A joint selective maintenance and multiple repair-person assignment problem

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***Abstract*—It is common in many industrial settings to have large multicomponent systems perform consecutive missions interspersed with finite breaks during which a limited set of component repairs or replacements can be carried out due to limited time, budget, or resources. The decision maker then has to decide which components to repair in order to guarantee a pre-specified performance level. This is known as the selective maintenance problem. This paper introduces a novel variant of the selective maintenance problem for a multi-component system performing consecutive missions separated by scheduled finite breaks by specifically modelling the assignment of the repair tasks to multiple repair-persons. Current models in the literature usually assume that only one repair channel is available or that the assignment optimization can be done at a subsequent stage. A novel integrated non-linear programming formulation is proposed and optimally solved. Numerical experiments show the benefits of jointly carrying out the assignment of the tasks to repair-persons and the selection of the components to be repaired.**

I. Introduction

It is common in many industrial settings to have large and complex multicomponent systems perform consecutive missions interspersed with finite breaks during which a limited set of component repairs or replacements can be carried out due to limited time, budget, or resources. In the airline industry for example, aircraft are subjected to overnight maintenance at night during a break interval typically varying from 2 to 8 hours. To prepare the system to successfully complete its next mission, its components must be properly maintained during the scheduled intermission break. Because of the built-in redundancies, and due to the limited duration of the scheduled breaks and scarce maintenance resources, only a limited set of components have and can be maintained during the breaks. It is therefore necessary to identify an optimal subset of components to maintain to meet the predetermined reliability level required for the next mission. In the maintenance literature, this is known as the selective maintenance problem (SMP).

The selective maintenance problem was initially introduced by Rice at al. [1]. The authors considered a series-parallel system in which subsystems are made of independent and identically distributed $(i.i.d)$ components with constant failure rate (CFR). The replacement of components at failure is the only maintenance option. To overcome the restrictive hypothesis of identical subsystem components in [1], Cassady et al. [2] developed a more general framework for selective maintenance formulation.

Cassady et al. [3] studied the SMP in a series-parallel system, where the components have Weibull distributed lifetimes. For each component, any of three potential maintenance actions can be carried out: minimal repair, corrective replacement of a failed component, and preventive replacement of a working component. The resulting SMP is solved using an enumeration method. However, enumeration solution methods become rapidly cumbersome when the number of the system components increases. To deal with the combinatorial complexity arising from large-size systems, four improved enumeration procedures are proposed in Rajagopalan and Cassady [4] to reduce the computation times. An exact method based on the branch-and-bound procedure and a Tabu search based algorithm are proposed in Lust et al. [5]. Khatab et al. [6] proposed two heuristic methods, adapted from those used to solve the redundancy allocation problem in [7], [8], [9]. Maaroufi et al. [10] studied the SMP for a system where some components are subject to both global failure propagation and isolation.

The development of SMP models has since increased and dealt with extensions such as imperfect maintenance, multistate systems. Imperfect maintenance in the selective maintenance setting is addressed by Liu and Huang [11], where the age reduction coefficient approach [12] is used to model imperfect maintenance. An imperfect selective maintenance model was also developed by Zhu et al. [13] and applied to a machining line system. Panday et al. [14] also studied the selective maintenance problem for binary systems under imperfect maintenance using the hybrid hazard rate approach introduced in [15]. In [11] and [14], a set of maintenance levels, ranging from minimal repair to replacement, are used to improve the reliability of a system component. Liu and Zhang [11] and Zhu et al. [13], the only parameter that determines the improvement in the component's health is the age reduction coefficient. In [14], however, both the age reduction coefficient and the adjustment coefficient impact the component's health. A more recent work [16] studied the selective maintenance problem when the quality of imperfect maintenance is stochastic. A nonlinear and stochastic optimization problem was proposed and solved for a series-parallel system.

