DEMB Working Paper Series

N. 123

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February 2018

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ISSN: 2281-440X online

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Private Information and the Commitment Value of Unobservable Investment[a]

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Abstract

The commitment value of unobservable investment with cost-reducing effects is examined in an entry model where the incumbent is privately informed about his costs of production. We show that when the price signals incumbent's costs, unobservable investment can not have any commitment value and the limit price does not limit entry. By contrast, if the price does not reveal costs, which is the more likely outcome, unobservable investment has a magnified value of commitment and a less aggressive limit price deters profitable entry.

JEL Classification Numbers: D42, D82, L12, L41.
Keywords: Commitment, entry deterrence, limit pricing, signaling.

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[a]We are grateful to Giacomo Calzolari and Michele Grillo for helpful comments on the related work Brighi and D'Amato (2014). We also wish to thank Marco Celentani, Albert Ma, Riccardo Martina, Massimo Morelli, Salvatore Piccolo and Régis Renault for helpful discussions.

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1 Introduction

In facing an entry threat, an incumbent may use strategically long term investments, or other irreversible actions with persistent effects on post-entry competition, in order to prevent entry. By undertaking investments in excess of the normal level, the incumbent commits itself to respond aggressively to entry, so that, upon observing incumbent's choice, potential entrants modify their decision and stay out. While the commitment value and the deterrence effect of *observable* investment decisions are well understood and have been thoroughly investigated in a well established strand of the literature,[1] less attention has been paid to the case where investment is *non observable.* In fact, there are a number of choices made by the incumbent which are unobservable to potential entrants but have effects on post-entry competition. For instance, decisions about investments in R&D and about the exploitation of the learning curve, or decisions about contracts, such as managerial compensation schemes or long run contracts with customers and suppliers. Nevertheless, only little research has focused on the effects of unobservable investments on entry decisions because, as pointed out by Bagwell (1995), if investment is not observed it can not have any commitment value and any impact on the scale and the occurrence of entry.

This view has been recently challenged in a recent contribution by In and Wright (2017), where it is shown that unobservable investment can still have an indirect value of commitment through the price. Indeed, a low price may force the incumbent to expand current output providing him with an incentive to invest in cost-reducing activities. If both the entrant and the incumbent realize that the price provides such an incentive to invest, the incumbent may use the price for strategic purposes, i.e. as an incentive to over-invest in order to lower costs and as a signal to convey this piece of information to the entrant and achieve a deterrence effect.

In and Wright (2017) derive this result in a context where the only kind of information that the price can convey is about unobservable investment. However, if other dimensions of private information are present, such as in the case where exogenous characteristics concerning costs or market demand are known to the incumbent but not to the entrant, it is not granted that the

[1] The reference is to the literature originated from the early contributions by Dixit (1979, 1980) and Spence (1977).

price is used to signal investment rather than other characteristics. For instance, if incumbent's costs are private information, as in the classical limit pricing model put forward by Milgrom and Roberts (1982), the price can also be used by the incumbent to convey information about costs to the potential entrant. However, if the price signals costs rather than investment, then unobservable investment can not have any commitment value. Hence, the crucial question arises as to which piece of information the price is able to convey in equilibrium. An answer to the above question can only be given by a careful analysis of the equilibrium in a fully fledged signaling model with hidden characteristics and unobservable actions.

The model developed in this paper is a first attempt to study the commitment value of unobservable investment in an entry problem with private information. We consider a two-period entry model where an incumbent who is privately informed about his marginal costs has the opportunity to invest in a cost-reducing technology. Investment is unobservable to potential entrants and affects current as well as future marginal costs. The incumbent decides how much to invest and the output to be produced (or equivalently the price to be charged) in the first period. After observing the output, but not the amount of investment, an entry decision by a potential entrant is made. If entry takes place the post-entry stage is the standard Cournot-Nash outcome with learning upon entry, otherwise the incumbent acts as a monopolist.

To simplify the analysis we assume that there are only two types of incumbent, the high cost type (inefficient incumbent) and the low cost type (efficient incumbent). We also assume that the efficiency gap between the two types affects the effectiveness of investment to cut costs. The efficient incumbent is supposed to be on the technological frontier, hence investment has no cost reducing effects for him. The high cost type, instead, has the opportunity to improve efficiency and reduce marginal cost of production by investing. Entry is supposed to be profitable to the potential entrant only in the face of the high cost type. There exists, however, a level of investment which allows the inefficient incumbent to lower his costs as much as needed to drive entrant's profits to zero.

The entry model is formalized as a non standard signaling game where the sender chooses an unobservable action as well as a signal. Compared to a standard signaling game the sender's choice will have to balance the incentives to manipulate information with the incentives to invest. On the other hand, the entrant will have to strive to infer both the incumbent's type and the incumbent's action, since both pieces of information are taken into account at the moment when entry is decided. The solution of the game is found by applying a notion of Perfect Bayesian Equilibrium in which the entrant formulates expectations not only about sender's type (costs), but also about sender's action (investment). Moreover, a modified version of Cho and Kreps (1987) Intuitive Criterion, restricting inferences about unobserved actions after observed deviations, is used to deal with the intrinsic multiplicity of equilibria.

The main results of our analysis can be summarized as follows. When the price can convey information about privately known costs and can provide incentive to undertake unobservable investments, a conflict arises between the incentives to signal costs and the incentive to invest. If in equilibrium the price conveys information about costs it cannot provide incentives to invest. The simple reason is that if the inefficient incumbent has a profitable opportunity to deter entry by setting a limit price which might persuade the entrant that a large amount of investment has been undertaken, then the efficient incumbent has always an incentive to mimic the inefficient type. Therefore, if the price reveals incumbent's costs it can not provide incentives to invest to the inefficient incumbent and unobservable investment can not have any commitment value.

Conversely, if in equilibrium the price does not signal costs it can be used to provide the incumbent with an incentive to invest. In such a case, as the entrant is left uncertain about incumbent's costs, the price not only signals unobservable investment but also magnifies its strategic effect. In other words, unobservable investment has a value of commitment which is magnified by the price. We find that the latter kind of equilibrium is the more likely outcome of the entry problem, so that we expect that the incumbent will be able to deter profitable entry with lower levels of investment and less aggressive limit prices than those that would result if the entrant were informed about incumbent's costs.

Being relevant for the study of the strategic role of investment, our analysis is very much related to the classical analysis of the 'battle' for market shares (Roberts, 1987). Our investigation suggests that results obtained in the large literature on entry barriers and irreversible investments (for a survey see Tirole, 1986 and Ordover and Saloner, 1989) can be extended to an environment where investments are more opaque and coexist with private information by incumbent firms. Moreover, our analysis is strictly related to the classical contribution by Milgrom and Roberts (1982). In fact, our work provides a complete equilibrium analysis of one of the few specifications of their 'generalized' limit pricing model (Milgrom and Roberts, 1982 sec. 3). We make two contributions to the analysis of the generalized model. On one hand, we show that when the price signals the type of incumbent it can restrain the strategic use of unobservable actions and this has non negligible effects on entry. On the other hand, our analysis supports with an explicit economic explanation Milgrom and Robert's view that pooling equilibria are the more likely outcome of the entry game in the generalized limit pricing model.

As already noticed, our work is also closely connected to the recent contribution by In and Wright (2017). We suggest that when private information about exogenous characteristics is introduced into the analysis, then the role played by prices in signaling private choices must be more carefully considered. In fact, in equilibrium, either unobservable investment has no commitment value because the price signal costs, or unobservable investment has a magnified value of commitment because the price leaves the entrant's uncertain about incumbent's costs. Under no circumstances the incumbent behaves as in the complete information case.

Finally, it is worth to mention that a first attempt to examine the commitment value of unobservable investment is also found in Brighi, D'Amato and Piccolo (2005). In an entry model with a continuum of types, they study the separating equilibria of a signaling game where an unobservable investment decision is available to the incumbent who has private information about costs. It is shown that limit pricing allows the incumbent to lower the probability of entry by restoring the commitment value of investment. Their analysis, however, does not extend to pooling equilibria.

The rest of paper is organized as follows. Section 2 sets out the model of entry and introduces the main assumptions. Section 3 provides the equilibrium analysis and main results. Section 4 examines the benchmark with complete information about costs and discusses the contributions of our model to the existing literature with particular reference to Milgrom and Roberts (1982) and In and Wright (2017). Finally, a summary and final remarks are offered in the last section. All the proofs are collected in the Appendix.

2 The entry model

We consider a standard two periods entry model where an incumbent firm faces the potential entry of a competing firm in a market for a homogeneous good. In the first period firm 1, the incumbent, who has private information about his costs of production, decides how much to produce, $q \geq 0$, and how much to invest in a cost reducing technology, $e \geq 0$. In the second period firm 2, the entrant, after observing first period output but not investment, decides whether to enter into the market. The entrant's choice is denoted by $y \in \{0, 1\}$, with $y = 1$ if entry takes place and 0 otherwise. If entry occurs the two firms compete, the entrant pays an entry cost, learns the incumbent's production costs (learning upon entry) and firms compete à la Cournot. Otherwise, firm 1 remains a monopolist and the potential entrant gains her outside option normalized to zero.

Market demand in each period is described by an inverse demand function, $p(q)$, which is assumed to be differentiable and strictly decreasing. The marginal costs of firm 1 and firm 2 are constant and the fixed costs of production are set to zero. There are only two types of incumbents, the L type with a low marginal cost (efficient incumbent) and the H type with a high marginal cost (inefficient incumbent). We denote by $\theta_t(e) \geq 0$, with $t = L, H$, the marginal cost of type t when the amount of investment is e. The symbol θ_t stands for $\theta_t(0)$. The prior probability that the incumbent is inefficient is denoted by β.

Investment is made by the incumbent in the first period and affects marginal costs in both periods. As the low cost incumbent is already on the technological frontier, we will assume that his investment activity has a purely dissipative nature so that no effect on marginal cost ensues, i.e. $\theta_L(e) = \theta_L$. Conversely, investment can allow the high cost incumbent to reduce the cost gap with respect to type L, even though it will never let the ranking of types, in terms of marginal costs, be overturned. Therefore, the cost reducing technology of the inefficient incumbent, $\theta_H(e)$, will be represented by a strictly decreasing and differentiable function of e, with $\theta_H(e) > \theta_L$ for all e. For future reference, some of the above assumptions are collected in

Assumption A.1. $\theta_L(e) = \theta_L$ *and* $\theta_H(e)$ *is strictly decreasing with* $\theta_H(e) > \theta_L$ *for all* e.

The per period incumbent's profits (gross of investment costs e) are given by $\Pi_t(e, q) \equiv [p(q) - \theta_t(e)]q$. The function $\Pi_t(e, q)$ is differentiable and *strictly quasi-concave* in q, so that, for each level of e, the incumbent's per period profit maximization problem has a unique solution, the monopoly quantity, denoted by $m_t(e)$. Monopoly profits, which are strictly positive, are denoted by $M_t(e) \equiv \Pi_t(e, m_t(e))$. For convenience of notation we define $m_t \equiv m_t(0)$ and $M_t \equiv M_t(0)$. It can be noticed that, by A.1, the L type monopoly quantity and profit are independent of investment, i.e. $m_L(e) = m_L$ and $M_L(e) = M_L$.

Incumbent's profits in the second period depend on the entry decision by firm 2. If she does not enter, the incumbent remains a monopolist and earns $M_t(e)$, while firm 2 makes zero profits. If entry occurs, the entrant pays an 'entry fee', learns private information about incumbent's costs, i.e. the type t and the amount of investment e, and the two firms compete *à la Cournot*. Incumbent's second period profits, if entry occurs, are denoted by $D_t(e) \geq 0$. Entrant's profits, net of entry fee, depend on the type of incumbent she faces and on investment, so they are denoted by $D_2(e, t)$. We define $D_t \equiv D_t(0)$ and $D_2(t) \equiv D_2(0, t)$. The function $D_H(e)$ is increasing and $D_2(e, H)$ is decreasing in e, whereas, by A.1, $D_L(e) = D_L$ and $D_2(e, L) = D_2(L)$.

