

**PROCEEDINGS OF THE 15TH INTERNATIONAL CONFERENCE OF DEA  
DEA2017 JUNE 2017, UNIVERSITY OF ECONOMICS, PRAGUE, CZECH REPUBLIC**



# **RECENT APPLICATIONS OF DATA ENVELOPMENT ANALYSIS**

Proceedings of the 15<sup>th</sup> International Conference of DEA

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**Title:** Recent Applications of Data Envelopment Analysis

**Subtitle (series):** Proceedings of the 15<sup>th</sup> International Conference on Data Envelopment Analysis

**Venue:** DEA2017, June 2017, University of Economics, Prague, Czech Republic.

**Edited by:** Ali Emrouznejad, Josef Jablonský, Rajiv Banker and Mehdi Toloo

**Date of Publication:** October 2017

**ISBN:** 978 1 85449 433 7

**Citation:** Emrouznejad, A., J. Jablonský, R. Banker and M. Toloo (2017), Recent Applications of Data Envelopment Analysis: Proceedings of the 15<sup>th</sup> International Conference of DEA, June 2017, University of Economics, Prague, Czech Republic, ISBN: 978 1 85449 433 7.

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# **Recent Applications of Data Envelopment Analysis**

Proceedings of the 15<sup>th</sup> International Conference of DEA, June 2017,  
University of Economics, Prague, Czech

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**October 2017**

**ISBN: 978 1 85449 433 7**

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## **PREFACE: A message from the local Organizers**

It is a great pleasure for me to welcome you on behalf of the Organising Committee, University of Economics, Prague, and the Czech Society of Operational Research to the 15th International Conference on Data Envelopment Analysis. This year the DEA conference takes place at the University of Economics in Prague, which is the largest school of economics and business in the Czech Republic, consisting of six faculties, and serving more than fifteen thousand students.

The high number of conference participants (almost 250) from more than 50 countries over the world shows that data envelopment analysis is an attractive scientific discipline with many new theoretical developments and practical applications. I believe that you find interesting the conference programme, including tutorials, plenary talks, invited and contributed parallel sessions as well as social events and tours. I hope that the conference can be the opportunity to meet colleagues, friends and establish new contacts for future research for all participants. Enjoy your time in the beautiful historical city of Prague.

I would like to express my personal thanks to all members of the Scientific and Organising Committees for their work in preparation of the conference programme.

**Josef Jablonský**

Chair of the Organizing Committee



# A DEA BASED FRAMEWORK FOR PROPOSED MERGER OF PUBLIC SECTOR BANKS IN INDIA

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## ABSTRACT

*The purpose of this paper is to suggest a framework for proposed merger of Public Sector Banks (PSBs) as initiated by Govt. of India to bolster the state-owned banks' capital base. The paper conducts two layers measurement of technical efficiency of PSBs before and after their hypothetical merger using DEA. The first layer consists of measuring technical efficiency before and after hypothetical merger of PSBs in alignment with Narasimham Committee's suggestion. 27 PSBs were merged hypothetically and reduced to 8 based on the premise of restructuring of weak bank by a big asset sized bank. The results indicate that the efficiency of large banks was relegated by weak banks after forced merger. In second layer we tried to consolidate banks according to their returns to scale (RTS) status. The banks in a particular RTS category are merged among themselves based on their capital adequacy. The results indicate that banks in increasing return to scale (IRS) and constant return to scale (CRS) category remained same after merger. Banks in decreasing return to scale (DRS) category upgraded to CRS and IRS category. Two banks in DRS category remained in same position even after merger.*

**Keywords:** DEA; Public Sector Banks; Merger & Acquisition (M&A); Returns to Scale (RTS); Bank Asset; Bank Consolidation

## INTRODUCTION

The financial sector, especially, the banking industry has undergone significant transformation in India since the onset of economic reform process in 1991. However, banking system continues to be dominated by Public Sector Banks (PSBs) which still have more than 70 per cent market share of the banking system assets. The PSBs follow similar organizational structure, business model and ownership structure. Therefore, it has been argued that a consolidation among PSBs with similar nature may result in benefits in terms of economies of scale and scope.

