GAS-LIQUID FLOW DISPERSION IN A VERTICAL TUBE

Valdemar R. Garcia and J. A. Sobrinho Teixeira

Instituto Politécnico de Bragança
Apartado 38, 5300 Bragança, Portugal
Email: sobrinho@ipb.pt

ABSTRACT

The main purpose of this work was to analyse the transport of a solute dissolved in a flowing liquid in upward direction at low velocity along a vertical column, in the bottom of which slugs are injected.

These experiments were carried out in a column with an internal diameter of 19 mm and height of 344.2 cm. Liquid flowrates of 1.0, 2.5 and 5.5 cm^3/s were utilized, and an enlarged variety of gas flowrates were injected; in these conditions the slugs were normally formed at a frequency between 0.78 and 88.1 s^{-1}. The scientific technics used in this study based on the measurement, at the top of the column, of a tracer solution injected in its bottom.

A simplified physical model was developed to explain the results obtained, which was based on two hypotheses: (i) the flow of the liquid between two slugs is laminar, being the respective profile of velocity approximate to the Poiseuille law; (ii) the stirring action provoked by each slug in the liquid is limited to a small region, the wake of slug, which can be taken as a "perfectly mixed tank".

One of the most relevant conclusions derived from this work and which at first analysis would appear paradoxical, is that, it is possible to proceed in such a way that the bubbling of gas through a liquid will provoke less dispersion in the residence time of liquid than which would be obtained in the absence of the introduction of gas.

INTRODUCTION

The problem of solute dispersion in flowing liquids has interested scientists and engineers for many years. Dispersion may be important in determining product yield in chemical reactors and it is certainly very important in determining levels of pollution in natural streams, following the discharge of toxic wastes. Gas-liquid contacting under conditions of slug flow occurs in many pratical situations, e.g. in air-lift systems, and it is likely to be more easily amenable to physical modelling.

The present paper deals with the problem of axial dispersion of the liquid in co-current gas-liquid slug flow, in vertical tubes. This problem has been given some attention by van Heuven and Beek[1] in their study of narrow gas lifts, although the treatment given by those authors was largely empirical. In the present paper new experimental data are reported and interpreted in terms of a detailed physical model.

EXPERIMENTAL

The diagram in Fig. 1 illustrates the main features of the experimental set up in this work. Liquid was supplied at a constant flowrate to the bottom of a vertical column (at A) and it was removed by overflow, through a side opening, near its top (at C). Air could be supplied to the bottom of this column, at a constant measured flowrate, in such a way that gas slugs formed regularly.

The test section of the column had an internal diameter of 19 mm and a height of 344.2 cm. Prior to each experimental run the column was washed and filled with clean water. Following this, air flow to the system was started and adjusted to the desired flowrate (measured by a calibrated flowmeter); the three way valve was then rotated to initiate the flow of tracer solution to the column, and this was maintained at a constant measured flowrate until the end of the experiment. A large number of small bottles (typically 5 cm^3) was used to collect samples of the liquid leaving the column, and the time of collection, t, relative to the beginning of tracer injection, was recorded for each sample. Tracer concentration in the samples was determined by spectrophotometry.

For each experimental run, slug frequency and liquid holdup in the column were also measured. Timing the passage of a set number of slugs at two locations, one near the top of the column and the other 0.5 m above its bottom, gave the corresponding frequencies. These measurements confirmed the impression given by visual inspection, that no coalescence took place in the main portion of the column, in nearly all the runs. At the end of each run, gas and liquid supply to the column were suddenly cut off and the holdup of liquid was measured after the eruption of the last slug at the free surface.

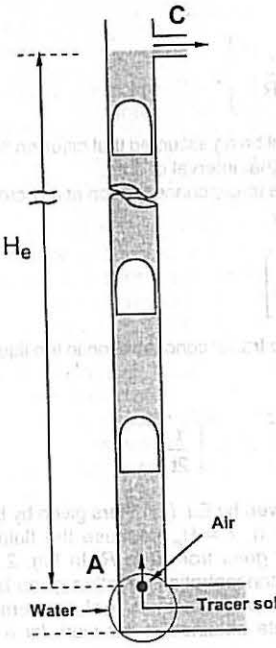

Fig. 1. Diagram of experimental setup.