The incumbent's decision about q and e, is based on *total profits* over the two periods. Assuming no time discounting, total profits are given by

$$V_t(e, q, y) = \Pi_t(e, q) - e + yD_t(e) + (1 - y)M_t(e). \tag{1}$$

The function $V_H(e, q, y)$ is assumed to be strictly quasi-concave in (e, q). If the incumbent behaves like a monopolist under no entry threat, he will play his monopoly output in both the first and the second period. In particular, the inefficient type will choose a level of investment maximizing his total profits of monopoly, i.e. $V_H(e, m_H(e), 0)$. The level of investment with no entry threat is denoted by $\bar{e}$ and the monopoly output by $\bar{m} \equiv m_H(\bar{e})$. Conversely, if entry is expected, the incumbent will produce the monopoly quantity in the first period and he will choose the optimal level of investment taking into account that duopoly profits will be made in the second period. The total profits of the two types when entry is accommodated are respectively given by $V_L^A = V_L(0, m_L, 1) = M_L + D_L$, for type L, and by

$$V_H^A = \max_e V_H(e, m_H(e), 1) \tag{2}$$

for type H. The level of investment that satisfies the maximization problem in (2), called *investment of accommodation*, is denoted by e_A and the associated monopoly quantity by $m_A \equiv m_H(e_A)$. For convenience we suppose that $e_A < \bar{e}$.

The entry decision by firm 2 depends on her expected profits. After observing first period market quantity or equivalently first period market price, the entrant makes an inference about incumbent's cost and investment. Let $\hat{\beta}(q)$ denote the entrant's *beliefs* about the probability of the incumbent being of type H and $\hat{e}_t(q)$ the *conjectures* about type t incumbent's investment choice. Notice that, by A.1, the level of conjectured investment undertaken by L is immaterial for the decision of entry as investment does not affect θ_L, therefore only conjectures about the high cost incumbent need to be considered. The entrant's expected profits in the event of entry are given by

$$\hat{\beta}(q) D_2(\hat{e}_H(q), H) + (1 - \hat{\beta}(q)) D_2(L). \tag{3}$$

Firm 2 enters if expected profits are strictly positive.

The next assumptions ensure that the entry threat is real and that a positive amount of investment is required in order to prevent entry. We assume that entry is profitable for firm 2 against type H if investment is zero. In addition, we also suppose that entry is profitable, if firm 2 faces uncertainty about incumbent's type and the inefficient incumbent invests as much as a monopolist in the absence of an entry threat. This condition rules out the case of blockaded entry.

Assumption A.2. $D_2(H) > 0$, $D_2(L) < 0$ *and* $\beta D_2(\bar{e}, H) + (1 - \beta) D_2(L) > 0$.

An immediate consequence of A.2 is that deterrence of profitable entry under uncertainty requires a level of investment higher than $\bar{e}$.[2] Our next assumption ensures that a level of investment exists for the inefficient incumbent which makes entry unprofitable to firm 2.

Assumption A.3. There exists a level of investment for type H, denoted by e_C, at which the entrant's post-entry profits are zero, i.e. $D_2(e_C, H) = 0$.

The amount e_C is called the *investment of commitment* because it is the level that would allow the inefficient incumbent to prevent entry if investment were observable by firm 2. The value of the parameter e_C is strictly related to the structural factors affecting market entry conditions, such as the magnitude of entry fees and the effectiveness of the cost-reducing technology. High entry fees and a 'steeper' technology $\theta_H(e)$ characterize unfavourable entry conditions, which result in lower levels of e_C.[3] Vice-versa, less unfavourable entry conditions for firm 2 are associated with higher values of e_C.

Finally, notice that, by Assumption A.2, entrant's profits at $\bar{e}$ are strictly positive, i.e. $D_2(\bar{e}, H) > 0$. As D_2 is strictly decreasing in e, Assumption A.3 implies that $e_C > \bar{e}$ and ensures that there exists a unique level of investment e_0, with $\bar{e} < e_0 < e_C$, which drives to zero entrant's *expected* profits, i.e. such that

$$\beta D_2(e_0, H) + (1 - \beta) D_2(L) = 0. \tag{4}$$

The *zero expected profit* investment e_0 will play an important role in the ensuing analysis.

The entry problem above is modeled as a non standard signaling game where the 'sender' chooses an unobservable action as well as a signal. The cost reducing technology $\theta_H(e)$, the marginal cost θ_L, the prior probability β, market demand $p(q)$ and duopoly profits $D_t(e)$ and $D_2(e, t)$ are common knowledge, whereas type t is private information to the incumbent. A notion of Perfect Bayesian equilibrium will be used as the solution concept and only equilibria in pure strategies will be considered. A modified version of Cho and Kreps (1987) Intuitive Criterion will be applied to

[2] Notice also that, as $e_A < \bar{e}$, assumption A.2 implies that entry is profitable to firm 2 if she faces uncertainty about costs and the incumbent chooses the investment of accommodation.

[3] This can be easily seen taking into account that higher entry fees cut entrant's profits and that $D_2(e, H)$ is a decreasing function of investment which is steeper the more effective the cost-reducing technology is.

refine the intrinsic multiplicity of equilibria. The following standard assumption will be used to guarantee the existence of separating equilibria

$$M_L - D_L \geq M_H(e) - D_H(e), \qquad \text{for all } e. \tag{5}$$

Condition (5) simply states that the low cost incumbent benefits from entry deterrence more than the high cost incumbent. Finally, in order to avoid trivial forms of separation, we assume that the high cost incumbent has an incentive to mimic the low cost one, i.e.

$$V_H(0, m_L, 0) > V_H^A. \tag{6}$$

The next section provides the equilibrium analysis of the entry model.

3 Deterrence with unobservable investment

Let us first set out the strategies and the solution concepts of the signaling game described in the previous section. A pure strategy for the incumbent is a function which associates with each type a level of investment and a first period quantity and consists of two pairs, (e_H, q_H) and (e_L, q_L). Non observability of investment along with its dissipative nature for type L (assumption A.1) imply that any incumbent's strategy with $e_L \neq 0$ is strictly dominated by a similar strategy where the L type chooses $e_L = 0$. The space of incumbent's strategies will thus be restricted by dropping strictly dominated strategies.

A strategy for the entrant, $y(q) \in \{0, 1\}$, specifies an entry decision for any first period quantity. After observing the signal q, the entrant makes an inference about incumbent's costs. The posterior probability that the incumbent has high costs, the entrant's belief, is $\hat{\beta}(q)$. The signal is also used by the entrant to make an inference about incumbent's investment choice. A conjecture $\hat{e}_t(q)$ is an estimate of the level of investment undertaken by type t held by the entrant after the quantity q is observed. Notice that, having removed strictly dominated strategies, the trivial entrant's conjecture about the investment undertaken by type L is $\hat{e}_L(q) = 0$.

For any given move of Nature and profile of strategies the incumbent's payoff is given by (1), whereas the entrant's payoff is $D_2(e,t)$, in the event of entry, and zero otherwise. The expected profits that firm 2 associates with entry are given by (3), where beliefs and conjectures are made by the entrant after observing q. The solution concept we use is the notion of Perfect Bayesian equilibrium (PBE) adapted to our context. The specification of conjectures, besides that of beliefs, is included in the definition.

Definition 1. *A profile of strategies (e_t, q_t) and $y(q)$, with $t = H, L$, is a PBE of the signaling game if there exist beliefs $\hat{\beta}(q)$ and conjectures $\hat{e}_t(q)$, such that the following conditions are satisfied:*

(i) The incumbent's strategy is optimal, i.e. for $t = H, L$,

$$(e_t, q_t) = \text{argmax}_{e,q}\ V_t(e, q, y(q))$$

(ii) The entrant's strategy is optimal, i.e. $y(q) = 1$ if and only if

$$\hat{\beta}(q) D_2(\hat{e}_H(q), H) + (1 - \hat{\beta}(q)) D_2(L) > 0$$

where, for any q,

$$\hat{e}_H(q) = \text{argmax}_e\ V_H(e, q, y(q)).$$

(iii) Beliefs are consistent with Bayes' rule and conjectures are consistent with the incumbent's strategy, i.e. $\hat{e}_t(q_t) = e_t$, for $t = H, L$.

As compared to a standard version of PBE, Definition 1.(ii) introduces a new condition in the spirit of subgame perfection requiring that the entrant's conjectures are optimal on and off the equilibrium path. It is supposed that, in forming her expectations, the potential entrant thinks as if the incumbent chooses his price first and then, accordingly, selects an optimal level of investment.[4] By

[4] In fact, $\hat{e}_H(q)$ can be seen as part of an optimal strategy by the incumbent in the 'continuation games' of a transformed version of the original game. The transformation is obtained by first applying a reordering of incumbent's moves, analogous to that recently proposed in In and Wright (2017), and then considering the reordered game as an example in the class of multi-stage games with observable actions and incomplete information described in Fudenberg and Tirole (1991, p. 331).

Definition 1.(iii), the entrant's inferences on type and investment must be consistent in equilibrium with Bayes' rule and the incumbent's strategy. Specifically, the entrant is required to correctly predict the level of investment undertaken by the incumbent which, by Definition 1.(i), must be a best reply to the entrant's equilibrium strategy.

As no restriction is placed on off-equilibrium beliefs, the signaling game exhibits a multiplicity of equilibria. The Intuitive Criterion originally proposed by Cho and Kreps (1987) can not be directly applied to refine the equilibria, unless the notion of *equilibrium domination* is modified to incorporate entrant's conjectures about investment. In this respect, recall that equilibrium domination postulates that the entrant compares, for each type of incumbent to whom a deviation can be ascribed, the equilibrium payoff and the *highest conceivable* payoff from that deviation. As the highest payoff is obtained in this game when entry does not take place and, in addition, when the incumbent makes his unobservable choice optimally, we will use this off-equilibrium conjecture to extend the notion of equilibrium domination. Notice that this conjecture is consistent with that used in Definition 1.

In order to arrive at a formal definition of the Intuitive Criterion, two functions have to be introduced. The *investment function* $\phi(q)$ is defined as the solution to the following maximization problem

$$\phi(q) = \text{argmax}_e \, V_H(e, q, 0). \tag{7}$$

It provides the optimal level of investment chosen by the high cost incumbent at a given quantity q if entry does not take place.[5] The *best profit function* $U(q)$ is the maximum value function associated with the above maximization problem, i.e.

$$U(q) = V_H(\phi(q), q, 0). \tag{8}$$

It provides the maximum total profit to type H contingent on quantity q being produced, $\phi(q)$ being invested and no entry. To check for equilibrium domination, let V_t^* denote the payoff to type t in a given equilibrium supported by q_t and consider a deviation $\tilde{q} \neq q_t$. The best payoff from

[5] The function ϕ is well defined, continuous and strictly increasing. See the Appendix, Lemma 1.