Narasimham Committee Report in 1991 put forward the broad pattern towards which the banking structure should evolve. The Report recommended a three tier banking structure in India through establishment of three large banks with international presence, eight to ten national banks and a large number of regional and local banks. Narasimham Committee Report in 1998 again focused on mergers of PSBs which according to their view should emanate from the managements of banks with the Government as the common shareholder playing a supportive role.

In the budget speech for 2016-17 in Parliament, Finance Minister Arun Jaitley mentioned that “our Public Sector Banks will have to be strong and competitive. The Bank Board Bureau will be operationalized during 2016-17 and a roadmap for consolidation of Public Sector Banks will be spelt out”. In pursuance of Govt.'s initiative to consolidate banks (Indradhanush action plan) cabinet of Indian govt. approved acquisition of five subsidiary banks by SBI on 16<sup>th</sup> August 2016.

To the best of the authors' knowledge, there are no previous studies in Indian context that deals with identifying exact basis of consolidation in order to improve cost and technical efficiency of merged entity. The paper makes an effort to identify the correct basis of consolidation by canvassing hypothetical merger situation using Data Envelopment Analysis (DEA). The main objectives of this paper are to: (a) appraise various basis of consolidation by portraying hypothetical merger situations (b) *identify* the basis of consolidation which may ensure the cost and technical efficiency of merged entity.

The rest of the paper is organised as follows: Section 2 discusses about a brief literature review on M&A of banks. Section 3 discusses about DEA and the models used in this paper and Section 4 discusses the selection of input and output variables and methodology used in the study. Section 5 elucidates selection of input and output variables while section 6 discusses the empirical results of the study and the final section summarises the findings and the conclusions from the study.

The paper is believed to be the first attempt to empirically examine scale efficiencies of banks with an intention to suggest proper merger policy that may be undertaken by Govt. of India.

## **METHODOLOGY**

We have made an effort to analyze the initiative of the govt. to consolidate the PSBs by portraying a hypothetical merger situation. The hypothetical merger situation is based on the premise of forced merger with an intention of restructuring of weak banks as in the case of recent decision of Govt. of India to merge SBI & its associate banks. There were innumerable instances of (e.g. Punjab National Bank acquired New Bank of India, Bank of India acquired Bank of Karad Ltd., State Bank of India acquired Kashinath Seth Bank, Oriental Bank of Commerce acquired Punjab Co-operative Bank Ltd and Global trust Bank., Union Bank of India acquired Sikkim Bank Ltd., Bank of Baroda acquired Benaras State Bank Ltd., IDBI Bank acquired United western Bank etc.) acquiring weak bank by a PSB. The assets of PSBs barring SBI were listed in descending order while SBI along with its associates were merged separately. Total number of PSBs except SBI & associates stood at 20. Following Narasimham committee I recommendation of establishing eight to ten national banks, 20 PSBs were reduced to 7 by way of consolidation. Since SBI and its associates were merged separately, total number of merged PSBs became 8.

The paper conducts two layer measurement of technical efficiency of PSBs before and after their hypothetical merger using DEA. The first layer consists of measuring technical efficiency before and after hypothetical merger of PSBs with an intention of restructuring of weak banks. The PSB with highest asset size has been hypothetically merged with lowest two PSBs in terms of asset size as at end March 2016. Hence the list of merged unit following hypothetical merger is given below.

DMU 1: Bank of Baroda, Vijaya Bank and Punjab & Sindh bank

DMU 2: Punjab National Bank, United bank of India & Dena bank

DMU 3: Bank of India, Andhra bank & Bank of Maharashtra

DMU 4: Canara bank, Central bank and Indian bank

DMU 5: IDBI bank, Oriental bank of commerce and UCO bank

DMU 6: Union bank, Syndicate bank and Allahabad bank

DMU 7: Central bank of India and Indian overseas bank

DMU 8: SBI & its associate banks

In the second layer, it is tried to consolidate banks according to their returns to scale (RTS) status. The banks in a particular RTS category with similar capital adequacy ratios are merged among themselves.

