

Hierarchization Process by Possibilistic Fuzzy Clustering of Fuzzy Rules

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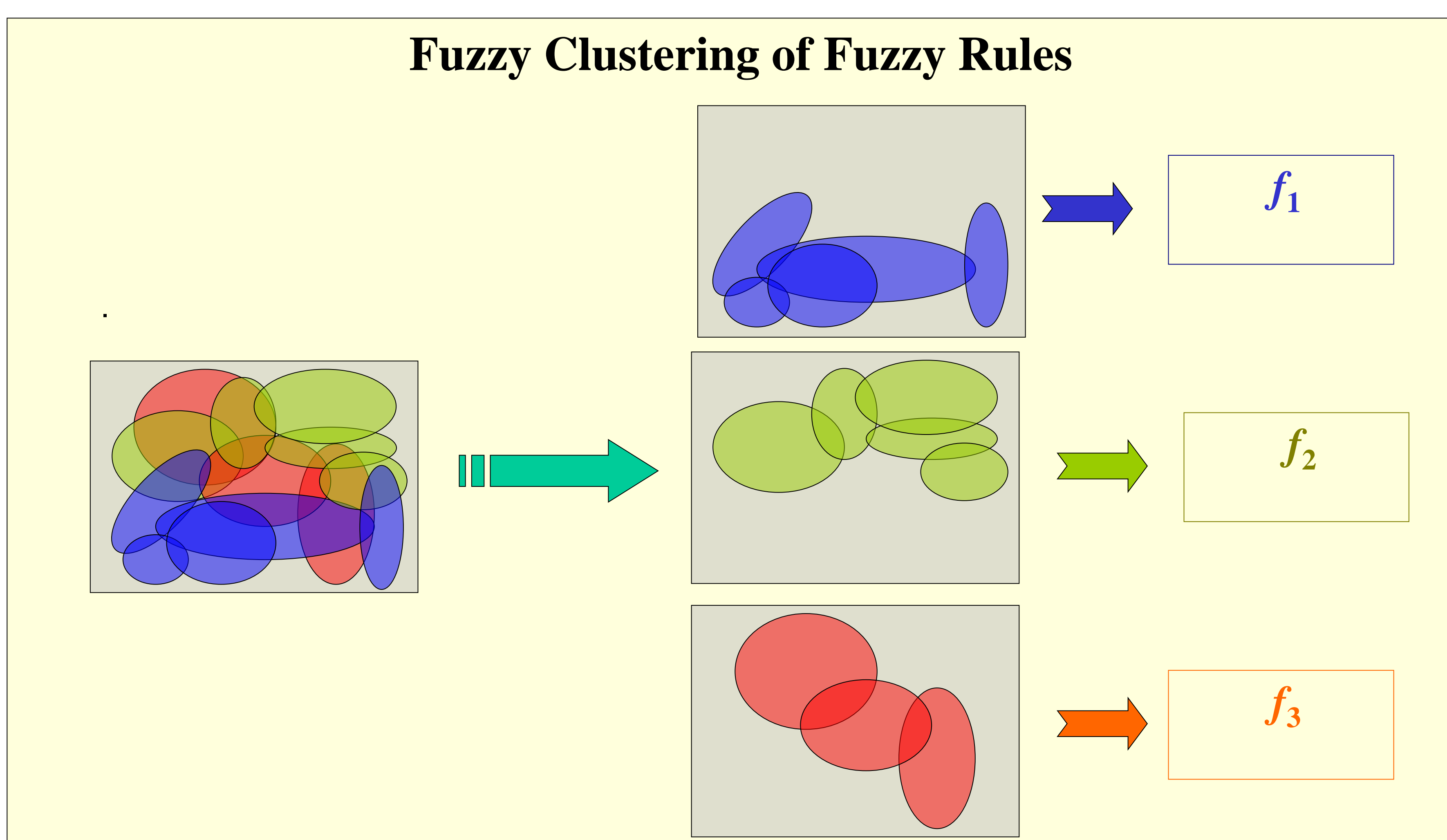
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Abstract

This paper presents a possibilistic fuzzy clustering algorithm that is applied to a multidimensional fuzzy set or fuzzy rules. This method can be used to decompose the fuzzy system into an hierarchical structure. The methodology presented leads to a fuzzy partition of the fuzzy rules, one for each cluster, which corresponds to a new set of fuzzy sub-systems. This technique is tested to organize the fuzzy model into a new and more comprehensive structure.



Fuzzy Clustering algorithms of fuzzy rules – FCAFR

Step 1– Sets: $X = \{x_1, \dots, x_n\}$ of points; $\mathfrak{F} = \{R_1, \dots, R_M\}$ of rules with relevance $\mathfrak{R}_l(x_k)$. c is the number of clusters. Initialize $U(0)$.

Step 2– On the r^{th} iteration, with $r = 0, 1, 2, \dots$, compute:

$$v_i^{(r)} = \frac{\sum_{l=1}^M u_{il}^m \cdot \sum_{k=1}^n (\mathfrak{R}_l(x_k) \cdot x_k)}{\sum_{l=1}^M u_{il}^m \cdot \sum_{k=1}^n \mathfrak{R}_l^m(x_k)}$$

Step 3– Compute the new partition matrix $U(r+1)$:

$$u_{il}^{(r+1)} = \frac{1}{1 + \left(\sum_{k=1}^n \mathfrak{R}_l^m(x_k) \cdot D_{ilk} / \eta_i \right)^{1/(m-1)}}$$

where $D_{ilk} = \left\| \mathbf{x}_k - \mathbf{v}_i \right\|^2 + \alpha_i \left\| \theta_l - h_i - \sum_{j=1}^e w_{ij} z_j(\mathbf{x}_k) \right\|^2$.

