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Convergence of Broadband, Broadcast, and Cellular Network Technologies

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Simulation of competition in NGNs with a Game Theory model

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ABSTRACT

Like in a real competitive market situation, Next Generation Networks (NGN) competitors need to adapt their strategy to face/react the strategies from other players. To better understand the effects of interaction between different players, we build a Game Theory model, in which the profit of each operator will be dependent not only on their actions, but also on the actions of the other operators in the market. This paper analyzes the impact of the price (retail and wholesale) variations on several output results: players' profit, consumer surplus, welfare, costs, service adoption, etc. We assume that two competing FTTH networks (incumbent operator and new entrant) are deployed in two different areas. We also propose an adoption model use in a way that reflects the competition between players and that the variation of the services prices of one player has an influence on the market share of all players. Finally, model use the Nash equilibrium to find the best strategies.

1 INTRODUCTION

The main objective of a game-theory model is providing a mathematical description of a social situation in which two or more players interact, and every player can choose from different strategies. (J. P. Pereira & Ferreira, 2012; Yongkang, Xiuming, & Yong, 2005) define game theory as a collection of mathematical models formulated to study situations of conflict and cooperation, and concerned with finding the best actions for individual decision makers. (Machado & Tekinay, 2008) argue that game theory is a theory of decision making under conditions of uncertainty and interdependence. The players compete for some good or reward, and often in business cases, the customer will be the aim of the competition (J. P. R. Pereira, 2013a; Verbrugge, Casier, Ooteghem, & Lannoo, 2009).

The object of study in game theory is the game, where there are at least two players, and each player can choose amongst different actions (often referred to as strategies). The strategies chosen by each player determine the outcome of the game - the collection of numerical payoffs (one to each player). So, the game has three main key parts (Easley & Kleinberg, 2010): a) a set of participants; b) each player has a set of options for how to behave; we will refer to these as the player's possible strategies; and c) for each choice of strategies, each player receives a payoff that can depend on the strategies selected by everyone (in our model, the payoff to each player is the profit each provider gets).

After the calculation of the several payoffs, game theoretic concepts can be used for retrieving the most likely (set of) interactions between the players (Verbrugge et al., 2009). There are several different equilibrium-definitions of which probably the Nash equilibrium is the most commonly known - A broad class of games is characterized by the Nash equilibrium solution. In 1950, John Nash demonstrated that finite games always have a Nash equilibrium, also called a strategic equilibrium (Yongkang et al., 2005). A Nash equilibrium is a list of strategies, one for each player, which has the property that no player can unilaterally change his strategy and get a better payoff - each player's strategy is an optimal response to the other players' strategies. Even when there are no dominant strategies, it should be expected that players use strategies that are the best responses to each other. This is the central concept of noncooperative game theory and has been a focal point of analysis since then. For example, if player 1 chooses strategy S1 and player 2 chooses S2, the pair of strategies (S1 and S2) is a Nash equilibrium if S1 is the best response to S2, and S2 is the best response to S1. So, if the players choose strategies that are best responses to each other, then no player has an incentive to turn to an alternative strategy, and the

system is in a kind of equilibrium, with no force pushing it toward a different outcome (Easley & Kleinberg, 2010).

One of the main goals of regulated access is to prevent the incumbent from abusing a dominant market position (J. P. R. Pereira, 2013b). It is necessary to make sure that alternative operators can compete effectively. It is fundamental that incumbent operators give access to the civil works infrastructure, including its ducts, and to give wholesale broadband access (bitstream) to the local loop (be it based on copper, new fiber, etc.). However, at the same time, alternative operators should be able to compete on the basis of the wholesale broadband input while they progressively roll out their own NGAN infrastructure. In some areas, especially with higher density, alternative operators have rolled out their own infrastructure and broadband competition has developed. This would result in more innovation and better prices to consumers (J. P. Pereira & Ferreira, 2011).

Many European incumbents and some alternative operators are starting to plan and in some cases deploy large-scale fiber investments, which has resulted in important changes for fixed-line markets (Amendola & Pupillo, 2007). The risk of alternative operators will take longer to deploy their own infrastructure and will give to incumbents the possibility to create new monopolies at the access level. The technologies used and the pace of development vary from country to country according to existing networks and local factors. Based on the different underlying cost conditions of entry and presence of alternative platforms, it may be more appropriate to geographically differentiate the access regulatory regime.

3 MODEL OVERVIEW

This part of the work focuses the development of a tool that simulates the impact of retail and wholesale price variation on provider's profit, welfare, consumer surplus, costs, market served, network size, etc. The programming language used to implement the model was C language. The application runs on several platforms and was developed to use multiprocessing capacity. The choice of C language was because the tool needs to compute higher quantity of data very quickly, and this programming language has the characteristics required (see Figure 1.).

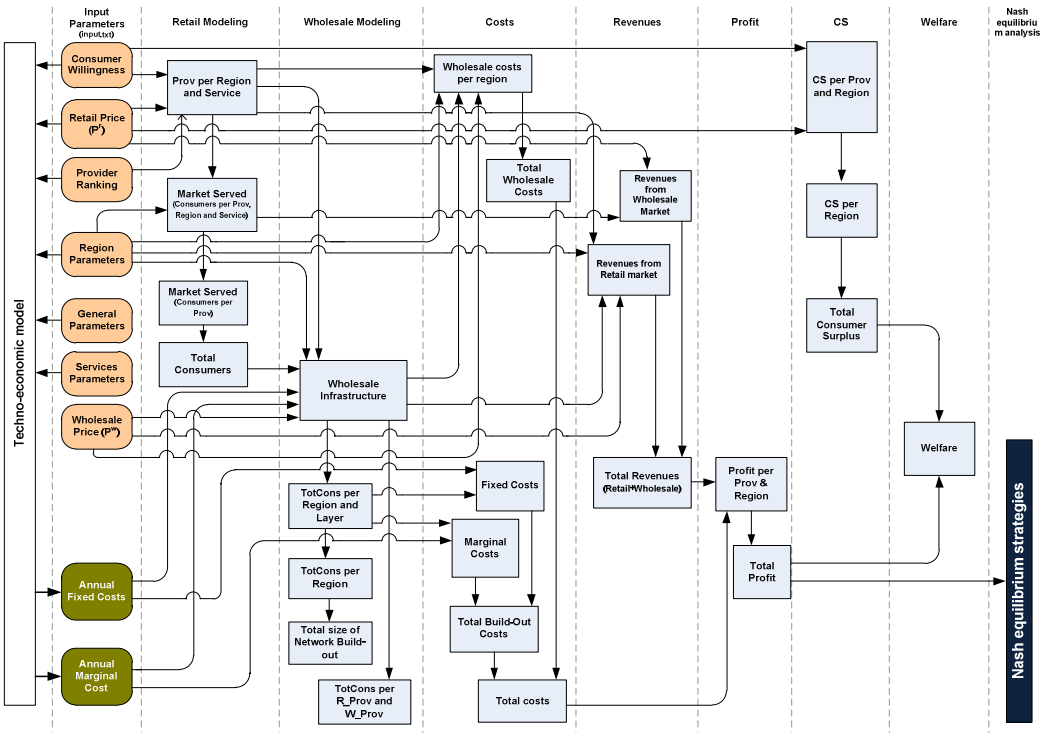

Figure 1. *Game-theoretic model structure*

In the proposed model, "Retail Prices" represents the set of retail prices charged by providers for each service to consumers in a given region/area. We assume that retail providers cannot price discriminate in the retail market. "Wholesale Prices" represents the prices that one provider charges to other provider to allow the later to use the infrastructure to reach consumers. We assume that wholesale price can be different in each area. Also, we assume that when a provider buys infrastructure access in the wholesale market, it cannot resell to another provider. The shared infrastructure consists of: conduit and collocation facilities; cable leasing (dark fiber requires active equipment to illuminate the fiber – for example repeaters); and bit stream.

For example, one or several wholesaler providers can sell Layer 0 access (conduit and collocation facilities) and/or Layer 1 access (cable leasing) or Layer 2 access (bitstream – network layer unbundling – UNE loop) only to retail providers and not directly to consumers. UNE loop is defined as the local loop network element that is a transmission facility between the central office and the point of demarcation at an end-user's premises.

The tool support scenarios with x providers (x>0), y regions (y>0), z services (z>0), and w infrastructure layers (3< x >0). However, because of the volume of calculus and data produced, some considerations are necessary. TABLE I. shows an example of a scenario with two regions, two providers, two services, and one infrastructure layers (Layer 0: Conduit – see TABLE VI.).

