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Non-Linear Carrying Capacity of Asymmetric Three-Dimensional Braced Steel Frames

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Abstract

Non-linear P-delta behaviour of three-dimensional frames with irregular plant geometry is studied, using a parametric variation of geometry and stiffness formerly chosen, by comparing results obtained with author's developed software and with established commercial software. Using the exact total stiffness formulation of non-linear geometric analyses in the developed software, allows surveying its degree of precision in selected calibration examples, as compared to the exact analytical results as well as to commercial software results. A parametric study of the critical load factor of asymmetric three-dimensional frames, un-braced and braced, permits to characterize their carrying capacity with respect to overall structural stability.

Keywords: non-linear geometric structural analysis, stability of asymmetric three dimensional frames, bracing of structures.

1 Introduction

Due to the increasing number of commercial software used in design and research in structural areas, there is a need to survey their use to guarantee the correct usefulness of such software and identify the problem-types for which it is possible to use them with more adequateness. Although the programmers verify their performance with classic calibration examples, it became imperative to evaluate for a large variety of non-conventional problems the way of operation, the consumption of resources in solutions determination and still the rigorousness of the gotten results. The present work contemplates two objectives: first, a verification or calibration frame ascertains the precision of results obtained with commercial software as compared with those obtained by a developed computational model (that in the sequel will be used for the second and main objective of the article); second, a parametric study related with the determination of the critical load factor of three-dimensional building structures, with asymmetries in plan.

Nowadays in the competitive software market dealing with structural design in civil engineering, much commercial software is available for different purposes with distinct approaches. The use of software *Cype* and *Tricalc* has been excluded in this study, since does not allow the calculation of critical parameters associated with buckling loads and instability modes. The use of the software *Robot Millennium* was discarded since it does not allow this type of analysis with great accuracy (when compared with the results here obtained by other equivalent software). Other more elaborated software used herein are *SAP-2000* and *ANSYS*, which were the selected available software used together with the author's developed software (*INST3D*) for the comparison of results of 3D frames stability analyses.

The realistic examples of 2D and 3D frames used in these analyses were pre-designed according to the regulations, namely RSA and EUROCODES 1 and 3, introducing the resistant characteristics as data in the diverse software namely *Cype* for concrete buildings pre-design, and *Tricalc* or *Robot* for metal buildings pre-design. The analysed 3D structures are composed of commercial metallic profiles and present reticulated geometry, compelling to a study of second order geometric effects. To introduce stabilizing effects in the structures, bracing elements are positioned for each parametric study of the non-linear variation of the critical load parameter with the geometry and space disposal of the structural elements.

Understanding the behaviour of each structural configuration of the braced structures, allows the choice of the best solution. To proceed with the parametric comparisons and characterization of the structures performance, use is made of a software of automatic calculation of the 2D or 3D frames carrying capacity (INST3D) already developed and presented by Cesar and Barros [1], based on the exact formulation of the structural stiffness matrix of bi-dimensional frame members in the displacement formulation of the incremental balance. The algorithm included in the software INST3D follows the methodology based on the formulation of the exact stiffness matrix with the stability functions proposed by Livesley and Chandler [2], and it allows getting the critical parameters for the diverse structural configurations studied as well as the respective instability modes.

As a form to illustrate the results comparatively between the diverse studied cases, the variation of the critical load factor with different parameters was represented in a graph, for a specific asymmetric un-braced 3D frame. The comparative study of the same frame with a specific brace disposition was also carried out, permitting to verify and ascertain the increase of the corresponding carrying capacity.

2 Matrix Formulation for the Determination of Critical Loads of 2D and 3D Frames

The practical study of the instability of structures corresponds to the determination of the critical load parameter λ and the corresponding instability or buckling mode. This methodology contemplates the knowledge of the exact stiffness matrix of the structural members that relates the acting forces with the structural deformations.

As the study is also applied to three-dimensional frames, two formulations can be used for the determination of the global structural stiffness matrix: use of exact 2D stiffness matrix formulations for each plane frame orientation but associated in the space assemblage by the structural members linking such frames, or direct use of exact 3D stiffness matrix for each member. Obviously that for each approach there is the possibility of the alternative use of approximate formulations based on simplifications of the stability functions of the exact stiffness matrix, through the linearization of these functions, what makes necessary to address the degree of bar or member modelling (member sub-structuring) in order to prevent errors due to the simplification used.

In the first approach, for the determination of the critical load a bi-dimensional exact stiffness matrix of a plane beam-column model is used for each prismatic bar of the plane frames ─ methodology in which is based the developed software (INST3D) ─ which is dependent on the stability functions *s*, *c* and *sc* originally developed by Livesley and Chandler [2] but that can easily be related also to the stability functions $\phi_j\,(j=1,2,3,4)$ used by other authors namely Reis and Camotim [3], Barros [4,5]. The formulation used in the developed software (INST3D) is based on the following exact stiffness matrix of a beam-column (Bazant and Cedolin [6]):

$$\begin{bmatrix} \frac{sEI}{L} & \frac{scEI}{L} & 0 & 0 & \frac{s(1+c)EI}{L^2} & \frac{-s(1+c)EI}{L^2} \\ \frac{scEI}{L} & \frac{sEI}{L} & 0 & 0 & \frac{s(1+c)EI}{L^2} & \frac{-s(1+c)EI}{L^2} \\ 0 & 0 & \frac{EA}{L} & \frac{-EA}{L} & 0 & 0 \\ 0 & 0 & \frac{-EA}{L} & \frac{EA}{L} & 0 & 0 \\ \frac{s(1+c)EI}{L^2} & \frac{s(1+c)EI}{L^2} & 0 & 0 & \frac{2s(1+c)EI}{mL^3} & \frac{-2s(1+c)EI}{mL^3} \\ \frac{-s(1+c)EI}{L^2} & \frac{-s(1+c)EI}{L^2} & 0 & 0 & \frac{-2s(1+c)EI}{mL^3} & \frac{2s(1+c)EI}{mL^3} \end{bmatrix} \tag{1}$$