The experimental data were organised in plots of C_e/C^0 vs. t/τ as shown in Figs. 2, 4 and 5; C^0 is the constant inlet tracer concentration and C_e is the corresponding variable outlet concentration. The average residence time of liquid in the column, τ, is the ratio between volume of liquid in the column and liquid flowrate through the column.

Experiments were performed with gas flowrates in the range 0 to 15 cm³/s and liquid flowrates between 1 and 5.5 cm³/s. The superficial velocities of liquid and gas were kept low in order to ensure laminar flow of the liquid between slugs and at the same time avoid coalescence of the slugs along the column.

THEORY AND INTERPRETATION OF EXPERIMENTAL DATA

It is convenient to start by considering separately the findings of two related studies, (i) and (ii) below, before discussing the detailed model for co-current flow of gas and liquid.

(i) Taylor dispersion for laminar flow along a tube

In the limit of zero gas flowrate, the experiments described above are similar to those performed by Taylor[2], and the velocity profile of the liquid in the tube is given by

$$u = u_0\left[1-\left(\frac{r}{R}\right)^2\right] \qquad (1)$$

where u is the velocity at distance r from the axis, u_0 is the velocity on the axis and R is the radius of the tube.

Following a step change in tracer concentration, from $C = 0$ to $C = C^0$ at $z = 0$ and $t = 0$, the marked fluid, with tracer concentration C^0, will occupy the region below the surface

$$z = u_0 t\left[1-\left(\frac{r}{R}\right)^2\right] \qquad (2)$$

at any instant t, it being assumed that diffusion of the tracer is negligible in that interval of time.

The average tracer concentration at any cross section is then given by

$$\frac{\bar{C}}{C^0} = \left[1-\frac{z}{u_0 t}\right] \qquad (3)$$

and the average tracer concentration in the liquid leaving the column is

$$\frac{C_e}{C^0} = 1-\left(\frac{H_e}{u_0 t}\right)^2 = 1-\left(\frac{\tau}{2t}\right)^2 \qquad (4)$$

The value given by Eq. (4) differs given by Eq. (3) for the outlet section, $z = H_e$, because the fluid velocity decreases as r goes from 0 to R. In Fig. 2, curve A represents the concentration variation given by Eq. (4) and it is seen to be in excellent agreement with experimental data obtained in this work for a range of liquid flowrates.

It is interesting to compare the concentration variation given by Eq. (4) with the corresponding curves for other flow conditions.

For plug flow of the liquid, the outlet concentration would follow curve B, while for perfectly mixed flow curve D would be observed, corresponding to

$$\frac{C_e}{C^0} = 1-e^{-t/\tau} \qquad (5)$$

Finally, for laminar flow of the liquid (with parabolic velocity profile) and appreciable molecular diffusion of the tracer, a curve like E or E' would be obtained.

A curious aspect of this result is that a higher diffusivity of the tracer leads to a lower value of the apparent dispersion coefficient; curve E in Fig. 2 refers to a solute of higher diffusivity than that corresponding to curve E'.

(ii) Mixing induced by gas slugs rising up a column of liquid

In the limit of zero liquid flowrate, the experiment illustrated in Fig. 1 corresponds to that described by Campos and Guedes de Carvalho[3], if a step distribution of tracer concentration is initially present in the column. Those authors found that the stirring action of the bubbling gas may be attributed to the recirculating flow in the wake of the individual slugs. These wakes may be conceived of as perfectly mixed 'tanks' occupying the whole cross section of the column and extending over a length l_w behind the bottom of each slug. Values of l_w were determined by Campos and Guedes de Carvalho[3] from experiments on mixing in columns with internal diameters $d = 19$ mm and $d = 32$ mm; the values $l_w = 2.3\ d$ and $l_w = 2.8\ d$, respectively, were found.

