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TECHNIQUES IN NUMERICAL DIFFERENTIATION OF EXPERIMENTALLY NOISY DATA

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SYNOPSIS

A comparison of different methodologies for spatial differentiation of experimental noisy data is presented. The transient displacement fields resulting from bending wave propagation on aluminum plate were measured with pulse TV-holography. A new method to perform the numerical spatial differentiation and a strategy for dealing with the border problem are presented. The method here presented results for the third order derivative and showed to be superior to the other methods tested. A good agreement between the new method and the Finite Elements Method was achieved.

INTRODUCTION

The spatial derivative of the displacements up to the third order is essential for stress calculation on thin plate (Timoshenko and Woinowsky-Krieger 1959). The sensors available can only permit discrete and localized curvature measurement (Wahyu Lestari 2005). The fact of these sensors being intrusive and expensive limited their use to some specific applications. Hence, an expedited methodology for numerical differentiation of experimental data becomes necessary. On the other hand, the numerical differentiation presents several limitations that should be overcome. The most important perturbation is due to the presence of noise in the experimental data which effect grows with the derivative order. A new methodology based on derivative and smoothing techniques was developed to reduce this problem. As the differentiation processes also uses several points to calculate each derivative (Babaud, Witkin et al. 1986; H.M.R. Lopes 2005) a smoothing procedure is already involved. In the study here described using three different differentiation techniques were tested and compared according to a methodology that can be described as follows:

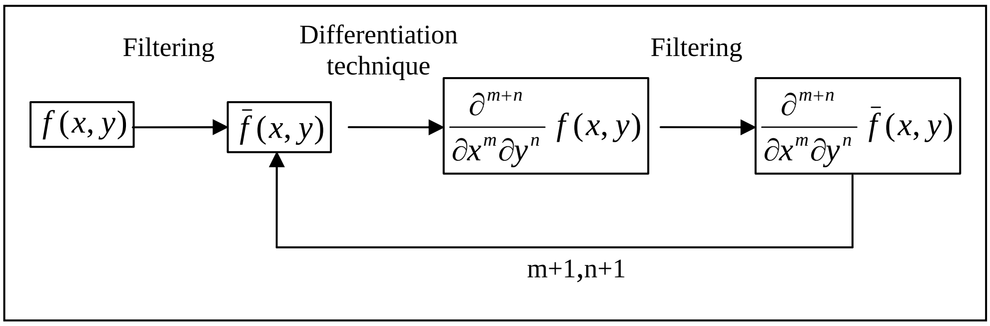

Figure 1 - Schematic diagram of the proposed methodology.

In this methodology, the noisy data is filtered each time the differentiation technique is applied. This procedure reduces the propagation of high frequency noise through the differentiation process and allows the method to perform high order derivatives without destroying the signal components. The filtering and the differentiation parameters are adjusted for best fit.

The experimental noisy displacement fields were obtained from the propagation of a transient bending wave on the surface of an aluminum thin plate. A non-contact, high sensitive and high resolution measurement was carried out using a double pulse TV holography.

EXPERIMENTAL SET-UP

The experimental tests were performed using the double-pulsed TV holography setup presented in figure 2. In this setup a LUMONICS Ruby LASER generates pairs of pulses with a time separation between 1 and 800 μs and the double pulse digital holograms are record by a CCD camera.

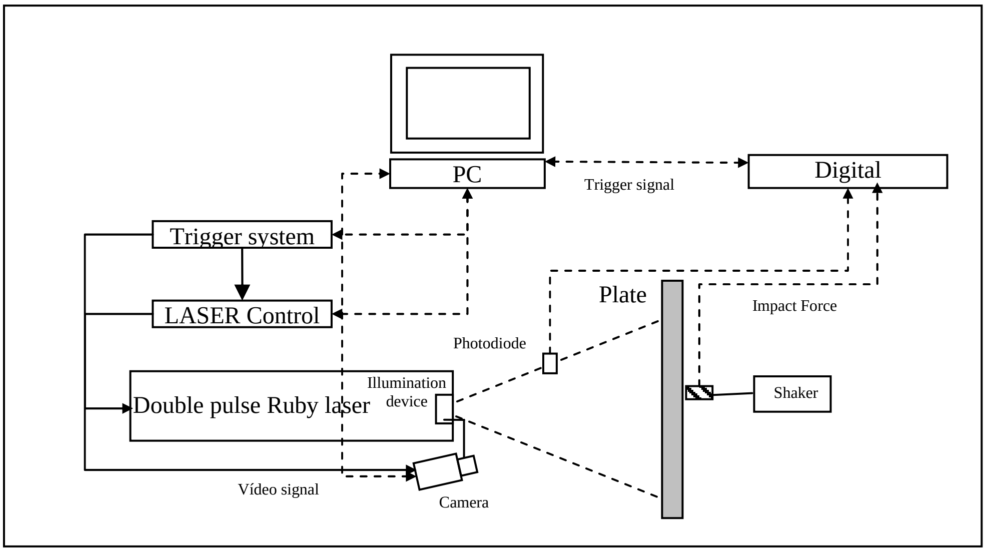

Figure 2- Schematic presentation of the experimental setup.

