INVERTED PENDULUM VIRTUAL CONTROL LABORATORY

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Abstract:
This paper describes a tool for interactive learning that can be used to improve control systems design. The developed system is ready to use and allows testing different control methods. It can be used by students for problem solving and individual learning. The virtual control laboratory was implemented as a teaching aid during lectures on control systems. As there is no need to do any special programming or debugging, the students can focus on the control items. Classical control methods such as PID and State-Space approaches are available and gains can be tuned. A friendly appearance based on openGL 3D shows a simulation of the real word: A cart with an inverted pendulum is "bumped" with a force. The dynamic equations of motion for the control system are linearized assuming that the pendulum does not move more than a few degrees away from the vertical allowing to apply linear control methods. Although, the simulated system is realistic and based on a Dynamics Engine.

Keywords: Control education, Interactive programs, Simulation

1. INTRODUCTION

Tasks like modeling, identification, analysis and simulation are common in control system design (Kroumov *et al.*, 2003). The developed application supports the simulation of an inverted pendulum system with its controller. The simulated world behavior and graphics are based on open source platforms. The objective of this project is to allow the students to learn and test control systems in an easy way. Students need a "ready to go" and friendly application to learn by playing control theory (Saco *et al.*, 2002).

Some virtual laboratories exist on-line on the web (W. and Tzes, 1999), presenting advantages like low cost, friendly user and supporting simultaneous multiple users, as examples: 'Laboratorio Virtual: pendulo invertido' (*Universidad de Valladolid (UVA), Laboratorio virtual: pendulo invertido*, 2006) and Fuzzy Pendulum Demo proposed by the Integrated Reasoning Group of the Institute for Information Technology of the National Research Council of Canada (Institute, 2006).

Tools like Matlab are powerful but sometimes hard to program. A student oriented application is presented in order to select and tune the controller that closes the loop. The final result is a friendly 3D interface scene allowing the user to move around the world.

This paper is organized as follows: Initially, the inverted pendulum system and some applications are described. Then, in section 3, system modeling equations where a State-Space approach is used is described. Section 4 presents the world construction, behavior and it is also shown how students can interact with the graphical interface, being possible to control the system manually. Then, in section 5, the real time closed loop simulation where results for different control methods are shown is presented. Finally, section 6 rounds up with conclusions and future work.

2. INVERTED PENDULUM OVERVIEW

An inverted pendulum is a pendulum having its center of gravity located above its pivot point. An inverted pendulum is therefore inherently unstable. When its center of gravity is directly above the pivot point, it may remain static. If the pivot is static and the center of gravity is slightly displaced from the vertical position, the pendulum will not return to its original position but will tend to find a new equilibrium position such that its center of gravity is at its lowest possible position. A feedback control system that positions the pivot may be used to balance an inverted pendulum (Krakow, 2005).

Some applications of the inverted pendulum are presented in the next subsections.

2.1 Segway

An inverted pendulum is present in devices such as a unicycle and a Segway Human Transporter (Figure 1). In these devices, the pivot of the pendulum is the axle of a wheel or pair of wheels. In a unicycle the wheel is powered by the rider. In a Segway Human Transporter, the wheel is powered by an electric motor. Motion of the wheel, or pair of wheel, is controlled so that the pendulum is dynamically balanced (*Segway web page*, 2006).

Fig. 1. Segway Human Transporter

There is also another version of Segway, the Segway Robotic Mobility Platform (RMP), that is a modified version of the Segway Human Transporter (HT) designed to provide scientists and engineers a mobile base for use in robotics research (*Segway Robotic Mobility Platform*, 2006).

It has been well established within the literature about agents and robotics that simulation can be a powerful tool for speeding up the development cycle for robot control systems (Go *et al.*, 2004).

2.2 Rocket Navigation

The inverted pendulum control model is similar to the rocket launch (Ogata, 2002). It was believed that, in flight, the rocket would "hang" from the engine like a pendulum hanging from a pivot. The weight of the fuel tank would keep the rocket flying straight up as long as the fuel lasted. However, this belief is incorrect, such a rocket will never fly in a straight line and will always turn and crash into the ground soon after launch. This is what happened to Goddard's rocket (*Wikipedia - The free encyclopedia*, 2006). In the present work, the pendulum is free to move in plane xy and the cart is able to slide across x axle only.