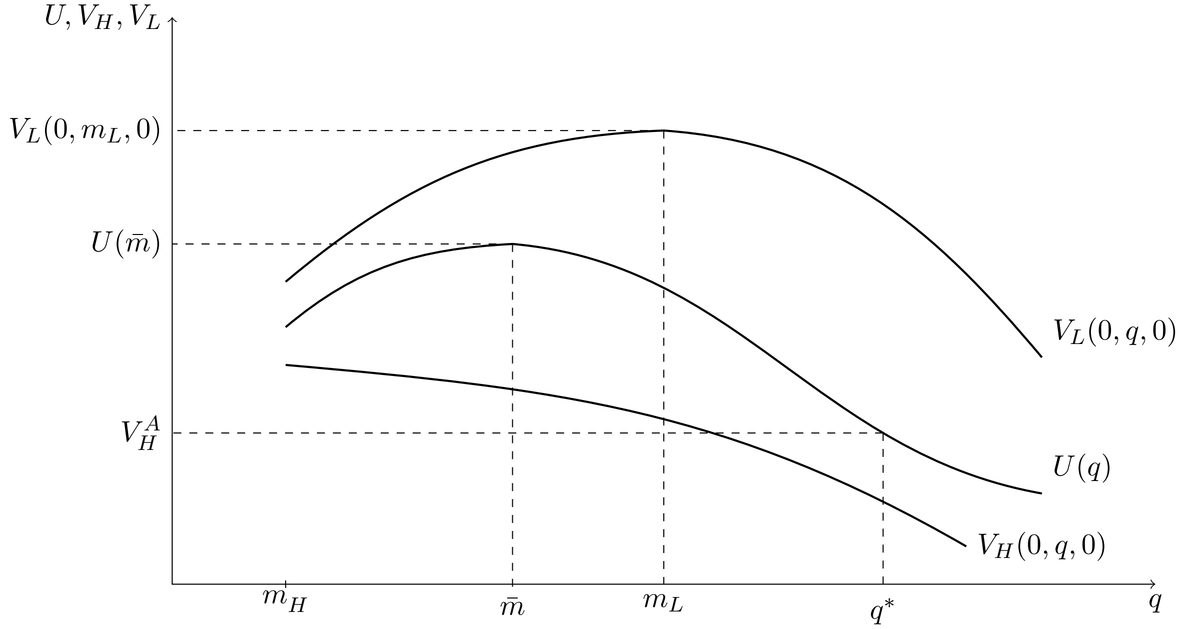

Figure 1: Total profits V_H, V_L and the value function $U(q)$

deviation to type H is given by $U(\tilde{q})$, because it is obtained when $e = \phi(\tilde{q})$, i.e. when investments are set to maximize total profits given $\tilde{q}$ and entry does not take place. Hence, a deviation $\tilde{q}$ is equilibrium dominated for type H if $V_H^* > U(\tilde{q})$. Equilibrium domination for type L follows the standard definition where the best payoff from deviation to type L is $V_L(0, \tilde{q}, 0)$. By using the above notation the formal definition for the Intuitive Criterion that will be applied to our context is the following:

Definition 2. *A PBE of the signaling game, (e_t, q_t) and $y(q)$ with $t = H, L$, satisfies the Intuitive Criterion if and only if there exists no deviation $\tilde{q} \neq q_t$ such that*

$$\begin{aligned} V_H(e_H, q_H, y(q_H)) &> U(\tilde{q}) \quad \textit{and} \\ V_L(0, q_L, y(q_L)) &< V_L(0, \tilde{q}, 0) \end{aligned}$$

The properties of $U(q)$, which are studied in the Appendix (Lemma 1), are important to prove our major result. Here it is sufficient to notice that $U(q)$ is strictly quasi-concave with a global maximum at $q = \bar{m}$, the monopoly output of the inefficient incumbent in the absence of any entry threat. The main characteristics of the best profit function are illustrated in Figure 1, where the graph of $U(q)$ is plotted along with the graphs of the total profit functions of both types of incumbent in the absence of entry and with zero investment.

Finally, let us define the threshold q^* as the level of first-period output at which type H total profits in the event of no entry of firm 2 are the same as those that he would earn by accommodating entry. The quantity threshold q^* is implicitly defined by

$$U(q^*) = V_H^A \qquad \text{with} \qquad q^* > m_L \tag{9}$$

and will be called the *threshold of accommodation profits*. Indeed, the amount q^* is the largest output that the inefficient incumbent is willing to sell in the first period if this behaviour would allow him to deter entry. Any larger quantity (or lower limit price) will not be as profitable as accommodating entry. The threshold q^*, which is also depicted in Figure 1, is well defined and unique (see Lemma 1.(iii) in the Appendix).[6]

Let us now turn to the analysis of pure strategy equilibria, i.e. separating and pooling equilibria.

3.1 Separating equilibrium

In a separating equilibrium different types of incumbent choose different quantities, i.e. $q_H \neq q_L$, and information about costs is revealed to the entrant. If the incumbent has low costs, firm 2 makes a loss in the event of entry (see A.2) and thus she remains out. The low cost type does not invest because investment is purely dissipative to him (see A.1) and he will charge a price that conveys information about his cost to the entrant.

If the incumbent has high costs, two potential outcomes need to be considered. Either entry is accommodated or it is deterred. In the former case, firm 2 will enter because she conjectures that

[6] Just to simplify the presentation of results, we also assume that the total profits of accommodation for type H are greater than his profits as a monopolist in a single period, i.e. $V_H^A > U(0)$.

investment is lower than the level of commitment e_C (see A.3). Hence, the H type will choose a monopoly output in the first period and a duopoly quantity in the second and his best choice will be $e_H = e_A$ and $q_H = m_A$, where e_A and m_A are as defined in section 2. In the latter case, firm 2 will not enter because she thinks that the inefficient incumbent has undertaken a sufficiently large level of investment. An incumbent aiming to deter entry will choose a quantity which allows firm 2 to infer the type and the level of investment undertaken. As players' choices are perfectly anticipated by opponents at equilibrium and as e_C is the minimum amount of investment which makes entrant's profits to vanish, the incumbent will choose the investment of commitment. Moreover, in order to let firm 2 correctly anticipate the level of investment, the incumbent will choose a quantity which provides a consistent incentive to invest. If firm 2 does not enter, the amount q_C, implicitly defined by the equality $\phi(q_C) = e_C$, is the minimum first-period output which allows the inefficient incumbent to signal to firm 2 that the investment of commitment e_C has been undertaken in the first period. Accordingly, q_C will be called the *quantity of commitment*.

The value of q_C depends on the value of e_C and reflects the entry conditions characterizing the industry. We shall focus on the case where $q_C > m_L$, because when the quantity of commitment is lower than the monopoly output of the efficient incumbent, only trivial cases of separation arise[7] and entry will never take place in equilibrium.

Moreover, notice that if the configuration of the parameters of the model are such that $q_C > q^*$, the inefficient incumbent will never decide to produce the quantity of deterrence because he would be better off by accommodating entry, as $U(q_C) < U(q^*) = V_H^A$. Although our analysis applies to general configurations of parameters, we will restrict attention to the more interesting case where the inefficient incumbent has the opportunity to profitably deter entry, i.e. the case where the quantity of commitment falls short of the threshold of accommodation profits q^*. In this case, also, the comparison between separating and pooling equilibria will result simpler and more clear. Hence, the following assumption will hold throughout the paper.

[7]If $q_C < m_L$, it can be easily seen that $q_H = q_C$ and $q_L = m_L$ support an intuitive separating equilibrium.

Assumption A.4. The quantity of commitment q_C, which is defined by the equality $\phi(q_C) = e_C$, exists, it is greater than the monopoly output of the efficient incumbent and lower than the threshold q^*, i.e. $m_L < q_C < q^*$.

Let us proceed to the analysis of the two candidate equilibrium outcomes, i.e. entry deterrence or entry accommodation. As at equilibrium the incumbent's quantities q_L and q_H must satisfy incentive compatibility requirements, i.e. each type of incumbent should prefer to choose his equilibrium quantity rather than that of his opponent, the incumbent's strategy must meet the two following conditions

$$U(q_L) \leq V_H(e_H, q_H, y(q_H)) \tag{10}$$

and

$$V_L(0, q_L, 0) \geq V_L(0, q_H, y(q_H)). \tag{11}$$

If the inefficient incumbent deters entry, his choices for e and q must be at least as large as those of commitment, namely $e_H \geq e_C$ and $q_H \geq q_C$. As firm 2 does not enter, the incentive compatibility conditions to be satisfied by incumbent's choices, i.e. (10) and (11), are respectively $U(q_L) \leq U(q_H)$ and $V_L(0, q_L, 0) \geq V_L(0, q_H, 0)$. As can be easily seen, given the lower bound for q_C set by assumption A.4 and the shape of functions U and V_L, the incumbent's choices of first period output must satisfy the inequalities $q_L < m_L$ and $q_H > m_L$. In other words, at equilibrium the inefficient incumbent would set a limit price, while the efficient type would charge an over limit price, i.e. a price above his monopoly level. As shown in the Appendix (Lemma 2.(i)), separating equilibria where the inefficient incumbent deters entry do not exist.

Let us consider then the case where, at equilibrium, the H type accommodates entry so that he sets $e_H = e_A$ and $q_H = m_A$. In such a case, the incentive compatibility condition (10) imposes on q_L the requirement that the maximum profits the high cost incumbent may earn if entry is avoided does not exceed his profit if entry is accommodated, i.e.

$$U(q_L) \leq V_H^A. \tag{12}$$

Moreover, condition (11) is not binding in this case and it holds as long as the participation condition for type L is satisfied, i.e.

$$V_L(0, q_L, 0) \geq V_L^A. \tag{13}$$

It is important to stress that the existence of a separating equilibrium is not granted for all configurations of parameters. To clarify this point, let us consider the *investment function given entry*

$$\psi(q) = \text{argmax}_e \, V_H(e, q, 1). \tag{14}$$

By applying the same type of arguments used for ϕ in Lemma 1 of the Appendix, it is easily seen that the function ψ is well defined and increasing. The first-period output that would induce the inefficient incumbent to undertake the investment of commitment even in the event of entry, the *quantity of sure commitment* q_{SC}, is implicitly defined as the solution to the equation[8]

$$\psi(q_{SC}) = e_C. \tag{15}$$

Under standard conditions about market demand and costs, the quantity of sure commitment is greater than the quantity of commitment, i.e. $q_C < q_{SC}$.[9] If there are profitable levels of first-period output that induce the H type to undertake the investment of commitment even if entry is expected, and precisely if $q_{SC} < q^*$, then no separating equilibrium exists in the present model. The basic idea behind this result is that accommodating entry can not be an optimal choice for the inefficient incumbent. Indeed, once firm 2 observes q_{SC}, she is sure that the H type has undertaken the investment of commitment and that her post-entry profits will be non positive, so that she will decide to stay out. As a result, the inefficient incumbent would rather set q_{SC} instead of m_A, because $U(q_{SC}) > V_H^A$, and no separating equilibrium exists (see Proposition 1.(i) below).

A separating equilibrium only exists provided that $q_{SC} \geq q^*$. In such a case, separating equilibria where the efficient type sets a limit price, i.e. where $q_L > m_L$, are characterized by the incumbent's

[8]For the sake of simplicity in the presentation of results, we assume that q_{SC} exists. Notice that, if q_{SC} did not exist, $\psi(q) < e_C$ for all q, and all the main results of our analysis hold true.

[9]It can be seen that, if entrant's marginal costs are θ_L and market demand is linear, $\phi(q) > \psi(q)$. Hence $e_C = \phi(q_C) > \psi(q_C)$, which by monotonicity of ψ yields $q_C < q_{SC}$.