## **SELECTION OF INPUT AND OUTPUT VARIABLES**

The toughest task to the researchers for estimating efficiency of banks through DEA methodology is to identify suitable and pertinent inputs and outputs. We have used the intermediation approach which considers banks as financial intermediaries that collect funds from units in form of deposits and then convert these funds into loans and other investments. This approach has been the preferred approach in most efficiency studies. Berger and Humphrey (1997) suggested that intermediation approach is appropriate for analyzing bank level efficiency whereas production approach for branch level efficiency. Therefore, extant study implements intermediation approach for selecting input and output variables for estimating bank level efficiency.

The following set of variables are used by the researchers in different combinations to study the efficiency banks, which are: (i) Fixed Assets, (ii) Number of Employees (iii) Advance, (iv) Net Interest Income (v) Non-interest Income, (vi) Net Profit, Return on advances, (vii) Return on investments, (viii) Number of bank branches. The present study uses following ratios as input variables: (i) cost of deposits, (ii) cost of borrowings, (iii) Number of bank branches (iv) Number of Employees and (v) return on advances (vi) return on investments, (vii) Profit after tax as output variables to examine the effect of M&A on the efficiency of banks. The rationale behind using the above ratios in this study are: the variables capture the financial performance of each banks and they reflect the spirit of variables used by other researchers as listed above.

## **EMPIRICAL RESULTS OF THE STUDY**

The following table depicts the technical and scale efficiency of major Schedule Commercial Banks (SCBs) before hypothetical merger takes place.

Table 1 depicts DEA score of major scheduled commercial banks which includes 26 PSBs and 13 private sector banks. It indicates that out of 26 PSBs 11 banks are efficient with OTE score equal to 1 and operating at constant return to scale (CRS), two banks are operating at sub-optimal level with increasing return to scale (IRS) while rest are operating with decreasing return to scale (DRS). That is, 42% of banks are operating at constant return to scale (CRS).

**Table 1: Pre merger efficiency scores of SCBs**

SCBs	OTE SCORE	OTIE (%)	Technical Efficiency	PTIE (%)	SE SCORE	SIE	RTS
Andhra Bank	0.926394	7.3606	0.966	3.4	0.959	4.1	drs
Allahabad Bank	1	0	1	0	1	0	crs
Bank of Baroda	1	0	1	0	1	0	crs
Bank of India	1	0	1	0	1	0	crs
Bank of Maharashtra	0.99	1	0.99	1	1	0	crs
Canara Bank	0.975	2.5	1	0	0.975	2.5	drs
Corporation Bank	0.884763	11.5237	0.891	10.9	0.993	0.7	drs
Central Bank	0.84994	15.006	0.91	9	0.934	6.6	drs
Dena Bank	1	0	1	0	1	0	crs
Indian Bank	1	0	1	0	1	0	crs
IDBI Bank	0.882375	11.7625	0.905	9.5	0.975	2.5	irs
Indian Overseas Bank	0.885417	11.4583	0.933	6.7	0.949	5.1	drs
Oriental Bank of Commerce	0.862557	13.7443	0.907	9.3	0.951	4.9	drs
Punjab national Bank	1	0	1	0	1	0	crs
Punjab Sindh Bank	0.875772	12.4228	0.918	8.2	0.954	4.6	drs
Syndicate Bank	0.970081	2.9919	0.973	2.7	0.997	0.3	drs
United Bank of India	0.958185	4.1815	0.963	3.7	0.995	0.5	irs
UCO Bank	0.935594	6.4406	0.946	5.4	0.989	1.1	drs
Union Bank	1	0	1	0	1	0	crs
Vijaya Bank	0.931221	6.8779	0.963	3.7	0.967	3.3	drs
SBBJ	1	0	1	0	1	0	crs
State Bank of Hyderabad	0.968	3.2	1	0	0.968	3.2	drs
State Bank of India	1	0	1	0	1	0	crs
State Bank of Maharashtra	0.958	4.2	0.958	4.2	1	0	crs
State Bank of Patiala	0.88266	11.734	0.94	6	0.939	6.1	drs
State Bank of Travancore	0.874458	12.5542	0.909	9.1	0.962	3.8	drs
Axis Bank	1	0	1	0	1	0	crs
DCB	1	0	1	0	1	0	crs
HDFC Bank	1	0	1	0	1	0	crs
ICICI Bank	1	0	1	0	1	0	crs
IndusInd Bank	1	0	1	0	1	0	crs
Kotak Mhindra Bank	1	0	1	0	1	0	crs
Yes Bank	1	0	1	0	1	0	crs
Federal Bank	0.972075	2.7925	0.975	2.5	0.997	0.3	irs
J&K Bank	1	0	1	0	1	0	crs
Karnataka Bank	0.89395	10.605	0.941	5.9	0.95	5	drs
Karur Vysa Bank	0.960351	3.9649	0.973	2.7	0.987	1.3	drs
South Indian Bank	0.910067	8.9933	0.947	5.3	0.961	3.9	drs
TMB	1	0	1	0	1	0	crs