SMPs have also been investigated for multi-state systems (MSS) in Chen et al. [17] where system components and the overall system may be found in more than two possible states. Liu and Huang [11] considered MSS where components have two operating states and are subjected to imperfect maintenance that can bring the condition of a component to an intermediate degradation level. Panday et al. [18] investigated the SMP for a MSS where the functioning of each component is modeled as a continuous-time Markov chain with more than two states. Dao et al. [19], [20] studied the SMP under economic and stochastic dependencies among MSS components. In [21], several levels of imperfect maintenance are considered and the system is composed of multi-state components modeled as by Pandey et al. [18]. In a recent paper, Dao and Zuo [22] investigated the SMP in a MSS with structural relationships between components.

All papers surveyed above do not consider the very common case where multiple repair-persons are available to carry out the maintenance actions. In the airline industry, when an aircraft undergoes overnight maintenance, several repair-persons are usually available and will be utilized as needed to inspect and perform repairs. Furthermore, given the size of the systems it is possible for several members of the repair crew to work simultaneously on the plane. Up to two crew members can be in the cockpit to deal with the avionics, while several others can take care of the structural components, the wings, the engines, etc. It is therefore interesting to jointly optimize the selective maintenance decisions and the assignment of the repairs to the repair crews.

The remainder of the article is organized as follows. Section II describes the multicomponent system under consideration and the resources available to carry out the repairs. In this section, we also present the development of the mathematical formulation of the joint selective maintenance and multiple repair-person assignment problem. A solution procedure is proposed in Section III along with several numerical experiments and the discussion of their results. Conclusions are drawn and future extensions discussed in the last section.

II. System description and problem formulation

Without loss of generality, the selective maintenance problem addressed in the present work concerns a series-parallel system S composed of n series subsystems S_i $(i = 1, \ldots, n)$ each of which is composed of N_i s-independent, and possibly, non-identical components/parts P_{ij} $(j = 1, \ldots, N_i)$ arranged in parallel. The system is assumed to have just finished a mission and then, turned off during the scheduled break of finite length and becomes available for possible maintenance activities. The system is thereafter used to execute the next mission of a given duration. The duration of the scheduled break is denoted as D and the duration of the next mission is denoted by U.

At the end of the current mission (i.e., at the beginning of the current break), the state variable X_{ij} describes the status of the component P_{ij}. Similarly, the state variable z_{ij} describes the status P_{ij} at the end of the current break (i.e., at the beginning of the next mission). These two state variables are then formally defined as:

$$X_{ij} : \begin{cases} 1 & \text{if } P_{ij} \text{ is working at the start of the break} \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

$$z_{ij} : \begin{cases} 1 & \text{if } P_{ij} \text{ is working at the end of the break} \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

The following assumptions are considered in this paper:

1) The system consists of multiple, repairable binary components (the components and the system are either functioning or failed).
2) During the break, system components do not age, i.e. the age of a component is operation-dependent.
3) No maintenance activity is allowed during the mission. Maintenance activities are allowed only during the break.
4) Multiple components can be worked on simultaneously without repairpersons colliding. This assumption is reasonable in large multicomponent systems and with modular design as previously discussed in the case of an aircraft.
5) The duration of the break is longer than the longest component repair time. This is reasonable as the contrary would automatically make the component non-eligible and out of consideration for optimization.

The following notation is used in the mathematical formulation of the RCSMP.

Indices

i : Index of subsystems, $\quad i = 1, ..., s$
j : Index of parts in subsystem i, $\quad j = 1, ..., N_i$
r : Index of potential repair-persons, $\quad r = 1, ..., m$

Parameters

D : Duration of the break
R_0 : Minimum required reliability level
k : Variable labour cost per unit of time
K_r : Fixed cost for hiring/utilizing repair-person r
t_{ijr} : Time to repair part P_{ij} by repair-person r
c_{ij} : Cost to repair/replace part to P_{ij}
X_{ij} : Status of P_{ij} at the start of the break
R_{ij} : Reliability of P_{ij} at the start of the break
m : Number of repair-persons available

Decision variables

x_{ijr} : Binary, $= 1$ if P_{ij} is repaired by repair-person r; 0, otherwise
z_{ij} : Binary, $= 1$ if P_{ij} is working at the end of the break; 0, otherwise
y_r : Binary, $= 1$ is repair-person r is hired/utilized; 0, otherwise.