Each line corresponds to a strategy of prices (St1, St2, Stn), and for each strategy the tool calculates the results (columns at the right side of the previous table). To calculate the number of strategies required, we use the following formula:

$$TS = TVS^{(TProv*(TServ+(TReg*TLay)))} \quad (1)$$

Where:
TS – Total strategies
TVS – Total values to simulate
TProv – Total providers
TServ – Total services
TReg – Total regions
TLay – Total layers

For the scenario presented in TABLE I. , and assuming that we want to simulate eight different prices (for retail and wholesale prices), we get 16.777.216 possible strategies or combinations (8 (2*(2+(2*1)))). The tool has been tested in two platforms: PC (Pentium i7 quad core with 4 GB of memory) running Windows 7 and in a cluster with 4 nodes (8 GB of memory for each node) running Linux.

TABLE I. *Structure of a scenario*

	Provider 1				Provider 2				Results						
	Retail price		Wholesale price		Retail price		Wholesale price								
	Service 1	Service 2	Region 1	Region 2	Service 1	Service 2	Region 1	Region. 2	Market Served	Size of Network	Total Costs	Total Revenues	Total Profit	Consumer Surplus	Welfare
			Layer 0	Layer 0			Layer 0	Layer 0							
St_1	$P^r_{(S1,V1)}$	$P^r_{(S2,V1)}$	$P^w_{(L0,V1)}$	$P^w_{(L0,V1)}$)	$P^r_{(S1,V1)}$	$P^r_{(S2,V1)}$	$P^w_{(L0,V1)}$	$P^w_{(L0,V1)}$)							
St_2	$P^r_{(S1,V1)}$	$P^r_{(S2,V1)}$)	$P^w_{(L0,V1)}$	$P^w_{(L0,V1)}$	$P^r_{(S1,V1)}$	$P^r_{(S2,V1)}$)	$P^w_{(L0,V1)}$	$\mathbf{P^w_{(L0,V2)}}$							
St_n	…														

4 INPUT PARAMETER ASSUMPTIONS

As we can see in Figure 1. , our tool has several input parameters (one of them came from the techno-economic model), computes several results and finds the strategies that are Nash equilibrium. The results

are represented in tables and graphics. In this section, we describe the inputs: fixed and marginal costs and retail/wholesale variation values.

4.1 Fixed and marginal costs

In our model, we assume that providers incur in fixed costs to build network infrastructure to provide access to a region and in marginal costs to connect each consumer separately. The fixed and marginal costs are calculated in the techno-economic tool.

The fixed costs are detailed by provider, region and infrastructure layer. So, we assume that the fixed costs of each provider can be different in different regions - for example, if a provider has part of the infrastructure deployed in a region, and in the other is required all the infrastructure, the costs are different. TABLE II. shows the structure used for fixed costs.

TABLE II. *Structure of fixed costs input parameter*

	Region1			Region2			…	Region r		
	Layer 0	Layer 1	Layer 2	Layer 0	Layer 1	Layer 2	…	Layer 0	Layer 1	Layer 2
Provider 1	$C^f_{(P1,R1,L0)}$	$C^f_{(P1,R1,L1)}$	$C^f_{(P1,R1,L2)}$	$C^f_{(P1,R2,L0)}$	$C^f_{(P1,R2,L1)}$	$C^f_{(P1,R2,L2)}$	…			
Provider 2	$C^f_{(P2,R1,L0)}$	$C^f_{(P2,R1,L1)}$	$C^f_{(P2,R1,L2)}$	$C^f_{(P2,R2,L0)}$	$C^f_{(P2,R2,L1)}$	$C^f_{(P2,R2,L2)}$	…			
…	…	…	…	…	…	…	…	…	…	…
Provider p							…			

For marginal costs, we assume that each provider has different costs for deployment in each infrastructure layer. In each region, the marginal cost could be different for each provider depending of the total number of subscribers – scale economies. This means that the marginal cost can decrease when a specific provider buys higher quantities of equipment, cable, etc. (see TABLE III.).

TABLE III. *Structure of marginal costs input parameter*

		Region 1				Region 2				…
Total Consumers		TotCons1	TotCons2	TotCons3	TotCons4	TotCons1	TotCons2	TotCons3	TotCons4	…
Provider 1	L0	$C^m_{(P1,R1,L0,V1)}$	$C^m_{(P1,R1,L0,V2)}$	$C^m_{(P1,R1,L0,V3)}$	$C^m_{(P1,R1,L0,V4)}$	$C^m_{(P1,R2,L0,V1)}$	$C^m_{(P1,R2,L0,V2)}$	$C^m_{(P1,R2,L0,V3)}$	$C^m_{(P1,R2,L0,V4)}$	
	L1	$C^m_{(P1,R1,L1,V1)}$	$C^m_{(P1,R1,L1,V2)}$	$C^m_{(P1,R1,L1,V3)}$	$C^m_{(P1,R1,L1,V4)}$	$C^m_{(P1,R2,L1,V1)}$	$C^m_{(P1,R2,L1,V2)}$	$C^m_{(P1,R2,L1,V3)}$	$C^m_{(P1,R2,L1,V4)}$	
	L2	$C^m_{(P1,R1,L2,V1)}$	$C^m_{(P1,R1,L2,V2)}$	$C^m_{(P1,R1,L2,V3)}$	$C^m_{(P1,R1,L2,V4)}$	$C^m_{(P1,R2,L2,V1)}$	$C^m_{(P1,R2,L2,V2)}$	$C^m_{(P1,R2,L2,V3)}$	$C^m_{(P1,R2,L2,V4)}$	
Provider 2	L0	$C^m_{(P2,R1,L0,V1)}$	$C^m_{(P2,R1,L0,V2)}$	$C^m_{(P2,R1,L0,V3)}$	$C^m_{(P2,R1,L0,V4)}$	$C^m_{(P2,R2,L0,V1)}$	$C^m_{(P2,R2,L0,V2)}$	$C^m_{(P2,R2,L0,V3)}$	$C^m_{(P2,R2,L0,V4)}$	
	L1	$C^m_{(P2,R1,L1,V1)}$	$C^m_{(P2,R1,L1,V2)}$	$C^m_{(P2,R1,L1,V3)}$	$C^m_{(P2,R1,L1,V4)}$	$C^m_{(P2,R2,L1,V1)}$	$C^m_{(P2,R2,L1,V2)}$	$C^m_{(P2,R2,L1,V3)}$	$C^m_{(P2,R2,L1,V4)}$	
	L2	$C^m_{(P2,R1,L2,V1)}$	$C^m_{(P2,R1,L2,V2)}$	$C^m_{(P2,R1,L2,V3)}$	$C^m_{(P2,R1,L2,V4)}$	$C^m_{(P2,R2,L2,V1)}$	$C^m_{(P2,R2,L2,V2)}$	$C^m_{(P2,R2,L2,V3)}$	$C^m_{(P2,R2,L2,V4)}$	
…	…	…	…	…	…	…	…	…	…	…

4.2 Pricing strategy

Both suppliers and consumers aim at maximizing the benefit or surplus they receive (ITU-T, 2008). The suppliers aim at maximizing the profit, which is the difference between revenue and cost. The consumers aim at maximizing the consumer surplus, which is the difference between consumer value (also known as utility or maximum willingness to pay) and price. As discussed previously, some of the factors that are important in the design of pricing scheme include technology risks, availability of resources, competition, supplier and consumer behavior, price discrimination and regulation.

Definition of the variation in retail prices

The definition of retail prices and trend was explained previously. For the game-theoretic tool, we need to define the variation in retail prices which we want to simulate. So, for each service, we define the price values we wish to simulate - the tool gives the possibility to simulate n values.

In the example presented in the next table, the tool simulates the results obtained when the value of service 1 is $P^r_{S1,Value1}$, $P^r_{S1,Value2}$, $P^r_{S1,Value3}$, and $P^r_{S1,Value4}$ for all players (providers).

TABLE IV. *Variation values for retail prices*

	Value 1	Value 2	Value 3	Value 4	…
Value for Service 1	$P^r_{S1, Value1}$	$P^r_{S1, Value2}$	$P^r_{S1, Value3}$	$P^r_{S1, Value4}$	
Value for Service 2	$P^r_{S2, Value1}$	$P^r_{S2, Value2}$	$P^r_{S2, Value3}$	$P^r_{S2, Value4}$	
…					

4.3 Definition of the variation in wholesale prices

For wholesale prices, we define the variation in wholesale price layers that we want to simulate. Similarly, for retail price, for each layer we define the price values we wish to simulate—the tool gives the possibility to simulate n values.

