

# Game Theoretic Modeling of NGANs: Impact of retail and wholesale services price variation

João Paulo R. Pereira

Polytechnic Institute of Bragança (IPB), Portugal

[jprp@ipb.pt](mailto:jprp@ipb.pt)

Pedro Ferreira

Technical University of Lisbon (IST), Portugal

[pedrof@cmu.edu](mailto:pedrof@cmu.edu)

**Abstract** - The increasing demand for broadband access leads operators to upgrade the existing access infrastructures (or building new access network). Broadband access networks require higher investments (especially passive infrastructures such as trenches/ducts and base station towers/masts), and before making any decision it is important to analyze all solutions. The selection of the best solution requires understanding the technical possibilities and limitations of the different access technologies, as well as understanding the costs of building and operating the networks. This study analyzes the effect of asymmetric retail and wholesale prices on operators' NPV, profit, consumer surplus, welfare, retail market, wholesale market, and so on. For that, we propose a techno-economic model complemented by a theoretic-game model. This tool identifies all the essential costs of building (and operating) access networks, and performs a detailed analysis and comparison of the different solutions in various scenarios. Communities, operators/service providers, and regulators can use this tool to compare different technological solutions, forecast deployment costs, compare different scenarios, and so on, and help them in making deployment (or regulatory) decisions. The game-theory analyses give a better understanding of the competition and its effect on the business case scenarios' economic results.

**Index Terms** - Next generation networks (NGN), Cost model, Game-theory model, Segmented regulation

## I. INTRODUCTION

Service providers, network operators, and Internet access providers are faced with the challenge of providing higher capacity access to the end user and offering wider services [1]. Consequently, new Internet infrastructure and technologies that are capable of providing high-speed and high-quality services are needed to accommodate multimedia applications with diverse quality of service (QoS) requirements. Until a few years ago, Internet access for residential users was almost exclusively provided via public switched telephone networks (PSTN) over the twisted copper pair [2]. The new quadruple play services (i.e., voice, video, data, and mobility), which require high-speed broadband access,

created new challenges for the modern broadband wireless/wired access networks [3]. The new services led to both the development of several different last-mile solutions to make the access network capable of supporting the requirements and a stronger integration of optical and wireless access networks.

The move toward next-generation networks (NGNs) has significant implications for the technical architecture and design of access network infrastructure, as well as the value chains and business models of electronic communications service provision [4]. This migration has begun to transform the telecommunication sector from distinct single-service markets into converging markets [5]. NGNs allow consumers to choose between different access network technologies to access their service environment. In our work, the NGN architecture will be limited to the developments of network architectures in the access network (local loop), referred to as the next-generation access network (NGAN).

Although the cost of bandwidth in the active layer has reduced significantly (and continually) in recent years, the cost of civil works (such as digging and trenching) represents a major barrier for operators to deploy NGA infrastructure. Studies and deployments [6] show that civil infrastructure is the largest proportion of the costs of fixed access deployment (up to 80%). Duct is a critical part of the next-generation access networks and its sharing would reduce or eliminate this capital cost and barrier to entry. However, duct access may need to be complemented by extra civil work to increase infrastructure capacity, the use of dark fiber (where available), or the use of conduits of alternative infrastructure providers. This also highlights that different and/or complementary regulatory tools may be required in different parts of the network [7].

## II. EFFECTS OF NGNS ON MARKET DEFINITION

The entry of new competitors can be based on the resale of services from the incumbent, on building up their own infrastructures, on renting unbundled infrastructure from incumbents, or, on the combination of the above elements. The availability of these options to competitors and price definition are generally determined by regulatory policies [8]. So, the introduction of NGNs

by telecommunication network operators obligates the national regulators adapt their access regulation regimes to the new technological conditions. Regulation and/or promotion of competition by regulatory measures need to be analyzed and compared.

The access network is usually the most expensive component in terms of capital investment (specifically passive infrastructure) and OA&M costs. Of the several costs, civil engineering costs are greatest when it is necessary to run a new fiber or copper connection to the cabinet, building, or home. Moreover, access to existing infrastructure, such as the ducts of the incumbent or other market players or sewage pipes, is critically important to avoid digging.

For [9], a local loop network can be divided into three main layers or segments: a service layer and two infrastructure layers (see Figure 1). Layer 1 includes passive infrastructures, such ducts and cables, and requires the greatest investment. Layer 2 consists of active infrastructures, such as the technical installations at the end of the fibers that send, receive, and manage the optical signals. Layer 3 includes several services that consumers buy from telecommunication operators.