Step 4 – Compute the clusters offsets, $h^{(r+1)}$:

$$h_i^{(r+1)} = \frac{\sum_{l=1}^M u_{il}^m \left(\bar{\mathfrak{R}}_l \theta_l - \sum_{j=1}^e w_{ij}^{(r)} \sum_{k=1}^n \mathfrak{R}_l(x_k) z_j(x_k) \right)}{\sum_{l=1}^M u_{il}^m \bar{\mathfrak{R}}_l}, \quad i=1, \dots, c \quad (6)$$

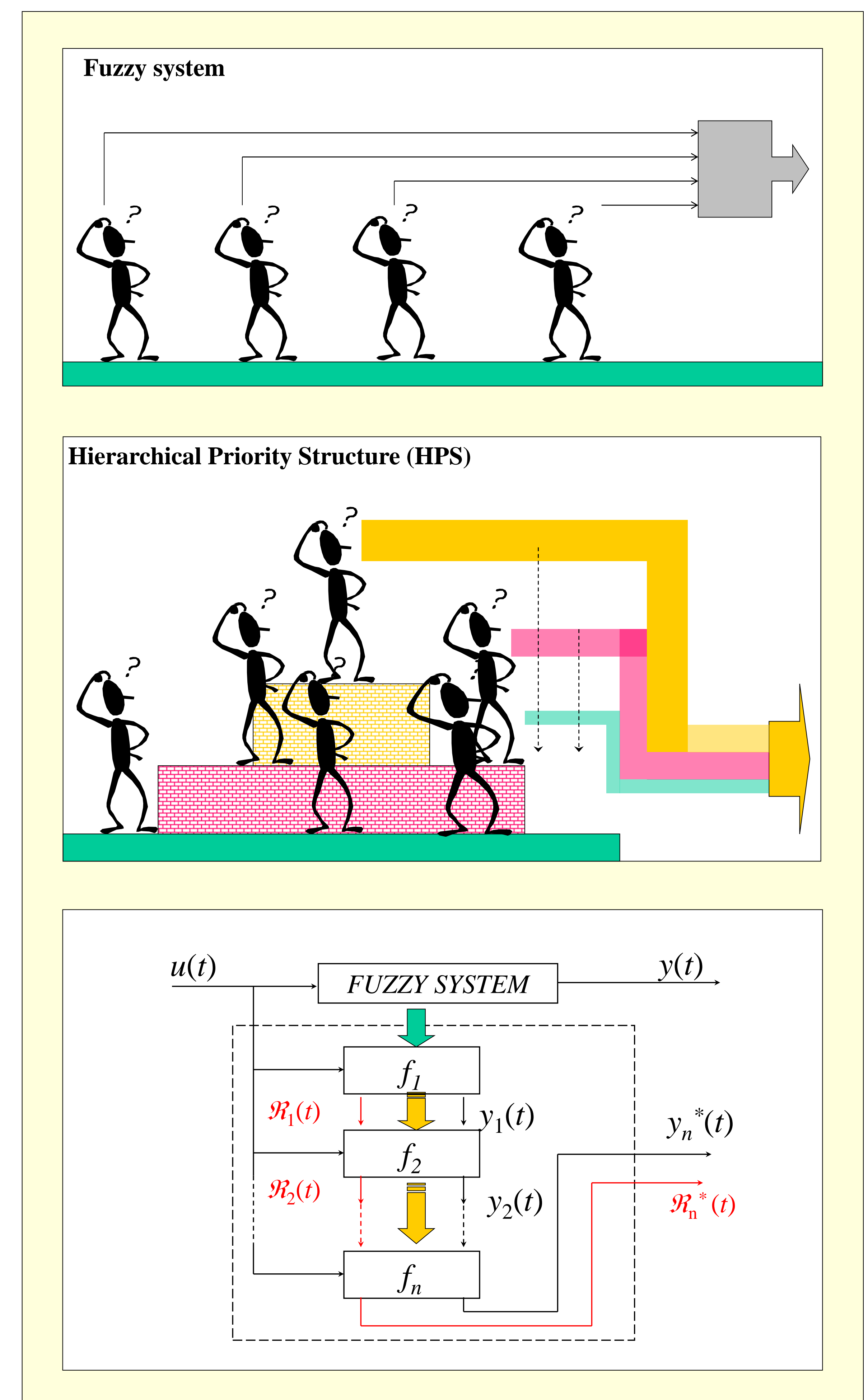
Step 5 – Compute the new partition matrix $W(r+1)$:

For each cluster i minimizes:

$$W_i^{(r+1)}: \text{Min}_{W_i} J = \text{Min}_{W_i} \sum_{k=1}^n \sum_{l=1}^M \left\| \theta_l - h_i^{(r+1)} - \sum_{j=1}^e w_{ij}^{(r)} z_j(x_k) \right\|_{Q^{(i)}}^2 \quad (7)$$

where $Q^{(i)}$ is the weigh matrix of the i^{th} cluster problem with non-null elements $Q_{lk}^{(i)} = u_{il}^m \mathfrak{R}_l(x_k)$.

Step 6– If $\|U(r+1) - U(r)\| < \varepsilon$ then the process ends. Otherwise let $r = r + 1$ and go to step 2.



Histogram + PFCFR Segmentation