Nowadays, Rockets use Inertial Navigation to solve this problem. Inertial navigation employs accelerometers and gyroscopes to determine speed and position. (*Japan Aerospace Exploration Agency (JAXA)*, 2006).

3. SYSTEM MODELING

Controlling the inverted pendulum is a classical problem in control laboratories because the pendulum dynamics is both nonlinear and unstable (Samad and Balas, 2003). The dynamic equations of motion for the control system are linearized, although the simulated system is realistic and based on a Dynamics Engine.

The inverted pendulum system consists of two moving parts:

- The pendulum
- The cart

The dynamic equations of motion for the system are linearized assuming that pendulum does not move more than a few degrees away from the vertical. One important issue to emphasize is that cart wheels dynamics is ignored, in other words, their mass and moment of inertia are considered null. The actuator and sensor dynamics are despised but a saturation nonlinearity can be applied to the actuator as shown in the section 5 where a fast dynamic controller is applied.

The modeled System is shown in Figure 2.

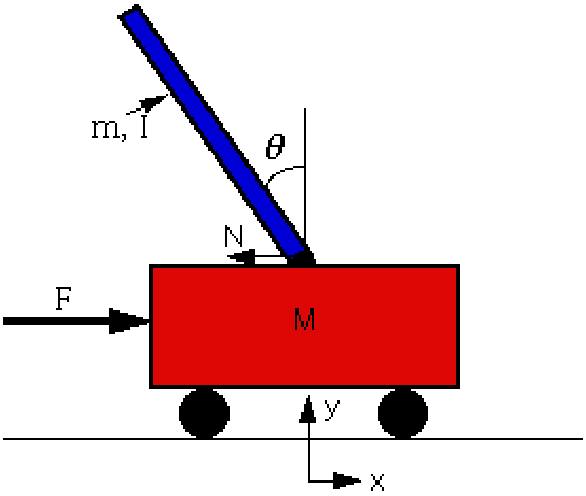

Fig. 2. Inverted pendulum system

The constants and variables for this case study are defined as follows:

- M mass of the cart 3 kg
- m mass of the pendulum 1.57 kg
- b friction of the cart 0.1 $N/m/s$
- l length to pendulum center of mass 2 m
- I inertia of the pendulum 8.37 kgm^2
- F force applied to the cart
- N reaction force applied to the cart
- x cart position coordinate
- θ pendulum angle from vertical
- g gravity acceleration 9.8 m/s^2

Where the pendulum moment of inertia is shown in equation (1).

$$I = \frac{1}{3} ml^2 \tag{1}$$

The well known Newton's second law of motion, equations (2) and (3), where F is the applied force, P the gravitational force and T the applied torque, allows to write the Lagrangian equations for the system described in equations (4) and (5) (*University of Michigan, Matlab control tutorial*, 2006).

$$\sum F = ma \tag{2}$$

$$\sum T = I\ddot{\theta} \tag{3}$$

$$F - N - b\dot{x} = M\ddot{x} \tag{4}$$

$$-Pl\sin(\theta) - Nl\cos(\theta) = I\ddot{\theta} \tag{5}$$

In order to apply linear control methods the system equations should be linearized, assuming that $\theta = \pi + \phi$, where ϕ represents a small angle from the vertical upward direction. Therefore, $\cos(\theta) \approx -1$, $\sin(\pi + \phi) \approx -\phi$, and $\dot{\theta}^2 \approx 0$ (*University of Michigan, Matlab control tutorial, 2006).*

The linearized system equations can be represented in State-Space form (Ribeiro, 2002), as shown in the next equations:

$$\begin{pmatrix} \dot{x} \\ \ddot{x} \\ \dot{\theta} \\ \ddot{\theta} \end{pmatrix} = A \begin{pmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta} \end{pmatrix} + B\,u \tag{6}$$

$$Y = C \begin{pmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta} \end{pmatrix} \tag{7}$$

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{-(I+ml^2)b}{\lambda} & \frac{m^2gl^2}{\lambda} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{-mlb}{\lambda} & \frac{mgl(M+m)}{\lambda} & 0 \end{pmatrix} \tag{8}$$

Where $\lambda = I(M+m) + Mml^2$.

$$B = \begin{pmatrix} 0 \\ \frac{(I+ml^2)b}{\lambda} \\ 0 \\ \frac{ml}{\lambda} \end{pmatrix} \tag{9}$$

$$C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \tag{10}$$

Resulting A in matrix (11) and B in matrix (12).