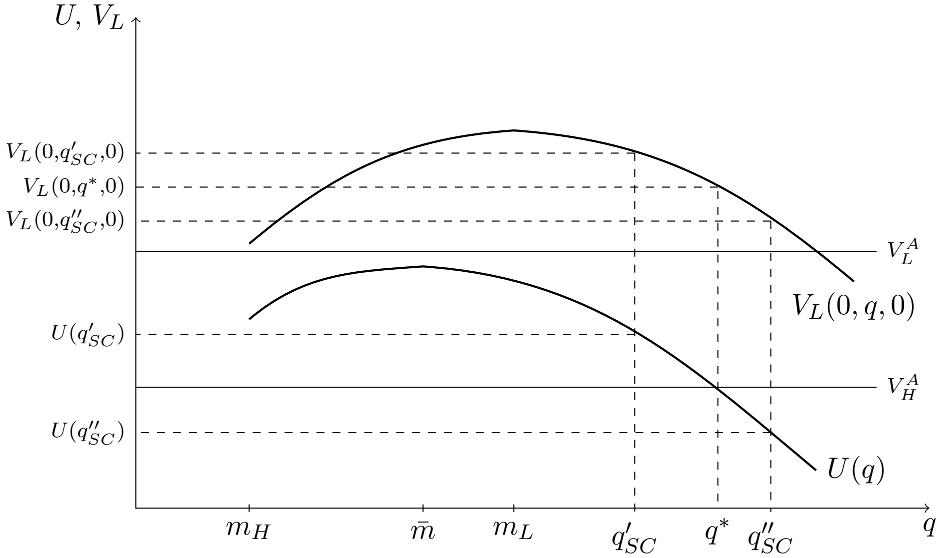

Figure 2: Existence of the intuitive separating equilibrium

strategies $(e_H, q_H) = (e_A, m_A)$ and (e_L, q_L) satisfying (12), (13) with $e_L = 0$ and $q_L \leq q_{SC}$ (see Lemma 2.(ii) in the Appendix). Moreover, it turns out that there exists a unique intuitive separating equilibrium.[10]

Proposition 1. *Let q^* and q_{SC} be as respectively defined by (9) and (15).*

(i) If $q_{SC} < q^*$, no separating equilibrium exists.

(ii) Conversely, if $q_{SC} \geq q^*$, there exists a unique separating equilibrium satisfying the Intuitive Criterion, which is the equilibrium supported by the incumbent's strategy $(e_H, q_H) = (e_A, m_A)$ and $(e_L, q_L) = (0, q^*)$.

Figure 2 provides a graphical illustration of both cases contemplated in Proposition 1. If the quantity of sure commitment is q'_{SC} the L type will never choose to separate with the quantity q^* because he would earn higher profits by mimicking the commitment behaviour of the H type. If,

[10]By uniqueness we mean that the same incumbent strategy is shared by any other equilibrium.

instead, the quantity of sure commitment is q''_{SC} both types of incumbent would rather comply with their respective separating choices which yield higher profits.

According to Proposition 1, the most likely separating equilibrium, if it exists, is the one where the inefficient incumbent sets his monopoly price and accommodates entry, whereas the efficient type sets a limit price to signal his cost to firm 2, who learns that entry would be unprofitable and stays out. Proposition 1 offers quite a surprising result as it states that, in equilibrium, the inefficient incumbent can not take advantage of the commitment value of investment. Indeed, he can not lower his price as much as needed, because otherwise the efficient type would find more profitable to raise his equilibrium price at the level set by the inefficient one. Actually, Proposition 1 reflects a general point. In entry problems with unobservable investment and private information a conflict that the price is not able to settle typically arises between the incentive to reveal costs and the incentive to invest. The price can not signal both the investment of commitment and the costs at the same time, because the efficient type is better off by mimicking the inefficient one. As a result, a separating equilibrium can only exists under particular configurations of parameters and whenever it exists, the incentive to invest must be dampened by the price so that unobservable investment can not have any value of commitment. As will be seen below, however, a separating equilibrium need not be the most likely outcome of the entry problem.

3.2 Pooling equilibrium

In a pooling equilibrium both types of incumbent choose the same quantity q_P, i.e. $q_H = q_L = q_P$, and the potential entrant stays out. Although the entrant does not learn any information about the type of incumbent she faces, her conjecture must be that the high cost type has made at least the zero expected profits level of investment e_0, given by (4), i.e. $\hat{e}_H(q_P) \geq e_0$. Indeed, any other conjecture yields positive expected profits and would induce firm 2 to enter. The investment choice by the H type must be optimal given q_P and, as entry does not take place, it must be given by the investment function, i.e. $e_H = \phi(q_P)$. As the entrant's conjecture is correct in equilibrium, i.e. $\hat{e}_H(q_P) = e_H$, the pooling equilibrium quantity must satisfy the condition $\phi(q_P) \geq e_0$. In other words, the equilibrium quantity must be able to induce type H to undertake a level of investment at least as large as e_0. In order to derive the constraints on q_P, let us introduce the *zero expected profits quantity* q_0 implicitly defined by the equality

$$\phi(q_0) = e_0. \tag{16}$$

The quantity q_0 is well defined, it lies between $\bar{m}$ and q^* and it is lower than the quantity of commitment, i.e. $\bar{m} < q_0 < q^*$ and $q_0 < q_C$.[11] By the above discussion, a pooling equilibrium quantity must satisfy the inequality $q_P \geq q_0$.

Other requirements to be satisfied in equilibrium are the incumbent's participation constraints. The inefficient incumbent has two alternative options. He may either accommodate entry, and make profits V_H^A, or he may choose the quantity of sure commitment and make the profit $U(q_{SC})$ by leaving the entrant out. His best alternative choice depends on the relative magnitude of q^* and q_{SC}. Given the shape of U, if $q^* < q_{SC}$, accommodation is better, because $V_H^A = U(q^*) > U(q_{SC})$. Vice-versa, if $q_{SC} < q^*$, the best option for type H is to choose the quantity of sure commitment. Thus the participation constraint for type H places an upper limit to the pooling equilibrium quantity which can not exceed the minimum between q^* and q_{SC}, i.e.

$$q_P \leq \min\{q^*, q_{SC}\}. \tag{17}$$

Similarly, for type L, the alternative options are either to accommodate entry or to deter entry by mimicking the inefficient incumbent and setting the quantity of sure commitment. As the equilibrium outcome must yield higher payoffs than those available with alternative choices, the participation constraint of the low cost incumbent can be written as follows.

$$V_L(0, q_P, 0) \geq \max\{V_L^A, V_L(0, q_{SC}, 0)\}. \tag{18}$$

As shown in the Appendix (Lemma 4), a pooling equilibrium can be characterized by an incumbent's strategy (e_H, q_H), (e_L, q_L), satisfying (17) and (18) with $q_H = q_L = q_P$, $q_P \geq q_0$, $e_H = \phi(q_P)$

[11] The inequality $q_0 < q_C$ holds because $e_0 < e_C$. Moreover, $q_0 < q^*$ by A.4. Notice also that, as $e_0 > \bar{e}$ and $\bar{e} = \phi(\bar{m})$ (see Lemma 1 in the Appendix) the zero expected profits quantity is larger than the monopoly output of an incumbent under no entry threat, i.e. $\bar{m} < q_0$.

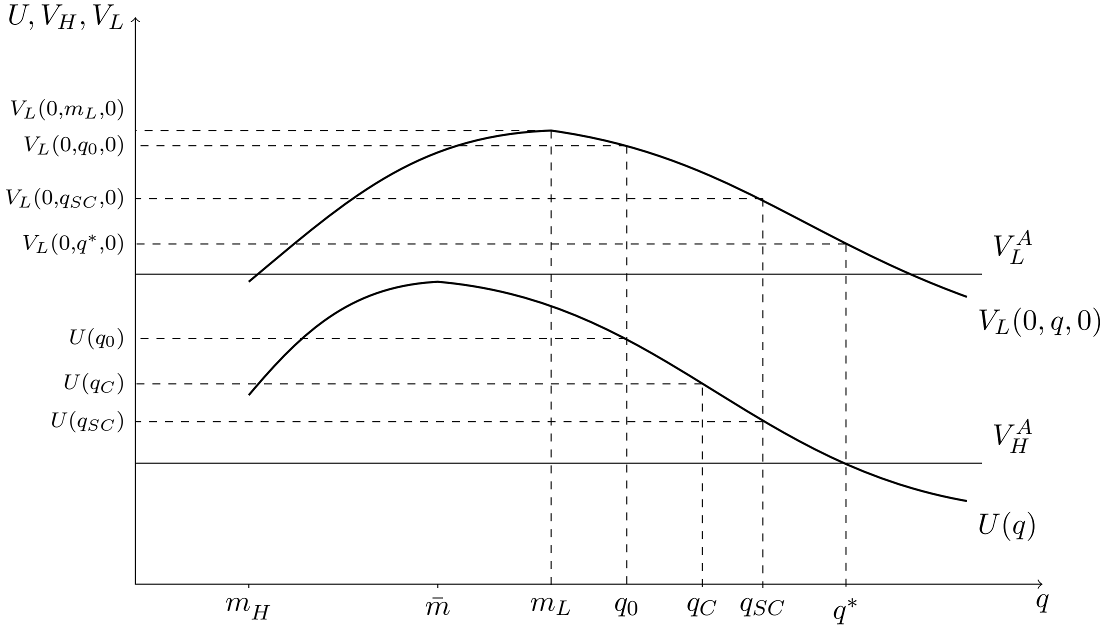

Figure 3: Intuitive pooling equilibria and no separating equilibria

and $e_L = 0$. Moreover, a pooling equilibrium always exists. In fact, the next result states that there always exists a unique intuitive pooling equilibrium which is *Pareto undominated* according to the incumbent's interim payoffs, i.e. such that none of the types of incumbent can be made better off in any other intuitive pooling equilibrium.

Proposition 2. *Let q_0 be the zero expected profits quantity defined by (16), and let q^* be the threshold quantity given by (9). There exists a unique pooling equilibrium satisfying the Intuitive Criterion which is Pareto undominated according to the incumbent's interim payoffs. This equilibrium is supported by the quantity $q_P = m_L$ if $q_0 < m_L$ and by the quantity $q_P = q_0$ if $q_0 \geq m_L$.*

At a pooling equilibrium the incumbent sets a price at or below the monopoly price of the efficient incumbent and the entrant stays out. The peculiar nature of this result is that limit pricing deters entry just because it does not convey any piece of information about the cost type, but only about the high cost incumbent investment behaviour. In fact, after observing the equilibrium price, the potential entrant decides not to enter because she does not know which type of incumbent is in the market, but she does know that the H type has undertaken investments large enough to induce non positive expected profits. As a result, in a pooling equilibrium, unobservable investment gains a 'magnified' value of commitment as compared to the case where the investment were directly observable by the potential entrant. Indeed, a smaller amount of investment is sufficient to deter entry because $e_0 < e_C$. Hence, profitable entry is deterred because firm 2 is left out when the incumbent is inefficient.

Proposition 2 also shows that pooling equilibria exists under more general configurations of parameters as compared to separating equilibria. Indeed, a pooling equilibrium exists even when the investment of sure commitment falls short of the threshold quantity of accommodation profits, i.e. when $q_{SC} < q^*$. This case is also illustrated in Figure 3, where a separating equilibrium does not exist. The L type will never separate from type H by setting the quantity q^*, because he will be better off by playing the quantity of sure commitment q_{SC} as firm 2 will stay out. On the other hand, a pooling equilibrium is seen to exist in which both types of incumbent prefer to play the less aggressive limit price associated with q_0.

The main conclusions from the overall analysis of the entry model can be summarized as follows. Proposition 1 and Proposition 2 provide us with two candidates for the solution to the signaling game, a separating and a pooling equilibrium, although the separating one only exists under a restricted set of parameter configurations. Both equilibria satisfy the Intuitive Criterion, but the pooling is also Pareto superior to the separating equilibrium from the point of view of the incumbent, as both types are better off. The fact that the outcome of the intuitive pooling equilibrium is strictly preferred by both types suggests that it is the most plausible solution.[12] Therefore, we conclude that the predicted outcome of the entry game is the intuitive pooling equilibrium characterized in Proposition 2, where the deterrence of profitable entry is supported by a limit price which is at or below the monopoly price of the efficient incumbent and by an amount of investment below the level of commitment.