Source: Author's own calculation

**Notes:** OTE, Overall technical efficiency; OTIE (%),  $(1 - \text{OTE}) \times 100$ ; PTE, Pure technical efficiency; PTIE %,  $(1 - \text{PTE}) \times 100$ ; SE, Scale efficiency; SIE %,  $(1 - \text{SE}) \times 100$ ; RTS, Returns to scale; IRS, Increasing returns to scale; CRS, Constant returns to scale; DRS, decreasing returns to scale.

The Government of India aims at merger of banks with the guiding principle as “larger asset size banks be merged with banks having smaller asset size”. Thus the hypothesis may be stated as follows:

$H_0$ =Merger of banks having higher assets with banks with lower assets will lead to efficiency in banks after merger

The DMUs in the category of PSBs have already been constructed based on asset size in section 4. The DEA of the DMUs after hypothetical merger is presented in table 2 below.

**Table 2: Post merger efficiency scores**



DMUs	OTE SCORE= PTE*SE	OTIE (%)	Technical Eff./PTE SCORE	PTIE (%)	SE SCORE	SIE	RTS
DMU1	0.912016	8.7984	0.952	4.8	0.958	4.2	drs
DMU2	0.991018	0.8982	0.994	0.6	0.997	0.3	irs
DMU3	0.972132	2.7868	0.994	0.6	0.978	2.2	drs
DMU4	0.94464	5.536	0.984	1.6	0.96	4	drs
DMU5	0.932195	6.7805	0.935	6.5	0.997	0.3	irs
DMU6	1	0	1	0	1	0	crs
DMU7	0.892944	10.7056	0.936	6.4	0.954	4.6	drs
DMU8	1	0	1	0	1	0	crs

Source: Author's own calculation

The merger based on asset size does not yield the desired result as only 2 of the post merged DMUs are operating at constant return to scale (CRS), two banks are operating at sub-optimal level with increasing return to scale (IRS) while rest are operating with decreasing return to scale (DRS). That is, 25% of total number of merged entity is operating at constant return to scale (CRS).

Thus the hypothesis stands rejected. That is, the alternative hypothesis “Merger of banks having higher assets with banks with lower assets will not lead to efficiency in banks after merger” stands accepted. However the question remains what should be the premise for merger of banks.

According to Federation of Indian Chambers of Commerce & Industry (FICCI) research report dated December 28, 2016 “The core idea behind exploring merger of banks is to enable creation of large sized banks of adequate capital base to enable disbursement of greater credit, especially for large developmental projects as well as for effective management of Non Performing Assets. Hence, the likely capital size of the merged entity needs to be considered while evaluating the decision for consolidation.” Capital adequacy ratio (CAR) is one of the key performance indicators and not the asset size. The reasons are obvious as a small bank with lower asset size can always be efficient compared with banks with higher asset size but performing below optimality. The data of the individual DMUs (pre-merger) were collected and a hypothetical merger based on the following two principles:

1. “banks with similar level of efficiency should be merged”
2. Merger of similarly firms (in terms of scale efficiency) should be based on capital adequacy ratio. That is, banks with higher CAR but operating in the same level of efficiency be merged.