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & -0.0257 & 1.6925 & 0 \\ 0 & 0 & 0 & 1.0000 \\ 0 & -0.0055 & 2.4632 & 0 \end{pmatrix} \tag{11}$$

$$B = \begin{pmatrix} 0 \\ 0.2566 \\ 0 \\ 0.0550 \end{pmatrix} \tag{12}$$

4. WORLD CONSTRUCTION AND BEHAVIOR

Dynamic engines like Newton Dynamics, YADE-Yet Another Dynamic Engine and ODE-Open Dynamics Engine are powerful tools that allow programmers to create a physic world composed by objects connected through joints and simulate them. The simulation dynamics are based on articulated rigid body dynamics and includes forces and collision treatment. The ODE base objects for the pendulum and the cart are a parallelepiped and a cylinder respectively. The parallelepiped is connected through an hinge joint to one end of the cylinder, allowing it to move freely in the xy plane. It is also connected through a slider joint allowing movements along x axle.

4.1 Interactive environment issues

Some time ago, simulation tools were mainly text based. Nowadays, computer graphics development allow us to make great scenarios like 3D rendering, camera positioning and freedom textures captivating people's attention. An interactive educational system should have the following requirements (Kroumov *et al.*, 2003):

- Graphical user interface: easy HMI (human machine interface) reducing as much as possible text typing.
- People want to easily reach results without much manual reading. Most students have more success experimenting than reading from books only.
- Simple terms are a must: to understand the complex control methods, students should have access to basic terms and concepts.
- The simulation system should be easily accessible.

The presented system aims to help students in control engineering courses. Concepts like feedback, stability, PID Tunning and State-Space approach are applied.

Tools like Simulink, make the simulation possible but they are designed to cover a wide engineering area and students must memorize some commands in order to use these powerful Software. In the developed system there are no commands but buttons instead with a friendly 3D graphical environment, as shown in Figure 3.

The graphical based interactive interface, allows users to change PID gains and closed loop poles parameters for the State-Space feedback controller approach. Random noise in the measure is also introduced in order to evaluate the system robustness.

5. REAL TIME CLOSED LOOP SIMULATION

Assuming that simulation time step is much faster than dynamics, it is possible to assure that it is a continuous time model. A time step of 20 ms is enough to validate this approach. There are three closed-loop control methods that can be used in this project: the user can manually control the applied force to the cart, the PID control

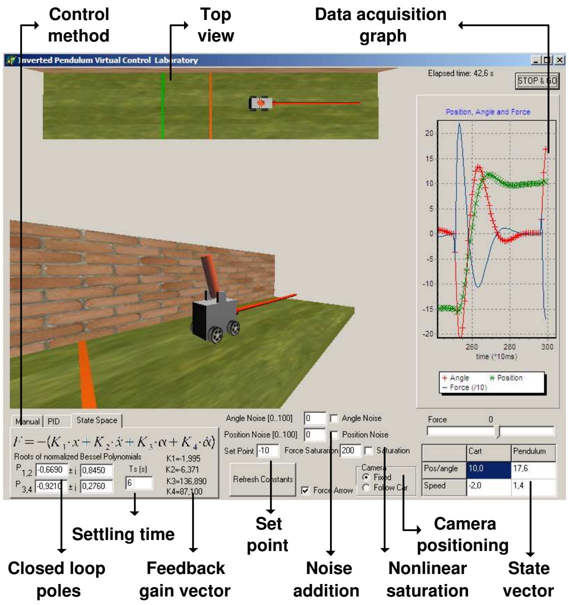

Fig. 3. Virtual Control Laboratory Screen Shot

where user can introduce proportional, integral and derivative gains and the State-Space feedback control where user can apply a pole placement approach or introducing the time settling, resulting in a feedback gain vector. A graphical time evolution shows the cart position, speed and pendulum angle and a zoom feature is also implemented in order to highlight some details. A top and a frontal view, where user can place the camera everywhere, shows the 3D real world. User can also define the set point and noise addition.