TABLE V. *Variation values for wholesale prices*

	Value 1	Value 2	Value 3	Value 4	…
Value for Layer 0	$P^w_{L0, Value1}$	$P^w_{L0, Value2}$	$P^w_{L0, Value3}$	$P^w_{L0, Value4}$	
Value for Layer 1	$P^w_{L1, Value1}$	$P^w_{L1, Value2}$	$P^w_{L1, Value3}$	$P^w_{L1, Value4}$	
Value for Layer 2	$P^w_{L2, Value1}$	$P^w_{L2, Value2}$	$P^w_{L2 ,Value3}$	$P^w_{L2, Value4}$	

For infrastructure, the definition of which layer or combination of layers we would like to simulate is also required (next table). For example, if a provider wants to use (lease) the conduit from another provider, we choose option 0.

TABLE VI. *Wholesale layers*

Wholesale Infrastructure	
0	Conduit
1	Cable
2	Bit-Stream (Conduit + Cable +Equipment)

5. SIMULATION MODEL (MODELING COMPETITION)

The simulation model can be sub-divided into seven main parts: retail and wholesale modeling, calculate total costs (build and lease infrastructure), calculate revenues (retail and wholesale market), calculate profit, calculate consumer surplus, and calculate welfare. The next sections describe all these parts.

5.1 Retail modeling

In our model, we assume that consumers choose the service from the provider with the lowest price. However, consumers only buy a service if the price is less than their willingness to pay. This means that if there are two or more providers, consumers choose the service from the provider with the lowest price. Moreover, if several providers have the same price, we use the provider ranking. We also assume that consumers have a different willingness to pay for each service (e.g., voice, video and data).

First, the tool identify the retail provider for each service in the regions in study using information from providers, retail prices, consumer willingness to pay, and provider rank. Next, as we know which provider will provide each service, we can compute the total subscribers per region, service, and provider (market segment). The structure used is presented in next figure.

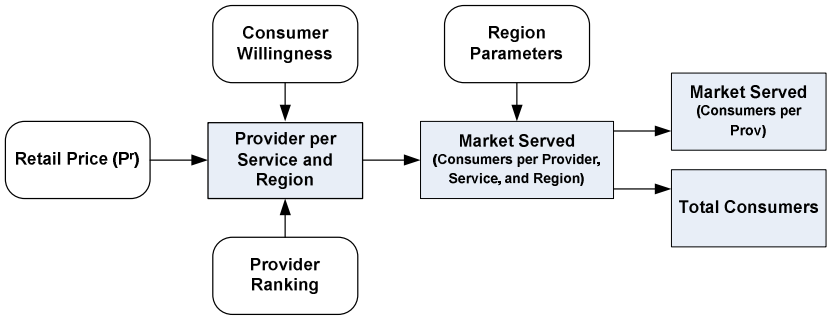

Figure 2. *Retail market modeling*

5.2 Wholesale modeling

In wholesale modeling, we determine the infrastructure chosen by each provider to reach consumers. To model the wholesale market, we assume that if a provider does not have infrastructure, it uses the infrastructure (or part of the infrastructure, such as a conduit cable) of another provider if the price charged to access it is lower than the cost to build an infrastructure. To achieve that goal, the algorithm uses information about wholesale prices, fixed costs, and marginal costs to identify the best solution (lease or build infrastructure) for each region and service. The algorithm also utilizes the information produced in retail modeling to determine which providers offer services to consumers in all the regions. The fixed and marginal costs are calculated in the techno-economic tool.

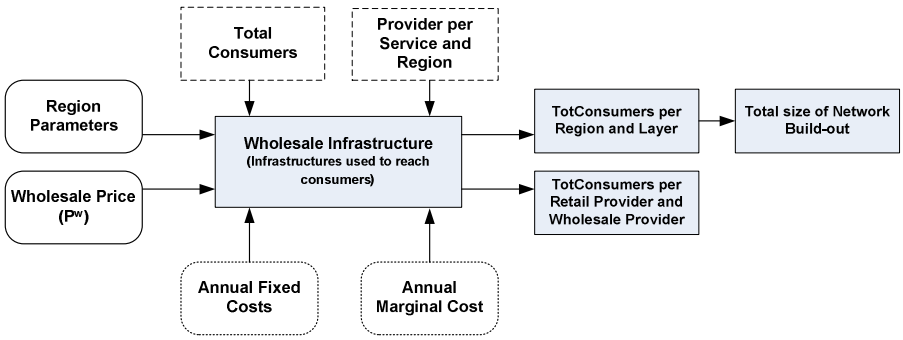

Figure 3. *Wholesale market modeling*

5.3 Calculate total costs (build and lease infrastructures)

The calculation of the total costs incurred by each provider is divided in two main parts: wholesale costs and build-out costs. As sees in next figure, in order to compute the total wholesale costs, we use the wholesale infrastructure design computed previously and the wholesale prices charged by the infrastructure owners (i.e., payments that a specific provider gives to the infrastructure owner to buy wholesale access in order to reach consumers). We assume that the network owner charges the same wholesale price to all providers.

To calculate the build-out costs, the algorithm uses the fixed and marginal costs parameters with region parameters to compute the total costs required to deploy an entire or part of an infrastructure. The total number of consumers per region and per provider is also used to add the effect of economies of scale. When a provider buys a large quantity of equipment, the probability of attaining better prices is higher.

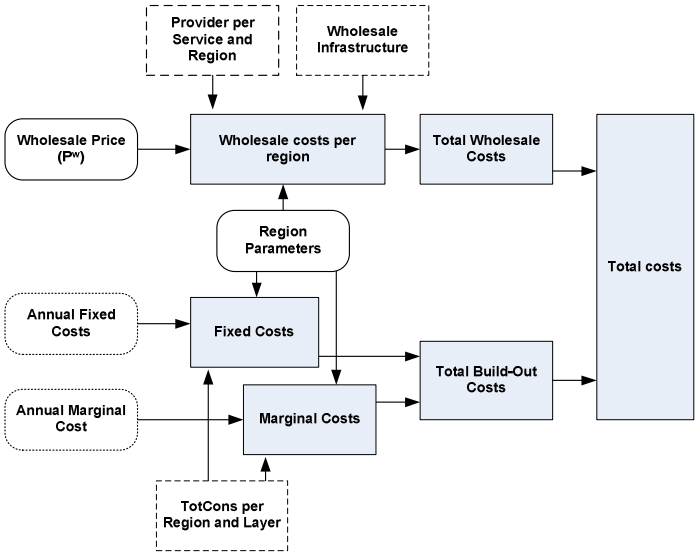

Figure 4. *Total costs calculation*

5.4 Calculate revenues (retail and wholesale market)

To compute the total revenues per provider, we first calculate the revenues from the retail market. These are primarily based on the retail prices charged by providers and the total number of consumers per provider and services computed in the retail modeling. Revenues from the retail market are equal to the product of the retail price of each service and the total customers of the service.

Next, we calculate the revenues from the wholesale market. The wholesale infrastructure provides information about the number of access leased. The revenues of a provider are the sum of all payments received from other providers that use its infrastructure to reach consumers. Finally, the total revenues of a given provider are the sum of the revenues from the retail and the wholesale market.

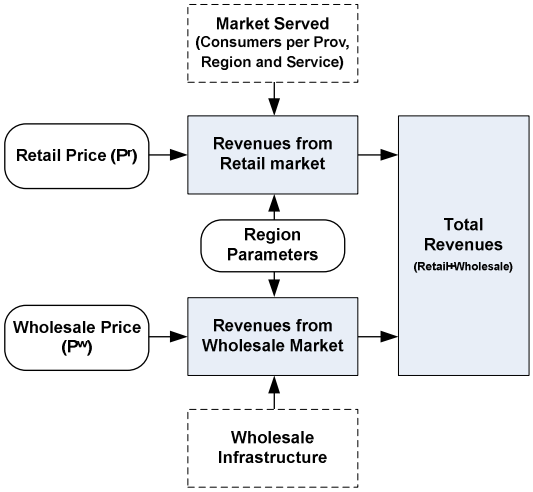

Figure 5. *Revenues calculation*

5.5 Calculate profit

After computing the total costs and revenues in the previous algorithms, the formula we use to calculate total profit is the difference between total revenues and total profit. The total profit is also used in the identification of the Nash equilibrium strategies.