**Table 3: CAR of Banks in CRS category (descending order)**

Kotak Mahindra Bank	14.71	Yes Bank	9.5
IndusInd Bank	13.78	State Bank of India	9.49
ICICI Bank	12.8	State Bank of Bikaner & Jaipur	9.11
DCB Bank	12.62	State Bank of Mysore	8.87
Axis Bank	12.23	Union Bank	8.23
HDFC Bank	11.08	Bank of India	8.2
Inian Bank	10.88	Allahabad Bank	8.05
Jammu & Kashmir Bank	10.86	Bank of Maharashtra	7.57
Bank of Baroda	10.13	Dena Bank	7.26
Punjab National Bank	9.76		

Source: RBI (2015), Statistical Tables Relating to Banks in India

Table 3, 4 and 5 reveal CAR of SCBs portrayed in table 1 as at end March 2015 grouped under the various returns to scales.

**Table 4: CAR of Banks in DRS category (descending order)**

Karur Vysa Bank	13.1	Vijaya Bank	8.54
SouthIndian Bank	12.05	Andhra Bank	8.52
Karnataka Bank	10.51	State Bank of Travancore	8.46
Canara Bank	9.77	Punjab & Sind Bank	8.38
State Bank of Hyderabad	9.25	Corporation Bank	8.33
Oriental Bank of Commerce	9.18	Central Bank of India	8.09
UCO Bank	9.06	State Bank of Patiala	8.02
Syndicate Bank	8.96	Indian Overseas Bank	7.8

**Source: RBI (2015), Statistical Tables Relating to Banks in India**

**Table 5: CAR of Banks in IRS category (descending order)**

Federal Bank	14.09
United Bank of India	8.40
IDBI Bank	7.68

*Source: RBI (2015), Statistical Tables Relating to Banks in India*

The suggested merger policy based on principles described is listed below. Since in first layer of asset size based merger three banks are merged together, in CAR based merger also same approach has been tried to maintain. While merging hypothetically PSBs and private banks are not differentiated.

Merged banks in CRS category

DMU1: Kotak Mahindra Bank, Indusind Bank and ICICI Bank

DMU2: DCB Bank, Axis Bank and HDFC Bank

DMU3: Indian Bank, Jammu & Kashmir Bank, and Bank of Baroda

DMU4: Punjab National Bank, Yes Bank and State Bank of India

DMU5: State Bank of Bikaner & Jaipur, State Bank of Mysore and Union Bank

DMU6: Bank of India, Allahabad Bank, Bank of Maharashtra and Dena Bank

Merged banks in DRS category

DMU7: Karur Vysya Bank, South Indian Bank and Karnataka Bank

DMU8: Canara Bank, State Bank of Hyderabad and Oriental Bank of Commerce

DMU9: UCO Bank, Syndicate Bank and Vijaya Bank

DMU10: Andhra Bank, State Bank of Travancore and Punjab & Sindh Bank

DMU11: Corporation Bank, Central Bank of India, State Bank of Patiala and Indian Overseas Bank

Merged banks in IRS category

DMU 12: Federal Bank, United Bank of India and IDBI Bank

DEA score of banks after merger based on above principle is shown in Table 6 below.

**Table 6: CAR based efficiency scores**

DMUs	OTE SCORE= PTE*SE	OTIE (%)	Technical Eff./PTE SCORE	PTIE (%)	SE SCORE	SIE	RTS
DMU1	1	0	1	0	1	0	crs
DMU2	1	0	1	0	1	0	crs
DMU3	1	0	1	0	1	0	crs
DMU4	1	0	1	0	1	0	crs
DMU5	1	0	1	0	1	0	crs
DMU6	1	0	1	0	1	0	crs
DMU7	1	0	1	0	1	0	crs
DMU8	1	0	1	0	1	0	crs
DMU9	0.974	2.6	1	0	0.974	2.6	irs
DMU10	0.982017	1.7983	0.983	1.7	0.999	0.1	drs
DMU11	0.90889	9.111	0.937	6.3	0.97	3	drs
DMU12	0.984	1.6	1	0	0.984	1.6	irs

*Source: Author's own calculation*

There are only two DMUs with DRS. Seven out of twelve banks (58%) are operating with constant return to scale. Hence the following proposition can be proposed.

Proposition 1: Banks with dissimilar returns to scale cannot yield efficient DMU if merged

Proposition 2: Capital adequacy ratio (CAR) is better KPI (key performance indicator) to evaluate the suitability of banks' strength and hence may be considered" in merger decisions as well.