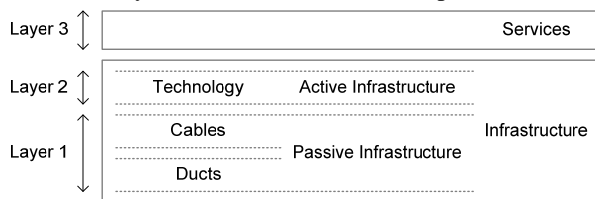


Figure 1. Network layers [9]

III. BUSINESS CASE DEFINITION

The definition of a business case implies a great number of assumptions, such as the penetration rate, components prices, and the market share rate. However, it is difficult to get an exact forecast of its performance. The utility of a business case is to offer a more approximated estimation that allows the construction of scenarios for the future. A business case should be as realistic as possible in order to be useful and reflect all the variables of interest of the market, as well as their evolution and expected behavior [10].

A. Territory and demography

The geographical areas considered are an area with high population density and an area with low population density and high coverage. For the rural area, the rollout strategy does not cover the whole area (1173 km<sup>2</sup>)—the target area is limited to 34.04 km<sup>2</sup> with 23,000 inhabitants (see next table). In our model, we consider the last 10 years to estimate the average rate of increase: 0.62% for the urban area and value of 0.01% for rural target area. The population density in the urban area is 3,748 inhabitants per square kilometer and 675 in the rural.

Parameters presented in next table are important to calculate the cost of trenches/ducts, which are the most

significant proportion of the costs of fixed access deployment.

TABLE I. AVERAGE LENGTHS ASSUMPTIONS

Segment	Region 1 - Urban	Region 2 - Rural
Feeder	750 m	1500 m
Distribution	300 m	750 m
Drop	15 m	25 m

Several studies and models [11-13] assume that in urban areas, the duct availability rate is about 60% for feeder segments, and 40% for the distribution segment. In rural areas, the duct availability rate is 25% for feeder and 0% for the distribution network. The report from [14] assumes that a substantial proportion (80% near to the CO and 30% nearer to the premises) of existing ducts can be re-used for fiber deployment [15].

B. Service profiles assumptions

In this business case, we define two different services: slow Internet browsing service with downstream throughput of 2 Mbps, and triple play service with 20 Mbps of downstream rate. The expected tariff evolution (the factor by which the tariff is expected to increase or decrease annually) is defined for both tariffs: connection and monthly fee (see next table).

The assumptions presented are based in the data from the review of the literature. We observe that several studies and deployments [11, 16-20] use the yearly price erosion of between 5% and 15%. The service price assumptions (prices and annual variation) are presented in next table.

TABLE II. SERVICE PROFILE CHARACTERISTICS: RETAIL PRICES

Service Profiles	One time Activation Fees (Connection)	Expected tariff evolution [%]	Monthly Subscription Fees	Expected tariff evolution [%]
Serv. 1	100 €	-10%	20 €/month	-5%
Serv. 2	100 €	-10%	50 €/month	-8%

C. Broadband market forecasts

Next figure shows the penetration forecast for DSL, HFC, fiber and WiMAX for urban areas. In 2020, for the urban area, the expected penetration rates for the fixed technologies are 1.5% for WiMAX, 14.25% for HFC, 22.71% for fiber, and 30.97% for DSL. In the rural area, the expected penetration rate in 2020 is 10.95% for HFC, 23.7% for DSL, 16.41% for fiber, and 7.5% for FWA. We also assume that in rural areas the FWA operator has higher market share than in urban areas.

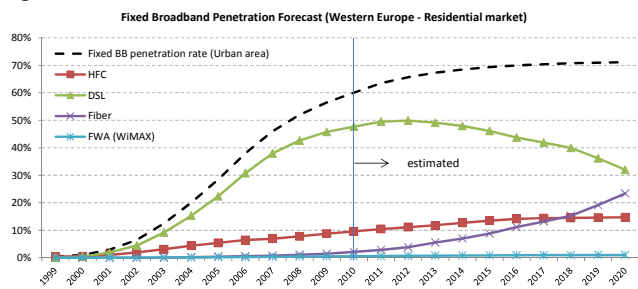


Figure 2. Fixed broadband penetration forecasts (2010-2020)

D. Competitive situation and operators market share

In this section, the market share (relative size) of all the firms (operators) is projected. As competition between operators is different in each area, we estimate the market share for each operator depending on the area, technology, service, and the market.