The mathematical formulation for the minimization of the total cost (model 1) is as follows:

$$\text{Min } \mathcal{C} = \sum_{i=1}^{s} \sum_{j=1}^{N_i} \left[c_{ij} \left(z_{ij} - X_{ij} \right) + k \sum_{r=1}^{m} t_{ijr} x_{ijr} \right] + \sum_{r=1}^{m} K_r y_r$$

Subject to:

$$\sum_{i=1}^{s} ln \left[1 - \prod_{j=1}^{N_i} \left(1 - R_{ij}\, z_{ij}\right) \right] \geq lnR_0 \qquad \forall i, j \qquad (3)$$

$$z_{ij} = X_{ij} + \sum_{r=1}^{m} \left(1 - X_{ij}\right) x_{ijr} \qquad \forall i, j \qquad (4)$$

$$x_{ijr} \leq 1 - X_{ij} \qquad \forall i, j, r \qquad (5)$$

$$\sum_{i=1}^{s} \sum_{j=1}^{N_i} t_{ijr}\, x_{ijr} \leq D\, y_r \qquad \forall r \qquad (6)$$

$$\sum_{r=1}^{m} x_{ijr} \leq 1 \qquad \forall i, j \qquad (7)$$

$$\sum_{j=1}^{N_i} z_{ij} \geq 1 \qquad \forall i \qquad (8)$$

$$x_{ijr},\ z_{ij},\ y_r \in \{0, 1\} \qquad \forall i, j, r. \qquad (9)$$

The objective function is the sum of three terms. The first term is the total cost to repair/replace the components that have been selected to undergo maintenance. The second term is the related labor cost. The third term is the cost to hire the repair-persons needed to carry out the repairs.

Constraint (3) ensures that the system reliability is at least equal to the minimum required reliability R_0. Since the system under consideration is a series arrangement of subsystems, the system reliability during the following mission is given by

$$R_s(U) = \prod_{i=1}^{s} (1 - \prod_{j=1}^{N_i} (1 - R_{ij}\, z_{ij})).$$

Constraints (4) defines the status of $P_{i,j}$ at the end of the break, which is its status at the start of the break plus any change that occurred during the break. A working $P_{i,j}$ does not need replacement. A failed $P_{i,j}$ may or may not be replaced.

Constraint (5) ensures that only failed parts are candidates for replacement/repair.

Constraint (6) ensures that if a repair-person is utilized then their total repair times is no more than the break duration. If the repair-person is not hired, then they cannot perform any repair.

Constraint (7) guarantees that each component will receive at most one repair.

Constraint (8) ensures that each subsystem has at least one working component at the start of the next mission. This constraint is introduced to lift the commonly used assumption that requires all subsystems to be operating at the end of the previous mission.

The last constraint defines the binary decision variables used in the formulation.

The above mathematical formulation contains one nonlinear constraint (3). It is optimally solved using the KNitro 9 solver [28] in MPL 5.0 [29]. In the following section, numerical experiments are carried out to show that our model yields valid maintenance decisions.

Note that an alternative formulation (model 2), if a maintenance budget $\mathcal{C}_0$ is pre-specified, is:

Max $R_s = \prod_{i=1}^{s}(1 - \prod_{j=1}^{N_i}(1 - R_{ij}\, z_{ij}))$

Subject to:

$$\sum_{i=1}^{s}\sum_{j=1}^{N_i}\left[c_{ij}\left(z_{ij} - X_{ij}\right) + k\sum_{r=1}^{m} t_{ijr}x_{ijr}\right] + \sum_{r=1}^{m} K_r y_r \le \mathcal{C}_0$$

$$z_{ij} = X_{ij} + \sum_{r=1}^{m}\left(1 - X_{ij}\right)x_{ijr} \qquad \forall i,j$$

$$x_{ijr} \le 1 - X_{ij} \qquad \forall i,j,r$$

$$\sum_{i=1}^{s}\sum_{j=1}^{N_i} t_{ijr}\, x_{ijr} \le D\, y_r \qquad \forall r$$