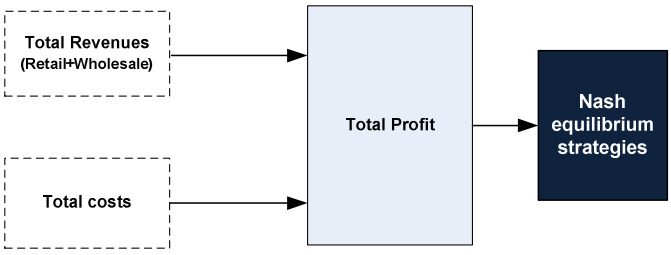

Figure 6. *Profit calculation*

5.6 Calculate consumer surplus

Consumer surplus (CS) is the difference between the total amount that consumers are willing and able to pay for each service and the total amount that they actually pay (i.e., the retail price). So, the CS of a specific market is the sum of the individual consumer surpluses of all those customers in the market who actually bought the service at the going retail price "Pr" (ACMA, 2009). To compute CS, we need information about consumer willingness to pay and retail prices for each service (next figure).

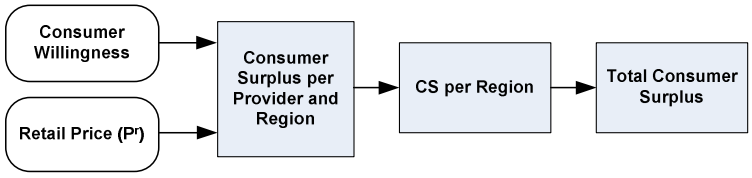

Figure 7. *Consumer surplus calculation*

5.7 Calculate total welfare

Total welfare is computed on base of the formula: welfare = consumer surplus + total profit. Like the previously calculations, the CS and the profit are computed in the algorithms presented above. The block diagram is presented in the next figure.

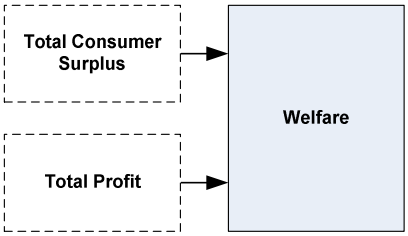

Figure 8. *Welfare calculation*

6 STRATEGIES AND MAIN ASSUMPTIONS

To analyze the impact of retail and wholesale services price variations, we propose two games (Figure 9.): (1) analysis the impact of retail price variation on NPV (wholesale prices are defined by regulator); and (2) analysis the impact of retail and wholesale price variations on profit, consumer surplus, welfare, and retail/wholesale market (different wholesale prices in each region). For the game-theoretic evaluation, the model calculates the NPV and operator's profit for both operators' pricing strategies. Operators' NPVs are used as payoffs for the players in the first and second game, and operators' profits for the third game.

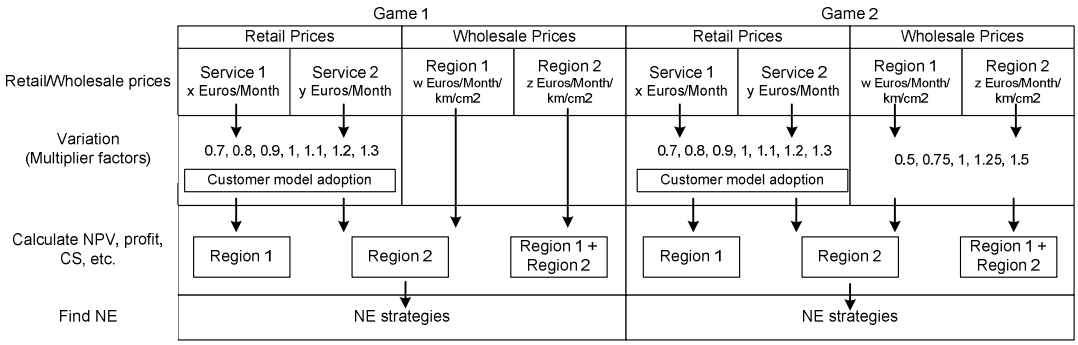

Figure 9. *Games proposed*

From the several assumptions, we posit: (a) the price that players charge for their services (retail and wholesale) will be varied; (b) the retail price setting will influence the market share of both players (resulting in a higher or lower market share); and (c) consumers only buy a retail service if the price is less than their willingness to pay.

As stated above, we assume that when one player increases/decreases the retail price, the market share of all players will be affected. For example, if one player offers cheaper services, it will be able to capture a higher market share. If a price decreases to nearly zero, everyone will use the service, and the market share of this operator will be close to 100% (total market). On the other hand, if an operator charges a higher price for a service, no one will subscribe to the service from this player, and its market share will decrease to 0%.

The impact of changing prices in the market share (i.e., the estimate of the impact of the price on the service adoption) is modeled using the Boltzmann equation:

$$Y = \frac{V_i - V_f}{(1 + e^{(x-x0)/dx})} + V_f \qquad (2)$$

in which the variables are defined as follows:

- X_0: is the mean base (or center)
- dx: is the width
- V_i: is the initial value of y
- V_f: is the final value of y

The next graph shows the s-curves of three functions. The slope (parameter b) of the market penetration should give an advantage or disadvantage to one of the players (Katsianis, Gyürke, Konkoly, Varoutas, & Sphicopoulos, 2007).

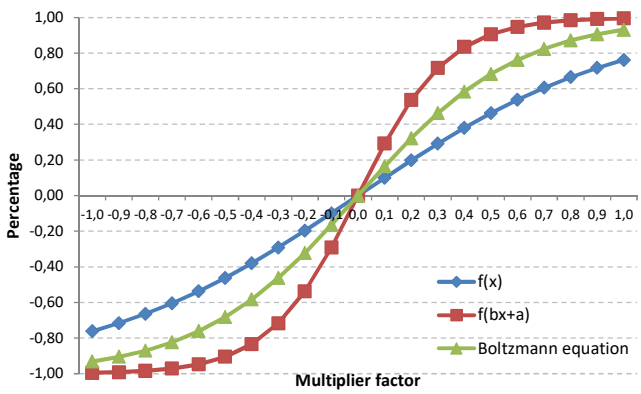

Figure 10. *Models to estimate the impact of the price on the service adoption (a=0.4, b=3, dx=0.3)*

6.1 Main assumptions

We assume that the willingness to pay for each retail service is different in both regions. In the urban area (region 1) the maximum amount subscribers would be willing to pay for service 1 is 26 euros and 65 euros for service 2. In the rural area we assume a willingness value of 22 euros for service 1 and 55 euros for service 2 (see TABLE VII.).

TABLE VII. *Willingness assumptions*

Parameters	Region 1 (Urban area)		Region 2 (Rural area)	
	Service 1	Service 2	Service 1	Service 2
Monthly Subscription Fee (Year1)	20€	50€	20€	50€
Willingness Value	26 €	65€	22€	55€
Willingness Multiplier	1.3	1.3	1.1	1.1

For the wholesale infrastructure we assume a duct availability of player 1 100% in the urban area and 90% in the rural area. We also assume that operator 2 (new entrant) leases 100% of the ducts available in the urban area and 100% of the ducts available (operator 1 has only 90% and the remaining 10% are deployed by operator 2) in the rural area from operator 1 (incumbent operator). In the other hand, player 1 leases the 10% remaining (in region 2) from operator 2. The wholesale prices assumptions are: 9.1€ (month / km / cm2) for urban area and 7.5€ (month / km / cm2) for the rural area. The wholesale infrastructure assumptions and described in TABLE VIII. .

TABLE VIII. *Wholesale infrastructure assumptions*

Parameters	Region 1 (Urban)		Region 2 (Rural)	
	Feeder segment	Distribution Segment	Feeder segment	Distribution Segment
Provider 1				
Duct Availability (# of ducts available for leasing)	100%	100%	90%	90%
Wholesale price charged to access owned ducts (€/Km)	€110	€110	€90	€90
Proportion of ducts leased	0%	0%	10%	10%
From operator	0	0	2	2
Provider 2				
Duct Availability (# of ducts available for leasing)	0%	0%	10%	10%
Wholesale price charged to access owned ducts (€/Km)	€110	€110	€90	€90
Proportion of ducts leased	75%	75%	100%	100%
From operator	1	1	1	1

The next sections present the three games results and analyses. In the first game, retail prices vary between tariff multiplier 0.7 and 1.3 (in increments of 0.1). For the second game, retail prices vary between 0.8 and 1.2, and wholesale prices between 0.5 and 1.5.