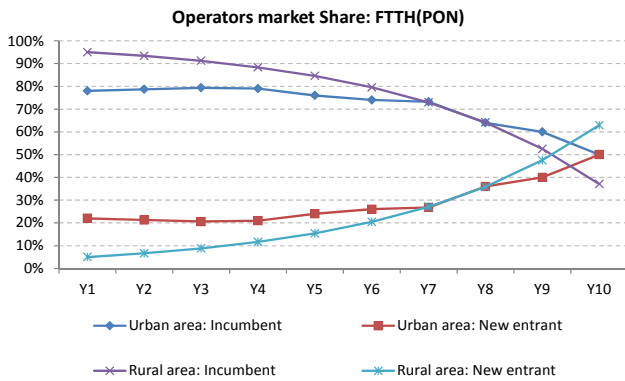


Figure 3. Market share per operator and region (FTTH market)

IV. GAME THEORY FOR COMPETING MODELING

With game theory, we want to understand the effects of the interaction between the different players defined in our business case. In the proposed games, the profit (outcome) of each operator (player) will be dependent not only on their actions, but also on the actions of the other operators in the market.

This section analyzes the impact of the price (retail and wholesale) variations on several output results: players' profit, consumer surplus, welfare, costs, service adoption, and so on. For that, two price-setting games are played (Figure 4. ). Players' profits and NPV are used as the payoff for the players in the games analyzed.

From the several markets presented previously, in this section we present the results for FTTH (PON) market. We assume that two competing FTTH(PON) networks (incumbent operator and new entrant) are deployed in both areas. For the game-theoretic model, it is necessary to change the adoption model used in the techno-economic model in a way that reflects the competition between players (see next Figure 5. ). We assume that the variation of the services prices of one player has an influence on the market share of all players (detailed in the next section).

In our model we also use the Nash equilibrium to find equilibrium. Proposed tools include a module to search the Nash equilibrium in the game. One strategy is a Nash equilibrium when both competitors play their best strategy related to the other strategies selected (players know each other's strategy in advance).

A. Strategies

To analyze the impact of retail and wholesale services price variations, we propose two games (see next figure): (1) analysis the impact of retail price variation on NPV (wholesale prices are defined by regulator); and (2) analysis the impact of retail and wholesale price variations on profit, consumer surplus, welfare, and

retail/wholesale market (different wholesale prices in each region). For the game-theoretic evaluation, the model calculates the NPV and operator's profit for both operators' pricing strategies. Operators' NPVs are used as payoffs for the players in the first and second game, and operators' profits for the third game.

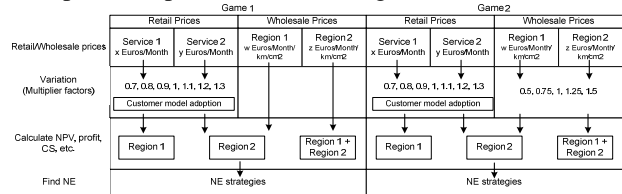


Figure 4. Games proposed

From the several assumptions, we posit: (a) the price that players charge for their services (retail and wholesale) will be varied; (b) the retail price setting will influence the market share of both players (resulting in a higher or lower market share); and (c) consumers only buy a retail service if the price is less than their willingness to pay.

As stated above, we assume that when one player increases/decreases the retail price, the market share of all players will be affected. For example, if one player offers cheaper services, it will be able to capture a higher market share. If a price decreases to nearly zero, everyone will use the service, and the market share of this operator will be close to 100% (total market). On the other hand, if an operator charges a higher price for a service, no one will subscribe to the service from this player, and its market share will decrease to 0%.

B. Adoption model

The impact of varying retail prices on market shares is estimated using the Boltzmann equation.

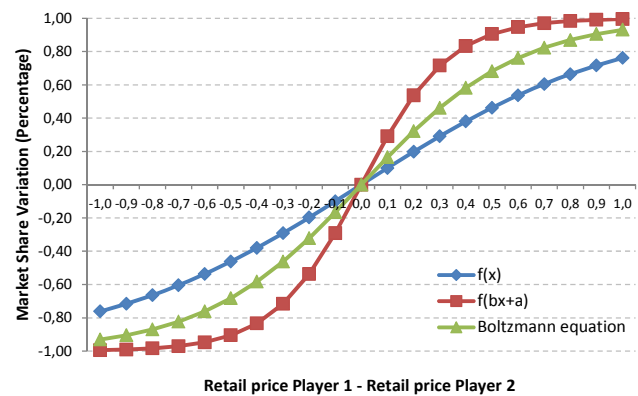


Figure 5. Models to estimate the impact of the price on the service adoption (a=0.4, b=3, dx=0.3)

C. Main assumptions

We assume that the willingness to pay for each retail service is different in both regions. In the urban area (region 1) the maximum amount subscribers would be willing to pay for service 1 is 26 euros and 65 euros for service 2. In the rural area we assume a willingness value of 22 euros for service 1 and 55 euros for service 2 (see TABLE III. ).