where when ρ > 0 (compressed member) the stability functions are:

$$\left.\begin{aligned} s &= \frac{(1-2\alpha\cdot\cot(2\alpha))\alpha}{\tan\alpha-\alpha} & m &= \frac{2s(1+c)}{2s(1+c)-\pi^2\cdot\rho} \\ c &= \frac{2\alpha-sen(2\alpha)}{sen(2\alpha)-2\alpha\cdot\cos(2\alpha)} & \rho &= \frac{P}{P_E}\ ;\ P_E=\frac{\pi^2 EI_z}{L^2}\ ;\ \alpha=\frac{\pi}{2}\sqrt{\rho} \end{aligned}\right\} \tag{2}$$

and when ρ < 0 (tensioned member) the stability functions are:

$$
\left.
\begin{aligned}
s &= \frac{(1-2\gamma\cdot\coth(2\gamma))\gamma}{\tanh\gamma-\gamma} \\
c &= \frac{2\gamma-senh(2\gamma)}{senh(2\gamma)-2\gamma\cdot\cosh(2\gamma)} \qquad \gamma=\frac{\pi}{2}\sqrt{-\rho}
\end{aligned}
\right\}
\tag{3}
$$

The *m* function introduced by Merchant [7] allows to represent the reduction of stiffness due to the relative transversal displacement between the ends of the beam-column, as result of the inclination that occurs between the bar centroidal axis and the direction of the axial forces P.

Appropriate assemblage of such member stiffness allows obtaining the global stiffness matrix in the developed software *INST3D*, from which the critical load is determined through an iterative process in the resolution of the incremental equilibrium equation. Such iterative determination of the critical parameter also involves the knowledge of the lower (for sidesway or displaceable joints portal frames) and upper (for un-sway, sidesway prevented or un-displaceable joints portal frames) load parameter bounds, between which the critical value is looked upon. In the analysis associated with INST3D, the laminar elements – floor slabs – of the portal frames are modelled as rigid diaphragms, a procedure which can also be modelled in the commercial software SAP 2000 by the use of diaphragm constraints (rigid links).

The commercial software of computational programs are based on universally known algorithms, then more easy to use and to understand and less prone to errors, to solve structural civil engineering problems. In one of the possible approaches use is made of the total stiffness matrix (elastic and geometric) for which care as to be drawn towards the modelling or discretization of the structural members. In the case of software SAP 2000 and in accordance with the program manual [8], for the solution of structural stability eigenvalues and eigenvectors (determination of buckling loads and buckling modes), the software uses the following equation:

$$
\left[K_E-\lambda\ K_G\right]\varphi=0 \tag{4}
$$

where K_E corresponds to the elastic stiffness matrix, K_G to the geometric stiffness matrix (dependent on the load vector associated with some load combination), λ are the load parameters or load factors (eigenvalues) and φ are the buckling modes or instability modes (eigenvectors). This formulation corresponds to the linearization of the stability functions (either *s*, *c*, *sc* and *m*; or ϕ_j for j=1,2,3,4). A 3D-formulation already presented by Almeida, Caraslindas and Barros [9], is based on the geometric stiffness matrix (corresponding to the member axial force P) of equation (5).

But with respect to a more precise second approach, diverse authors have developed three-dimensional exact stiffness matrices K ─ like the one represented in equation (6) ─ namely Eock, Yosuk and Hyu [10].

$$K_G = \frac{P}{L}\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
 & \frac{6}{5} & 0 & 0 & 0 & \frac{L}{10} & 0 & -\frac{6}{5} & 0 & 0 & 0 & \frac{L}{10} \\
 & & \frac{6}{5} & 0 & -\frac{L}{10} & 0 & 0 & 0 & -\frac{6}{5} & 0 & -\frac{L}{10} & 0 \\
 & & & \frac{I}{A} & 0 & 0 & 0 & 0 & 0 & -\frac{I}{A} & 0 & 0 \\
 & & & & \frac{2L^2}{15} & 0 & 0 & 0 & \frac{L}{10} & 0 & -\frac{L^2}{30} & 0 \\
 & & & & & \frac{2L^2}{15} & 0 & \frac{L}{10} & 0 & 0 & 0 & -\frac{L^2}{30} \\
 & & & & & & 1 & 0 & 0 & 0 & 0 & 0 \\
 & & & & & & & \frac{6}{5} & 0 & 0 & 0 & -\frac{L}{10} \\
 & & (Symmetric) & & & & & & \frac{6}{5} & 0 & \frac{L}{10} & 0 \\
 & & & & & & & & & \frac{I}{A} & 0 & 0 \\
 & & & & & & & & & & \frac{2L^2}{15} & 0 \\
 & & & & & & & & & & & \frac{2L^2}{15}
\end{bmatrix} \tag{5}$$

$$K = \begin{bmatrix}
S_1k_{1,1} & 0 & 0 & 0 & 0 & 0 & S_1k_{7,1} & 0 & 0 & 0 & 0 & 0 \\
 & S_7k_{2,2} & 0 & 0 & 0 & S_6k_{6,2} & 0 & S_7k_{8,2} & 0 & 0 & 0 & S_6k_{12,2} \\
 & & S_9k_{3,3} & 0 & S_8k_{5,3} & 0 & 0 & 0 & S_9k_{9,3} & 0 & S_8k_{11,3} & 0 \\
 & & & k_{4,4} & 0 & 0 & 0 & 0 & 0 & k_{10,4} & 0 & 0 \\
 & & & & S_4k_{5,5} & 0 & 0 & 0 & S_8k_{9,5} & 0 & S_5k_{9,3} & 0 \\
 & & & & & S_2k_{6,6} & 0 & S_6k_{8,6} & 0 & 0 & 0 & S_3k_{2,6} \\
 & & & & & & S_1k_{7,7} & 0 & 0 & 0 & 0 & 0 \\
 & & & & & & & S_7k_{8,8} & 0 & 0 & 0 & S_6k_{12,8} \\
 & & & (Sim.) & & & & & S_9k_{9,9} & 0 & S_8k_{11,9} & 0 \\
 & & & & & & & & & k_{10,10} & 0 & 0 \\
 & & & & & & & & & & k_{11,11} & 0 \\
 & & & & & & & & & & & k_{12,12}
\end{bmatrix} \tag{6}$$