7 RESULTS

Based on the numerous input parameters described, our tool computes several results, including profit, consumer surplus, welfare, market served, network size, costs, and revenues, and finds the strategies that are Nash equilibriums. The results are saved in text files (see Figure 11.). The final results with all strategies computed are saved in a file named “results_f.txt”. In addition, the Nash equilibriums are saved in a file named “equilib.txt”.

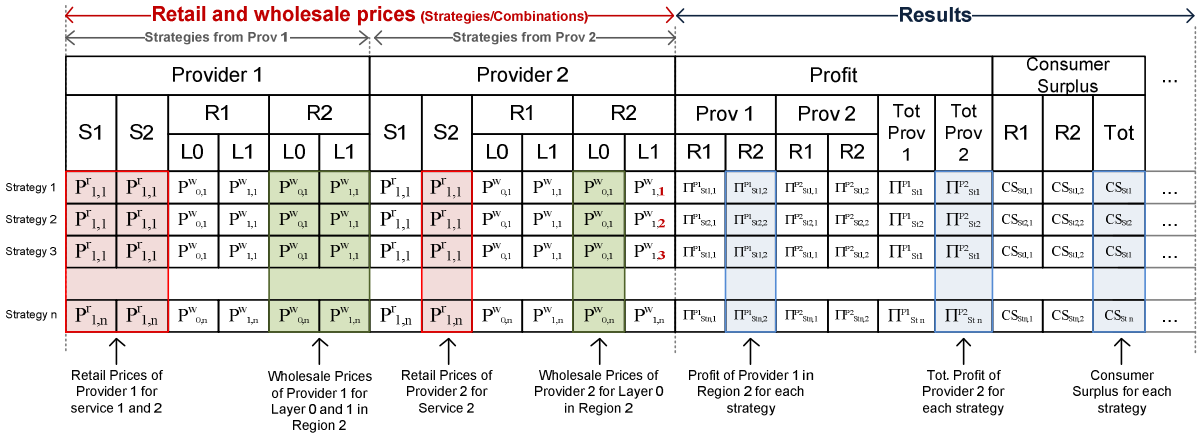

Figure 11. *Structure of the results produced (output from tool)*

Figure 11. show the structure of the results that correspond to a scenario of two providers, two retail services, two infrastructure layers, and two regions. Each line is a strategy. We consider a strategy to be a set of retail and wholesale prices. For each combination of prices, the tool calculates profit, CS, welfare, market served, network size, and total costs.

In addition to the results presented in the tables, the tool creates several types of graphs. For that, we incorporated a Gnuplot program in our C code. Gnuplot is a portable command-line-driven graphing utility for Linux, OS/2, MS Windows, OSX, VMS, and many other platforms. The source code is copyrighted but freely distributed.

Next figures show two examples of the graphs produced. The graph shows the impact on profit of both providers and variation in wholesale and retail prices. This representation gives users a tool to gain a better perspective of the results.

7.1 Game 1: Impact of retail prices variation on NPV

In this game we assume that wholesale prices are fixed and that operators choose retail prices to maximize their profit. The impact of varying retail prices on market shares is estimated using the Boltzmann equation (described above). The main goal of this analysis is to determine the optimal retail price strategy for both players. The retail prices vary between –30% and 30%, with increasing steps of 10% (next table).

TABLE IX. *Retail prices variation values*

Tariff multiplier factor	0.7 (-30%)	0.8 (-20%)	0.9 (-10%)	1 (0%)	1.1 (10%)	1.2 (20%)	1.3 (30%)
Service 1 price	14	16	18	20	22	24	26
Service 2 price	35	40	45	50	55	60	65

The combination of the two retail prices and seven multiplier factors leads to 49 possible strategies for each player (49x49 matrix) in each region (2,401 total strategies). The next table presents the structure of the combinations and calculated NPV.

TABLE X. *Structure of combinations and results for Game 1*

Strategies	Player 1	Player 2	NPV			
	Retail Price	Retail Price	Player 1	Player 2	Total Player1	Total Player2

	R1& R2		R1 &R 2		R1	R2	R1	R2	R1+R2	R1+R2
	S1	S2	S1	S2						
1	0.7	0.7	0.7	0.7	…	…	…	…	..	…
2	0.7	0.7	0.7	0.8	…	…	…	…	…	…
n	…	…	…	…	…	…	…	…	…	…

The results (payoff matrix) of this game are presented in TABLE XI. for region 1 (urban area). TABLE XIII. shows the sum of the payoffs of each player in both regions. The following tables present the NPV for both players for each possible combination of strategies (one strategy for each player); Nash equilibrium strategies are also identified.

The first two rows represents the prices multiplier factor of player 2 (for services 1 and 2) and the first two columns show the variation (multiplier factors) of player 1. Each cell contains two values: The left value corresponds to the NPV of player 1, and the value on right side corresponds to the NPV of player 2. For example, the first value calculated (16,512,089 €) corresponds to the NPV of player 1 when the strategy of player 1 is to decrease the price of service 1 and service 2 by about 30% (multiplier factor 0.7), and the strategy of player 2 is also to decrease the price of service 1 and service 2 by about 30%.

TABLE XI. *Impact of retail prices variation (0.7 to 1.3) on NPV (€)—Region 1 (urban area)*

		Player 2 (New entrant) strategies															
	Price S1	0,70					...	1,30									
	Price S2	0,70		...	1,30		...	0,70		...	1,10		...	1,30			
Player 1 (Incumbet operator) strategies	0,70	0,7	16.512.089	-12.544.009		28.802.318	-24.111.799		18.536.768	-14.477.627		26.229.241	-19.253.255		30.826.996	-26.045.417	
		0,8	15.251.728	-8.366.277		33.887.508	-24.111.799		17.276.406	-10.299.894		28.901.992	-16.816.923		35.912.186	-26.045.417	
		0,9	13.130.606	-4.410.360		38.972.698	-24.111.799		15.155.285	-6.343.978		30.820.067	-13.978.016		40.997.376	-26.045.417	
		1	10.487.574	-853.341		44.057.888	-24.111.799		12.512.253	-2.786.958		31.955.838	-10.820.742		46.082.566	-26.045.417	
		1,1	7.648.610	2.199.270		49.143.078	-24.111.799		9.673.288	265.652		32.367.961	-7.486.434		51.167.756	-26.045.417	
		1,2	-5.677.393	11.011.259		-5.564.353	-24.006.800		-3.652.715	9.077.642		-3.652.715	28.947.191		-3.539.674	-25.940.418	
		1,3	-5.439.184	11.011.259		-5.326.143	-24.006.800		-3.414.505	9.077.642		-3.414.505	28.947.191		-3.301.465	-25.940.418	
	...	...															
	0,90	0,7	15.905.526	-11.114.024		28.195.755	-22.681.814		20.201.491	-14.477.627		27.893.964	-19.253.255		32.491.720	-26.045.417	
		0,8	14.645.164	-6.936.292		33.280.944	-22.681.814		18.941.129	-10.299.894		30.566.716	-16.816.923		37.576.910	-26.045.417	
		0,9	12.524.043	-2.980.375		38.366.134	-22.681.814		16.820.008	-6.343.978		32.484.790	-13.978.016		42.662.100	-26.045.417	
		1	**9.881.011**	**576.645**		43.451.324	-22.681.814		14.176.976	-2.786.958		33.620.561	-10.820.742		47.747.289	-26.045.417	
		1,1	7.042.046	3.629.255		48.536.514	-22.681.814		11.338.012	265.652		34.032.684	-7.486.434		52.832.479	-26.045.417	
		1,2	-6.283.956	12.441.245		-6.170.916	-22.576.815		-1.987.991	9.077.642		-1.987.991	28.947.191		-1.874.951	-25.940.418	
		1,3	-6.045.747	12.441.245		-5.932.707	-22.576.815		-1.749.782	9.077.642		-1.749.782	28.947.191		-1.636.742	-25.940.418	
	...	...															
	1,20	0,7	14.473.931	-9.509.528		26.764.160	-21.077.318		22.698.576	-14.477.627		30.391.049	-19.253.255		34.988.805	-26.045.417	
		0,8	13.213.570	-5.331.796		31.849.350	-21.077.318		21.438.214	-10.299.894		33.063.801	-16.816.923		40.073.995	-26.045.417	
		0,9	11.092.448	-1.375.879		36.934.540	-21.077.318		19.317.093	-6.343.978		34.981.875	-13.978.016		45.159.184	-26.045.417	
		1	8.449.416	2.181.140		42.019.730	-21.077.318		16.674.061	-2.786.958		36.117.646	-10.820.742		50.244.374	-26.045.417	
		1,1	5.610.452	5.233.751		47.104.920	-21.077.318		13.835.097	265.652		36.529.769	-7.486.434		55.329.564	-26.045.417	
		1,2	-7.715.551	14.045.741		-7.602.510	-20.972.319		509.094	9.077.642		509.094	28.947.191		622.134	-25.940.418	
		1,3	-7.477.342	14.045.741		-7.364.301	-20.972.319		747.303	9.077.642		**747.303**	**28.947.191**		860.343	-25.940.418	
	1,30	0,7	12.709.232	-8.402.733		24.999.461	-19.970.523		12.693.783	-14.481.984		20.386.256	-19.257.613		24.984.012	-26.049.774	
		0,8	11.448.871	-4.225.001		30.084.651	-19.970.523		11.433.421	-10.304.252		23.059.008	-16.821.281		30.069.202	-26.049.774	
		0,9	9.327.750	-269.084		35.169.841	-19.970.523		9.312.300	-6.348.335		24.977.082	-13.982.373		35.154.392	-26.049.774	
		1	6.684.717	3.287.935		40.255.031	-19.970.523		6.669.268	-2.791.316		26.112.853	-10.825.099		40.239.582	-26.049.774	
		1,1	3.845.753	6.340.546		45.340.221	-19.970.523		3.830.304	261.295		26.524.976	-7.490.792		45.324.771	-26.049.774	
		1,2	-9.480.250	15.152.536		-9.367.209	-19.865.524		-9.495.699	9.073.285		-9.495.699	28.942.834		-9.480.250	-26.045.417	
		1,3	-9.242.041	15.152.536		-9.129.000	-19.865.524		-9.257.490	9.073.285		-9.257.490	28.942.834		-9.242.041	-26.045.417	

