should start at spin-parity 6^+. This is an intriguing observation. If the tidal wave description is to be useful, then the yrare lower spin extensions of the phonon bands will need to be identified.

In conclusion, the rotational band built on the $0_2{}^+$ state has been extended in ^{150}Sm and, uniquely, additional intra-band $E1$ transitions between the $0_2{}^+$ bands and octupole bands in both ^{150}Sm and ^{152}Gd have been observed. The relative strengths of the $E1$ transitions and the behaviour of both $0_2{}^+$ bands argues for, but does not prove, the Chasman interpretation [37] of the ground state being quadrupole

deformed whereas the $0_2{}^+$ state has an additional octupole deformation forming a simplex $s = 1$ alternating parity band with the lowest negative parity band. The tidal wave scenario, painted by Frauendorf [41], has similarities with the structures in ^{150}Sm but less so with those in ^{152}Gd. The differences with proton number between the lowest rotational bands in the $N = 88$ nuclei is in need of convincing explanation. Coulomb excitation of beams of ^{150}Sm and ^{152}Gd, in the manner of the experiments on ^{148}Nd [17,18], would give information on $B(E3)$ strengths as well as on quadrupole deformations. Detailed calculations along the lines of those by Chasman [37] are called for.

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