[12]The selection of the intuitive pooling equilibrium can also be justified on more formal grounds by resorting to the notion of 'defeated' equilibrium proposed by Mailath, Okuno-Fujiwara and Postlewaite (1993).

4 Complete information benchmark and discussion of results

In order to better evaluate our contribution to the existing literature we will compare the results of section 3 with those obtained in a benchmark with complete information. Accordingly, we will slightly modify the model of section 2 by assuming that, after Nature's move, the type of incumbent is revealed to both players, the incumbent and the entrant. Hence, we consider a game with complete information and unobservable investment. Choice variables and payoffs are as specified in section 2. The only change is that here the entrant knows exactly the type of incumbent she is facing. The comparison with the model of section 3 is focused on the case where both separating and pooling equilibria exist, i.e. the case in which $q_{SC} > q^*$.

The outcome of the benchmark when the incumbent has low costs is trivial. Type L sets his monopoly output and firm 2 will stay out because she knows entry is unprofitable. The behaviour of the inefficient incumbent, instead, must be obtained as the solution to the complete information game between the incumbent H and the entrant. This game has multiple Nash equilibria. The two pure strategy equilibria which are particularly relevant for our analysis are, respectively, the equilibrium with an outcome of accommodation and that with a deterrence outcome. In the former equilibrium, the incumbent accommodates entry and firm 2 enters. The profile of strategies is $(e_H, q_H) = (e_A, m_A)$ and $y(q) = 1$ for all $q \leq q^*$ and zero otherwise. As is easily seen, both players' strategies are best replies, indeed, the accommodation quantities are best choices for the incumbent given entry and, on the other hand, firm 2's entry decision is optimal given that investment is below the level of commitment.

In the latter Nash equilibrium, the incumbent sets a price below his monopoly level and deters entry. The profile of strategies is $(e_H, q_H) = (e_C, q_C)$, $y(q) = 1$ if $q < q_C$ and $y(q) = 0$ if $q \geq q_C$, where e_C and q_C are, respectively, the commitment quantities of investment and output. Firm's 2 strategy is a best reply given that the incumbent undertakes the commitment level of investment e_C. On the other hand, the incumbent's choice of investment is optimal given q_C, as $e_C = \phi(q_C)$, and his choice of output is optimal because it allows him to avoid entry.

The Nash equilibrium with the deterrence outcome is the equilibrium selected by applying the

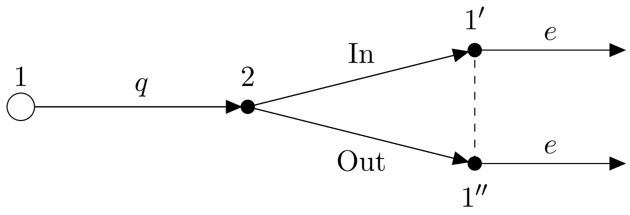

Figure 4: Sequence of moves in the reordered game

analysis put forward by In and Wright (2017). In fact, the complete information game between the incumbent H and the entrant is an example of the 'endogenous signaling games' studied by In and Wright, where incumbent's 'private information' about costs is determined endogenously by his unobservable action, and not by Nature, and where the incumbent has the opportunity to signal his private choice through the price. In and Write suggest to study this class of games by performing a reordering of incumbent's moves which does not affect the information structure of the game. In the specific application to entry problems, they suggest to proceed as if the incumbent first announces the first period output to firm 2 and commits to it and then the two players move simultaneously: the H type decides the amount of unobservable investment to undertake and the entrant makes her choice. This reordered game has proper sub-games which are associated with the observed first period output, so that the selection criterion of sub-game perfection can be applied. The sequence of moves in the reordered game is outlined in the diagram of Figure 4. Type H and the entrant are, respectively, player 1 and player 2, while the dashed line identifies an information set of player 1.

Let us consider the sub-game originating from the information node reached by q_C. There are two Nash equilibria associated with this node. In the former, firm 2 enters and the H type chooses a level of investment below e_C, i.e. $y(q_C) = 1$ and $e_H(q_C) = \psi(q_C) < e_C$. In the latter equilibrium of the sub-game, firm 2 stays out and the incumbent chooses the investment of commitment, i.e. $y(q_C) = 0$ and $e_H(q_C) = \phi(q_C) = e_C$. As suggested by In and Wright (2017), the application of a forward induction argument[13] allows us to select the equilibrium with no entry in the sub-game originating

[13] If the incumbent had expected entry he would have not chosen q_C in the first place, because his profits would have been greater at $q_H = m_A$. Hence, the incumbent does not expect entry and plays $e_H = e_C$ and firm 2 stays out.

from the node reached by q_C. Taking this result into consideration, it is not difficult to see that the reordered game has a unique sub-game perfect equilibrium which is the one with the outcome of deterrence, characterized by the profile of strategies $(e_H, q_H) = (e_C, q_C)$ and $y(q) = 1$ if $q < q_C$ and $y(q) = 0$ otherwise. Therefore, we conclude that, applying In and Wright (2017) analysis to the benchmark model with complete information, the most plausible solution is the equilibrium where the incumbent H sets the limit price associated with q_C, undertakes the investment of commitment e_C and firm 2 stays out.

The comparison between the results of section 3 and those of the benchmark will help to highlight the main contributions of the present paper to the existing literature, with particular reference to the work by In and Wight (2017) and Milgrom and Roberts (1982).

In section 3, both a separating and a pooling equilibrium satisfying the Intuitive Criterion were characterized. The benchmark and our model provide different predictions if the comparison is carried out at the separating equilibrium. In fact, in the intuitive separating equilibrium (Proposition 1) the incumbent H sets his monopoly price and accommodates entry, while in the benchmark a limit price is used by type H to deter entry. This appears to be quite a surprising result if one think that in a separating equilibrium the entrant learns exactly the cost of the incumbent, so that one might expect that the separating equilibrium outcome and the benchmark outcome for type H be the same. The difference of results is due to the fact that, in the presence of private information, the price can not simultaneously perform two tasks. Indeed, the price can not settle the conflict between the incentive to signal costs and the incentive to invest for commitment purposes. A model without private information, such as the benchmark or In and Wright (2017)'s class of models, does not take this conflict into account and this is why the price can be freely used by the incumbent to convey information about investment and to restore its commitment value. When private information is present, instead, the inefficient incumbent has not any more this freedom in the use of price, because now the efficient type may have an incentive to mimic the strategic behaviour of type H. It is this conflict that prevents the inefficient incumbent from using the price to signal investment as a commitment mechanism to deter entry.

A striking difference of predictions between the benchmark and the model of section 3 also emerges if the comparison is carried out at the pooling equilibrium. In the benchmark, entry deterrence is obtained by the inefficient type through a limit price associated with quantity q_C. There is no deterrence of profitable entry because in equilibrium investment is e_C. In the model of section 3 with private information, instead, the inefficient type will deter entry by charging the limit price associated with q_0. This limit price is higher than that set under complete information. Moreover, it is profitable entry which is deterred because the inefficient incumbent invests $e_0 < e_C$. As under conditions of private information deterrence is obtained through a higher limit price and a lower level of investment than that of the benchmark we conclude that private information confers on unobservable investment a deterrence effect on entry which is stronger than the effect that investment has under complete information. Hence, our model shows that unobservable investment not only has a commitment effect, but it also has an *extra strategic effect* relative to the complete information case. In the class of models analysed by In and Wright (2017), this extra strategic effect of unobservable investment is missed, so that the qualitative predictions offered by the the two models of entry are quite different. In particular, in the incomplete information setting the incumbent is able to deter profitable entry with higher limit prices. This has important different implications for the analysis of competition policy, which should take seriously into consideration the fact that entry deterrence is more likely and has more negative effects on welfare when information about incumbent's costs is private rather than when it is publicly available.

Our model also contributes to the classical analysis by Milgrom and Roberts (1982) who consider a generalization of their limit pricing model in which the presence of unobservable actions is allowed. As Milgrom and Roberts do not carry out a complete analysis, but only provide some 'implications of equilibrium for firms' behaviour', the model of section 3 can be considered as one of the few specifications of the generalized classic limit pricing model where a complete analysis of equilibria is offered. We make two contributions to the analysis of the generalized model. The first refers to separating equilibria. Milgrom and Roberts (1982) claim that if the type of incumbent is perfectly inferred from the signal, entry occurs in precisely the same circumstances *as if* the cost of the incumbent had been directly announced (see p. 456). This need no be the case. Milgrom and Roberts fail to recognise that when both an action and an exogenous characteristic are not observed by the entrant, the potential conflict that the price is asked to settle in order to signal these two pieces of information might not be resolved. Thereby, the effect on entry cannot be as if the cost is announced, because if the cost is directly announced the price can be freely used to signal investment, while if the cost is signaled by price, then the price can not be used to signal investment and the effect on entry is quite different. Indeed, if the type is announced, as in the benchmark, entry never occurs because the price is able to convey information about investment restoring its commitment value. Conversely, if the price is inferred by the signal, as in the separating equilibrium of our model, entry only occurs when the incumbent is inefficient, i.e. with probability β.

Our second contribution to the analysis of the generalized limit pricing model relates to pooling equilibria. Milgrom and Roberts (1982) suggest that, due to the unrestricted dimensionality of exogenous characteristics and actions, it is unlikely that in equilibrium the observation of the signal permits a precise inference of the type of incumbent. In other words, in equilibrium the price signal is not able to convey enough information about unknown characteristics and actions. We identify here a precise mechanism by which unrestricted dimensionality acts in a generalized limit pricing model in order to make unlikely separating equilibria. The price may not be able to perform its informative tasks because of the conflict between the incentive to signal exogenous characteristics by the best type and the incentive to signal actions with long term effects by the worst type. Specifically, the type can not be inferred from the signal because the best type prefers to mimic the action of the worst type. For some configurations of parameters, these incentives can not be made compatible by prices and no separating equilibrium exists, as was shown in Proposition 1. On the other hand, for other configurations of parameters, incentives are made compatible because prices weaken the incentives to invest. As we have seen in the analysis of section 3, pooling equilibria exist under more general conditions as compared to separating equilibria and are more plausible as solutions to the generalized limit pricing model, which is a result in accordance with the prediction by Milgrom and Roberts (1982).

5 Conclusions

In this paper we studied the commitment value of unobservable investment in an entry model where an incumbent firm has private information about his costs of production. We found that the value of commitment of unobservable investment is closely related to the ability of prices to reveal information about incumbent's costs. On one hand, unobservable investment can not have any commitment value if the price is used to signal costs. Indeed, in such a case if the price would also convey information that a level of investment large enough to drive entrant's profits to zero has been undertaken, the efficient incumbent would be better off by mimicking the inefficient type. On the other hand, we show that unobservable investment has a magnified value of commitment if the price does not signal costs. In this case, the price affects the entry decision not only because it modifies the entrant's conjecture about the incumbent's choice of investment, but also because it leaves the entrant uncertain about incumbent's costs. Both the deterrence effect of limit pricing and the commitment value of investment are magnified because a less aggressive limit price and a smaller amount of investment than those needed under complete information are sufficient to deter entry.

This latter case is the more likely outcome and has relevant implications for competition policy and business strategy analysis. The incumbent will never play a 'top dog' strategy, by pricing very aggressively in order to persuade the entrant that an amount of investment high enough to cut down her post-entry profits to zero has been undertaken. Instead, the incumbent will set a higher limit price which leaves the entrant uncertain about incumbent's costs, but at the same time warns her that entry conditions are worsened enough to advise her against entry. As a result, the effects of unobservable investments on social welfare are less desirable than those which would obtain if incumbent's costs where observable to the entrant.

From the point of view of competition policy recommendations, our results suggest that either more accurate information about costs or more transparency about investment should be required especially in those markets where entry conditions appear to be less favourable, for example, because of high entry fees or strong cost reducing effects of investment.