TABLE III. WILLINGNESS ASSUMPTIONS

Parameters	Region 1 (Urban area)		Region 2 (Rural area)	
	Serv. 1	Serv. 2	Serv. 1	Serv. 2
Monthly Subscription Fee (Year1)	20€	50€	20€	50€
Willingness Value	26 €	65€	22€	55€
Willingness Multiplier	1.3	1.3	1.1	1.1

For the wholesale infrastructure we assume a duct availability of player 1 100% in the urban area and 90% in the rural area. We also assume that operator 2 (new entrant) leases 100% of the ducts available in the urban area and 100% of the ducts available (operator 1 has only 90% and the remaining 10% are deployed by operator 2) in the rural area from operator 1 (incumbent operator). In the other hand, player 1 leases the 10% remaining (in region 2) from operator 2. The wholesale prices assumptions are: 9.1€(month / km / cm2) for urban area and 7.5€(month / km / cm2) for the rural area. The wholesale infrastructure assumptions and described in next table.

TABLE IV. WHOLESALE INFRASTRUCTURE ASSUMPTIONS

Parameters	Region 1 (Urban)		Region 2 (Rural)	
	Feeder segment	Distribution Segment	Feeder segment	Distribution Segment
<b>Provider 1</b>				
Duct Availability	100%	100%	90%	90%
Wholesale price charged to access ducts (€Km)	€10	€10	€0	€0
Proportion of ducts leased	0%	0%	10%	10%
From operator	-	-	2	2
<b>Provider 2</b>				
Duct Availability	0%	0%	10%	10%
Wholesale price charged to access ducts (€Km)	€10	€10	€0	€0
Proportion of ducts leased	75%	75%	100%	100%
From operator	1	1	1	1

The next sections present the three games results and analyses. In the first game, retail prices vary between tariff multiplier 0.7 and 1.3 (in increments of 0.1). For the second game, retail prices vary between 0.8 and 1.2, and wholesale prices between 0.5 and 1.5.

*Game 1: Impact of retail prices variation on NPV*

In this game we assume that wholesale prices are fixed and that operators choose retail prices to maximize their profit. The impact of varying retail prices on market shares is estimated using the Boltzmann equation (described above). The main goal of this analysis is to determine the optimal retail price strategy for both players. The retail prices vary between -30% and 30%, with increasing steps of 10% (next table).

TABLE V. RETAIL PRICES VARIATION VALUES

Tariff multiplier factor	0.7	0.8	0.9	1	1.1	1.2	1.3
Service 1 price	14	16	18	20	22	24	26
Service 2 price	35	40	45	50	55	60	65

The combination of the two retail prices and seven multiplier factors leads to 49 possible strategies for each player (49x49 matrix) in each region (2,401 total strategies). The next table presents the structure of the combinations and calculated NPV.

TABLE VI. STRUCTURE OF COMBINATIONS AND RESULTS FOR GAME 1

Strategies	Player 1		Player 2		NPV					
	Retail Price		Retail Price		Player 1		Player 2		Tot. P1	Tot. P2
	R1 & R2		R1 & R2		R1	R2	R1	R2	R1 + R2	R1 + R2
	S1	S2	S1	S2						
1	0.7	0.7	0.7	0.7	...	...	...	...	..	...
2	0.7	0.7	0.7	0.8	...	...	...	...	...	...
n	...	...	...	...	...	...	...	...	...	...

The results (payoff matrix) of this game are presented in Table 11- shows the sum of the payoffs of each player in both regions. This table presents the NPV for both players for each possible combination of strategies (one strategy for each player); Nash equilibrium strategies are also identified.

The first two rows represents the prices multiplier factor of player 2 (for services 1 and 2) and the first two columns show the variation (multiplier factors) of player 1. Each cell contains two values: The left value corresponds to the NPV of player 1, and the value on right side corresponds to the NPV of player 2. For example, the first value calculated (15831024€) corresponds to the NPV of player 1 when the strategy of player 1 is to decrease the price of service 1 and service 2 by about 30% (multiplier factor 0.7), and the strategy of player 2 is also to decrease the price of service 1 and service 2 by about 30%.

From these results presented in Table 11 we find three pure NE strategies (black cells) that are described in the next table. The next table shows the NE strategies that maximize the profit of both players. To maximize profit, in the first equilibrium strategy, operator 1 increases retail prices by 10%. Operator 2, in face of the imposed wholesale prices, decreases the price of service 1 and service 2 by 30% and 20%, respectively. A new entrant has to pay the wholesale to the incumbent, but if increase the retail prices their market share will decrease (see model above).