We have analyzed the different strategies of the two players to find the game's Nash equilibria. (Our model includes a function for searching NE in the games). The NE strategies are formatted with a black background. From the analysis of these results, we find two NE strategies that are detailed in the next table.

TABLE XII. *Pure NE strategies for region 1 (urban area)*

Strategies	**Player 1 (Incumbent operator)**		**Player 2 (New entrant)**		**NPV € Player 1**	**NPV € Player 2**
	Retail service 1	Retail service 2	Retail service 1	Retail service 2		
1	0.9 (18€)	1 (50€)	0.7 (14€)	0.7 (35€)	9.881.001	576.645
2	1.2 (24€)	1.3 (65€)	1.3 (26€)	1.1 (55€)	747.303	28.947.191

The next graph shows the impact of service 2 price variation on the NPV of both operators (for the urban area).

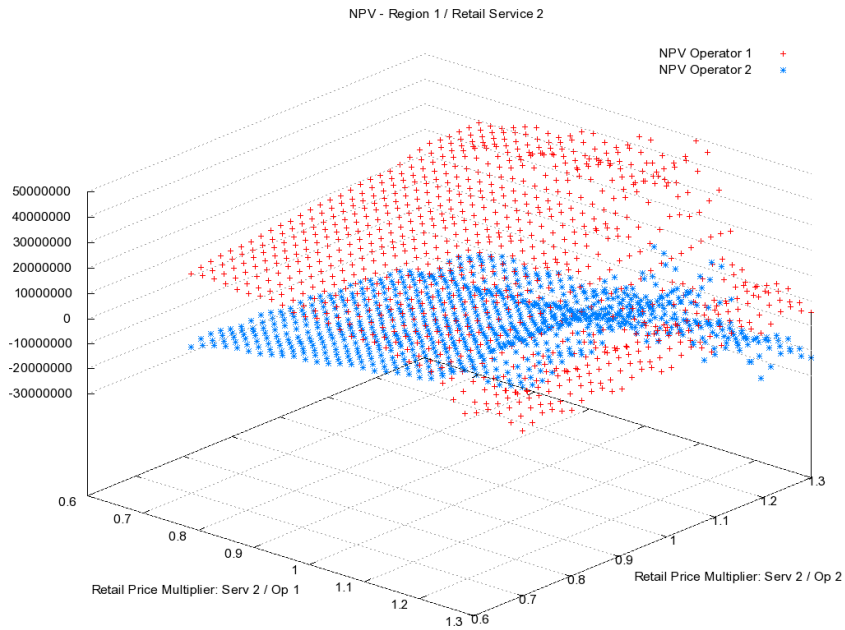

Figure 12. *NPV variation: Provider 1 and 2/Region 1/Retail service 2*

In the previous tables and graphs we presented the results for region 1 and region 2 when isolated. However, operators are also interested in the results for both regions. So, the next table analyzes the sum of the payoffs of each player in both regions.

TABLE XIII. *Impact of retail prices variation (0.7 to 1.3) on NPV (€)—Region 1 and 2*

Player 2 (New entrant) strategies

Price S1		0,70										1,20										1,30									
	Price S2	0,70		0,80		,,,	1,30		,,,	0,70		,,,	1,30		,,,	0,70		,,,	1,10		,,,
Player 1 (Incumbet operator) strategies																					
0,70	0,7	15831024	-18582287	18183087	-19363781		28936533	-30770826		17496173	-19795217		30601681	-31983756		18113915	-20768607		26428738	-25938555	
	0,8	14472132	-14083788	19556293	-16693988		34299456	-30618352		16137281	-15296719		35964604	-31831283		16755023	-16270109		29303695	-23304216	
	0,9	12185918	-9824136	17582209	-11580297		39612688	-30427158		13851067	-11037066		41277837	-31640088		14468808	-12010456		31362952	-20234582	
	1	9338223	-5994009	14713466	-6738115		44866176	-30195243		11003372	-7206939		46531325	-31408173		11621113	-8180329		32577289	-16820707	
	1,1	6280874	-2707019	11341982	-2384199		50053039	-29925006		7946023	-3919949		51718187	-31137937		8563765	-4893339		33014991	-13215411	
	1,2	-7258790	6298197	-6933535	11368737		-4619767	-29519466		-5593641	5085267		-2954619	-30732396		-4975899	4111877		-3280895	23715662	
	1,3	-7216198	6452291	-6935595	11588571		-4402383	-29202072		-5551049	5239361		-2737234	-30415002		-4933308	4265971		-3375080	24186698	
,,,	,,,																				
1,10	0,7	14054700	-15642494	16406763	-16423988		27160208	-27831033		19064485	-17729134		32169993	-29917673		21886956	-20768607		30201779	-25938555	
	0,8	12695808	-11143995	17779969	-13754195		32523131	-27678559		17705593	-13230635		37532916	-29765199		20528064	-16270109		33076737	-23304216	
	0,9	10409594	-6884343	15805885	-8640503		37836364	-27487364		15419379	-8970983		42846148	-29574004		18241850	-12010456		35135994	-20234582	
	1	7561899	-3054215	12937141	-3798322		43089852	-27255449		12571684	-5140855		48099637	-29342089		15394155	-8180329		36350330	-16820707	
	1,1	4504550	232775	**9565657**	**555595**		48276714	-26985213		9514335	-1853865		53286499	-29071853		12336806	-4893339		36788033	-13215411	
	1,2	-9035114	9237991	-8709859	14308530		-6396091	-26579672		-4025329	7151351		-1386307	-28666312		-1202858	4111877		492146	23715662	
	1,3	-8992522	9392085	-8711919	14528364		-6178707	-26262279		-3982737	7305445		-1168922	-28348918		-1160266	4265971		397962	24186698	
1,20	0,7	13511514	-15140682	15863576	-15922176		26617022	-27329221		19138606	-17069339		32244114	-29257878		22830217	-20768607		31145040	-25938555	
	0,8	12152622	-10642183	17236783	-13252383		31979945	-27176747		17779714	-12570840		37607037	-29105404		21471325	-16270109		34019997	-23304216	
	0,9	9866407	-6382531	15262698	-8138692		37293177	-26985553		15493499	-8311188		42920269	-28914210		19185110	-12010456		36079254	-20234582	
	1	7018712	-2552404	12393955	-3296511		42546666	-26753638		12645805	-4481060		48173758	-28682294		16337416	-8180329		37293591	-16820707	
	1,1	3961364	734586	9022471	1057406		47733528	-26483401		9588456	-1194070		53360620	-28412058		13280067	-4893339		37731293	-13215411	
	1,2	-9578300	9739802	-9253046	14810342		-6939278	-26077861		-3951208	7811146		-1312186	-28006517		-259597	4111877		**1435407**	**23715662**	
	1,3	-9535709	9893896	-9255106	15030176		-6721894	-25760467		-3908617	7965240		-1094801	-27689124		-217006	4265971		1341222	24186698	
1,30	0,7	11507889	-13885393	13859952	-14666887		24613397	-26073932		11507889	-9292724		24613397	-21481263		11492251	-20732168		19807074	-25902116	
	0,8	10148997	-9386894	15233158	-11997094		29976320	-25921458		10148997	-4794225		29976320	-21328789		10133359	-16233669		22682032	-23267776	
	0,9	7862782	-5127242	13259073	-6883402		35289552	-25730264		7862782	-534573		35289552	-21137595		7847145	-11974017		24741289	-20198142	
	1	5015088	-1297114	10390330	-2041221		40543041	-25498349		**5015088**	**3295555**		40543041	-20905679		4999450	-8143889		25955625	-16784268	
	1,1	1957739	1989876	7018846	2312695		45729903	-25228112		1957739	6582545		45729903	-20635443		1942101	-4856899		26393328	-13178971	
	1,2	-11581925	10995092	-11256671	16065631		-8942903	-24822571		-11581925	15587761		-8942903	-20229902		-11597563	4148317		-9902559	23752102	
	1,3	-11539334	11149186	-11258730	16285465		-8725518	-24505178		-11539334	15741855		-8725518	-19912509		-11554971	4302411		-9996743	24223138	

