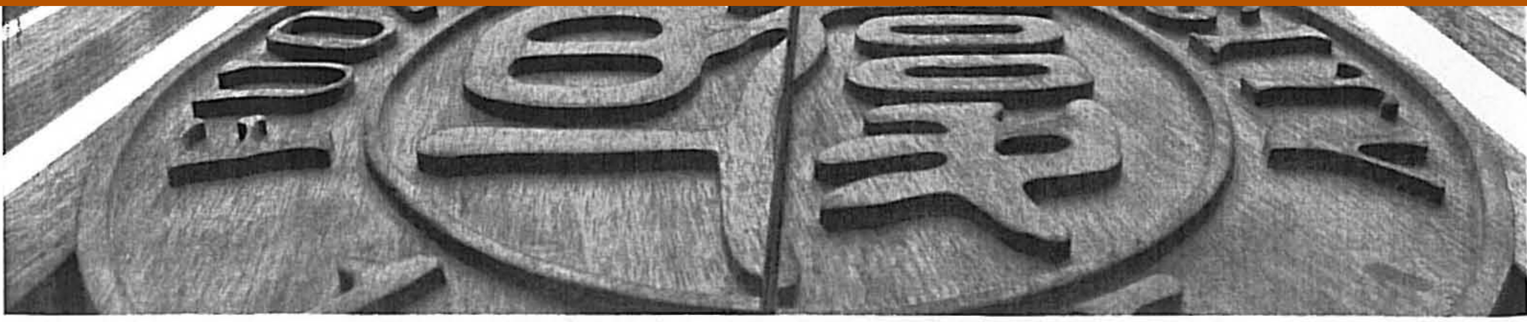

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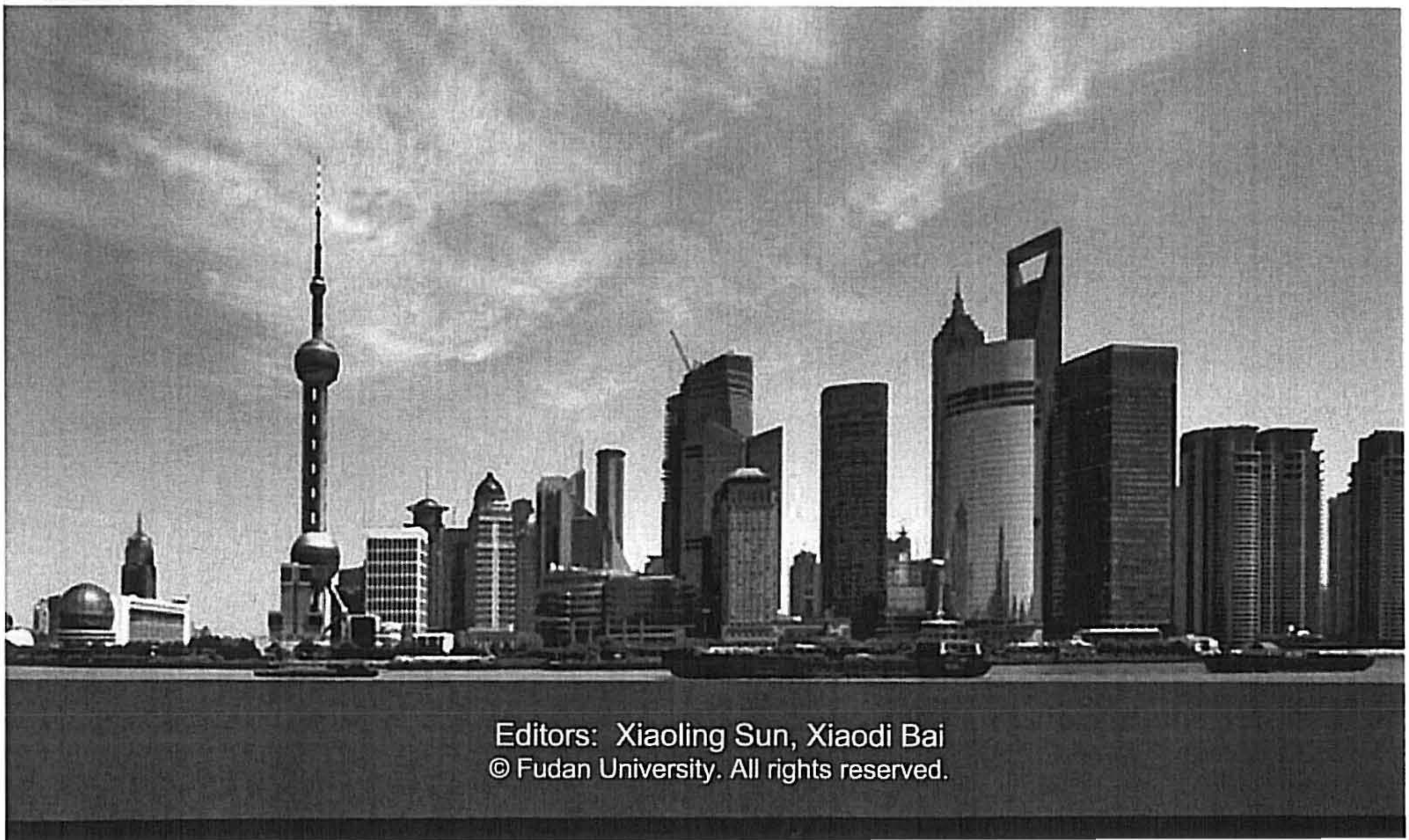