TABLE VII. PURE NE STRATEGIES FOR BOTH REGIONS

Strategy	Player 1 (Incumbent)		Player 2 (New entrant)		NPV K€ Player 1	NPV K€ Player 2
	Retail Serv. 1	Retail Serv. 2	Retail Serv. 1	Retail Serv. 2		
1	1.1 (22€)	1.1 (55€)	0.7 (14€)	0.8 (40€)	9.565	555
2	1.2 (24€)	1.2 (60€)	1.3 (26€)	1.1 (55€)	1.435	23.715
3	1.3 (26€)	1 (50€)	1.2 (24€)	0.7 (35€)	5.015	3.295

The next figure shows the impact of service 2 variation on NPV of both operators. From the analysis of the next figure we can conclude that the variation of retail prices of service 2 has a greater influence in the NPV than the variation of service 1 price. Service 2 price variation can

drop the NPV of operator 1 to negative. On the other hand, operator 2 can turn the NPV positive when the tariff of service 2 increases.

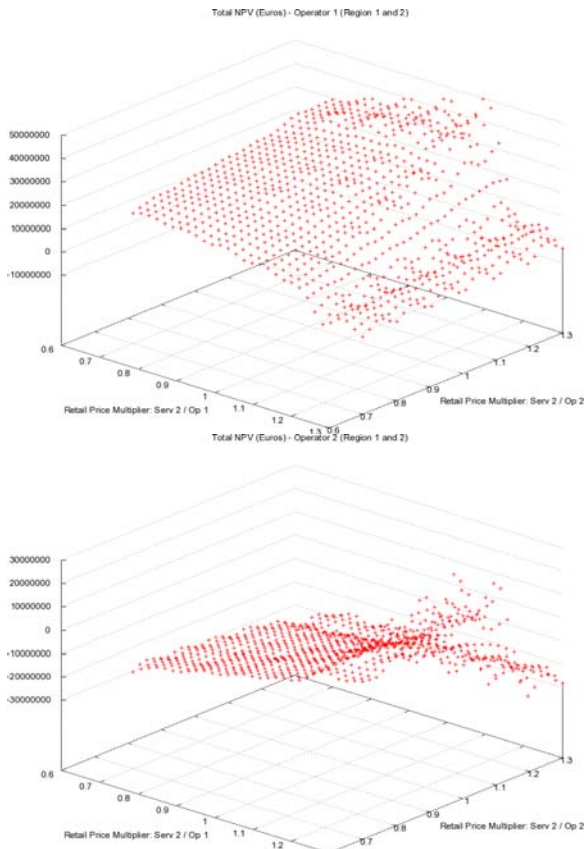


Figure 6. NPV variation: Operator1 and 2/Retail service 2

*Game 2: Impact of retail and wholesale prices variation on NPV*

In this game we assume that wholesale prices are not pre-imposed and we investigate what is the reaction of operators when they can also choose different wholesale prices in different regions (see next table). In game 2 we assume that has the same variation for both regions. Retail prices vary between 0.8 (-20%) and 1.2 (20%) (in increments of 0.1). For wholesale price we assume a variation between 0.5 and 1.5 (in increments of 0.25).

TABLE VIII. RETAIL AND WHOLESALE PRICES VARIATION VALUES FOR GAME 2

Service	Tariff multiplier factor				
	0.8	0.9	1	1.1	1.2
Retail price	0.8	0.9	1	1.1	1.2
Wholesale price	0.5	0.75	1	1.25	1.5

In this context, the combination of the three prices and variation multipliers (described in the previous table) leads to  $625(5^4)$  possible strategies for each player ( $625 \times 625$  matrix) in each region (390625 strategies in both regions) - TABLE IX. shows the structure used.

As the matrix is to bigger, for this game we decide to present the NE strategies (players profit is used as payoff) and the graphs that show the impact of variation in the several results (presented in TABLE XII. ).

TABLE IX. STRUCTURE OF COMBINATIONS AND RESULTS FOR GAME 2

Strategies	Player 1		Wholesale Price		PI 2	Results - NPV					
	Retail Price		Wholesale Price			Player 1		Player 2		Tot P1	Tot P2
	R1 & R2		R1	R2	...	R1	R2	R1	R2	R1 + R2	R1 + R2
	S1	S2	Duct Access		...						
1	0.8	0.8	0.8	0.8	...						
2	0.8	0.8	0.8	0.8	...						
n	...	...	...	...	...						

The analysis of the results finds five NEs strategies. As player 2 do not operates in the wholesale market of region 1, the variation of this price is not significant (see next table). We conclude that, in the business case defined, when operators can charge different retail and wholesale prices, they choose to increase wholesale prices. To maximize profits, operators increase wholesale prices and decrease retail prices. However, the increase in wholesale prices precludes entry of new operators into the market.

