ANGULAR MOMENTUM TRANSPORT IN THE SUN THROUGH MERIDIAN CIRCULATION

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ABSTRACT

We investigate the transport of angular momentum in the radiative interior of the Sun through the mechanism described by Zahn (1992), namely the meridian circulation driven by the solar wind and the turbulence caused by the shear instabilities due to the differential rotation.

The dynamical equations are coupled to the evolutionary code CESAM (Morel 1996) and we calculate the resulting internal rotation profile. The advective nature of the meridian circulation leads to a transport which is even less efficient than that produced by the rotation-induced turbulent diffusion considered by Pinsonneault et al. 1989. Both fail to extract enough angular momentum from the deep interior, and hence we conclude that another mechanism must be present to enforce the low differential rotation observed through helioseismology.

1. Introduction and Governing Equations

The evolution of the rotation profile in stellar interiors through meridian circulation and turbulent diffusion was established by Zahn (1992). Previously the angular momentum transport has been modeled as a diffusion process (Endal & Sofia 1978; Pinsonneault et al. 1989), and the circulation velocity entering in the diffusion coefficient was evaluated assuming that the star rotates like a solid body. In our treatment, the angular momentum is advected by the circulation, and the outcome is that the angular velocity obeys to a "hyper-diffusion" equation.

The main assumption of both treatments is the presence of a highly anisotropic turbulence, with a horizontal transport leading to a state of shellular rotation. When the suitable horizontal averages are performed, one obtains the following equation for the vertical transport of angular momentum:

$$\rho\frac{d}{dt}[r^2\Omega] = \frac{1}{5r^2}\frac{\partial}{\partial r}\left[\rho r^4 U(r)\Omega\right] + \frac{1}{r^2}\frac{\partial}{\partial r}\left[\rho\nu_v r^4\frac{\partial\Omega}{\partial r}\right], \tag{1}$$

where the vertical component of the meridian velocity $U(r)$ is given by

$$U(r) = \frac{L}{Mg}\left(\frac{P}{C_P\rho T}\right)\frac{1}{\nabla_{ad}-\nabla}\left[E_\Omega(r) + E_\mu(r)\right]. \tag{2}$$

We have used the classical notations for all physical parameters, and the expressions for E_Ω and E_μ are given in Zahn (1992). E_Ω depends mainly on the rotation profile: it comprises the classical Eddington-Sweet term (Sweet 1950) and a baroclinic component which depends on the angular velocity gradient, whereas E_μ involves the relative amplitude of the horizontal variation of molecular weight.

We consider the shear instability as the only source of vertical turbulence, and ν_v is then given by

$$\nu_v = \frac{4}{15}Ri_c K\left(\frac{r}{N}\frac{\partial\Omega}{\partial r}\right)^2, \tag{3}$$

with K being the thermal diffusivity, N the Brunt-Väisälä frequency and $Ri_c \approx 1/4$ the critical Richardson number.

2. Numerical Procedures and Boundary Conditions

The stellar evolutionary code CESAM (Morel 1996) was used to compute a sequence of solar main-sequence models. The nonlinear 4^{th} order partial differential equation in r which governs the angular momentum transport was decomposed in four first order equations and solved by finite differences. We used intermediate time steps for the angular momentum, and this is particularly convenient at the beginning of the evolution, when the spin-down time is particularly short. All spatial derivatives were expressed by second-order accuracy centered differences and time integrations were semi-implicit. The difference equations were solved at each time step by a quadratically convergent Newton-Raphson iteration. The typical grid size was 300 points.

Four boundary conditions are required, with two concerning the flux of angular momentum. Since there is no source at the center, $\mathcal{F}_J = 0$. At the surface the flux of angular momentum matches that which is carried away by the solar wind, and which is usually described by a power law in Ω, $\mathcal{F}_J = -W_k\Omega^n$. The wind constant W_k is calibrated to yield the equatorial surface velocity which is observed in the present Sun ($\approx$ 2 km/s). The wind exponent n is adjusted so that the surface velocity decreases as $t^{-1/2}$, to comply with the observations. We further impose $\partial\Omega/\partial r = 0$ at the center of the star and at the bottom of the convective zone, to ensure continuity of the temperature, and we assume that the convective zone rotates like a solid body.