Appendix

Lemma 1 *Let $V_H(e, q, 0)$, $\phi(q)$ and $U(q)$ as defined respectively by (1), (7) and (8). The following properties hold.*

(i) $\phi(q)$ *is well defined and strictly increasing.*

(ii) $V_H(e, q, 0)$ *has a global maximum at* $(\bar{e}, \bar{m})$, *where* $\bar{e} = \phi(\bar{m})$ *and* $\bar{m} = m_H(\bar{e})$.

(iii) $U(q)$ *is strictly quasi-concave and has a global maximum at* $q = \bar{m}$; *specifically,* $U(q)$ *is strictly increasing for* $q \leq \bar{m}$ *and strictly decreasing for* $q \geq \bar{m}$. *Moreover, the threshold* q^*, *defined by (9), and the quantity* q^{**} *implicitly defined by*

$$U(q^{**}) = V_H^A \qquad \text{with} \qquad q^{**} < m_L \tag{19}$$

exist and are unique.

(iv) 'Single crossing'. $V_L(0, q'', 0) - V_L(0, q', 0) > U(q'') - U(q')$ for $q'' > q'$.

Proof of Lemma 1

(i) A solution to the problem (7) exists because e is bounded from below by 0 and from above by the maximum total profit of type L. The solution is unique by strict-quasi concavity of V_H. Hence, $\phi(q)$ is well defined. Let $q'' > q'$, $e'' = \phi(q'')$ and $e' = \phi(q')$. We have to show that $\phi(q'') > \phi(q')$. Comparing the first order conditions of (7) it is easily seen that $e' \neq e''$. By definition of e' and e'' and strict quasi-concavity of $V_H(e, q, 0)$ we have

$$\begin{aligned}
\Pi_H(e', q') - e' + M_H(e') &> \Pi_H(e'', q') - e'' + M_H(e'') \\
\Pi_H(e'', q'') - e'' + M_H(e'') &> \Pi_H(e', q'') - e' + M_H(e')
\end{aligned}$$

Adding the two inequalities yields

$$\Pi_H(e'', q'') + \Pi_H(e', q') > \Pi_H(e', q'') + \Pi_H(e'', q')$$

By rearranging terms we obtain $[\theta_H(e') - \theta_H(e'')](q'' - q') > 0$, thus $\theta_H(e') > \theta_H(e'')$ and by assumption A.1 we have $e'' > e'$, so that $\phi(q)$ is strictly increasing.

(ii) By continuity, $V_H(e,q,0)$ has a global maximum in a compact set and by strict quasi-concavity the maximum is unique. Let (e',q') be the maximum, then by definition of ϕ, $e' = \phi(q')$ because $V_H(\phi(q'),q',0) \geq V_H(e,q',0)$ for all e. Moreover, $q' = m' = m_H(e')$. Indeed, suppose $q' \neq m'$ thus $V_H(e',q',0) > V_H(e',m',0)$ implies $\Pi_H(e',q') > \Pi_H(e',m')$, which is impossible because m' is the monopoly quantity.

(iii) Let $q' \neq q''$ and $q_\lambda = \lambda q' + (1-\lambda)q''$, with $\lambda \in (0,1)$. We have to show that $U(q_\lambda) > \min\{U(q'),U(q'')\}$. Let $e_\lambda = \lambda\phi(q') + (1-\lambda)\phi(q'')$, then by strict quasi-concavity of V_H we have

$$V_H(e_\lambda,q_\lambda,0) > \min\{V_H(\phi(q'),q',0)\,,\,V_H(\phi(q''),q'',0)\}$$

and, by definition of U,

$$V_H(e_\lambda,q_\lambda,0) > \min\{U(q')\,,\,U(q'')\}. \tag{20}$$

By (7) and (8), $U(q_\lambda) \geq V_H(e,q_\lambda,0)$ for all e, thus from (20) we obtain $U(q_\lambda) > \min\{U(q'),U(q'')\}$. Next, let us show that $\bar{m}$ is a global maximum for $U(q)$. By (ii), $V_H(\bar{e},\bar{m},0) \geq V_H(e,q,0)$ for all (e,q), then $V_H(\bar{e},\bar{m},0) \geq V_H(\phi(q),q,0)$ for all q, and by definition of U we obtain $U(\bar{m}) \geq U(q)$ for all q. Finally, let us show that that $U(q)$ is strictly increasing for $q \leq \bar{m}$. Let $q' < q'' < \bar{m}$, then there exists $\lambda \in (0,1)$ such that $q'' = \lambda q' + (1-\lambda)\bar{m}$. By strict quasi-concavity of U we have $U(q'') > \min\{U(q'),U(\bar{m})\} = U(q')$. The proof that $U(q)$ is strictly decreasing for $q \geq \bar{m}$ is similar.

To complete the proof of point (iii), notice that total profits $U(q)$ become negative for sufficiently large quantities, thus there exists $\hat{q} > m_L$ such that $U(\hat{q}) = 0$. Moreover, by (6), $U(m_L) > V_H^A$. Since $U(q)$ is continuous and strictly decreasing in $[m_L,\hat{q}]$ and $V_H^A > 0$, the solution to (9) exists and is unique in $[m_L,\hat{q}]$. By using similar arguments and the assumption $U(0) < V_H^A$ it is easily seen that also q^{**} exists and is unique.

(iv) First notice that

$$V_L(0,q'',0) - V_L(0,q',0) > V_H(e,q'',0) - V_H(e,q',0) \tag{21}$$

for $q'' > q'$ and for all e. Inequality (21) is a consequence of $\theta_H(e) > \theta_L$ and can be derived from $[\theta_H(e) - \theta_L]q'' > [\theta_H(e) - \theta_L]q'$ after simple manipulations.

Next let $e'' = \phi(q'')$ so that $V_H(e'', q'', 0) = U(q'')$. Replacing e with e'' in (21) yields

$$V_L(0, q'', 0) - V_L(0, q', 0) > U(q'') - V_H(e'', q', 0). \tag{22}$$

By definition of U, $U(q') \geq V_H(e'', q', 0)$ so that $-U(q') \leq -V_H(e'', q', 0)$ and

$$U(q'') - V_H(e'', q', 0) \geq U(q'') - U(q'). \tag{23}$$

Finally, the result follows from (22) and (23). Q.E.D.

Lemma 2

(i) No separating equilibrium exists in which the inefficient incumbent deters entry, i.e. in which $y(q_H) = 0$.

(ii) Let $q_{SC} > q^*$*, where* q_{SC} *is defined by (15) and* q^* *by (9). An incumbent's strategy* (e_H, q_H)*,* (e_L, q_L) *supports a separating equilibrium if and only if* $e_H = e_A$*,* $q_H = m_A$*,* $e_L = 0$*,* q_L *satisfies (12), (13), with either* $q_L > m_L$ *and* $q_L \leq q_{SC}$ *or* $q_L < m_L$ *and* $V_L(0, q_L, 0) \geq V_L(0, q_{SC}, 0)$*.*

Proof of Lemma 2.

(i) Suppose there exists a separating equilibrium in which $y(q_H) = 0$ and derive a contradiction. In a separating equilibrium, $\hat{\beta}(q_H) = 1$ so that Definition 1.(ii) and $y(q_H) = 0$ imply $D_2(\hat{e}_H(q_H), H) \leq 0$. Hence, $\hat{e}_H(q_H) \geq e_C$. By Definition 1.(ii) the conjecture must be $\hat{e}_H(q_H) = \phi(q_H)$ and as $e_C = \phi(q_C)$, we have $\phi(q_H) \geq \phi(q_C)$ and thus, by monotonicity of ϕ, $q_H \geq q_C (> m_L)$. Moreover, in a separating equilibrium, it holds $\hat{\beta}(q_L) = 0$ so that, by Definition 1.(ii) we have $y(q_L) = 0$. By Definition 1.(i) for type H, it must hold $U(q_H) > U(q_L)$, which, given the properties of U, means that either $q_L > q_H$ or $q_L < m_L < q_H$. In the former case, as V_L is decreasing for $q > m_L$, we have $V_L(0, q_H, 0) > V_L(0, q_L, 0)$ and this contradicts Definition 1.(i) for type L. In the latter case, we have $U(q_H) - U(q_L) > 0$ and $q_H > q_L$ so that Lemma 1.(iv) yields $V_L(0, q_H, 0) - V_L(0, q_L, 0) > 0$, which, again, contradicts Definition 1.(i) for type L. These contradictions establish the claim in (i).

(ii) Let $(e_H, q_H) = (e_A, m_A)$ and (e_L, q_L) satisfy (12) and (13) with $e_L = 0$, and consider, in turns, the two cases, (a) $q_L > m_L$ and (b) $q_L < m_L$. We have to show that the incumbent's strategy supports a separating equilibrium.

(a) As $q_L > m_L > m_A$ we have $q_L > q_H$. Take the beliefs $\hat{\beta}(q) = 0$ if $q \geq q_L$ and $\hat{\beta}(q) = 1$ if $q < q_L$ and choose the entrant's strategy $y(q) = 0$ if $q \geq q_L$ and $y(q) = 1$ if $q < q_L$. Set $\hat{e}_H(q) = \phi(q)$ for $q \geq q_L$ and $\hat{e}_H(q) = \psi(q)$ for $q < q_L$, where ψ is as defined by (14). If $y(q) = 1$, then $q < q_L$, thus $\hat{e}_H(q) = \psi(q)$ and $\hat{e}_H(q) < e_C$ as $q < q_{SC}$ and ψ is increasing. As $\hat{\beta}(q) = 1$, firm 2's expected profits are $D_2(\hat{e}_H(q), H) > 0$. Vice-versa, if expected profits are positive at q then $\hat{\beta}(q) = 1$ and $q < q_L$ so that $y(q) = 1$. Thus, we have shown that 2's strategy satisfies Definition 1.(ii). As for incumbent's strategy, the choice of $e_L = 0$ and q_L is optimal for type L, because for $q > q_L$ his total profits are decreasing and for $q < q_L$ firm 2 enters and incumbent's best profits are lower or equal to V_L^A; thus, by (13), $V_L(0, q_L, 0) > V_L(0, q, y(q))$ for all $q < q_L$. As for type H, for $q > q_L$ the function U is decreasing so that, by (12), his profits are lower than V_H^A. For $q < q_L$, firm 2 enters and the total profits at e_A, m_A are the highest by definition. Thus we have shown that Definition 1.(i) holds. Finally, notice that Bayes'rule holds and $\hat{e}_H(m_A) = \psi(m_A) = e_A$, so that also Definition 1.(iii) is satisfied.

(b) Let us turn to the case where $q_L < m_L$. As $V_H(0, m_H, 0) > V_H(0, m_L, 0)$ and by (6), condition (12) implies $q_L < m_H (< m_A)$. Hence $q_L < q_H$. Take the beliefs $\hat{\beta}(q) = 0$ if $q \leq q_L$ and $\hat{\beta}(q) = 1$ if $q > q_L$ and $y(q) = 0$, if $q \leq q_L$ or $q \geq q_{SC}$, and $y(q) = 1$ if $q_L < q < q_{SC}$. Set $\hat{e}_H(q) = \psi(q)$ for $q_L < q < q_{SC}$ and $\hat{e}_H(q) = \phi(q)$ for $q \leq q_L$ or $q \geq q_{SC}$. By similar arguments as those used above, this profile of strategies, with beliefs and conjectures is seen to satisfy Definition 1.