The bending waves are created by impact loads generated with an electromagnetic shaker. A force transducer is used to measure the impact force time history. The force vs time recordings were stored with a digital oscilloscope to be used in the numerical simulation. An accurate trigger system was developed to capture the impact force between holographic recordings. The trigger time and amplitude force were adjust to measure the bending waves when they reach the plate’s border.

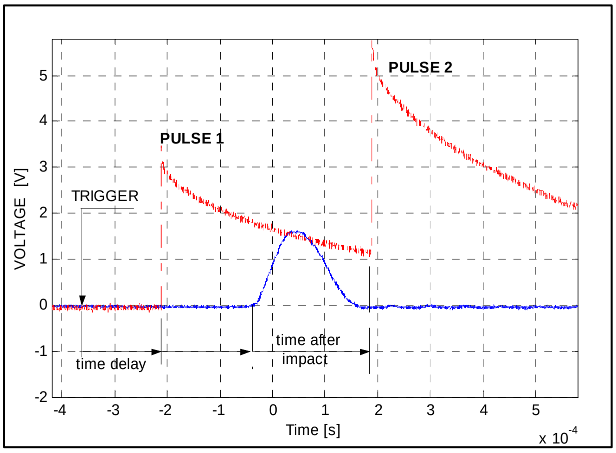

Figure 3- Recorded force and double pulse LASER signal.

The phase distribution of each holographic recording was obtained by demodulation of the spatial carrier introduced in the primary fringes. The final phase map is extract by subtracting the phase map of two recorded LASER pulses. The phase filter and unwrapping algorithms were implemented to compute bending wave displacements from the phase map. The displacement field computation process is presented in figure 4.

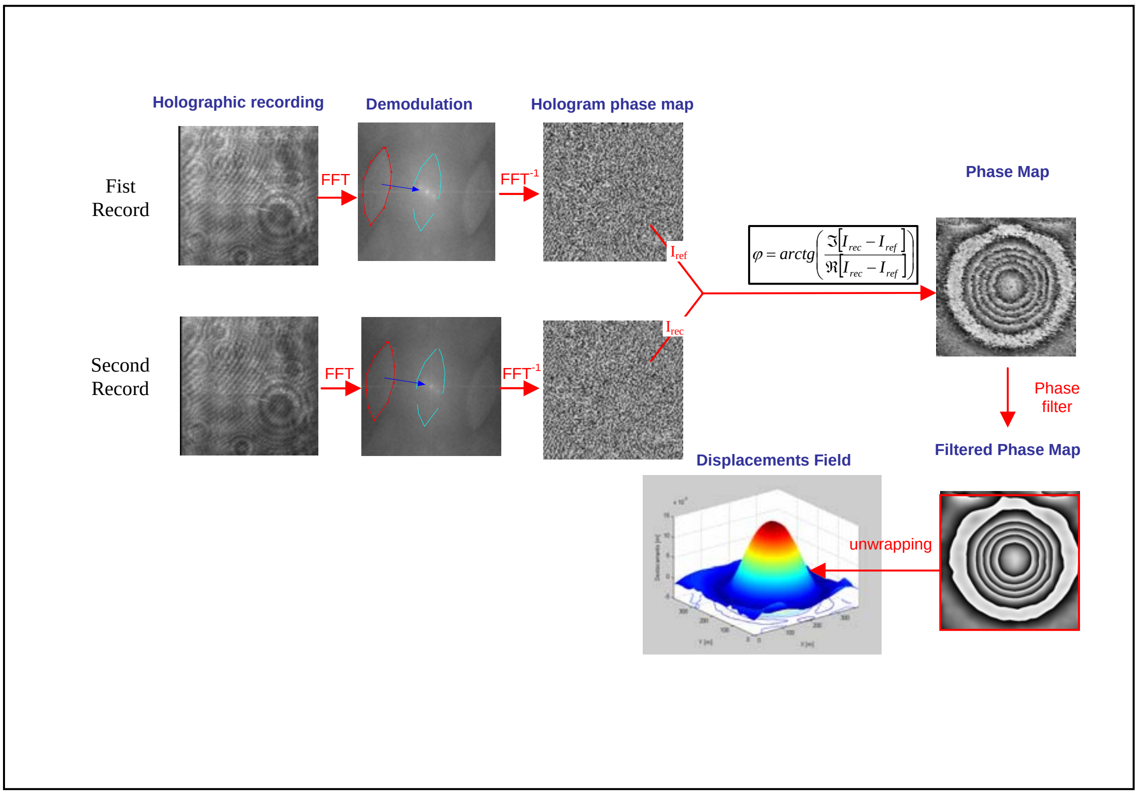

Figure 4- Extracting the displacement field from the acquired data.