TABLE X. PURE NE STRATEGIES IN BOTH REGIONS (GAME 2)

Player 1 (Incumbent operator)				Player 2 (New entrant)				Profit (K€) Player 1	Profit (K€) Player 2
Retail		Wholesale		Retail		Wholesale			
S1	S2	R1	R2	S1	S2	R1	R2		
0.8	0.8	1.25	1.25	0.8	0.8	0.50 0.75 1 1.25 1.5	1.25	22 402	101
0.8	0.9	1.25	1	0.8	0.8	0.50 0.75 1 1.25 1.5	1.25	19 543	6.198

The main results of this game are summarized in the next figures. In the graphs we can see the impact of retail prices (Figure 7. ) and wholesale prices (Figure 8. ) on players profit. We can verify that both prices can turn profit positive/negative.

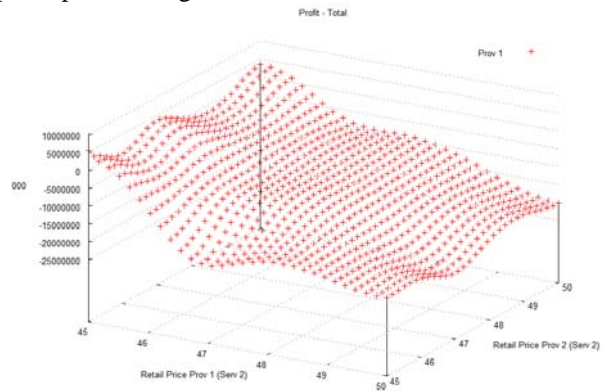


Figure 7. Profit variation: Retail service 2

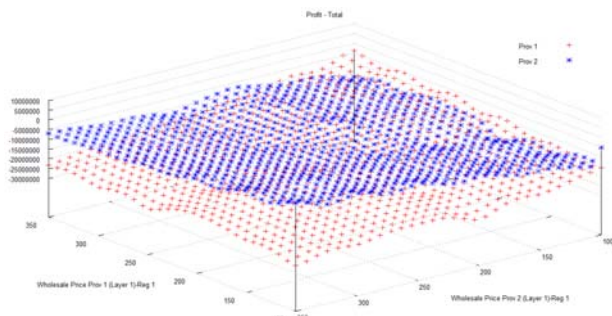


Figure 8. Profit variation: Wholesale service

As expected, consumer surplus decreases with the increase of prices (Figure 9. ). As also predictable and modeled above the impact of retail prices variation has higher influence in the market share of competitors (see Figure 10. ).

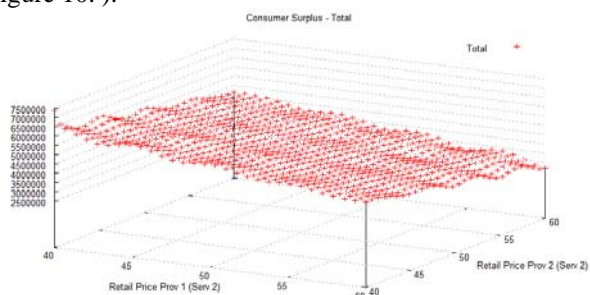


Figure 9. Consumer Surplus variation

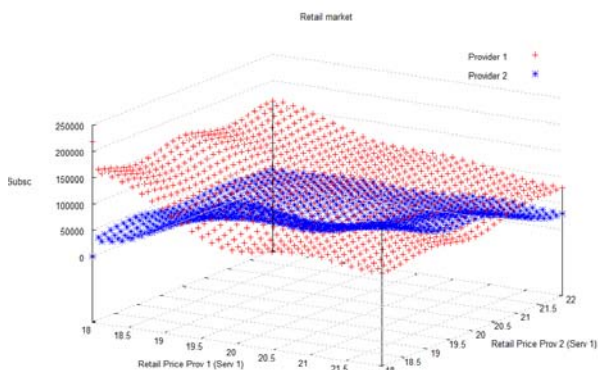


Figure 10. Retail market variation

The comparison of the two games above shows that when the regulator defines wholesale prices, operators increase retail prices to maximize profit. However, when wholesale prices are not regulated, operators maximize profit by decreasing retail prices and increasing wholesale prices. However, without regulation, the higher wholesale prices will limit the entrance of new competitors.

V. CONCLUSIONS

The European Commission argues that infrastructure-based competition is the best and fastest way for broadband development. The arguments are that infrastructure-based competition provides efficiency incentives to operators, reduces prices, increase penetration, stimulates innovation, and so on. On the other hand, service-based competition implies that the

new entrants (alternative operators) are dependent on the incumbent. However, because of the high costs of deploying infrastructures (especially trenching and ducting), service competition has been used as a substitute or complement to infrastructure competition. In regions with lower numbers of existing access infrastructures, new entrants are obligated to build their own infrastructure. In this way, infrastructure sharing can stimulate the construction of new access infrastructures that can be leased to other operators.