[If $y(q) = 1$, then $q_L < q < q_{SC}$, thus $\hat{e}_H(q) = \psi(q)$ and $\hat{e}_H(q) < e_C$. As $\hat{\beta}(q) = 1$, firm 2's expected profits are $D_2(\hat{e}_H(q), H) > 0$. Vice-versa, if expected profits are positive at q then $\hat{\beta}(q) = 1$ and $q_L < q < q_{SC}$ so that $y(q) = 1$. Thus, we have shown that 2's strategy satisfies Definition 1.(ii). As for incumbent's strategy, the choice of $e_L = 0$ and q_L is optimal for type L, because for $q < q_L$ his total profits are decreasing, for $q_L < q < q_{SC}$ firm 2 enters and incumbent's best profits are lower or equal to V_L^A, and for $q > q_{SC}$ $V_L(0, q_L, 0) > V_L(0, q_{SC}, 0) > V_L(0, q, 0)$ because V_L is decreasing, hence, by (13), $V_L(0, q_L, 0) > V_L(0, q, y(q))$ for all $q > q_L$. As for type H, for $q < q_L$ U is increasing so that, by (12), his profits are lower than V_H^A. For $q_L < q < q_{SC}$, firm 2 enters and the total profits at e_A, m_A are the highest by definition and, finally, as $q_{SC} > q^*$, $U(q) < V_H^A$ for $q \geq q_{SC}$. Thus we have shown that Definition 1.(i) holds. Finally, notice that Bayes'rule holds and $\hat{e}_H(m_A) = \psi(m_A) = e_A$, so that also Definition 1.(iii) is satisfied.]

This completes the first part of the proof.

Let us show the converse and suppose that (e_H, q_H) and (e_L, q_L) support a separating equilibrium. We show, first, that $(e_H, q_H) = (e_A, m_A)$, $e_L = 0$ and q_L satisfies (12) and (13). As the incumbent's strategy supports a separating equilibrium, it must hold $\hat{\beta}(q_H) = 1$ and $\hat{\beta}(q_L) = 0$. Since $D_2(L) < 0$, by Definition 1.(ii), we must have $y(q_L) = 0$. As for type H, by Lemma 2.(i) it holds $y(q_H) = 1$. Definition 1.(i) for type H implies $V_H(e_H, q_H, 1) \geq V_H(e_A, m_A, y(m_A)) \geq V_H(e_A, m_A, 1)$. Moreover, by definition of e_A and m_A we have $V_H(e_A, m_A, 1) \geq V_H(e_H, q_H, 1)$. Hence, $V_H(e_A, m_A, 1) = V_H(e_H, q_H, 1)$ and by strict quasi-concavity of V_H we obtain $(e_H, q_H) = (e_A, m_A)$.[14]

Turning to e_L and q_L, notice first that, by Definition 1.(i) for type L it must be $e_L = 0$. Moreover, it is not difficult to see that q_L satisfies (12) and (13). Indeed, if (12) is violated then $U(q_L) = V_H(\phi(q_L), q_L, y(q_L)) > V_H(e_A, m_A, 1) = V_H(e_H, q_H, y(q_H))$, which contradicts Definition 1.(i) for type H. Also, by Definition 1.(i) for type L, $V_L(0, q_L, 0) \geq V_L(0, m_L, y(m_L)) \geq V_L(0, m_L, 1) = V_L^A$ and (13) is satisfied.

Next, let us show that if $q_L > m_L$ then $q_L \leq q_{SC}$. Let us suppose that $q_L > q_{SC}$ and derive a contradiction. First notice that as $\phi(q_{SC}) \geq \psi(q_{SC}) = e_C$, by Definition 1.(ii) the conjecture at q_{SC} must be $\hat{e}_H(q_{SC}) \geq e_C$ and $y(q_{SC}) = 0$ because entrant's expected profits are negative. Thus, as $q_{SC} > m_L$ (by A.4), we have $V_L(0, q_{SC}, 0) > V_L(0, q_L, 0)$, so that q_L is not optimal and Definition 1.(i) is violated. Hence $q_L \leq q_{SC}$.

Finally, it is easily seen that if $q_L < m_L$ then $V_L(0, q_L, 0) \geq V_L(0, q_{SC}, 0)$. Indeed, as $y(q_{SC}) = 0$, as shown above, if $V_L(0, q_{SC}, 0) > V_L(0, q_L, 0)$ then Definition 1.(i) for type L would be violated. Hence, the result follows and this completes the proof of the lemma. Q.E.D.

Lemma 3 *Let $q_{SC} \geq q^*$. The incumbent's strategy $(e_H, q_H) = (e_A, m_A)$ and (e_L, q_L) supports an intuitive separating equilibrium if and only if $e_L = 0$, $q_L > m_L$ and q_L is a solution to the following*

[14] In fact, the maximum of $V_H(e, q, 1)$ is unique by strict quasi-concavity.

maximization problem:

$$\begin{aligned} \max_{q} \quad & \Pi_L(q) \\ \textit{subject to} \quad & U(q) \leq V_H^A \\ & \Pi_L(q) \geq D_L \end{aligned}$$

Proof of Lemma 3

Let $e_L = 0$ and $q' > m_L$ be a solution to the maximization problem. We have to show that $(e_L, q_L) = (0, q')$ supports an intuitive separating equilibrium. First, notice that at q' the first constraint of the maximization problem must be binding, i.e. $U(q') = V_H^A$. Indeed, if $U(q') < V_H^A$ given that U and Π_L are decreasing for $q > m_L$, Π_L can be increased without violating any constraint. Thus, $U(q') = V_H^A$ and by Lemma 1.(ii) $q' = q^*$. As $q^* < q_{SC}$, by Lemma 2.(ii) the choice $(e_L, q_L) = (0, q')$ supports a separating equilibrium, since $(0, q')$ satisfies (12) and (13). To show that the equilibrium satisfies the Intuitive Criterion, let us suppose that there exists a deviation $\tilde{q}$ from q' such that $U(\tilde{q}) < V_H^A$ and $V_L(0, \tilde{q}, 0) > V_L(0, q', 0)$. But, then $\Pi_L(\tilde{q}) > \Pi_L(q')$ and q' can not be a solution to the maximization problem. This contradiction completes the first part of the proof.

Let us show the converse and suppose that (e_L, q_L) supports an intuitive separating equilibrium. We have to show that $e_L = 0$, $q_L > m_L$ and that q_L is a solution to the maximization problem. First, notice that by Lemma 2.(ii) $e_L = 0$. Next, to show that $q_L > m_L$, notice that, given the properties of U (Lemma 1.(iii)), $U(q) \leq V_H^A$ only for $q \geq q^*$ or $q \leq q^{**}$, where q^{**} is given by (19) and $U(q^*) = U(q^{**}) = V_H^A$. Let us suppose that $q_L < q^{**}$ and consider the deviation $\tilde{q} = q^*$. By Lemma 1.(iv)

$$V_L(0, \tilde{q}, 0) - V_L(0, q_L, 0) > U(\tilde{q}) - U(q_L) \geq 0$$

Thus $V_L(0, \tilde{q}, 0) > V_L(0, q_L, 0)$. As V_L and U are continuous for $q > m_L$, there exists a deviation $\tilde{q}' > \tilde{q}$ sufficiently close to $\tilde{q}$ which violates Definition 2. Hence, $q_L < m_L$ cannot support an intuitive equilibrium and we conclude that $q_L > m_L$.

Finally, let us proceed by contradiction and suppose that q_L supports an intuitive separating equilibrium, but it is not a solution, i.e. there exists $q' \neq q_L$ such that $\Pi_L(q') > \Pi_L(q_L)$ and $U(q') \leq V_H^A$. First, it follows that $V_L(0, q', 0) > V_L(0, q_L, 0)$. Next, if $U(q') < V_H^A$ the equilibrium is not intuitive, contrary to the assumption, therefore, it must be $U(q') = V_H^A$. By Lemma 2.(ii), $U(q_L) \leq U(q')$ and as $q_L > m_L$ (see above) and U is decreasing for $q > m_L$, it must hold $q_L > q'$. By continuity of Π_L and U, there exists $q'' > q'$ sufficiently close to q' such that $U(q'') < U(q') = V_H^A$ and $\Pi_L(q'') > \Pi_L(q_L)$ or $V_L(0, q'', 0) > V_L(0, q_L, 0)$. Thus, the deviation $\tilde{q} = q''$ from the equilibrium q_L violates Definition 2 and $(0, q_L)$ does not support an intuitive equilibrium, contrary to the assumption. This contradiction completes the proof. Q.E.D.

Proof of Proposition 1.

(i) Let us suppose that a separating equilibrium exists and derive a contradiction. First, notice that by Definition 1.(ii) the conjecture of type H at q_{SC} must be $\hat{e}_H(q_{SC}) \geq e_C$, since $\phi(q_{SC}) > \psi(q_{SC}) = e_C$. Hence, $y(q_{SC}) = 0$ because entrant's expected profits are non positive. Next, notice that, by Lemma 2.(i), only separating equilibria where the inefficient incumbent accommodates entry are possible. By the incentive compatibility condition (12), $U(q_L) \leq V_H^A = U(q^*)$. If $q_L > m_L$, by the properties of U it follows $q_L \geq q^*$, while if $q_L < m_L$ it follows that $q_L \leq q^{**}$, where q^* and q^{**} are respectively given by (9) and (19) and $U(q^*) = U(q^{**})$. Applying Lemma 1.(iv) for $q^* > q^{**}$ yields $V_L(0, q^*, 0) > V_L(0, q^{**}, 0)$. Thus, as $q_{SC} < q^*$ we obtain $V_L(0, q_{SC}, 0) > V_L(0, q^*, 0)$. As either $q_L \geq q^*$ or $q_L \leq q^{**}$ we have $V_L(0, q_{SC}, 0) > V_L(0, q_L, 0)$. Therefore, q_L can not satisfy optimality for type L and definition 1.(i) is violated.

(ii) To carry out the proof we use Lemma 3 and show that $q^* > m_L$ is a solution to the maximization problem $\max_q \Pi_L(q)$ subject to $U(q) \leq V_H^A$ and $\Pi_L(q) \geq D_L$. By Lemma 1.(iii), for any q in the open interval $]q^{**}, q^*[$ it holds $U(q) > V_H^A$. Thus the solution must lie either in the interval $q \geq q^*$ or in the interval $q \leq q^{**}$. Notice, first, that $U(q^*) - U(q^{**}) = 0$, $q^* > q^{**}$ and Lemma 1.(iv) imply $V_L(0, q^*, 0) - V_L(0, q^{**}, 0) > 0$ and $\Pi_L(q^*) > \Pi_L(q^{**})$. Thus, as Π_L is strictly increasing for $q < m_L$, we have $\Pi_L(q^*) > \Pi_L(q)$ for all $q \leq q^{**}$. Next, as Π_L is strictly decreasing for $q > m_L$, we have $\Pi_L(q^*) > \Pi_L(q)$ for $q > q^*$. Thus q^* maximizes Π_L subject to the first constraint and it is the unique maximizer by strict quasi-concavity of Π_L. To complete the proof we have to show that q^*

also satisfies the constraint $\Pi_L(q^*) \geq D_L$. By definition of q^*, we have

$$\Pi_H(e^*, q^*) - e^* + M_H(e^*) = M_H(e_A) - e_A + D_H(e_A), \tag{24}$$

where $e^* = \phi(q^*)$. By definition of V_H^A it must hold

$$M_H(e_A) - e_A + D_H(e_A) \geq M_H(e^*) - e^* + D_H(e^*). \tag{25}$$

Thus, (24) and (25) imply $\Pi_H(e^*, q^*) \geq D_H(e^*)$ and subtracting D_L to both sides and rearranging yields

$$D_L - D_H(e^*) \geq D_L - \Pi_H(e^*, q^*). \tag{26}$$

Next, notice that

$$\begin{aligned} \Pi_L(q^*) - \Pi_H(e^*, q^*) &= (\theta_H(e^*) - \theta_L)q^* \\ &> (\theta_H(e^*) - \theta_L)m_L \\ &= M_L - \Pi_H(e^*, m_L) \\ &> M_L - M_H(e^*), \end{aligned} \tag{27}$$

where the first inequality follows from $q^* > m_L$ and assumption A.1, and the last inequality from $M_H(e^*) > \Pi_H(e^*, m_L)$. By condition (5) we have $M_L - M_H(e^*) \geq D_L - D_H(e^*)$, therefore (26) and (27) yield $\Pi_L(q^*) - \Pi_H(e^*, q^*) > D_L - \Pi_H(e^*, q^*)$ and, finally, $\Pi_L(q^*) > D_L$. Thus, we conclude that q^* is the unique solution to the maximization problem, so that by Lemma 3 , $(e_L, q_L) = (0, q^*)$ supports the unique intuitive separating equilibrium and this completes the proof of Proposition 1.