## CONCLUSIONS

While consolidation of PSBs is a top priority agenda of Government of India and merger of SBI with its associates has already been taken place, it is tried to prescribe a policy for proposed merger. The study indicates that consolidation of banks considering asset size and ignoring scale category does not generate a successful merger. Rather adequacy of capital may be used as a basis for consolidation in similar scale category. We sincerely hope that this study unlocks a broad scope for further researches to appraise the relative efficiency of banks lining up for amalgamation, and in turn will contribute for the development of Indian banks. The future research may broaden our efforts by considering synergies, efficiency and cost saving aspects of banks.

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# **A MULTIOBJECTIVE WAY TO SELECT THE BEST SETTINGS USING SUPER-EFFICIENCY SBM DEA MODELS TO DELIVER NETWORK VIRTUALIZATION SERVICES – A STOCHASTIC CASE OF STUDY**

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## **ABSTRACT**

*This paper brings a new way to choose the most efficient settings to deliver virtual network services. This paper conducts measurements using type II hypervisors running Virtual Machines with a lightweight virtualization called container-based virtualization. To the assembly of the environments were used five Linux's distributions, as well as the emulated network interfaces by hypervisors, besides the container-based tools Docker and LXC. The experiments followed the IETF experimentation's methodology named Request for Comments 2544 and 6815. The assessments were performed using TCP/IP's protocols on Layers 2 and 3. The stochastic processes were referring to the virtualized traffic point out to Long-Range Dependency, that it has memory, it is contagious and causes high variability, i.e. irregular delivery of services, bringing the Self-Similarity's (SS) concept. Hurst parameter (a SS measure) and Confidence Intervals for the TCP bandwidth mean were used to compose the Objective Functions (OF). Were evaluated 50 DMUs, where Super-efficiencySBM DEA models were used to select the most efficient settings, as well as seek to minimize SS's effects. The results have quite similar behavior with or without CI.*

**Keywords:** Stochastic Process; Self-Similarity; Performance Evaluation; Virtual Networks; SDEA models.

## **INTRODUCTION**

Virtualization technologies implement an abstraction concept which allows splitting physical resources (e.g. memory, CPU, network) into multiple virtual machines (VM) or slices. Thus, the tradeoff between performance and isolation of virtual resources running on a shared physical substrate is a concern for delivering stable services in cloud computing environments. Particularly, deployment of virtual networks (VN) through virtual links and routers over a physical infrastructure are of paramount importance for connecting local or remote resources in datacenters. Evaluation of strategies for activating open virtual routers (VR) showed that the most prominent virtualization technologies are those that use container approach (Rathore, Hidell, & Sjödin, 2013). Hence, container technology is the key to achieving the scalability and performance towards scaling up the computer nodes number representing an economy of scale. This work proposes to extend cited work to compare both container-based strategies as Docker versus LXC on VN, as well as increase the accuracy of experiments. The proposed experimentation

framework methodology was based on Request for Comments (RFCs) 2544<sup>1</sup> and 6815<sup>2</sup>. Also serving like an updating of these documents to make a whole turning point for a benchmarking tool to offer better services on VN. It was made an in-depth analysis of TCP traffic using both type II hypervisors VMWARE and LXC, with its own emulated network interfaces forming DMUs. Five of the most popular Linux distributions (Arch, Fedora, OpenSuse, Ubuntu 14-04 and Ubuntu 16-04) were used to mount VN settings to be evaluated with its container tool, hypervisor, and emulated network interfaces per VM.

The work of (Jatoh, Gangadharan, & Fiore, 2016) has an approach with ours because used a modified Stochastic Super-Efficiency DEA (SDEA) model plus Analytic Network Process (ANP) to weight the variables to select the most efficient Cloud Service Providers. Thus, some performance metrics used that are not directly correlated with the problem to be solved; another drawback is that variables neither considered real measured data nor stochastic ones. Their variables were obtained at site <http://cloudharmony.com>. After exploratory analyses of the continuous random variables (CRV) of our experiments, it was noticed the presence of the Self-Similarity (SS) with Long-Range Dependency (LRD) over time. To measure the SS degree of CRV were calculated the Hurst parameters per DMU. The central question to be raised is that any setting used to mount TCP experiments had an unlikely changing of performance between in each of them. As the TCP traffic behavior maintain this pattern over time, then the decision maker could choose the DMUs that at the same time seek to minimize the Hurst parameter and a maximize the mean of TCP bandwidth. To achieve this goal, several SDEA models were analyzed.