The results of this investigation show that the sharing of passive infrastructures (e.g., ducts, trenching, base station sites, antenna masts, etc.) is a viable strategy, particularly in the context of new building (in scenarios with developed access infrastructure). When an operator deploys an access network, the access to existing civil engineering significantly reduces the investment. There are strong arguments to be made for allowing infrastructure sharing.

In this context, regulators must guarantee new entrant operators access to civil engineering; this will stimulate investment in new networks. The reduction of the barriers to new infrastructure investment by opening passive existing infrastructure would be key in the future. This study has shown that in rural areas, characterized by a small number of developed access infrastructure, the access to civil engineering does not make the scenario economically viable for the operator.

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TABLE XI. GAME 1 RESULTS - SUMMARY

Price S1		Player 2 (New entrant) strategies														
		0.70				1.20				1.30						
		0.70	0.80	1.30	...	0.70	1.20	1.30	...	0.70	1.30	1.10	...			
0.70	0.7	15831024	-18582287	18183087	-19363781	28936533	-30770826	17496173	-19795217	30601681	-31983756	18113915	-20768607	26428738	-25938555	
	0.8	14472132	-14083788	1956293	-16693988	34299456	-30618352	16137281	-15296719	35964604	-31831283	16755023	-16270109	29303695	-23304216	
	0.9	12185918	-9824136	17582209	-11580297	39612688	-30427158	13851067	-11037066	41277837	-31640088	14468808	-12010456	31362952	-20234582	
	1	9338223	-5994009	14713466	-6738115	44866176	-30195243	11003372	-7206939	46531325	-31408173	11621113	-8180329	32572789	-16820707	
	1.1	6280874	-2707019	11341982	-2384199	50053039	-29925006	7946023	-3919949	51718187	-31137937	8563765	-4893339	33014991	-13215411	
	1.2	-7258790	6298197	-6933535	11368737	-4619767	29519466	-5593641	5085267	-2954619	-30732396	-4975899	4111877	-3280895	23715662	
	1.3	-7216198	6452291	-6935595	11588571	-4402383	-2920272	-5551049	5239361	-2737234	-30415002	-4933308	4265971	-3375080	24186698	
	...	0.7	14054700	-15642494	16406763	-16423988	27160208	-27831033	19064485	-17729134	32169993	-29917673	21886956	-20768607	30201779	-25938555
	...	0.8	12695808	-11143995	17779969	-13754195	32523131	-27678559	17705593	-13230635	37532916	-29765199	20528064	-16270109	33076737	-23304216
	...	0.9	10409594	-6884343	15805885	-8640503	37836364	-27487364	15419379	-8970983	42864148	-29574004	18241850	-12010456	35135994	-20234582
1.10	1	7561899	-3054215	12337141	-3798322	43089852	-27255449	12571684	-5140855	48099637	-29342089	15394155	-8180329	36350330	-16820707	
	1.1	4504550	232775	9565657	5555555	48276714	-26985213	9514335	-1853865	53286499	-29071853	12336806	-4893339	36789033	-13215411	
	1.2	-9035114	9237991	-8709859	14308530	-6396091	-26579672	-4025329	7151351	-1386307	28666312	-1160266	4265971	492146	23715662	
	1.3	-8992522	9382085	-8711919	14528364	-6178707	-26262279	-3982737	7305445	-1168922	28348918	-1160266	4265971	397962	24186698	
	...	0.7	13511514	-15140682	15863576	-15922176	26617022	-27329221	19138806	-7069339	32244114	-29257878	22830217	-20768607	31145040	-25938555
	...	0.8	12152622	-10642183	17236783	-13252383	31979945	-27167447	17779714	-12570840	37670737	-29105404	21471325	-16270109	34019997	-23304216
	...	0.9	9866407	-6382531	15262698	-8138692	37291777	-26985553	15493499	-8311188	42920269	-28914210	19185110	-12010456	36079254	-20234582
	...	1	7018712	-2524404	12393955	-3296511	42546666	-26753638	12645805	-4481060	48173758	-28682294	16337416	-8180329	37293591	-16820707
	...	1.1	3961364	734586	9022471	1057406	47733528	-26483401	9589456	-1194070	53360620	-28412058	13280067	-4893339	37731293	-13215411
	...	1.2	-9578300	9739802	-9253046	14810342	-6939278	-26077861	-3951208	7811146	-1312186	28006517	-2959597	4111877	1435407	23715662
1.20	1.3	9535709	9893896	-9255106	15030176	-6721894	-25766047	-3908617	7965240	-1094801	-27689124	-217006	4265971	1341222	24186698	
	0.7	11507889	-13885393	13859952	-14666887	24613397	-26073922	11507889	-9292724	24613397	-21481263	11492251	-20732168	19807074	-12590216	
	0.8	10148997	-9386894	15233158	-11997094	29976320	-25921458	10148997	-4794225	29976320	-21328789	10133359	-16233669	22682023	-23267776	
	0.9	7862782	-5127424	13259073	-6883402	35289552	-25730264	7862782	-534573	35289552	-21137595	7847145	-11974017	24174289	-20198142	
	1	5015088	-1297114	10390330	-2041221	40543041	-25498349	5015088	-3295555	40543041	-20905799	4999450	-8143889	29595625	-16784268	
	1.1	1957739	1989876	7018846	2312695	45729903	-25228112	1957739	6582545	45729903	-20635443	1942101	-4856899	26393328	-13178971	
	1.2	-11581925	10995092	-11256671	16065631	-8942903	-24822571	-11581925	15587761	-8942903	-20229902	-11597563	4148317	-9902559	23752102	
	1.3	-11539334	11149186	-11258730	16285465	-8725518	-24505178	-11539334	15741855	-8725518	-19912509	-11554971	4302411	-9996743	24223138	