(iii) Co-current flow of gas and liquid

In the more general situation of co-current flow of liquid and gas slugs, it is important to consider the combined effects of tracer transport in the recirculating wakes and laminar dispersion of tracer in the liquid between each wake and the following slug.

The simplified picture given in Fig. 3 is the basis of the model to be developed. In the wake of a slug (between sections A and B) the liquid recirculates vigorously and it may be assumed to be fully mixed. Turbulence dies out

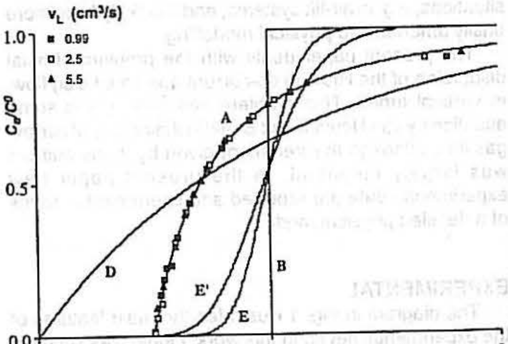

Fig. 2. Outlet tracer concentration for different types of flow. A – laminar flow; B – plug flow; D – perfectly mixed flow; E and E' – plug flow with dispersion.

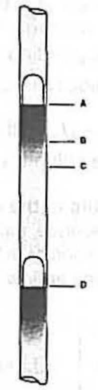

Fig. 3. Diagram of slugs and their wakes.

gradually from B to C and eventually a parabolic velocity profile is established in the liquid, corresponding to laminar flow at a flowrate ($v_L + v_G$) where v_L and v_G are the flowrates of liquid and gas, respectively, fed to the bottom of the column. Laminar flow is observed in the region between C and D.

In the simplified model to be presented, the liquid in the column is assumed to be in developed laminar flow, except over the portions of length l_w behind each gas slug, where it is assumed to be fully mixed. This model accounts for the two significant contributions to mixing. If at any one instant a higher concentration of tracer is present in the lower part of the column, there will be a net upward transport of tracer preferentially near the column axis, in the region of laminar flow between the wakes of two successive slugs. As the trailing slug rises through the liquid, the recirculating wake behind it will mix the contents of the column over the entire cross section, thus destroying the radial concentration gradients which were previously set up as a result of the parabolic velocity profile of the liquid. In some sense the wake of a slug may be seen as imparting locally a high diffusivity to the tracer, thus attenuating the radial concentration gradients which would otherwise be set up by forward laminar convection alone. The injection of a few slugs may then be expected to bring the residence time distribution of the liquid in the column closer to that for plug flow, in very much the same way that a tracer of high diffusivity would, as found by Taylor[2]. This result is surprising at first, in that gas injection is often used to replace mechanical stirring of a liquid. Obviously, if the frequency of slug injection is increased sufficiently, the extent of mixing in the axial direction will also increase due to the finite length of the wakes and, within the limit, the residence time distribution of the liquid in the column will approach that for a perfectly mixed tank. The detailed mathematical modelling of the physical situation is presented in the Appendix and the curves from the model are given in Figs. 4 and 5, alongside the experimental data.

Each of those figures reports data for one value of the liquid flowrate and three values of the gas flowrate. In reality six or seven runs were performed (at different gas flowrates), for each value of the liquid flowrate, but only three are represented for the sake of clarity. The theoretical lines corresponding to liquid progression in plug flow

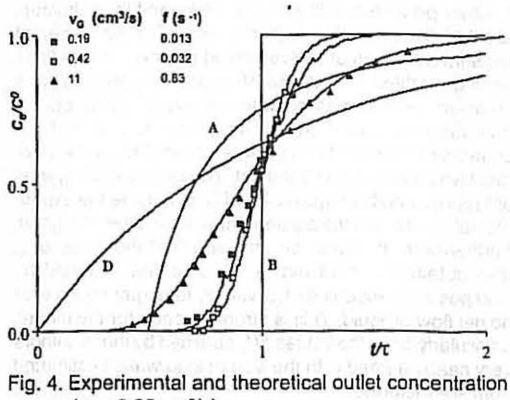

Fig. 4. Experimental and theoretical outlet concentration curves (v_L = 0.99 cm³/s).