5.1 PID

The analog PID control has been used successfully in many industrial control systems for over half a century. The basic principle of the PID control scheme is to act on the variable to be manipulated trough a proper combination of three control actions: proportional control action (where the control is proportional to the error signal, which is the difference between the input and the feedback signal), integral control action (where the control action is proportional to the integral error signal) and derivative control action (where the control action is proportional to the derivative of the error signal). In its most simple form, PID involves three mathematical control functions working together: Proportional, Integral and Derivative, as shown in equation (13).

$$m(t) = Kp[e(t) + \frac{1}{Ti}\int_0^t e(t) + Td\frac{de(t)}{dt}] \quad (13)$$

As an example, is shown in Figure 4 the closed loop system response to a disturbance in the pendulum, where the cart position is not controlled.

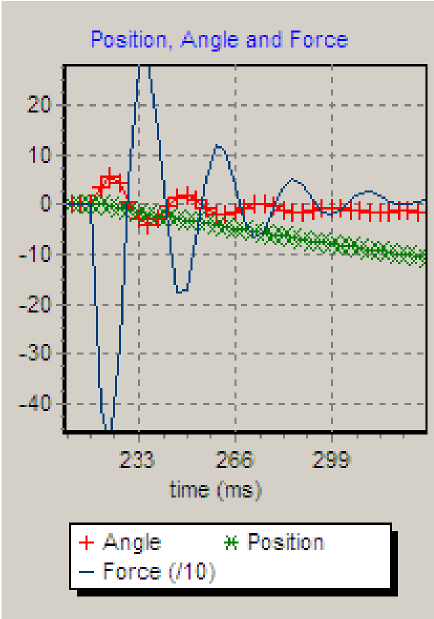

Fig. 4. PID closed loop system with Kp=8, Ki=0 and Kd=8 data graph

5.2 State-Space feedback controller

The force applied to the cart can be given by the state feedback vector presented in equation (14).

$$F = -\begin{pmatrix} K_1 & K_2 & K_3 & K_4 \end{pmatrix} \begin{pmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta} \end{pmatrix} \quad (14)$$

Having the system described in State-Space, the K values (closed loop gains) can be found by equation (15) (Vaccaro, 1995).

$$K = \begin{pmatrix} 0 & 0 & 0 & 1 \end{pmatrix} Q^{-1} \alpha(A) \quad (15)$$

Where $\alpha(s)$, the characteristic equation, presented in equation (17) and Q the controllability matrix, given by equation (16).

$$Q = \begin{pmatrix} B & AB & A^2B & A^3B \end{pmatrix} \quad (16)$$

$$\alpha(s) = (s - p_1)(s - p_2)(s - p_3)(s - p_4) \quad (17)$$

The desired closed loop poles are presented in equations (18) and (19).

$$poles_{1,2} = a \pm ib \quad (18)$$

$$poles_{3,4} = c \pm id \quad (19)$$

As result, the State-Space feedback vector is given by the next equations:

$$K_1 = -1.8553(c^2 + d^2)(b^2 + a^2)$$

$$K_2 = (3.7106a)d^2 + 3.7106c^2a +$$
$$3.7106\ \mathrm{b}^2c + 3.7106a^2c - 0.1$$

$$K_3 = (18.182 + 8.6560a^2 + 8.6560b^2)d^2$$
$$+72.727\ \mathrm{c}\ \mathrm{a} + 18.182\ \mathrm{c}^2 + 18.182b^2 + 18.182a^2$$
$$+8.6560\ \mathrm{b}^2c^2 + 8.6560a^2c^2 + 44.786$$

$$K_4 = (-17.312a)d^2 - 36.364a - 36.364c$$
$$-17.312\ \mathrm{c}^2a - 17.312cb^2 - 17.312ca^2$$

A State-Space fast dynamic controller, based on a fourth order Bessel prototype with a time settling of 4 seconds (Vaccaro, 1995), is shown in Figure 5, where a=−1.00, b=1.27, c=−1.38 and d=0.41 with the feedback gain vector represented by:

$$K = \begin{pmatrix} -10.09 & -21.27 & 278.20 & 185.51 \end{pmatrix}.$$

An example of nonlinear saturation feature of the input force is also shown.