TABLE XII. GAME 2 RESULTS - SUMMARY

R Price S1		Player 2 strategies												
		0.50				1.00				0.75				
		0.50	0.75	1.00	1.25	1.50	0.50	0.75	1.00	1.25	1.50	0.50	0.75	
0.5	0.5	20981654	1704052	20954077	1728871	20926500	1753691	20989223	1778510	20871345	1803330	20981654	1704052	
	0.75	21232678	1425137	21205100	1449956	21177523	1474776	21149946	1499595	21122369	1524415	21232678	1425137	
	1	21483701	1146222	21456124	1171041	21428547	1195861	21400966	1220680	21373392	1245500	21483701	1146222	
	1.25	21734724	867307	21707147	892127	21679570	916946	21651993	941766	21624416	966585	21734724	867307	
	1.5	21985748	588392	21958171	613212	21930593	638031	21903016	662851	21875439	686780	21985748	588392	
	0.75	0.5	21113446	1576216	21085869	1582436	21058292	1607255	21030715	1632075	21003137	1656894	21113446	1576216
		0.75	21364470	1278701	21336892	1303521	21309315	1328340	21281738	1353160	21254161	1377979	21364470	1278701
		1	21615493	999786	21587916	1024606	21560339	1049425	21532761	1074245	21505184	1099064	21615493	999786
		1.25	21866516	720872	21838939	745691	21811362	770511	21783785	795330	21756207	820150	21866516	720872
		1.5	22117540	441957	22089963	466776	22062385	491596	22034808	516415	22007231	541235	22117540	441957
1		0.5	21245238	1411181	21217661	1436000	21190084	1460820	21162506	1485639	21134929	1510459	21245238	1411181
		0.75	21496261	1132266	21468684	1157085	21441107	1181905	21413530	1206724	21385953	1231544	21496261	1132266
		1	21747285	853351	21719708	878171	21692130	902990	21664553	927809	21636976	952629	21747285	853351
		1.25	21998308	574436	21970731	599256	21943154	624075	21915577	648895	21887999	673714	21998308	574436
		1.5	22249332	295521	22221754	320341	22194177	345160	22166600	369980	22139023	394799	22249332	295521
	1.25	0.5	21377030	1264745	21349453	1289565	21321876	1314384	21294298	1339204	21266721	1364023	21377030	1264745
		0.75	21628053	985830	21600476	1010650	21572899	1035469	21545322	1060289	21517745	1085108	21628053	985830
		1	21879077	706916	21851500	731735	21823922	756555	21796345	781374	21768768	806194	21879077	706916
		1.25	22130100	428001	22102523	452820	22074946	477640	22047369	502459	22019791	527279	22130100	428001
		1.5	22381124	149086	22353546	173905	22325969	198725	22298392	223544	22270815	248364	22381124	149086
1.5		0.5	21508822	1118310	21481245	1143129	21453668	1167949	21426090	1192768	21398513	1217588	21508822	1118310
		0.75	21759845	839395	21732268	864214	21704691	889034	21677114	913853	21649536	938673	21759845	839395
		1	22010869	560480	21983291	585300	21955714	610119	21928137	634939	21900560	659578	22010869	560480
		1.25	22261892	281565	22234315	306385	22206738	331204	22179161	356023	22151583	380843	22261892	281565
		1.5	22512915	2650	22485338	27470	22457761	52289	22430184	77109	22402607	101928	22512915	2650