From these results presented in the previous table, we find three pure NE strategies (black cells) that are described in the next table. The next table shows the NE strategies that maximize the profit of both players. To maximize profit, in the first equilibrium strategy, operator 1 increases retail prices by 10%. Operator 2, in face of the imposed wholesale prices, decreases the price of service 1 and service 2 by30%

and 20%, respectively. A new entrant has to pay the wholesale to the incumbent, but if increase the retail prices their market share will decrease (see model above).

TABLE XIV. *Pure NE strategies for both regions*

Strategy	**Player 1 (Incumbent operator)**		**Player 2 (New entrant)**		**NPV € Player 1**	**NPV € Player 2**
	Retail service 1	Retail service 2	Retail service 1	Retail service 2		
1	1.1 (22€)	1.1 (55€)	0.7 (14€)	0.8 (40€)	9.565.657	555.595
2	1.2 (24€)	1.2 (60€)	1.3 (26€)	1.1 (55€)	1.435.407	23.715.662
3	1.3 (26€)	1 (50€)	1.2 (24€)	0.7 (35€)	5.015.088	3.295.555

The next figure shows the impact of service 1 variation on NPV of both operators. We can verify that the variation of the retail price of service 1 does not have the same impact on NPV that it has on service 2.

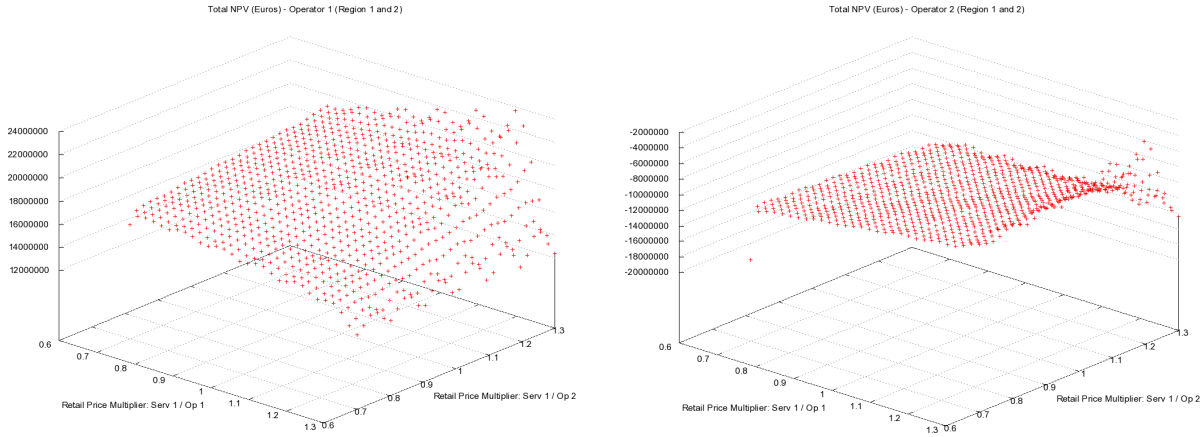

Figure 13. *NPV variation: Operator 1 and 2/Retail service 1*

From the analysis of the next figure we can conclude that the variation of retail prices of service 2 has a greater influence in the NPV than the variation of service 1 price. Service 2 price variation can drop the NPV of operator 1 to negative. On the other hand, operator 2 can turn the NPV positive when the tariff of service 2 increases.

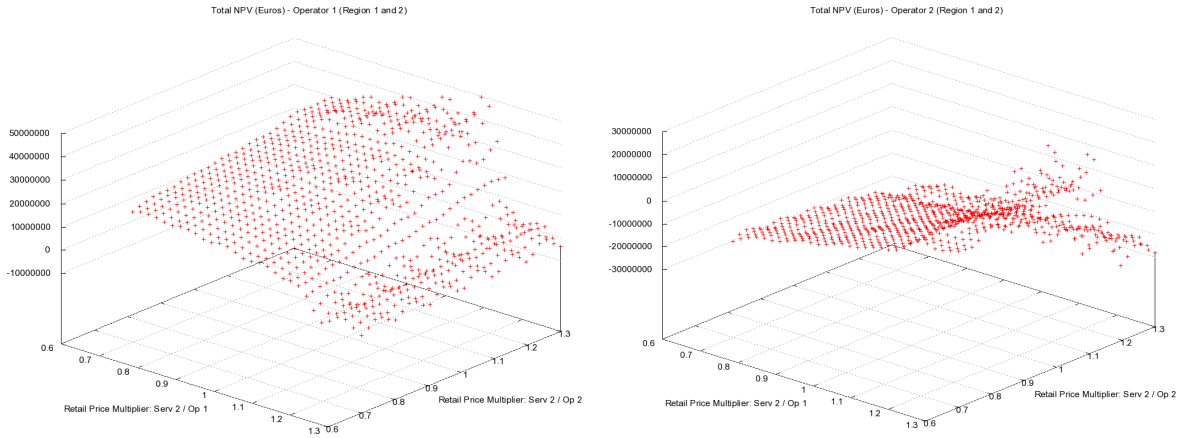

Figure 14. *NPV variation: Operator1 and 2/Retail service 2*

7.2 Game 2: Impact of retail and wholesale prices variation on NPV

In this game we assume that wholesale prices are not pre-imposed and we investigate what is the reaction of operators when they can also choose different wholesale prices in different regions (see next table). In game 2 we assume that has the same variation for both regions. Retail prices vary between 0.8 (-20%) and 1.2 (20%) (in increments of 0.1). For wholesale price we assume a variation between 0.5 and 1.5 (in increments of 0.25).

TABLE XV. *Retail and wholesale prices variation values for game 2*

Service	Tariff multiplier factor				
Retail price	0.8	0.9	1	1.1	1.2
Wholesale price	0.5	0.75	1	1.25	1.5

In this context, the combination of the three prices and variation multipliers leads to 625(5^4) possible strategies for each player (625x625 matrix) in each region (390625 strategies in both regions).

TABLE XVI. *Structure of combinations and results for Game 2*

Strategies	Player 1				Player 2				NPV						…
	Retail Price		Wholesale Price		Retail Price		Wholesale Price		Player 1		Player 2		Total Player1	Total Player2	Other Results: Profit, Consumer surplus, Welfare, Retail & Wholesale market, ...
	R1& R2		R1	R2	R1 &R 2		R1	R2	R1	R2	R1	R2	R1+R2	R1+R2	
	S1	S2	Duct Access		S1	S2	Duct Access								
1	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8							
2	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.9							
n	…	…	…	…	…	…	…	…							

As the matrix is to bigger, for this game we decide to present the NE strategies (players profit is used as payoff) and the graphs that show the impact of variation in the several results (presented in the previous table). The analysis of the results finds five NEs strategies. As player 2 do not operates in the wholesale market of region 1, the variation of this price is not significant.