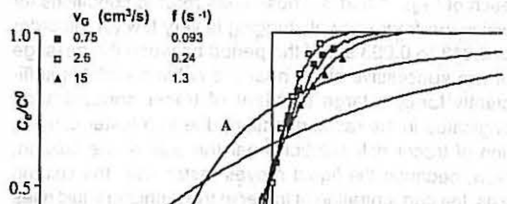

Fig. 5. Experimental and theoretical outlet concentration curves (v_L = 5.5 cm³/s).

(line B), perfectly mixed flow (line D, equation (5)) and laminar flow with a parabolic velocity profile (line A, equation (4)) are also represented, together with the predictions from the model detailed in the Appendix. The 2.5 cm³/s experiment is not represented because their results are similar to find in Figs. 4 and 5.

In both of this two figures it may be seen that injection of gas at the lowest flowrate reduces liquid dispersion significantly below that corresponding to no gas injection. Some increase in the flowrate of injected gas is seen to further reduce dispersion of the liquid, but a minimum in the extent of dispersion is reached as the gas flowrate is increased; a pronounced increase in the gas flowrate helps promoting axial mixing. The three or four experimental runs not represented for each liquid flowrate correspond to intermediate values of the gas flowrate; if the points obtained in those runs were represented in the plots, they would fall very close to the full squares, thus showing that there is a relatively wide range of gas flowrates for which mixing of the liquid is close to the minimum.

In order to compare experimental values with theoretical predictions it is important to understand how the lines in each of Figs. 4 and 5 were drawn. The lines from the model detailed in the Appendix depend on only one adjustable parameter, namely the length of the slug wake,

l_w; other parameters, like slug velocity and liquid holdup, are known from the experiments. For each set of experimental runs at a given liquid flowrate, model lines were generated for a range of values of l_w and the lines obtained were compared with the experimental points. The value of l_w was chosen for which the sum of the squares of the deviations between prediction and experiment was a minimum for the set. The corresponding lines are represented in Figures 4 and 5, alongside the experimental points, and the agreement is seen to be very good. Furthermore, it should be stressed that the value of l_w thus obtained l_w = 44 mm = 2.3 d is that reported by Campos and Guedes de Carvalho[3], for experiments with no net flow of liquid. This is strong support for the model, particularly since the values of l_w obtained by those authors very nearly agreed with the size of slug wakes estimated from still pictures.

Special reference should be made to the indented form of the line corresponding to the lowest gas flowrate, in each of Figs. 4 and 5. Those lines refer to conditions for which the frequency of slugging is very low (of the order of 0.013 to 0.093 s^{-1}). If the period between the passage of two successive slugs near the column outlet is sufficiently long, a large gradient of tracer concentration originates in the radial direction, due to a faster convection of tracer rich solution near the axis of the column. Now, because the liquid moves faster near the column axis, the concentration of tracer in the outgoing liquid rises significantly above its average concentration over the cross section at the column outlet; when the following slug passes at that level the liquid is completely mixed over the cross section and the resulting local concentration is also that observed in the outgoing liquid stream. The experimental points follow the indented lines very closely and this, again, is strong support for the model.

CONCLUSIONS

The present work gives detailed insight into the process of liquid dispersion during gas liquid slug flow in vertical tubes. Liquid dispersion is shown to result from the combined action of Taylor dispersion in the flowing liquid and intense recirculation in the wakes of the rising slugs. These two effects combine in such a way that for a given flowrate of liquid, gas injection leads first to a decrease in dispersion and then to an increase, as the frequency of slugging is increased. This is a somewhat surprising finding in that aeration is often used to promote mixing.

A detailed model of the mixing process is presented and its predictions are shown to closely agree with the experimental results obtained in a 19 mm i.d. column. The model developed is used to study the influence of a wider range of parameters than could be done in the experiments and this suggests the adoption of a simpler model of mixing, based on the idea of plug flow of the liquid with superimposed axial dispersion.