Q.E.D.

Lemma 4 *An incumbent's strategy* (e_H, q_H), (e_L, q_L), *supports a pooling equilibrium if and only if it satisfies (17) and (18) with* $q_H = q_L = q_P$, $q_P \geq q_0$, $e_H = \phi(q_P)$ *and* $e_L = 0$.

Proof of Lemma 4

Let the incumbent's strategy (e_H, q_H), (e_L, q_L), with $q_H = q_L = q_P$, $q_P \geq q_0$, $e_H = \phi(q_P)$ and $e_L = 0$ satisfy (17) and (18). We have to show that it supports a pooling equilibrium. Let us consider the following two cases in turn, (a) $q_P \geq m_L$ and (b) $q_P < m_L$.

(a) Let $q_P \geq m_L$ and set the beliefs as follows: $\hat{\beta}(q) = \beta$ if $q \geq q_P$ and $\hat{\beta}(q) = 1$ if $q < q_P$. Moreover, let $\hat{e}_H(q) = \phi(q)$ if $q \geq q_P$, $\hat{e}_H(q) = \psi(q)$ if $q < q_P$ and take the entrant's strategy $y(q) = 1$ if $q < q_P$ and $y(q) = 0$ if $q \geq q_P$. It is easily seen that this profile of strategies, along with beliefs and conjectures, satisfy Definition 1.(iii) and that the requirement about the conjecture $\hat{e}_H(q)$ in Definition 1.(ii) is met. Let us check that the entrant's strategy is optimal. If $y(q) = 1$ then $q < q_P$, the beliefs are $\hat{\beta}(q) = 1$ and $\hat{e}_H(q) = \psi(q) < \psi(q_{SC}) = e_C$ by (17). Thus the entrant's expected profits are positive. Vice-versa, if at q the entrant's expected profits are positive, the beliefs are $\hat{\beta}(q) = 1$. Indeed, if $\hat{\beta}(q) = \beta$, then $q \geq q_P$, $\hat{e}_H(q) = \phi(q) \geq e_0$, because $q_P \geq q_0$, and entrant's profits would be non positive. Hence, it must be $\hat{\beta}(q) = 1$, so that $q < q_P$ and $y(q) = 1$. Thus, Definition 1.(ii) is satisfied. As for the incumbent's strategy, let us show that the L type choice is optimal. In fact, for $q < q_P$, $V_L(e, q, y(q)) = V_L(e, q, 1) \leq V_L^A$ and, by (18), $V_L(0, q_P, 0) \geq V_L(e, q, y(q))$. If $q > q_P$, $y(q) = 0$ and $V_L(0, q_P, 0) > V_L(0, q, 0)$ because V_L is strictly decreasing for $q > m_L$. Thus, the L type choice is optimal. The same conclusion holds for type H. For $q < q_P$, $y(q) = 1$ and, by definition of V_H^A, $V_H^A \geq V_H(e, q, 1) = V_H(e, q, y(q))$. For $q = q_P$, $y(q_P) = 0$, $e_H(q_P) = \phi(q_P)$, so that $V_H(\phi(q_P), q_P, 0) = U(q_P)$. Since, by (17), $q_P < q^*$, $U(q_P) \geq V_H^A$ and $U(q_P) \geq V_H(e, q, y(q))$ for $q < q_P$. On the other hand, if $q > q_P$, $y(q) = 0$ and by definition of U, $U(q) \geq V_H(e, q, y(q))$. As U is strictly decreasing (Lemma 1.(iii)) $U(q_P) > U(q) \geq V_H(e, q, y(q))$ and the choice of type H is optimal.

(b) Let us consider the case where $q_P < m_L$ and set $\hat{\beta}(q) = \beta$ if $q = q_P$ and $\hat{\beta}(q) = 1$ otherwise. Moreover, let $\hat{e}_H(q) = \phi(q)$ and $y(q) = 0$ if $q = q_P$ or $q \geq q_{SC}$, and $\hat{e}_H(q) = \psi(q)$ and $y(q) = 1$ otherwise. Also recall that by (17) $q_P < q_{SC}$ and by (18) $V_L(0, q_P, 0) \geq V_L(0, q_{SC}, 0)$. It is easily seen that this profile of strategies, along with beliefs and conjectures, satisfy Definition 1.(iii) and that the requirement about the conjecture $\hat{e}_H(q)$ in Definition 1.(ii) is met. Let us check that the entrant's strategy is optimal. If $y(q) = 1$ then $q < q_P$ or $q_P < q < q_{SC}$ and $\hat{\beta}(q) = 1$ so that $\hat{e}_H(q) = \psi(q) < \psi(q_{SC}) = e_C$. Hence, entrant's expected profits are positive. Vice-versa, let the expected profits be positive. Clearly, if $\hat{\beta}(q) = \beta$ then $q = q_P$, $\hat{e}_H(q_P) = e_0$ and entrant's expected profits are non positive. Thus, it must be $\hat{\beta}(q) = 1$ and $q \neq q_P$. If $q \geq q_{SC}$ then

$\hat{e}_H(q) = \phi(q) > \psi(q_{SC}) = e_C$ and the expected profits would be non positive. Thus, it must be $q < q_{SC}$ and $q \neq q_P$ so that $y(q) = 1$. Thus, Definition 1.(ii) is satisfied. As for the incumbent's strategy, the choice of q_P is optimal for type L, because $V_L(0, q_P, 0) \geq V_L(0, q, y(q))$ for all q. Indeed, for $q \geq q_{SC}$ the above inequality holds because V_L is decreasing and because of (18); for $q \neq q_P$ with $q < q_{SC}$, we have $y(q) = 1$ and by (18), $V_L(0, q_P, 0) \geq V_L^A \geq V_L(0, q, 1)$. Similarly, q_P is optimal for type H. Indeed, $U(q_P) \geq U(q)$ for $q \geq q_{SC}$, as $q_{SC} > \bar{m}$ and U is decreasing; moreover, by (17), $U(q_P) \geq V_H^A \geq V_H(e, q, 1)$ for all $q < q_{SC}$, and the choice of type H is optimal.

This completes the first part of the proof, namely we have shown that the above conditions are sufficient for a pooling equilibrium.

To show the converse, let the incumbent's strategy support a pooling equilibrium. We have to show that (17) and (18) hold with $q_H = q_L = q_P$, $q \geq q_0$, $e_H = \phi(q_P)$ and $e_L = 0$. By definition of pooling equilibrium, $q_H = q_L = q_P$ and, by Definition 1.(iii), we have $\hat{\beta}(q_P) = \beta$ and $\hat{e}_H(q_P) = e_H$. If $y(q_P) = 1$ both types would rather choose their respective monopoly quantities in the first period and then accommodate entry, so that q_P can not be a pooling equilibrium. Thus, $y(q_P) = 0$ must hold and by (4) and Definition 1.(ii), $\hat{e}_H(q_P) \geq e_0$ and $\hat{e}_H(q_P) = \phi(q_P)$. It follows that $e_H = \phi(q_P)$, as required, and $\phi(q_P) \geq e_0 = \phi(q_0)$, which implies $q_P \geq q_0$ as requested.

Next, by the optimality of the L type choice, Definition 1.(i), it must be $e_L = 0$. By optimality of q_P we have $V_L(0, q_P, 0) \geq V_L(0, m_L, y(m_L)) \geq V_L(0, m_L, 1) = V_L^A$. Moreover, notice that at q_{SC} it must be $y(q_{SC}) = 0$. Indeed, by Definition 1.(ii) the conjecture must be $\hat{e}_H(q_{SC}) \geq e_C$, so that the entrant's expected profits must be non positive. Thus, by optimality of q_P for type L we have $V_L(0, q_P, 0) \geq V_L(0, q_{SC}, y(q_{SC})) = V_L(0, q_{SC}, 0)$ and (18) holds.

Optimality of choice by type H implies that $V_H(e_H, q_P, 0) \geq V_H(e_A, m_A, y(m_A)) \geq V_H(e_A, m_A, 1) = V_H^A = U(q^*)$. As $e_H = \phi(q_P)$, $V_H(e_H, q_P, 0) = U(q_P)$ and, owing to the above inequality, we have $U(q_P) \geq U(q^*)$ so that $q_P \leq q^*$. Moreover, by optimality, $U(q_P) \geq V_H(e, q_{SC}, y(q_{SC}) = V_H(e, q_{SC}, 0)$, because $y(q_{SC}) = 0$ by the above argument. As the above inequality is true for all e, it must also hold $U(q_P) \geq U(q_{SC})$ so that $q_P \leq q_{SC}$ and (17) is satisfied. This completes the proof of the lemma. Q.E.D.

Proof of Proposition 2.

Let $q_0 < m_L$ and consider the incumbent's strategy with $q_H = q_L = q_P = m_L$, $e_H = \phi(m_L)$ and $e_L = 0$. It is easily seen that this strategy satisfies (17) and (18) so that, by Lemma 4, it supports a pooling equilibrium. Moreover, this strategy supports an intuitive pooling equilibrium. Indeed, it trivially satisfies Definition 2, because no deviation from $q_P = m_L$ is strictly preferred by type L, as m_L maximizes $V_L(0, q, 0)$. Moreover, $q_P = m_L$ is interim Pareto undominated because type L is worse off at any other equilibrium. Next, let us show that this equilibrium is unique. As both functions $U(q)$ and $V_L(0, q, 0)$ are strictly decreasing for $q \geq m_L$, any intuitive pooling equilibrium supported by $q_P > m_L$ is Pareto dominated by m_L. Moreover, no intuitive pooling equilibria is supported by $q_P < m_L$. Indeed, a pooling equilibrium supported by $q_P < m_L$ would violate Definition 2, because the deviation $\tilde{q} = m_L$ would be equilibrium dominated for type H and strictly preferred by type L.

Let us turn to the case $q_0 \geq m_L$. It is easily seen that the incumbent's strategy with $q_H = q_L = q_P = q_0$, $e_H = \phi(q_0)$ and $e_L = 0$ satisfies (17) and (18) so that, by Lemma 4, it supports a pooling equilibrium. Any deviation $\tilde{q} > q_0$ is equilibrium dominated for both types of incumbent as U and V_L are decreasing for $q > m_L$. Consider next a deviation $\tilde{q} < q_0$. If $\tilde{q}$ is equilibrium dominated for type H, i.e. $U\tilde{q}) < U(q_0)$, then, by Lemma 1.(iv), $V_L(0, \tilde{q}, 0) < V_L(0, q_0, 0)$ so that $\tilde{q}$ is also equilibrium dominated for type L. Hence, Definition 2 is satisfied and the pooling equilibrium supported by q_0 is intuitive. Finally, by noting that any pooling equilibrium is supported by $q_P > q_0$ and that the functions $U(q)$ and $V_L(0, q, 0)$ are strictly decreasing for $q \geq m_L$, it is easily seen that the equilibrium supported by q_0 is the unique intuitive equilibrium interim Pareto undominated. This completes the proof of Proposition 2. Q.E.D.

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