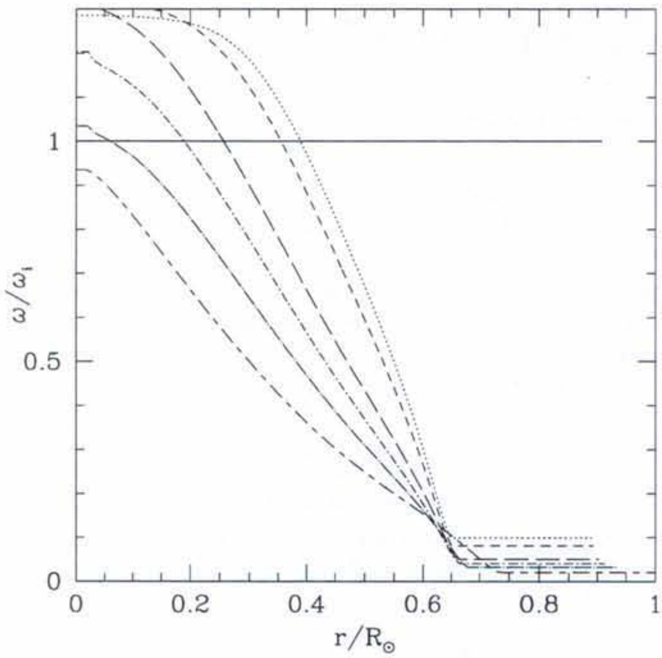

Figure 1: Variation with depth of the angular velocity (normalized with the angular velocity which corresponds to an initial equatorial velocity of 100 km/s). The profiles are shown for increasing age: ZAMS, 50, 100, 500, 1000, 2000, 4550 Myrs.

The $E\mu$ component of the meridian circulation has been neglected here, and this seems to be a reasonable approximation early in the evolution, before the molecular weight gradients are built either by nuclear reactions in the core and by gravitational settling at the bottom of the convective zone.

3. Results

If the Sun were not losing angular momentum through its wind, the advective and diffusive transport would tend to compensate each other, and the rotation profile would evolve to a stationary state. During this adjustment phase, the meridian circulation would transport angular momentum towards the interior, building a differential rotation which does not exceed 10-20 %.

In the real case, the Sun loses angular momentum, which is transported towards the surface by the combined action of the meridian circulation and the turbulent diffusivity. The evolution of the rotation profile is represented in Fig. 1.

We start with an initial surface velocity of 100 km/s and with a ZAMS structure model. The torque applied to the Sun is very efficient in slowing down the convection zone, and the motions induced in the radiative interior begin to carry angular momentum upwards. As evolution proceeds, the profile of angular velocity changes slowly, indicating that a fraction of the angular momentum is extracted from the core. On closer inspection, the advection through the meridian flow dominates over the diffusive transport provided by the shear instability, except in a very thin region adjacent to the convective zone. However, as can be seen in Fig. 1, these transports leave behind a steep gradient of angular velocity, implying that they are unable to extract sufficient angular momentum from the deep interior to flatten the rotation profile.

The same conclusion was reached by Pinsonneault et al. (1989) and more recently by Chaboyer et al. (1995), using their diffusive description of the momentum transport.

We performed similar calculations starting the evolution from PMS solar models, and their results differed only slightly.

4. Discussion and Conclusions

Our calculations clearly demonstrate that even with a better description of the meridian circulation, as that proposed by Zahn (1992), the predicted solar rotation profile disagrees with that deduced from helioseismology. This leads us to conclude that although meridian circulation is certainly present in stars, and does probably play an important role in the transport of chemical elements, it is not the mechanism responsible for the flat rotation profile detected in the radiative interior of the Sun. For this reason we have examined another transport process, namely that by internal gravity waves, and the results will be reported in a forthcoming paper (Zahn, Talon & Matias 97) (see also the poster presented at this symposium).

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