APPENDIX: Detailed model of liquid mixing

The calculations may be performed on a 'gas free' basis, since the tracer is confined to the liquid phase, but a correction of the velocities is needed to accommodate the contraction in length scale.

The average residence time of liquid in the column is $\tau = HA/v_L = A\lambda H_e/v_L$, where H is the height of liquid on a 'gas-free' basis. The average liquid velocity is then $\bar{u} = H/\tau$ and the corresponding centre-line velocity is $u_0 = 2\bar{u}$. The slug velocity is corrected according to $u_s = U_s' H / H_e$, where U_s' is the absolute velocity of the slug relative to the column wall.

(i) Concentration profile in the column

On a frame of reference moving with a slug (and associated wake) the velocity profile of the liquid in the region of developed laminar flow is given by

$$u_r = u_s - u_0\left[1 - \left(\frac{r}{R}\right)^2\right] \qquad \text{(A1)}$$

The rising slug may thus be viewed as generating flow at a rate

$$Q = \int_0^R 2\pi r u_r dr = \left(u_s - \frac{u_0}{2}\right)\pi R^2 \qquad \text{(A2)}$$

through a perfectly mixed tank (the wake) of volume $V_w = Al_w$. At time t, the rate at which tracer enters a given wake is given by $Q\overline{C}_{in}(t)$ where $C_{in}(t)$ is the distribution of tracer concentration at the level corresponding to the inlet to the wake.

The material balance over the perfectly mixed wake may then be written as

$$Q\overline{C}_{in}(t) = QC_{out}(t) + V_w \frac{dC_{out}(t)}{dt} \qquad \text{(A3)}$$

where C_{out} represents the concentration of tracer at the wake outlet.

Now, slug formation is assumed to be instantaneous, the position of the newly formed wake being at the instant of its formation. At time t after its formation, a wake will have rised a distance $z = u_s t$; liquid will be entering the wake at level $z + l_w$ and leaving it at level z. Thus, converting the temporal derivative to a spatial derivative, Eq. (A3) may be rewritten as (by substitution of Eq. (A2)).

$$\overline{C}_{in}(z+l_w) = C_{out}(z) + l_w \frac{u_s}{u_s - \frac{u_0}{2}} \frac{dC_{out}(z)}{dz} \qquad \text{(A4)}$$

Integration of this differential equation with the appropriate boundary condition gives the concentration of tracer at section z, $C_{out}(z)$, immediately alter the passage of a slug at that sectoon, if the profile $C_{in}(z, r)$ prior to its injection is known. (Note that $C_{out}(z)$ is not a concentration profile. It represents the concentration left at some level z at the moment the base of a wake is there).

First slug

Following Eqs. (2) and (A2), the average inlet concentration to the first wake, at level $z + l_w$ is, upon integration,

$$\overline{C}_0^{\bullet}(z + l_w) = \tag{A5}$$

$$\begin{cases} \dfrac{C^0}{u_s - \frac{u_0}{2}}\left(1 - \dfrac{z + l_w}{u_0 t}\right)\left[u_s - u_0 + \dfrac{u_0}{2}\left(1 - \dfrac{z + l_w}{u_0 t}\right)\right], & 0 < \dfrac{z + l_w}{u_0 t} < 1 \\ 0 & , \dfrac{z + l_w}{u_0 t} \geq 1 \end{cases}$$

If the first slug is formed at time t_1 after initiation of the flow of tracer solution, then we have in Eq. (A5)

$$t = t_1 + \frac{z}{u_s} \tag{A6}$$

the second term on the r.h.s. accounting for the time of rise of the wake.

Assuming that during slug formation the portion of liquid within $0 < z < l_w$ is fully mixed, the initial outlet concentration $C_1(z = 0)$ is determined by, upon integration

$$C_1(z = 0) = C^0 \frac{1}{l_w}\left(1 - \frac{l}{2 u_0 t_1}\right) \tag{A7}$$

where $C(z)$ is the average tracer concentration at section z (Eq. (3)) and $l = \min(l_w, u_0 t_1)$.