$$\sum_{r=1}^{m} x_{ijr} \le 1 \qquad \forall i,j$$

$$\sum_{j=1}^{N_i} z_{ij} \ge 1 \qquad \forall i$$

$$x_{ijr},\ z_{ij},\ y_r \in \{0,1\} \qquad \forall i,j,r.$$

III. Numerical examples

In this section, three sets of numerical experiments are conducted. Table I lists all the parameters used. These are the same ones used by Cassady et al. [2]. Because, we are introducing multiple repair-persons instead of one as considered by [2], we have created 3 additional columns with repair times. Repair-persons 2 and 3 are considered to have standard skills and have the same repair times as in [2]. Repair-person 1 is more skilled, therefore their repair times are shorter but they are more expensive to hire and use. Repair-person 4 would be equivalent to a rookie or trainee. Thus, their repair times are longer than the standard ones but they are cheaper to hire and use.

TABLE I. Parameter values adapted from Cassady [2]

Part	**Reliability**	**Time to repair per repair-person**				**Unit cost**	**Status**
$P_{i,j}$	$R_{i,j}$	1	2	3	4	$c_{i,j}$	$S_{i,j}$
$P_{1,1}$	0.80	6	7	7	8	2	1
$P_{1,2}$	0.70	3	4	4	5	3	0
$P_{1,3}$	0.85	5	6	6	7	1	1
$P_{2,1}$	0.65	2	3	3	4	4	0
$P_{2,2}$	0.50	1	2	2	3	2	0
$P_{2,3}$	0.70	5	6	6	7	1	1
$P_{2,4}$	0.75	6	7	7	8	3	1
$P_{2,5}$	0.60	3	4	4	5	4	0
$P_{3,1}$	0.55	4	5	5	6	3	1
$P_{3,2}$	0.75	2	3	3	4	5	1
$P_{3,3}$	0.60	6	7	7	8	2	0
$P_{3,4}$	0.70	3	4	4	5	2	0

A. Experiment #1: Model 1 - All repair-persons have the same skills

For this experiment all repair-persons have the same standard skills and costs. The results in Table II are obtained for $K_r = 60$, $k_r = 4$, $t_{ijr} = t_{ij2}$, and $D = 11$. This is equivalent to the first experiment in Cassady et al. [2] when $R_0 = 0.9475$. Our results are identical to Cassady et al. [2] for that case: one repair-person has to be hired and the following components have to be replaced: $P_{1,2}$, $P_{2,1}$, $P_{2,5}$ and $P_{3,4}$. The results table also lists the computation times (CPUt) in seconds, which are negligible.

As the required minimum reliability increases, the number of components to be replaced increases and more repair-persons are needed.

TABLE II. Results for different values of R_0

R_0	m^*	CPUt	$\mathcal{C}$($)	Components replaced
0.8	1	0.14s	66	$P_{2,2}$
0.9	1	0.17s	80	$P_{2,2}$; $P_{3,4}$
0.9475	1	0.25s	117	$P_{1,2}$; $P_{2,1}$; $P_{2,5}$; $P_{3,4}$
0.97	2	1.88s	209	Rep.1: $P_{2,1}$; $P_{3,3}$; $P_{3,4}$ Rep.2: $P_{1,2}$; $P_{2,2}$; $P_{2,5}$

B. Experiment #2: Model 1 - Repair-persons have different skills

For this experiment, the repair-persons have different skills as listed in Table I. The results in Tables III and IV are obtained respectively for $R_0 = 0.9475$ and $R_0 = 0.97$. The duration of the break is varied. The other parameter values are: $K_1 = 60$, $K_2 = K_3 = 50$, $K_4 = 30$, $k_1 = 4$, $k_2 = k_3 = 2$, and $k_4 = 1$.

TABLE III. Results when varying D - Case of $R_0 = 0.9475$

D	m^*	CPUt	Repair-persons				$\mathcal{C}$($)	Parts replaced
			1	2	3	4		
11	1	0.25s			✓		117	Rep.3: $P_{1,2}$; $P_{2,1}$; $P_{2,5}$; $P_{3,4}$
9	2	0.74s			✓	✓	118	Rep.3: $P_{1,2}$; $P_{2,5}$ Rep.4: $P_{2,1}$; $P_{3,4}$
8	2	0.89s	✓			✓	140	Rep.1: $P_{1,2}$; $P_{2,1}$; $P_{2,5}$ Rep.4: $P_{3,4}$