DIFFERENTIATION METHODS

The spatial derivatives up to the third order are computed from the measured displacements using the here proposed method, schematically presented in figure 1. The high frequency noise is removed through the process by average filtering convolution. The differentiation procedure was performed by Cubic spline, Fast Fourier Transform and Gaussian derivative techniques. All of the three algorithms will be presented ahead:

On the first algorithm the derivative map is extracted from cubic spline coefficients S_i using a least-squares spline approximation of the filtered data. The spatial derivative with Fast Fourier technique is computed in the frequency domain. In this case the low frequency components of the filtered data are selected for the derivative calculation. The spatial derivative using the Gaussian derivative kernel convolution is performed by the third algorithm described in figure 5. As a result of the numerical convolution process, the spatial derivative cannot be obtained up to the boundary. In some cases, this problem can be overcome by extrapolation of the information beyond the image limits. Unfortunately, this cannot always be applied and some times the results have poor quality. A more successful approach is done interpolating the information with cubic spline functions. Afterwards, the spatial derivative is calculated and no data is lost by the convolution process.

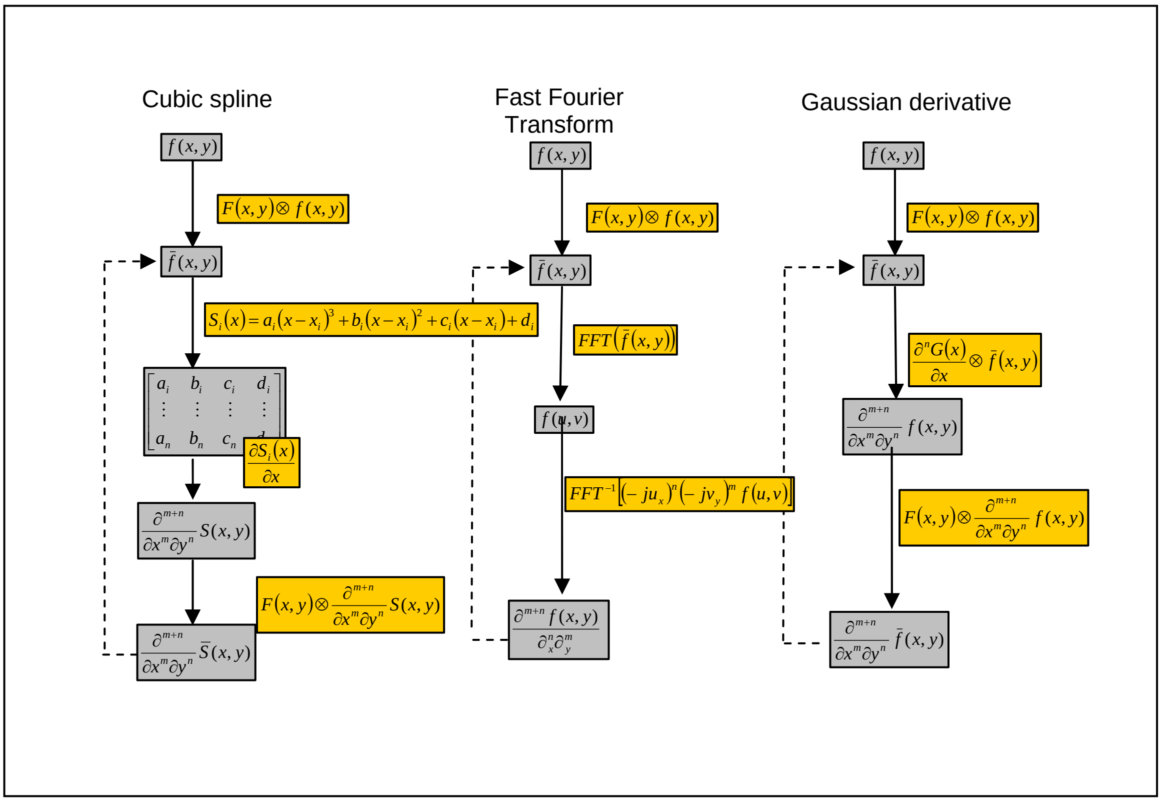

Figure 5- Spatial derivatives using different techniques.