This work is organized into sections. The next section presents SS, its role in computer networks evaluation, and shows an Experimentation Scenario for assessment. Results and discussion serve to summarize and interpret CRV captured by setting, to choose the most efficient DMUs to obtain more stable TCP traffic over time using SDEA models. Conclusion resumes the work and guides future works.

## METHODS

The SS was presented in work of (Mandelbrot, 1965) which proposed a model of random perturbations in clusters and bursts present in telecommunication. It was observed that memoryless channels are unable to account for the behavior of telecom circuits of high quality or low rate of errors. Although these errors were occasionally encountered, many of them appear to be always grouped, which keep grouped itself in bursts over time. Thus, SS is a "contagion" between occurrences of errors in adjacent symbols better reflecting real network traffic behavior than Markov's theory. One of the main features of the SS traffic is the absence of intermittent streams (burstiness), so the SS traffic is known by not have a stable mean value. Therefore, these significant differences in mean values are one of the reasons because traffic that exhibits SS with LRD is harder to control than the standard traffic (shaped). The work of (Liu, Yan, Dong, & Tang, 2012) brings a survey of state of the art of SS with LRD on traditional computer networks.

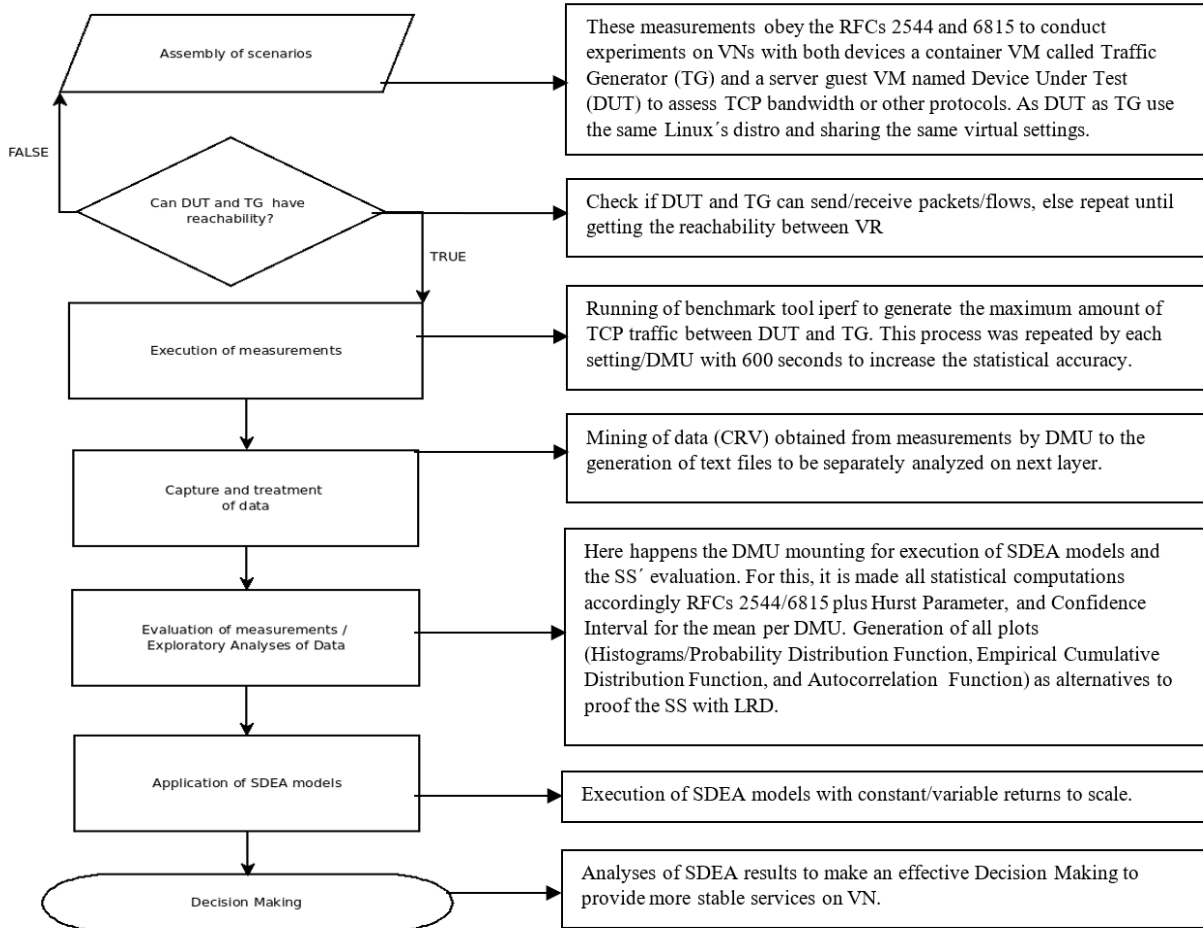
One of some ways of finds the SS degree of a time series is computing the Hurst Parameter ( $H$ ), it uses the linear regression method called rescaled range analysis (R/S). The work of (Weron, 2002) shows how to calculate this burstiness' index on stochastic processes. Thus, when  $H < 0.5$  these CRV exhibit Short-