TABLE IV. Results when varying D - Case of $R_0 = 0.97$

D	m^*	CPUt	Repair-persons				$\mathcal{C}$($)	Parts replaced
			1	2	3	4		
11	2	0.79s	✓			✓	162	Rep.1: $P_{1,2}$; $P_{2,1}$; $P_{2,5}$; $P_{3,4}$ Rep.4: $P_{2,2}$; $P_{3,3}$
8	3	1.13s	✓		✓	✓	205	Rep.1: $P_{2,1}$; $P_{2,2}$; $P_{2,5}$ Rep.3: $P_{1,2}$; $P_{3,4}$ Rep.4: $P_{3,3}$

The results in Tables III and IV show that when the duration of the break reduces, more repair-persons are needed. At first, the trainee is used to help the standard employee. At some point, the skilled repair-person is utilized to help speed up the repairs and make all required repairs fit in the interval of time available. Here again, we see that computation times are very small, showing that the joint selective maintenance and assignment problem is not more complex to solve with a proper formulation.

C. Experiment #3: Model 2 - Repair-persons have different skills

This experiment is designed to maximize the system reliability when a predetermined budget $\mathcal{C}_0$ is made available to the decision maker. Solving model 2 using the parameter values from Table I and varying the values of $\mathcal{C}_0$, the results in Table V are obtained for $D = 8$ units of time.

TABLE V. RESULTS FOR VARYING MAINTENANCE BUDGET $\mathcal{C}_0$

$\mathcal{C}_0$	m^*	CPUt	Repair persons 1	2	3	4	R^*	Parts replaced by
205	3	0.23s	✓	✓		✓	0.972	R.1: $P_{1,2}$; $P_{2,1}$; $P_{2,2}$ R.2: $P_{2,5}$; $P_{3,4}$ R.4: $P_{3,3}$
200	3	0.43s		✓	✓	✓	0.967	R.2: $P_{1,2}$; $P_{2,1}$ R.3: $P_{2,5}$; $P_{3,4}$ R.4: $P_{3,3}$
150	2	0.11s	✓			✓	0.952	R.1: $P_{1,2}$; $P_{2,1}$; $P_{3,4}$ R.4: $P_{3,3}$
125	2	0.26s			✓	✓	0.945	R.3: $P_{1,2}$; $P_{3,4}$ R.4: $P_{2,1}$; $P_{2,2}$
100	1	0.23s	✓				0.925	R.1: $P_{2,1}$; $P_{2,2}$; $P_{3,4}$
70	1	0.07s		✓			0.913	R.2: $P_{2,1}$; $P_{3,4}$
60	1	0.11s				✓	0.902	R.4: $P_{2,2}$; $P_{3,4}$

A clear trend can be seen from the above results. As the budget decreases, the number of repair-persons that can be hired and utilized decreases along with the number of components that can be replaced within the duration of the break. Thus, the maximum achievable reliability decreases with the budget.

When the budget allows it, the model will always give priority to the skilled repair-person as they are capable to repair more components in the fixed repair window. Additional repair-persons are added as permitted by the budget to complement the work of the skilled repair-person.

This model is capable of finding the best trade-offs between component costs, repair durations and labor costs to achieve the highest reliability possible while performing the assignment of repairs to workers.

IV. Conclusion

In this paper, we introduced a novel variant of the selective maintenance problem for a multi-component system performing consecutive missions separated by scheduled finite breaks by specifically modelling the assignment of the repair tasks to multiple repair-persons. Current models in the literature usually assume that only one repair channel is available or that the assignment optimization can be done at a subsequent stage. A novel integrated nonlinear programming formulation was proposed and optimally solved. Numerical experiments show the benefits of jointly carrying out the assignment of the tasks to repair-persons and the selection of the components to be repaired.

Future extensions that the authors are working on include a generalization to k-out-of-n subsystems with imperfect maintenance and repair-person eligibility constraints on the subsystems. Most models study the selective maintenance problem with reliability as the performance indicator. It would be of great value to consider system availability. Therefore, we are planning to study the trade-offs between system availability and the hiring of repair-persons using a bi-objective optimization model.

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