Book of abstracts of the 7th International Conference
on Advanced Computational Methods
in Engineering, ACOMEN 2017
18–22 September 2017.

A Novel Calderón Preconditioner for the Simulation of Conductive and High-Dielectric Contrast Media

Michiel Gossye[1], Dries Vande Ginste[1] and Hendrik Rogier[1]

[1] *Electromagnetics Group, Department of Information Technology, Ghent University*

e-mails: Michiel.Gossye@UGent.be, Dries.VandeGinste@UGent.be, Hendrik.Rogier@UGent.be

Abstract

We propose a novel Calderón preconditioner, based on the Poincaré-Steklov operator, to solve scattering problems involving (lossy) conductive and high-dielectric contrast media. The resulting system matrix is well-conditioned, independent of the specific material characteristics of the scattering media, as demonstrated by a numerical example.

Key words: Calderón preconditioning, electromagnetic scattering, Poincaré-Steklov operator, high-dielectric contrast media, boundary integral equations.

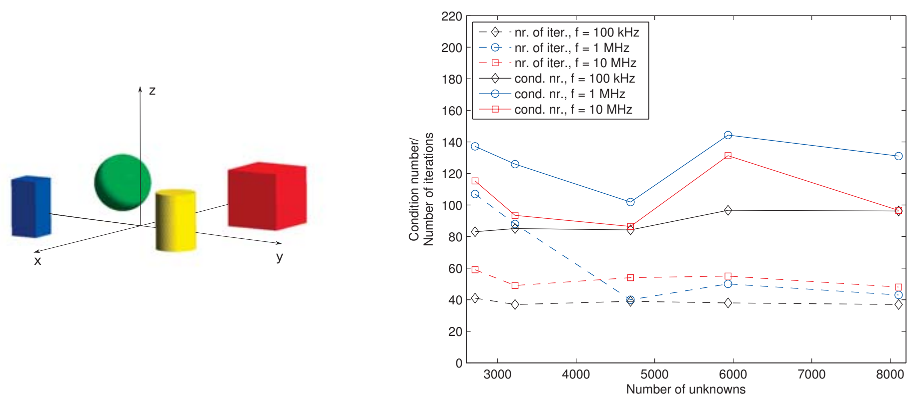
1 Formulation

The computational solution of electromagnetic scattering problems involving (piecewise) homogeneous structures can be obtained by the Method of Moments (MoM). The popularity of this method stems from the small system matrix dimensions when compared to other full-wave simulation algorithms. In general, this leads to a faster solution. However, the MoM system matrices notoriously suffer from dense-mesh and low-frequency breakdown, which worsens the convergence time and accuracy of the iterative solution. The introduction of a Calderón preconditioner in earlier research [1] solves this problem only partially, since the presence of conductive or high-dielectric contrast media still leads to bad conditioning [2]. For this reason, we introduce a Calderón preconditioned single-source equation that does not suffer from breakdown in these problematic cases.

The proposed method allows to model scattering problems with N bounded, disjunct domains $\mathcal{D}_l \in \mathbb{R}^3$, $l = 1,\dots,N$ with boundaries $\mathcal{S}_l$, characterized by a (complex) permittivity ε_l and permeability μ_l. These domains reside in a homogeneous background medium $\mathcal{D}_0$, defined by ε_0 and μ_0. We introduce an incident electromagnetic field $(\mathbf{e}^i, \mathbf{h}^i)$ with $e^{j\omega t}$ time dependency. In general, the field $(\mathbf{e}^s, \mathbf{h}^s)$, scattered by the media in $\mathcal{D}_l$, $l = 1,\dots,N$ cannot be determined analytically. Therefore, an equivalent problem is introduced, in which the medium inside each scattering domain is replaced by that of the exterior region. We impose fictitious magnetic currents $\mathbf{m}_l$, residing on $\mathcal{S}_l$, that generate the same scattered fields as in the original problem. This can be achieved by introducing a Poincaré-Steklov operator $\mathcal{P}_l$ that maps the tangential electric field on the tangential magnetic field at each boundary $\mathcal{S}_l$, and by imposing continuity of the tangential fields along $\mathcal{S}_l$. However, after discretization, the resulting matrix equation is ill-conditioned and computationally expensive to solve. Hence, a Calderón preconditioner based on the electric field integral operator [1] is introduced. It can be shown that the accumulation points of the eigenvalue distribution of the preconditioned matrix are given by $\frac{1}{2} \pm \frac{1}{2}\sqrt{\frac{\varepsilon_0}{\varepsilon_l}}j$ and $\frac{1}{2} \pm \frac{1}{2}\sqrt{\frac{\mu_l}{\mu_0}}j$. When the permeability remains bounded, the system is well conditioned regardless of dielectric contrast, as long as $\varepsilon_l \nrightarrow -\varepsilon_0$ and $\mu_l \nrightarrow -\mu_0$.

2 Numerical Results

First, the method was verified by comparing the scattered fields of a single sphere to the Mie series solution. Second, electromagnetic scattering at a configuration consisting of four objects is considered to numerically validate the novel Calderón preconditioner, see Fig. 1(a). From Fig. 1(b), it is clear that dense-mesh breakdown does not occur, despite the presence of high-dielectric and conductive media.



(a) Geometry of the scattering problem. (b) Conditioning and number of iterations.

Figure 1: Scattering at a configuration consisting of a sphere ($\epsilon_r = 4$, radius = 1 m), cube ($\epsilon_r = 15$, length = 2 m), cuboid ($\epsilon_r = 100$, height = 2 m, length = 1 m) and cylinder (copper, height = 2 m, radius = 0.5 m). In (a), the geometry is visualized. In (b), the conditioning and number of iterations to reach convergence (relative error $\leq 10^{-5}$) are given as a function of the number of unknowns, for different frequencies.

Acknowledgements

The authors thank the Research Foundation Flanders (FWO) for supporting this research.

References

[1] K. Cools, F. P. Andriulli and E. Michielssen, *A Calderón multiplicative preconditioner for the PMCHWT integral equation*, IEEE Trans. Antennas Propag. **59** (2011) 4579–4587.

[2] P. Ylä-Oijala and S. P. Kiminki, *Challenges in developing efficient Calderón preconditioners for resonating or high material contrast penetrable objects*, J. Comput. Appl. Math. **289** (2015) 296–305.