Editors: Xiaoling Sun, Xiaodi Bai

Experiences with reduction method to solve semi-infinite programming problems

Ana I. Pereira
Department of Mathematics - ESTiG, Polytechnic Institute of Braganca, Campus de Sta Apolónia, 5301-854 Braganca, Portugal
apereira@ipb.pt

Edite M.G.P. Fernandes
Department of Production and Systems, University of Minho, Campus de Gualtar, 4710-057 Braga, Portugal
emgpf@dps.uminho.pt

Abstract

In this talk, some variants of reduction-type method combined with a line search filter method to solve nonlinear semi-infinite programming problems are presented. We use the stretched simulated annealing method and the branch and bound technique to compute the maximizers of the constraint. The filter method is used as an alternative to merit functions to promote convergence from poor starting points.

Keywords: Semi-infinite programming. Reduction method, Line search filter method.

1 Introduction

The semi-infinite programming (SIP) problem is considered to be of the form

$$\min f(x) \text{ subject to } g(x,t) \leq 0, \text{ for every } t \in T \tag{1}$$

where $T \subseteq \mathbb{R}^m$ is a nonempty set defined by $T = \{t \in \mathbb{R}^m : a \leq t \leq b\}$. Here, we assume that the set T does not depend on x. The nonlinear functions $f : \mathbb{R}^n \to \mathbb{R}$ and $g : \mathbb{R}^n \times T \to \mathbb{R}$ are twice continuously differentiable with respect to x and g is continuously differentiable functions with respect to t.

There are many problems in the engineering area that can be formulated as SIP problems. For a review of some applications, the reader is referred to [9, 5, 10].

The numerical methods that are mostly used to solve SIP problems generate a sequence of finite problems. Some examples can be found in [9, 2, 3, 4, 8, 10].

This work aims to describe a reduction method for SIP. Conceptually, the method is based on the local reduction theory. The focus of our proposal is to compare the perfomance of some methods in global programming theory: the simulated annealing method and the branch and bound method. This work comes in the sequence of a previous reduction-type method presented in [6].

2 Reduction method

A reduction approach, based on the local reduction theory, describes the feasible set of the SIP locally by finitely many inequality constraints. Thus, the SIP problem can be locally reduced to a finite one (at least conceptually, see [4]). Given a feasible point $x_k \in \mathbb{R}^n$, consider the so-called lower level problem $O(x_k)$:

$$\max_{t \in T} g(x_k, t), \tag{2}$$

where $T_k = \{t_1, \ldots, t_{L_k}\}$ is the set of its local solutions satisfying the following condition

$$|g(\bar{x}, t_l) - g^*| \leq \delta^{ML}, \; l = 1, \ldots, L_k, \tag{3}$$

where L_k represents the cardinality of T_k, δ^{ML} is a positive constant and g^* is the global solution value of (2).

Under some conditions, it is possible to replace the infinite set of constraints by a finite set that is locally sufficient to define the feasible region. Thus the problem (1) is locally equivalent to the so-called reduced optimization problem

$$\min_{x \in U_k} f(x) \text{ subject to } g_l(x) \equiv g(x, t_l(x)) \leq 0, \, l = 1, \ldots, L_k. \quad (4)$$

where U_k is a open neighborhood of x_k.

A reduction method then emerges when any method for finite programming is applied to solve the locally reduced problem (4). Thus, at each iteration k, the main procedures of the algorithm are:

1. based on an approximation to the SIP problem, x_k, compute the set T_k, solving problem (2), with condition (3);
2. based on the set T_k, implement at most $i^{\max}$ iterations to get an approximation $x_{k,i}$, by solving the reduced problem (4);
3. use a globalization technique to compute a new approximation x_{k+1} that improves significantly over x_k;
4. use termination criteria to decide if the iterative process should terminate.

The remaining part of this work presents our proposals for the four steps of a global reduction method for SIP. An algorithm to compute the set T_k is known in the literature as a multi-local procedure. Our proposals are the stretched simulated annealing method and branch and bound method. To solve the reduced problem (4) an interior point method is proposed. Finally, convergence of the overall reduction method to a SIP solution is encouraged by implementing a filter line search technique.

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