In this case the stability functions, for axial compressive loads, are expressed by:

$$S_1 = \frac{1}{1 + \frac{EA}{4P^3L^2}(H_y + H_z)} \tag{6a}$$

$$S_2 = \frac{1}{4}\alpha L \frac{\sin\alpha L - \alpha L\cos\alpha L}{2-2\cos\alpha L - \alpha L\sin\alpha L} \tag{6b}$$

$$S_3 = \frac{1}{2}\alpha L \frac{\alpha L - \sin\alpha L}{2-2\cos\alpha L - \alpha L\sin\alpha L} \tag{6c}$$

$$S_4 = \frac{1}{4}\beta L \frac{\sin\beta L - \beta L\cos\beta L}{2-2\cos\beta L - \beta L\sin\beta L} \tag{6d}$$

$$S_5 = \frac{1}{2}\beta L \frac{\beta L - \sin\beta L}{2-2\cos\beta L - \beta L\sin\beta L} \tag{6e}$$

$$S_6 = \frac{1}{6}\frac{\alpha^2 L^2(1-\cos\alpha L)}{2-2\cos\alpha L - \alpha L\sin\alpha L} \tag{6f}$$

$$S_7 = \frac{1}{6}\frac{\alpha^2 L^2(1-\cos\alpha L)}{2-2\cos\alpha L - \alpha L\sin\alpha L} - \frac{\alpha^2 L^2}{12} \tag{6g}$$

$$S_8 = \frac{1}{6}\frac{\beta^2 L^2(1-\cos\beta L)}{2-2\cos\beta L - \beta L\sin\beta L} \tag{6h}$$

$$S_9 = \frac{1}{6}\frac{\beta^2 L^2(1-\cos\beta L)}{2-2\cos\beta L - \beta L\sin\beta L} - \frac{\beta^2 L^2}{12} \tag{6i}$$

where $\alpha^2 = \frac{P}{EI_z}$ and $\beta^2 = \frac{P}{EI_y}$. This matrix can also be linearized giving rise to a geometric stiffness matrix in the form of the previous equation (5).

Due to its apparent difficulty such matrices are ignored in developing versatile commercial software capable of handling effectively structural instability, and have been used solely in research. In the future it is the authors' intention to include them in the developed software, to improve the evaluation of critical parameters of structural instability and of the respective buckling modes.

3 Exact Analysis *versus* Approximate Analysis

Before presenting the determination of the critical load parameter of an asymmetric three-dimensional frame through software INST3D and SAP 2000, it becomes necessary to survey the accuracy of each one in reaching the final result. As a form to calibrate the mentioned software two 2D steel frames with modulus of elasticity of 200 GPa (one un-sway frame, the other sidesway frame) built-in at the base, as represented in Figure 1, were hand-calculated to obtain analytically their critical load parameter λ_{cr} . Reis and Camotim [3] present equations for the analytical calculation of the critical load parameter in this type of structures, with which for the sidesway prevented frame (or un-displaceable joints) λ_{cr}=10835,206 and for the sidesway free frame (or displaceable joints) λ_{cr}=3010,538.

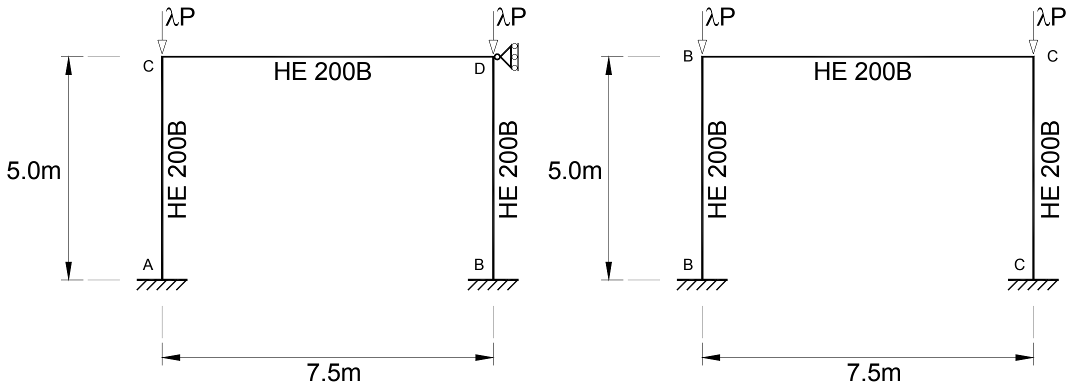

Figure 1: Calibration frames (left: sidesway prevented; right: sidesway free)

Thereafter the corresponding structural data was introduced in the authors developed software INST3D as well as in SAP 2000 and ANSYS 8.0 [11], to compare the accuracy of the results as well as to ascertain the degree of discretization required in member sub-structuring in bars. One should notice that software INST3D uses an exact formulation of the (bi-dimensional) stiffness matrix in the non-linear eigenvalue problem of structural stability, SAP 2000 uses an approximate formulation for matrices K_E and K_G in the linear eigenvalue problem, and ANSYS uses an exact formulation of the elastic stiffness matrix of three-dimensional prismatic bars (including flexural-twisting effects) as proposed by Przemieniecki [12] and the same geometric stiffness matrix as used in SAP 2000.

The computational calculation of the critical parameter using INST3D (without any member sub-structuring) allowed to determine the critical load parameter for the sidesway prevented frame as λ_{cr}=10822,652 and for the sidesway free frame as λ_{cr}=3009,226. It was then verified that with the software INST3D the results practically coincide with the analytical results, validating the formulation used, since were obtained with relative errors of 0.001%.