Integration of Eq. (A4) for the first slug can now be performed, with the boundary condition expressed in (A7) and with $\overline{C}_{in}(z + l_w) = \overline{C}_0^{\bullet}(z + l_w)$ (Eq. (A5)). The region of integration is $0 < z < (H - l_w)$ since there is no flow into the wake when it reaches the free surface. This integration has to be carried out numerically and a Runge-Kutta method was used. The result $C_{out}(z) = C_1(z)$ is the concentration left at section z immediately after the passage of the first wake. For $z < 0$ and $(H - l_w) < z < H$, we have

$$C_1(z) = \begin{cases} C_1(H - l_w), & H - l_w < z < H \\ C^0 & , z < 0 \end{cases} \tag{A8}$$

Second slug

For a frequency of slug injection f, the second slug is formed at time $t = t_1 + T$, with $T = 1/f$. When the top of this second wake reaches level z, the radial concentration profile at that level will be

$$C_1^{\bullet}(z, r) = C_1(z - \Delta z) \tag{A9}$$

where Δz satisfies

$$\Delta z = u_0\left(T + \frac{\Delta z - l'}{u_s}\right)\left[1 - \left(\frac{r}{R}\right)^2\right] \tag{A10}$$

with

$$l' = \begin{cases} l_w & , H - z \geq l_w \\ H - z, & H - z < l_w \end{cases} \tag{A11}$$

Indeed, due to the parabolic velocity profile, the liquid elements at level z at time $t = t_1 + T + z - l_w / u_s$ have originated from different levels and have therefore had different times of flow since they left the wake of the previous slug. Eq. (A10) may be solved for Δz, leading to

$$\Delta z = \frac{u_0\left(T - \dfrac{l'}{u_s}\right)\left[1 - \left(\dfrac{r}{R}\right)^2\right]}{1 - \dfrac{u_0}{u_s}\left[1 - \left(\dfrac{r}{R}\right)^2\right]} \tag{A12}$$

The average concentration which enters the second wake is given by

$$\overline{C}_1^{\bullet}(z + l_w) = \frac{1}{Q}\int_0^R 2\pi r u_r C_1^{\bullet}(z, r)\, dr \tag{A13}$$

The outlet concentration $C_2(z)$ is obtained by solving the tracer material balance, Eq. (A4) (with $C_{out}(z) = C_2(z)$ and $\overline{C}_{in}(z + l_w) = C_1^{\bullet}(z + l_w)$ for the second wake). The appropriate boundary condition is

$$C_2(z = 0) = \frac{1}{\pi R^2 l_w}\int_0^{l_w}\int_0^R 2\pi r C_1(z - \Delta z')\, dr\, dz \tag{A14}$$

where

$$\Delta z' = u_0\left(T + \frac{\Delta z' - z}{u_s}\right)\left[1 - \left(\frac{r}{R}\right)^2\right]$$

$$\Delta z' = \frac{u_0\left(T - \dfrac{z}{u_s}\right)\left[1 - \left(\dfrac{r}{R}\right)^2\right]}{1 - \dfrac{u_0}{u_s}\left[1 - \left(\dfrac{r}{R}\right)^2\right]} \tag{A15}$$

Numerical integration of Eq. (A4) is once again performed in the domain $0 < z < (H - l_w)$, while

$$C_2(z) = \begin{cases} C_2(H - l_w), & H - l_w < z < H \\ C^0 & , z < 0 \end{cases} \tag{A16}$$

i-th slug

The concentration $C_i(z)$ left at level z after the passage of slug i can be determined in much the same way as outlined above for the second slug. Equation (A6) has to be numerically integrated i times to yield the final calculated concentration, $C_i(z)$.