RESULTS

The forces on the plate can be obtained from the displacement field measurements by using the proposed differentiation methodologies. The most critical case are shear forces, which involves spatial derivative of displacement up to the third order (Timoshenko and Woinowsky-Krieger 1959). The shear force along x axis (Q_x) was used in our study to compare the results of proposed methodologies with the Finite Element Method (FEM). From the thin plate's theory the shear force can be express by the following equation:

$$Q_x(x,y,t) = -D\left(\frac{\partial^3 w(x,y,t)}{\partial x^3} + \frac{\partial^3 w(x,y,t)}{\partial x \partial y^2}\right) \quad (1)$$

In figure 5 are depicted the shear forces computed using the proposed methodologies. An iterative process was applied to adjust the algorithm parameters and obtain the best results. The Gaussian derivative and Fast Fourier Transform methods proved to be closer to the FEM prediction, Further the Gaussian derivative depicted worse results along the border due to its stronger smooth effect.

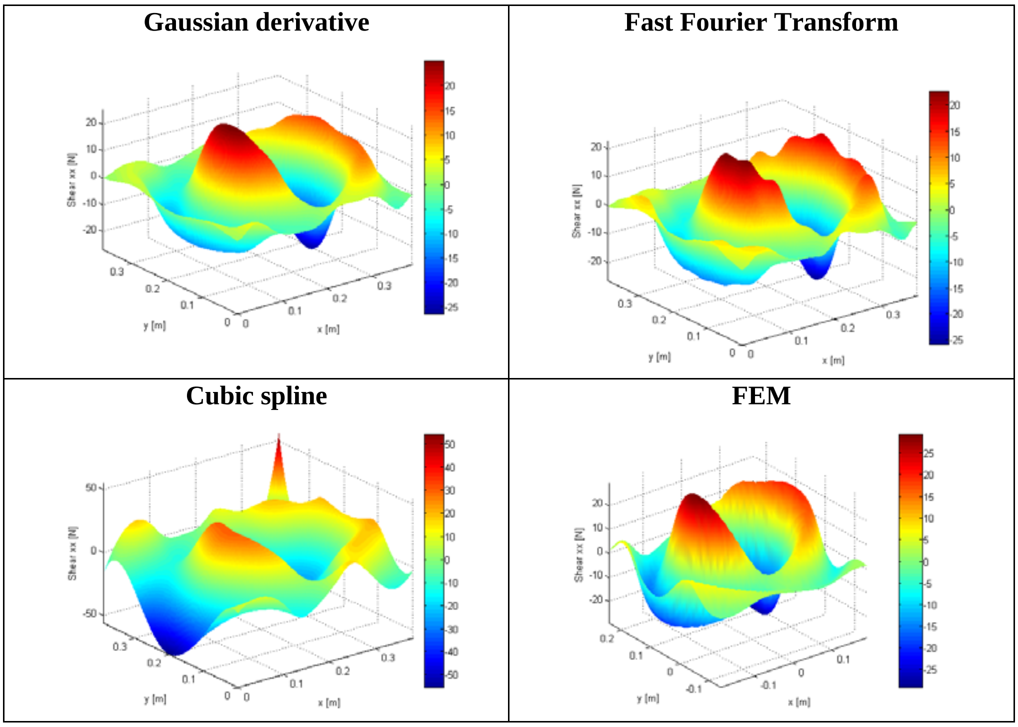

Figure 5 – The shear forces along x axis by the propose methodologies and by FEM.

CONCLUSIONS

A good agreement between the proposed methods and the Finite Elements Method was achieved. The methodology proved to be very robust in presence of experimental noisy data. The noise was gradually reduced by the filtering process minimizing the signal attenuation for high order derivatives calculation. The differentiation up to the border was solved by an interpolation process based on spline cubic functions. These results intended to prove that the Gaussian derivative and Fast Fourier methods lead to better results than the cubic spline. In general, the proposed methodologies proved to be suitable to be applied for force calculation in thin plates.

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REFERENCES

Babaud, J., A. P. Witkin, et al. (1986). "Uniqueness of the Gaussian Kernel for Scale-Space Filtering." Ieee Transactions on Pattern Analysis and Machine Intelligence 8(1): 26-33.

H.M.R. Lopes, R. M. G., M.A. Vaz (2005). "An Improved Mixed Numerical-Experimental Method for Stress Field Calculation." submitted to Optics & Laser Technology.

Timoshenko, S. and S. Woinowsky-Krieger (1959). Theory of plates and shells. New York,, McGraw-Hill.

Wahyu Lestari, P. Q. (2005). "Damage detection of fiber-reinforced polymer honeycomb sandwich beams." Composite Structures 67: 365–373.