In these frames the 2D buckling in plan XZ was considered and the frames were prevented to buckle in the perpendicular direction YZ. The degrees of freedom U_x, U_Z and R_y have been used, according to the notation of SAP 2000. Relatively to the commercial software used it was verified that the results obtained with SAP 2000 present errors due to the approximate formulation used by the program. As a means to diminish the error, one increases the degree of discretization sub-structuring the structural members in bars, as presented in the results of Table 1. The errors by excess are related with insufficient degree of discretization of the member structural elements that compose the structure; for the case of the sidesway prevented frame, with an element per member, the committed error is very great which indicates that a special care needs to be taken to handle this type of analysis. The errors by defect are related to the significant digits used in the software by different PC's as well as to the degree of precision used in the sectional properties of the structural members. A reference discretization of 4 elements per member is herein considered as 'correct', since to it correspond acceptable relative errors for all practical purposes.

#	Sway frame					Un-sway frame				
	SAP 2000	ANSYS	INST3D	Relative Error S	A	SAP 2000	ANSYS	INST3D	Relative Error S	A
1	3315	3034	3009	10.1%	0.8%	1043895	18224	10823	9545%	68%
2	3115	3016	3009	3.5%	0.5%	12613	11090	10823	16.5%	2.4%
3	3046	3010	3009	1.2%	0.0%	11677	10902	10823	7.9%	0.7%
4	2996	3009	3009	-0.4%	0.0%	10988	10852	10823	1.5%	0.3%
8	2946	3009	3009	-2.0%	0.0%	10258	10828	10823	-5.2%	0.0%
16	2933	3009	3009	-2.5%	0.0%	10072	10826	10823	-6.9%	0.0%
32	2930	3009	3009	-2.6%	0.0%	10026	10826	10823	-7.3%	0.0%
64	2929	3009	3009	-2.6%	0.0%	10014	10826	10823	-7.4%	0.0%

Table 1: Critical parameter and relative errors, by distinct software and modeling

The results obtained with ANSYS 8.0 for these calibration frames are practically coincident with those obtained with INST3D, emphasizing a bigger precision and simplicity of structural modelling in relation to SAP 2000.

4 Non-Symmetric 3D Frames (Un-braced and Braced)

In this section results of a parametric study will be presented, related with non-linear geometric analysis of 3D non-symmetric irregular frames, five stories in high. The irregular frames are space repetitions of a three-dimensional frame – symmetric base frame (Figure 2) – that allows comparing the results of the parametric study with diverse geometric characteristics of the structure. The geometric parameters allow optimizing the configuration of the structure, by determining structural solutions for modular designs (in elevation and plan) with better carrying capacity.

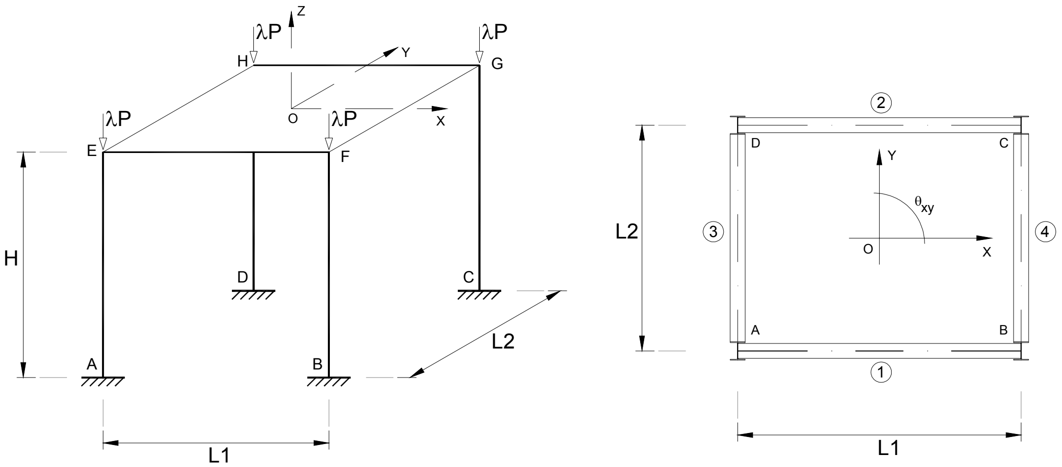

Figure 2: Un-braced 3D base frame, of four 2D local frames

In the sway frames the lateral displacements can occur by column bending and in this case the use of diagonal bracing (or equivalent, like chevron bracing) becomes important to prevent the loss of stability due to change in geometry; the brace axial stiffness absorb part of the forces that cause frame lateral displacement. Alternatively, at different costs, a compromise between the members inertias could also bound the frame horizontal displacements to pre-established requirements.

The bars used in the diagonal bracing are modelled with null bending stiffness, and axial stiffness smaller than the one of the columns and beams; in the matrix formulation, this corresponds to the addition of the contribution of axial stiffness of the inclined bars solely in the floors with diagonal bracing.

In the case of frames only partially braced in some floors, since it is not known if the frame is a displaceable joints sidesway-free frame or un-displaceable joints sidesway-prevented frame, it is necessary to make the two analyses determining the respective critical parameters λ_{dis} and λ_{undis}. This is achieved using the bi-dimensional matrix of exact stiffness relating the acting forces with the resultant deformations and obtaining, through an incremental process, the critical parameter and the corresponding buckling mode (deformation just prior to the loss of stability).

As previously presented by Cesar and Barros [1], the formulation used in program INST3D is applied to rectangular frames of constant section prismatic bars connected at rigid joints, with constant number of columns in each floor, columns along the same column-line built-in at the base, acted upon by vertical loads concentrated at the joints. In this work, and without any loss of generality, the results of the analysis of a five-storey asymmetric structure are presented: either without bracing elements or with a more efficient bracing distribution.