(ii) Concentration history of the liquid leaving the column

At time t after initiation of the flow of tracer solution, the number of wakes which have left the column is given by

$$i = \mathrm{INT}\frac{t - \left(t_1 + \frac{H - l_w}{u_s}\right)}{T} + 1 \qquad (A17)$$

Therefore, the concentration of tracer at the outlet of the column can be calculated by

$$C_e(z = H) = \frac{1}{v_L}\int_0^R 2\pi r u C_i(H - \Delta z'')\,dr \qquad (A18)$$

where

$$\Delta z'' = \frac{u_0\left(t_{is} - \frac{H - l_w}{u_s}\right)\left[1 - \left(\frac{r}{R}\right)^2\right]}{1 - \frac{u_0}{u_s}\left[1 - \left(\frac{r}{R}\right)^2\right]} \qquad (A19)$$

$C_i(z)$ is the concentration at level z after passage of slug i and t_{is} is the time interval elapsed from the moment wake was formed until the instant of sampling, that is $t_{is} = t - (t_1 + (i - 1)T)$.

NOMENCLATURE

A	cross-sectional area of column, m^2
C	concentration of tracer, kg m^{-3}
$\bar{C}$	average tracer concentration at any cross section, kg m^{-3}
$\bar{C}_0^*$	average inlet concentration to the first wake, kg m^{-3}
C_1	concentration of tracer at level z in the column, immediately after passage of first slug at that level, kg m^{-3}
C_1^*	radial concentration profile of tracer at inlet to second wake, kg m^{-3}
$\bar{C}_1^*$	average tracer concentration at inlet to second wake, kg m^{-3}
C_2	concentration of tracer at level z in the column, immediately after passage of second slug at that level, kg m^{-3}
C_i	concentration of tracer at level z in the column, immediately after passage of the i-th slug at that level, kg m^{-3}
C_{in}	radial concentration profile of tracer at inlet to the wake, kg m^{-3}
$\bar{C}_{in}$	average tracer concentration at inlet to the wake, kg m^{-3}
C_e	average tracer concentration in the liquid leaving the column, kg m^{-3}
C^0	initial concentration of tracer, kg m^{-3}
C_{out}	concentration of tracer at outlet from wake, kg m^{-3}
d	internal diameter of column, m
f	frequency of slug formation, s^{-1}
H	height of liquid in the column in the absence of gas bubbles, m
H_e	height at which liquid leaves the column, m
l_w	length of column equivalent to fully mixed wake, m
Q	flowrate of liquid through the wake, m^3 s^{-1}
r	radial coordinate, m
R	internal radius of column, m
t	time, s
t_1	instant of formation of first slug, s
t_{is}	time elapsed from the moment wake i was formed until the instant of sampling, s
T	period of slug formation, s
u	liquid velocity on a 'gas free' basis, m s^{-1}
$\bar{u}$	average liquid velocity on a 'gas free' basis, ms^{-1}
u_0	velocity of liquid over the axis on a 'gas free' basis, m s^{-1}
u_r	velocity of rise of slug relative to the liquid on a 'gas free' basis, m s^{-1}
u_s	velocity of rise of slug on a 'gas free' basis, ms^{-1}
U_s'	velocity of rise of slug relative to the column, ms^{-1}
v_G	flowrate of gas fed to column, m^3 s^{-1}
v_L	flowrate of liquid fed to column, m^3 s^{-1}
V_w	volume of wake, m^3
z	coordinate along the column, m
ε	fractional holdup of gas
λ	fractional holdup of liquid
τ	average residence time of liquid in the column, s

Subscripts

i	order of slug
l	variable in Eq. (A7)
l'	variable defined in Eq. (A11)
$\Delta z, \Delta z'$	displacement of liquid elements during the time elapsed between the passage of two consecutive slugs (Eqs. (A10) and (A15)), m
$\Delta z''$	displacement of liquid elements during the time elapsed from the moment of passage of slug until the instant of sampling (Eq. (A19)), m

REFERENCES

1. J. W. van Heuven and W. J. Beek, Gas absorption in narrow gas lifts, *Chem. Eng. Sci.*, 18, 377-389 (1963).
2. G. I. Taylor, Dispersion of soluble matter in solvent flowing slowly through a tube, *Proc. Roy. Soc.*, A219, 186-194 (1953).
3. J. B. L.. M. Campos and J. R. F. Guedes de Carvalho, Mixing induced by air slugs rising in narrow columns of water, *Chem. Eng. Sci.*, 43, 1569-1581 (1988).