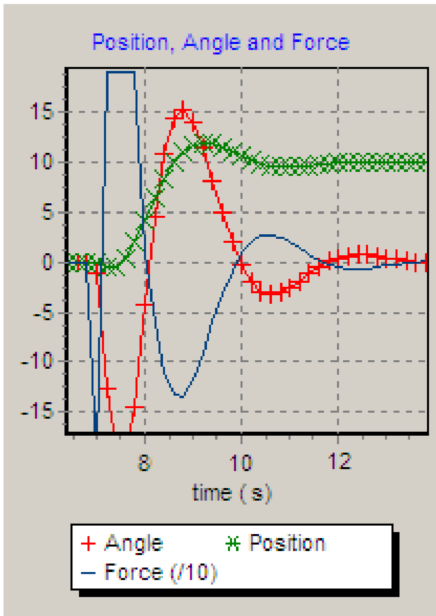

Fig. 5. State-Space fast dynamic controller with saturation data graph

A slower dynamic controller, based on a fourth order Bessel prototype with a time settling of 10 seconds (Vaccaro, 1995), presented in Figure 6, is implemented where a=−0.40, b=0.51, c=−0.55 and d=0.17, with the feedback gain vector represented by:

$$K = \begin{pmatrix} -0.26 & -1.46 & 75.80 & 41.03 \end{pmatrix}.$$

5.2.1. Noise disturbance feature Data acquisition in Physical Systems suffers problems such as signal noise, cumulative errors, electromagnetic interference and temperature drift.

Random noise addition in pendulum angle and cart position measurement allows to illustrate this

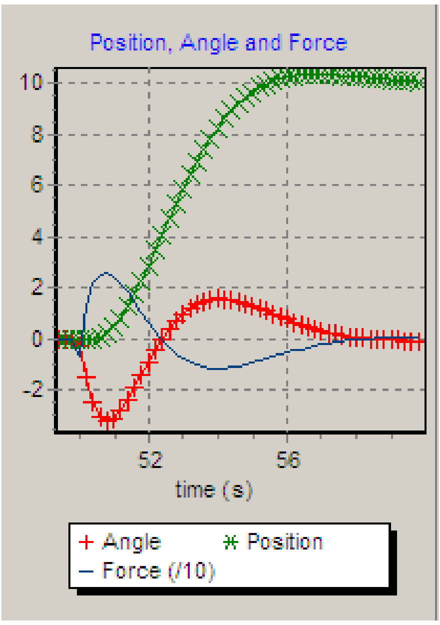

Fig. 6. State-Space slow dynamic controller data graph

problem. The Data acquisition is shown in Figure 7.

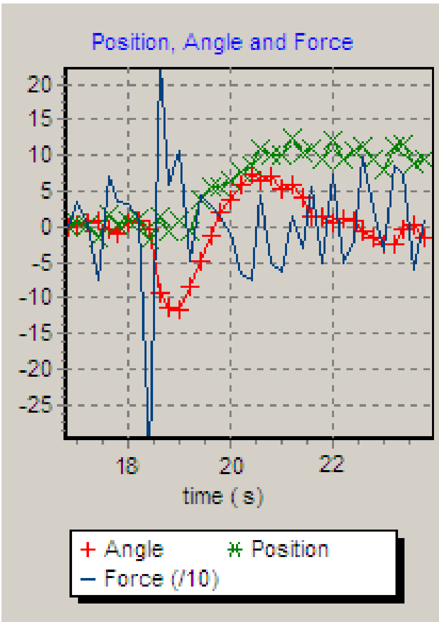

Fig. 7. State-Space controller data graph with noise

6. CONCLUSIONS AND FUTURE WORK

This paper has introduced an interactive learning tool for automatic control courses, where it is desired to control an inverted pendulum. The developed system allow students to focus on the control theory, helping them to improve their skills. The 3D visualization and animation effects are also an advantage to students for a better understanding of the physical systems. The real time graphic, where are shown pendulum angle, cart position and applied force, presents data in a way that can be easily decoded by students.

As future work, a two dimensional inverted pendulum, placed in an omnidirectional cart, is being developed.

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