The 3D frame with plant asymmetries (Figures 3 and 4) refers to a hypothetical residential steel building with equal nodal vertical loads λP at the top of the columns, pre-designed in accordance to the Portuguese regulations (RSA – *Regulamento de Segurança e Acções*) and *Eurocodes 1 and 3* in order to obtain real design characteristics. The corresponding live loads are presented in Table 2. Since this parametric study of the structural global stability is related to the carrying capacity of the 3D frames, no effects of wind or earthquakes have been considered.

	Live Load
Floors	2.0 kN/m^2
Roof	1.0 kN/m^2

Table 2: Live load in buildings – RSA (Portugal)

The floor slabs are considered as behaving like rigid diaphragms in their plan, and were pre-designed with a slab thickness of 0.15 m. Steel profiles of series HEA common in the design of these metallic structures where used for both the beams and the columns, to minimize diversities in the interpretation of results. The diagonal braces in the structural model possess an area of 11 cm^2, lower than the one adopted for columns and beams, and correspond to metallic profiles of series UPN (UPN-80). The steel used has a modulus of elasticity of 210 GPa.

Some geometric parameters of the (2D and) 3D frames have been realistically varied, namely the span between the columns (L) and the floor height (H). Parameter L varies between 3 m and 8 m, with increments of 1.0 m; while parameter H varies between 3 m and 4 m, with increments of 0.5 m. For each parametric case, the critical load factor was determined as well as the corresponding buckling mode.

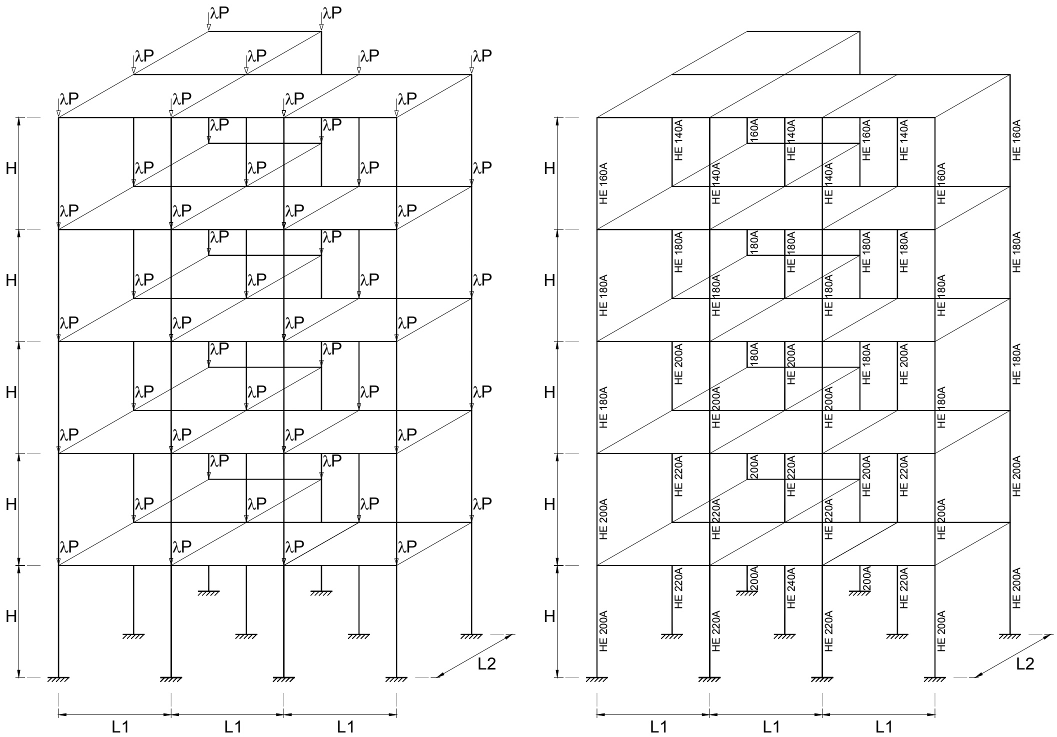

Figure 3: Un-braced 3D frame (equal nodal vertical loads λP and perspective view)

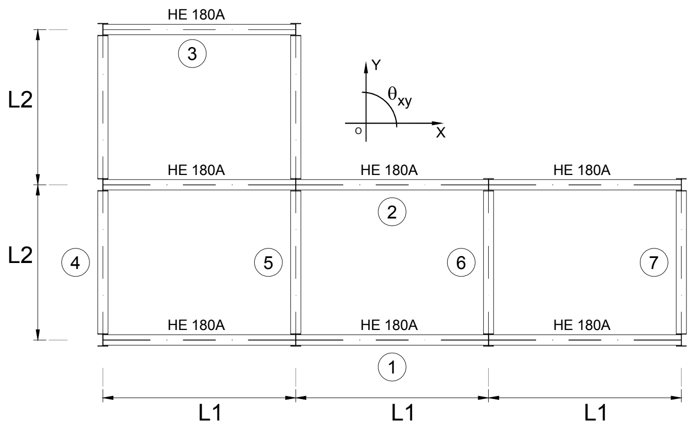

Figure 4: Un-braced 3D frame (plan view)

The study of three-dimensional frames involves the knowledge of the performance of the bi-dimensional frames that once associated constitute the 3D structure. To verify the performance and to control the error associated with the 3D modelling versus the model 2D, the 2D frames were studied for both the mobile and the fixed configurations: displaceable joints sidesway-free frame or un-displaceable joints sidesway-prevented frame. The families of results of the critical load factor obtained by distinct software for the frames of Figures 3 and 4, for one of the parametric cases studied (L1=L2= 4.0 m and H=3.0 m; discretization of 4 bars per member), are shown in Table 3 with their relative errors.