TABLE XVII. *Game 2 results - summary*

Player 2 strategies

R Price S1				0,80												...
	R Price S2			0,80												...
		W Price R1		0,50										0,75		...
			W Price R2	0,50		0,75		1,00		1,25		1,50		0,50	...	
Player 1 Strategies	0,8	0,8	0,5	0,5	20981654	1704052	20954077	1728871	20926500	1753691	20898923	1778510	20871345	1803330	20981654	1704052	
				0,75	21232678	1425137	21205100	1449956	21177523	1474776	21149946	1499595	21122369	1524415	21232678	1425137	
				1	21483701	1146222	21456124	1171041	21428547	1195861	21400969	1220680	21373392	1245500	21483701	1146222	
				1,25	21734724	867307	21707147	892127	21679570	916946	21651993	941766	21624416	966585	21734724	867307	
				1,5	21985748	588392	21958171	613212	21930593	638031	21903016	662851	21875439	687670	21985748	588392	
			0,75	0,5	21113446	1557616	21085869	1582436	21058292	1607255	21030715	1632075	21003137	1656894	21113446	1557616	
				0,75	21364470	1278701	21336892	1303521	21309315	1328340	21281738	1353160	21254161	1377979	21364470	1278701	
				1	21615493	999786	21587916	1024606	21560339	1049425	21532761	1074245	21505184	1099064	21615493	999786	
				1,25	21866516	720872	21838939	745691	21811362	770511	21783785	795330	21756207	820150	21866516	720872	
				1,5	22117540	441957	22089963	466776	22062385	491596	22034808	516415	22007231	541235	22117540	441957	
			1	0,5	21245238	1411181	21217661	1436000	21190084	1460820	21162506	1485639	21134929	1510459	21245238	1411181	
				0,75	21496261	1132266	21468684	1157085	21441107	1181905	21413530	1206724	21385953	1231544	21496261	1132266	
				1	21747285	853351	21719708	878171	21692130	902990	21664553	927809	21636976	952629	21747285	853351	
				1,25	21998308	574436	21970731	599256	21943154	624075	21915577	648895	21887999	673714	21998308	574436	
				1,5	22249332	295521	22221754	320341	22194177	345160	22166600	369980	22139023	394799	22249332	295521	
			1,25	0,5	21377030	1264745	21349453	1289565	21321876	1314384	21294298	1339204	21266721	1364023	21377030	1264745	
				0,75	21628053	985830	21600476	1010650	21572899	1035469	21545322	1060289	21517745	1085108	21628053	985830	
				1	21879077	706916	21851500	731735	21823922	756555	21796345	781374	21768768	806194	21879077	706916	
				1,25	22130100	428001	22102523	452820	22074946	477640	22047369	502459	22019791	527279	22130100	428001	
				1,5	22381124	149086	22353546	173905	22325969	198725	22298392	223544	22270815	248364	22381124	149086	
			1,5	0,5	21508822	1118310	21481245	1143129	21453668	1167949	21426090	1192768	21398513	1217588	21508822	1118310	
				0,75	21759845	839395	21732268	864214	21704691	889034	21677114	913853	21649536	938673	21759845	839395	
				1	22010869	560480	21983291	585300	21955714	610119	21928137	634939	21900560	659758	22010869	560480	
				1,25	22261892	281565	22234315	306385	22206738	331204	22402607	101928	22151583	380843	22261892	281565	
				1,5	22512915	2650	22485338	27470	22457761	52289	22430184	77109	22402607	101928	22512915	2650	
	...	...															
...	...																

We conclude that, in the business case defined, when operators can charge different retail and wholesale prices, they choose to increase wholesale prices. To maximize profits, operators increase wholesale prices and decrease retail prices. However, the increase in wholesale prices precludes entry of new operators into the market.

TABLE XVIII. *Pure NE strategies in both regions (Game 2)*

Strategy	**Player 1 (Incumbent operator)**				**Player 2 (New entrant)**				**NPV €** **Player 1**	**NPV €** **Player 2**
	Retail		Wholesale		Retail		Wholesale			
	S1	S2	R1	R2	S1	S2	R1	R2		
1-4	0.8	0.8	1.5	1.25	0.8	0.8	0.50 0.75 1 1.25 1.5	1.25	22 402 606	101 928
5-9	0.8	0.9	1.25	1	0.8	0.8	0.50 0.75 1 1.25 1.5	1.25	19 543 660	6.198.799

The comparison of the two games above shows that when the regulator defines wholesale prices, operators increase retail prices to maximize profit. However, when wholesale prices are not regulated, operators maximize profit by decreasing retail prices and increasing wholesale prices. However, without regulation, the higher wholesale prices will limit the entrance of new competitors.

The main results of this game are summarized in the next figures. In the first two graphs we can see the impact of retail prices (left) and wholesale prices (right) on players profit. We can verify that both prices can turn profit positive/negative.

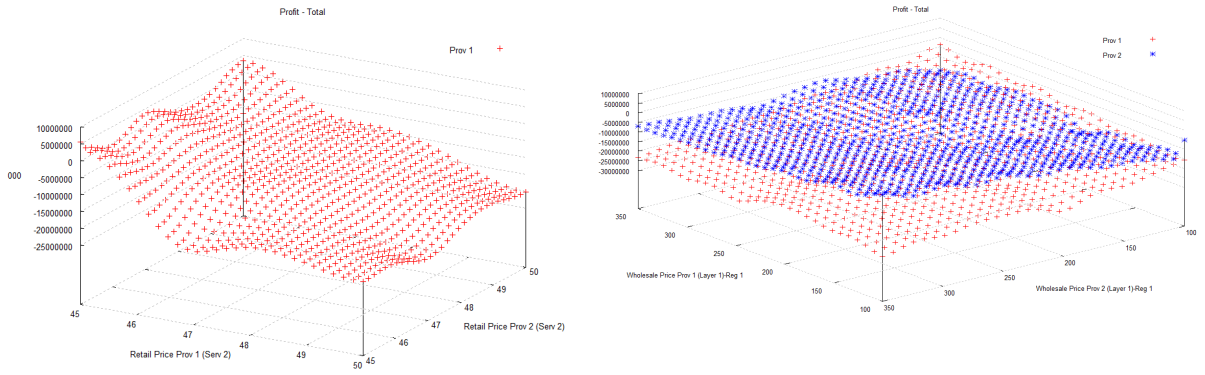

Figure 15. *Profit variation: Retail service 2 and wholesale service*

Consumer surplus decreases with the increase of prices (left graph). As also expected and modeled above the impact of retail prices variation has higher influence in the market share of competitors (see next figure).

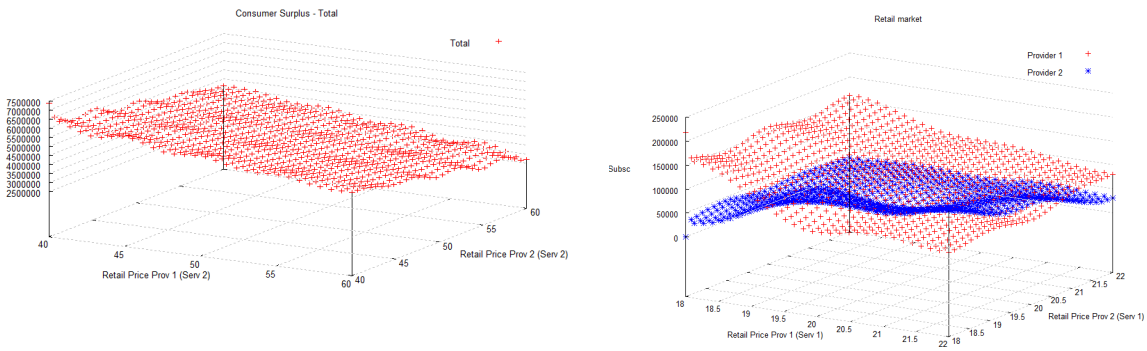

Figure 16. *Consumer Surplus retail market variation*

8 CONCLUSION

Sensitivity analysis shows the impact that changes in a certain parameter will have on the model's outcome. As the interaction between all the players is important, we put the competition component in the business case. With game theory, we want to understand the effects of the interaction between the different players defined in our business case. In the proposed games, the profit (outcome) of each operator (player) will be dependent not only on their actions, but also on the actions of the other operators in the market.

The impact of the price (retail and wholesale) variations on several output results: players' profit, consumer surplus, welfare, costs, service adoption, and so on. For that, two price-setting games are played. Players' profits and NPV are used as the payoff for the players in the games analyzed.

In our model we also use the Nash equilibrium to find equilibrium. Proposed tools include a module to search the Nash equilibrium in the game. One strategy is a Nash equilibrium when both competitors play their best strategy related to the other strategies selected (players know each other's strategy in advance).

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