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A Leapfrog Alternating-Direction Hybrid Implicit-Explicit FDTD Method for Local Grid Refinement

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Abstract

We propose a novel curl splitting technique to enhance the alternating-direction finite-difference time-domain (ADI-FDTD) method, as such allowing for a higher resolution in one or two dimensions. As our advocated approach leverages a hybrid implicit-explicit (HIE) update scheme it is named "leapfrog ADHIE-FDTD". The hybridization yields a time step that is solely bounded by the spatial steps in preferred dimensions.

Key words: Alternating-direction finite-difference time-domain (ADI-FDTD), hybrid implicit-explicit (HIE), electromagnetic theory

1 Formulation

The standard leapfrog ADI-FDTD method relies on a smart way to split the curl in Maxwell's equations without breaking the symmetry [1, 2]. Here, we propose a new type of curl splitting, resulting in a HIE scheme. Compared to ADI-FDTD, the novel scheme allows resolving structures that are fine in one or two dimensions, whilst explicit updating is used for the remaining coarsely discretized dimension(s), leading to increased accuracy and computational speed-up. Compared to the standard leapfrog Yee-FDTD and owing to the implicitization, the ADHIE-FDTD features a less stringent stability limit, rendering it computationally very efficient.

Suppose we want to resolve an object that is thin along the x-axis. Then, we propose to eliminate the x-dependence from the Courant limit by splitting the curl $C = C_0 + C_1 + C_2$ into the following three components:

$$C_0 = \begin{bmatrix} 0 & -\partial z & \partial y \\ \partial z & 0 & 0 \\ -\partial y & 0 & 0 \end{bmatrix}, \qquad C_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \partial x & 0 \end{bmatrix}, \qquad C_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -\partial x \\ 0 & 0 & 0 \end{bmatrix}. \tag{1}$$

The occurring derivatives are discretized by central differences on the conventional Yee lattice. The novel leapfrog ADHIE-FDTD update scheme is given by

$$\begin{bmatrix} \left(1 + \frac{\sigma Z \Delta\tau}{2}\right) I + \frac{\Delta\tau^2}{4\alpha} C_1 C_1^T & -\Delta\tau\, C \\ 0 & I + \frac{\Delta\tau^2}{4\alpha} C_2^T C_2 \end{bmatrix} \begin{bmatrix} Z^{-1}\mathbf{e}^{n+1} \\ \mathbf{h}^{n+0.5} \end{bmatrix} = \begin{bmatrix} \left(1 - \frac{\sigma Z \Delta\tau}{2}\right) I + \frac{\Delta\tau^2}{4\alpha} C_1 C_1^T & 0 \\ -\Delta\tau\, C^T & I + \frac{\Delta\tau^2}{4\alpha} C_2^T C_2 \end{bmatrix} \begin{bmatrix} Z^{-1}\mathbf{e}^{n} \\ \mathbf{h}^{n-0.5} \end{bmatrix}, \tag{2}$$

where σ is the electrical conductivity, $Z = (\mu/\epsilon)^{1/2}$ the wave impedance, $\Delta\tau = c\Delta t$ the time step rescaled by the phase velocity $c = (\epsilon\mu)^{-1/2}$, and α is a tunable parameter.

It can be proven, following the reasoning described in [3], that a *sufficient* condition for numerical stability of the proposed scheme is given by

$$\Delta\tau < \frac{1-\alpha}{\sqrt{\frac{1}{\Delta y^2} + \frac{1}{\Delta z^2}}}\ , \quad \alpha \in]0, 1[\ . \tag{3}$$

Owing to the curl splitting (1), Δx has been eliminated from the stability limit (3). From (2) and (3), it is now clear that the parameter α controls the trade-off between accuracy and simulation speed. One the one hand, the smaller α, the larger the perturbation introduced by ADI and the larger the numerical error will be. On the other hand, a smaller α yields a larger maximum allowed time step $\Delta\tau$.

2 Results

To test the advocated method, we simulate the shielding effectiveness (SE) [4] of a thin metallic sheet of infinite dimensions placed in the yz-plane. The sheet has a thickness of 10 μm in the x-dimension and a conductivity $\sigma = 10^7$. It is illuminated by a z-oriented electric dipole placed at a distance of 150 mm before the shield. The simulation domain is terminated by means of perfectly matched layers (PML) and the grid contains $16 \times 16 \times 164$ cells. In ADHIE-FDTD, the cell size is given by $\Delta x = \Delta y = \Delta z = 1.875$ mm everywhere, except in the thin sheet, where $\Delta x_{\text{sheet}} = 0.15873$ μm. Consequently, locally, the refinement ratio along the x-axis is very large, i.e. $\Delta x/\Delta x_{\text{sheet}} > 10^4$.

When sweeping the parameter α from 0.25 to 0.90, we observe that the relative accuracy on the SE at a frequency of 10 GHz varies from 5% to a few tens of one percent, where standard Yee-FDTD was used as a reference method. This validates the accuracy of our technique. Moreover, whereas in Yee-FDTD, the time step has to be chosen at $c\Delta t = 159$ nm to resolve the thin sheet, in our ADHIE-FDTD this time step can be chosen between 0.1 mm up to 1 mm. As the required CPU time scales as $1/\Delta t$, the novel scheme is clearly much faster than traditional methods.

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