Frame number	Sidesway-free			Sidesway-prevented		
	SAP 2000	INST3D	Relative error	SAP 2000	INST3D	Relative error
1	535.67	540.63	-0.91%	1563.43	1496.04	4.50%
2	568.10	608.75	-6.67%	1581.29	1533.12	3.14%
3	402.19	409.47	-1.77%	1380.67	1318.77	4.69%
4	782.68	830.75	-5.78%	2993.84	3206.13	-6.62%
5	828.85	860.25	-3.65%	3056.78	3267.11	-6.43%
6	713.02	761.39	-6.35%	3620.87	3859.97	-6.19%
7	624.62	665.26	-6.10%	2798.57	2978.42	-6.03%

Table 3: Critical load factors (and relative errors) associated with 2D frames (part of the 3D asymmetric frame)

The relative errors shown on the determination of the critical load factors are associated with the approximate formulation in SAP2000 (and to a smaller extent on the number of significant digits used for describing the properties of the sections).

In Figure 5 the un-braced 3D frame, analysed by software SAP 2000 and INST3D, is presented with the deformation associated with the first buckling mode. It is clear that this un-braced asymmetric 3D frame loses stability by global deformation in a standard deformation pattern typical of a displaceable joints sidesway-free frame, corresponding to a critical load factor in the order of $\lambda_{cr} = 409$.

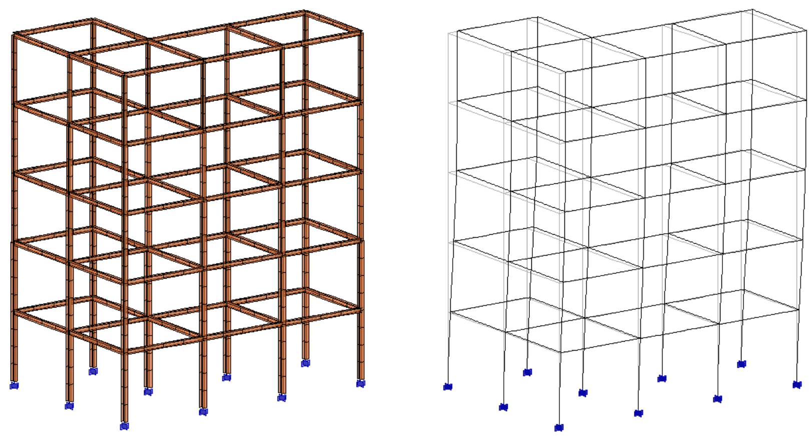

Figure 5: Structural model and buckling mode of an un-braced asymmetric 3D frame

The parametric representation of the variation of the critical load factor of this un-braced asymmetric 3D frame is shown in Figure 6, where it is noticeable the transition of behaviours from the stiffer frames to the more flexible frames.

As a means to reduce the mobility of the structure, several bracing configurations were studied. The bracing elements used were metallic profiles of series UPN (UPN-80) with area of 11 cm^2, modelled as bi-articulated members as mentioned before.

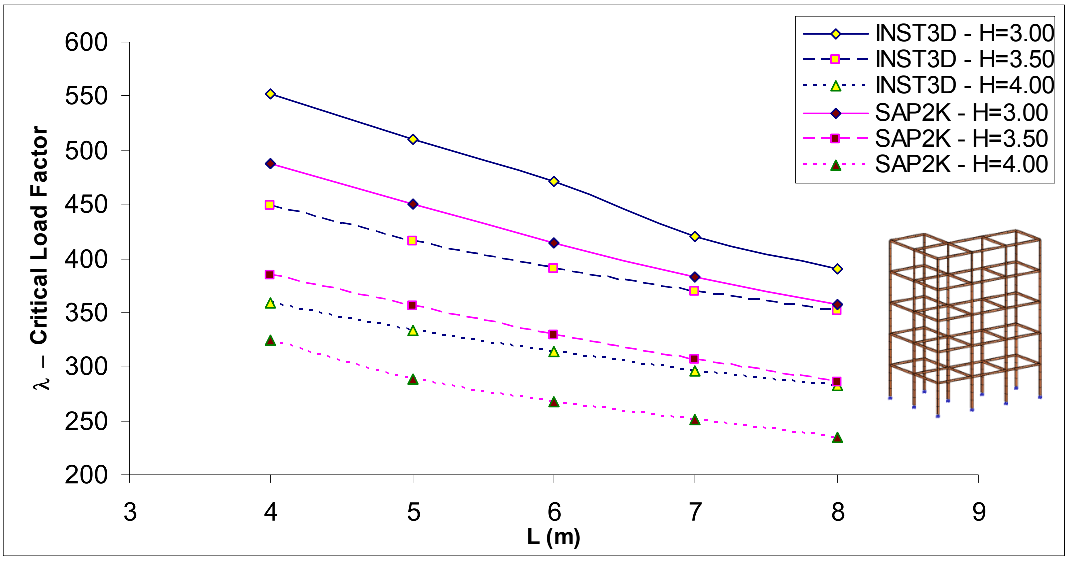

Figure 6: Parametric results of the critical load factor obtained by INST3D and SAP2000, for the un-braced frame with equal nodal vertical loads λP

In Figure 7 is represented the same asymmetric 3D frame considered before in this parametric study (L1=L2= 4.0 m and H=3.0 m; discretization of 4 bars per member), but now with a bracing distribution as specified (in bold lines) that conditions a loss of stability in frame number 3. The deformation pattern, for both the bi-dimensional frame (# 3) and the three-dimensional asymmetric structure, corresponds to an un-displaceable joints sidesway-prevented frame. The associated critical load factor of such 3D asymmetric frame is λ_{cr} = 1398.72 (analysis with SAP 2000) and is λ_{cr} = 1347.62 (analysis with INST3D); therefore the former is determined with a relative error of 3.9 %. For this parametric case it was observed almost a three times bigger carrying capacity, revealing the great importance of an efficient bracing system of three-dimensional metallic structures.

