REDUCTION METHOD WITH SIMULATED ANNEALING FOR SEMI-INFINITE PROGRAMMING

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Semi-infinite programming (SIP) problems are characterized by a finite number of variables and an infinite number of constraints. The class of the reduction methods is based on the idea that, under certain conditions, it is possible to replace the infinite constraints by a finite set of constraints, that are locally sufficient to define the feasible region of the SIP problem.

We propose a new reduction method based on a simulated annealing algorithm for multi-local optimization and the penalty method for solving the finite problem.

SIP PROBLEM

The semi-infinite programming problem can be defined as: minimize $f(x)$ subject to $g(x,t) \le 0$, for $x \in IR^n$ and $\forall t \in T = \{t \in IR^m : h_j(t) \ge 0, \text{ for } j = 1,\dots,r\}$.

REDUCTION METHOD

The reduction method consists of two phases. First we must find all global maxima of the constraint function, for a fixed x^k

$$\text{global max } g(t) \equiv g(x^k, t),\ \forall t \in T. \qquad (1)$$

Let T^* be the optimal set containing all global maxima.

In the second phase, we solve a finite constrained optimization problem

$$\min f(x) \text{ subject to } g(x, t^i) \le 0,\ \forall t^i \in T^*.$$

This problem is solved by a penalty method based on the exponential function.

To find all global solutions of the problem (1), we use a simulated annealing algorithm [1] combined with a stretching technique [2].

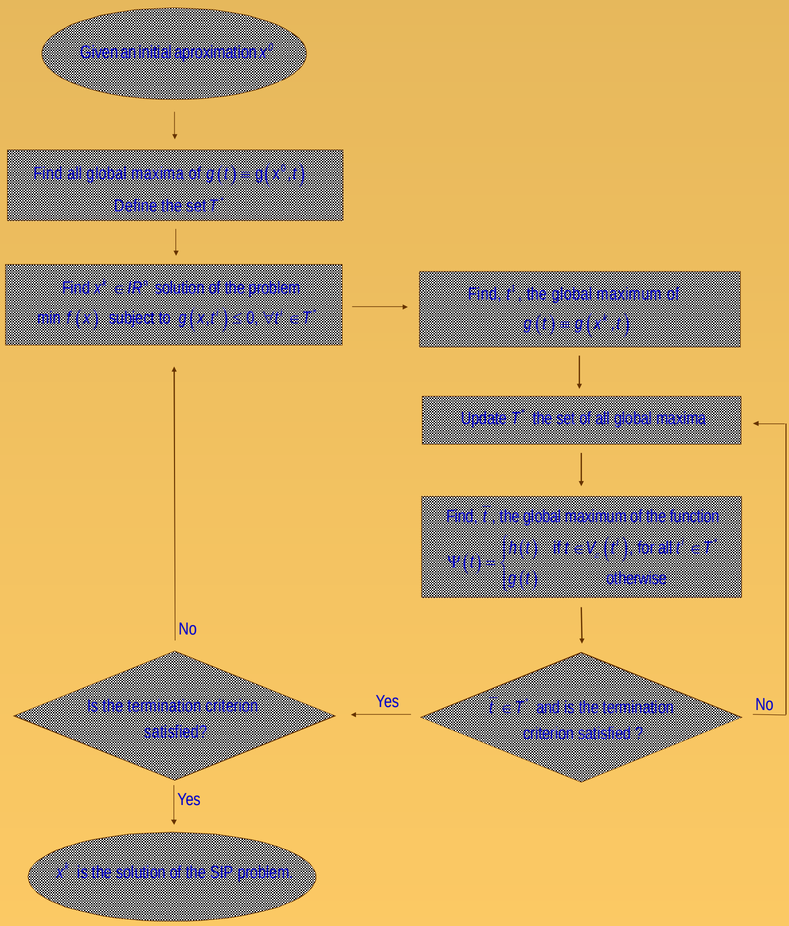
The main idea of this multi-local algorithm is to replace the constrained problem (1) by

$$\max \Psi(t) = \begin{cases} h(t) & \text{if } t \in V_\varepsilon(t^i), \text{ for all } t^i \in T^* \\ g(t) & \text{otherwise} \end{cases}$$

where $h(t) = w(t) - \dfrac{\delta_2\left[\operatorname{sgn}\left(g(t^i) - g(t)\right) + 1\right]}{2\tanh\left(\eta\left(w(t^i) - w(t)\right)\right)}$ and

$w(t) = g(t) - \dfrac{\delta_1}{2}\left[\operatorname{sgn}\left(g(t^i) - g(t)\right) + 1\right], \delta_1, \delta_2, \eta > 0.$

If a new global maximum is found, then it is added to the optimal set T^*. The multi-local algorithm stops when the optimal set does not change for a fixed number of iterations.



PRELIMINARY NUMERICAL RESULTS

Problem	n	m	k_{RM}	k_{ML}	k_{PM}	$\bar{k}_{RM}$	$\bar{k}_{ML}$
Problem 2	2	1	1	4	7	8	10
Problem 3	3	1	2	4	6	11	23
Problem 7	3	2	4	4	8	9	14

These numerical results were obtained with problems 2, 3 and 7 of [3]. k_{RM}, k_{ML} and k_{PM} represent the number of iterations needed by reduction method algorithm, multi-local optimization and penalty method, respectively, of presented algorithm. $\bar{k}_{RM}$ and $\bar{k}_{ML}$ represent the number of iterations registered in [3].

REFERENCES

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