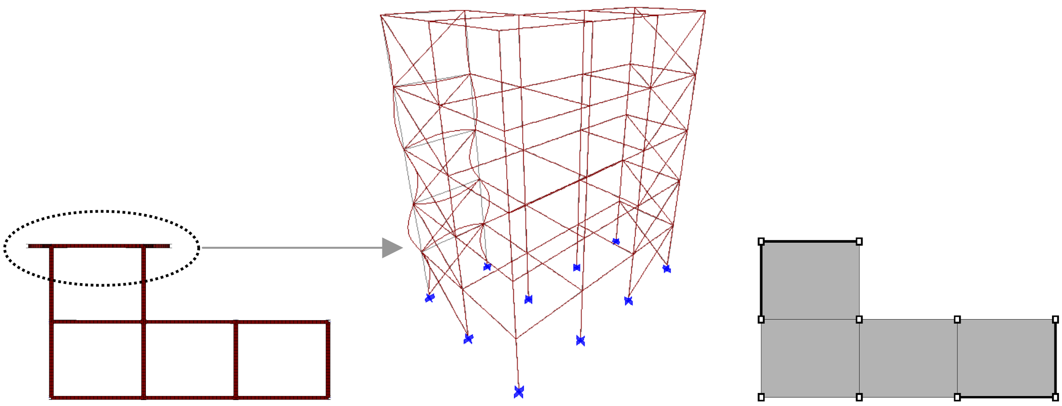

Figure 7: Asymmetric braced 3D frame (SAP 2000): deformation of 1st buckling mode for equal nodal loads and plan view of the bracing distribution

For this bracing configuration the frame number 3 controls design, for both software used. Other configurations of selected four bracing planes were also analysed, leading to the same critical load factor when calculated with INST3D (without torsion-bending effects) and slightly different load factors (but of same order of magnitude) when calculated with SAP 2000.

The parametric representation of the variation of the critical load factor of this braced asymmetric 3D frame is now shown in Figure 8, where it is also noticeable the transition of behaviours from the stiffer frames to the more flexible frames [13].

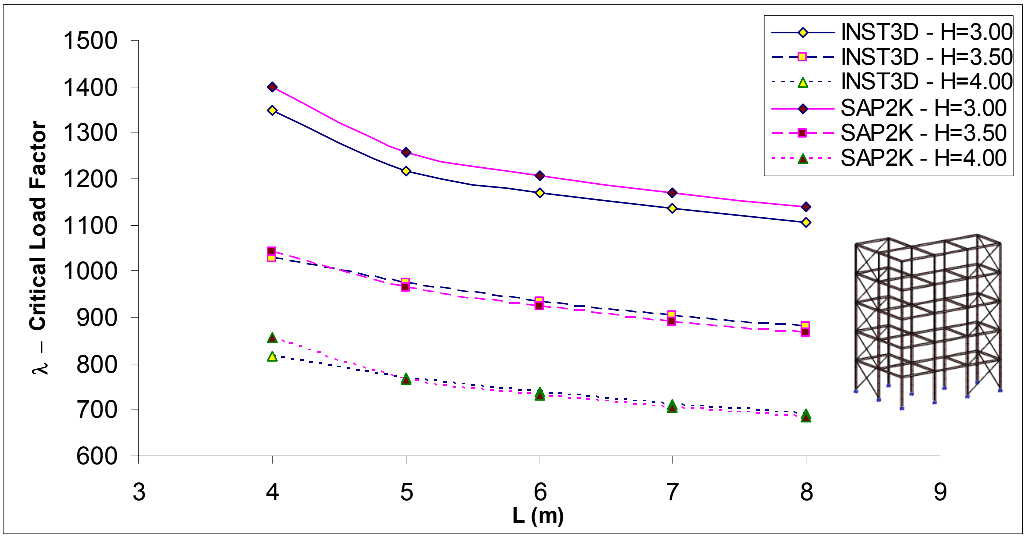

Figure 8: Parametric results of the critical load factor obtained by INST3D and SAP2000, for the braced frame with equal nodal vertical loads λP

It was verified that results obtained with the authors INST3D software were always conservative since they estimate global carrying capacities (global critical load factors) of this asymmetric 3D frame about 3-4% lower than the SAP 2000 results. Moreover both methodologies are close enough, justifying the use of the diaphragm constraint option to model rigid plane diaphragm in SAP 2000.

Finally a parametric study was performed for a similarly braced asymmetric 3D frame, but now with a more realistic axial load distribution on top of each column taking into account the relative weight of the nodal vertical loads on the basis of load influence zone at every node (Figure 9). This is equivalent to use real design vertical loads of the residential building, along the corresponding column lines.

As expected the critical load factor diminished (was practically halved) and the performance of the asymmetric 3D frame changed considerably: now it is the 2D frame with more nodal vertical loads – frame 2 – that controls overall stability of this 3D frame. The parametric representation of the variation of the critical load factor of this braced asymmetric 3D frame is now shown in Figure 10. The deformation pattern observed for the buckling mode corresponds to an un-displaceable joints sidesway-prevented frame, indicating that the bracing elements distribution shown has a good performance under several load conditions.

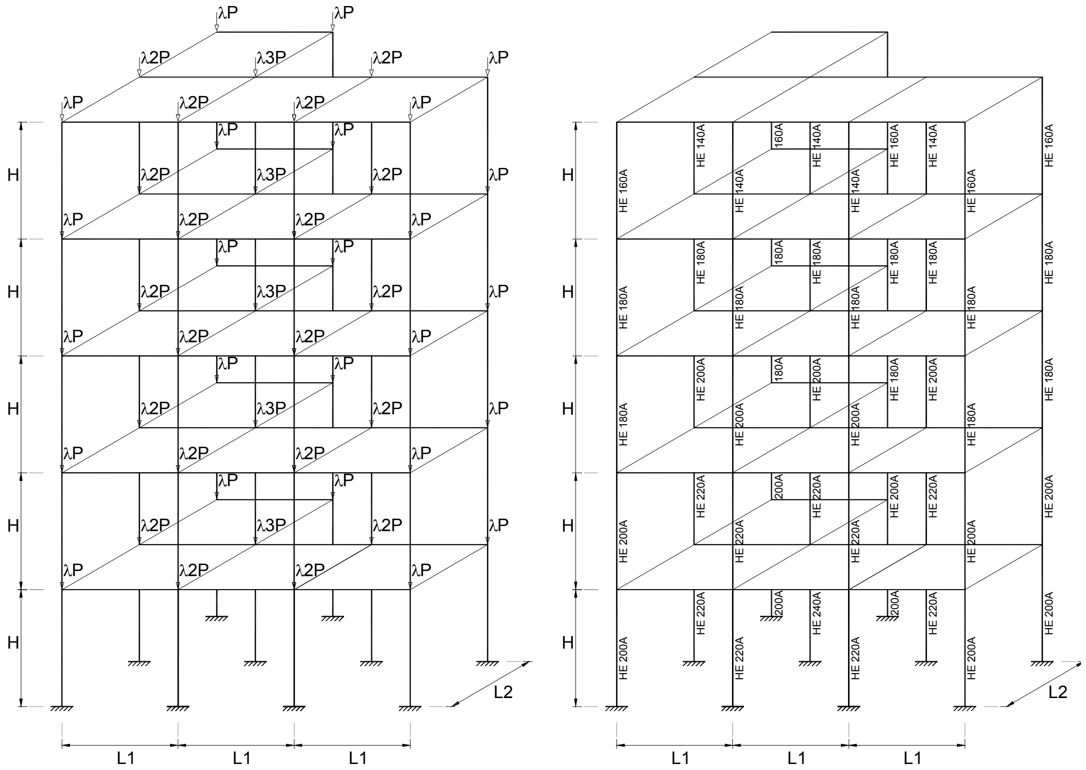

Figure 9: Braced 3D frame (uneven nodal vertical loads λP and perspective view)

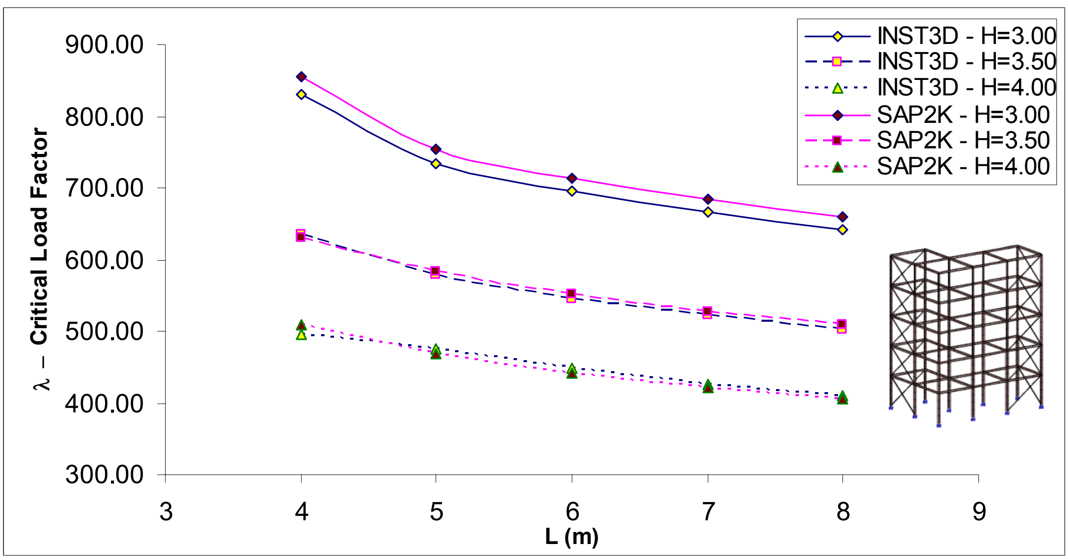

Figure 10: Parametric results of the critical load factor obtained by INST3D and SAP2000, for the braced frame with uneven nodal vertical loads λP

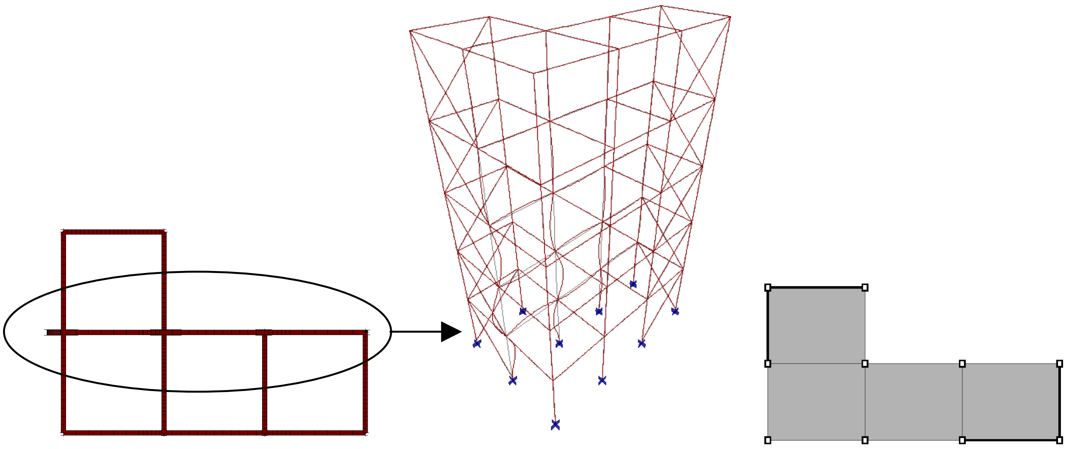

Figure 11: Asymmetric braced 3D frame (SAP 2000): deformation of 1st buckling mode for uneven nodal loads and plan view of the bracing distribution

5 Conclusions

Some exact and approximated methodologies for the study of the stability of two and three-dimensional frames have been detailed and used extensively. For a given calibration frame, the accuracy of developed and available software was ascertained. A parametric study of the load carrying capacity of a certain asymmetric 3D frame was successfully conducted, which also permitted to ascertain the two methodologies used. The critical load factors of these 3D frames were calculated, for both un-braced and braced configurations. It was concluded that the positioning of the braces could change considerably the structural behaviour and the carrying capacity of the 3D asymmetric frames with respect to the buckling load. The rational use of the bracing elements controls structural performance of distinct structural designs.

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