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<sup>1</sup>RFC 2544 – Available at the URL: <https://www.ietf.org/rfc/2544.html>

<sup>2</sup>RFC 6815 –Available at the URL: <https://www.ietf.org/rfc/6815.html>

Range Dependency (SRD), i.e. they are unlikely to happen due memoryless, also called anti-persistent process as known as Noah's effect. When stochastic processes have  $0.5 > H < 1$  indicate which this pattern remain with memory, for this is named of a persistent process also called of Joseph's effect. As only persistent measurements have memory over time, just DMUs that carried this pattern should be considered to assessment with SDEA models, anti-persistent DMUs should be excluded. Our work used the same thinking of (Janowski & Papir, 2005) that used Token Bucket (TB) Algorithm that got smooth network traffic using the TB curve to decrease the impact of SS with LRD on TCP traffic. The TB Algorithm used the TCP bandwidth mean, Hurst Parameter, and variance as its parameters. In our paper, Hurst Parameter is an input variable that should be minimized seeking to smooth the network traffic. The output variables are the confidence interval (CI) for the TCP transfer rate mean with 95% of confidence. The variance was initially considered as the input variable, but their results were not considered as good as without it. The variance is contained on  $H$  calculus, thus has lower variance when  $H \rightarrow 0.5$ , evidencing a smooth effect. Higher variance when  $H \rightarrow 1$ , carrying burstiness effect or infinite variance syndrome.



**Figure 1: Proposed SS' Evaluation Framework**

After selection of CRV, it is necessary to explain the SS' Evaluation Framework depicted in Figure 1, which improves the delivery of more stable network services, not only regarding VN but for any network.

It will create new directions on network performance evaluation because prior works that proved the SS with LRD on computer networks made the demonstration of concept using just a unique setting/device. However, this research goes beyond and shows what SS with LRD remains of different manner and performance per setting. Hence, choosing DMUs with the smallest values of  $H$  and a considerable average of TCP bandwidth is so important to deliver better network services. The SDEA models with constant returns to scale (CRS) used to decision making (DM) were Super CCR-I, Super-CCR-O, Super SBM-I-C, Super SBM-O-C, and Super SBM-C. The SDEA models with variable returns to scale (VRS) used were Super BCC-I, Super-BCC-O, Super SBM-I-V, Super SBM-O-V, and Super SBM-V. Explanations and formalisms referred to these models are in (Cooper, Seiford, & Tone, 2007).

## RESULTS AND DISCUSSIONS

Table 1 shows the DMUs appraised on SDEA assessment. Two DMUs were excluded due they have  $H < 0.5$ . Note which there is a great statistical difference among some DMUs on Table 1, e.g. the biggest TCP bandwidth mean is from DMU19 that is 463.313% bigger than smaller one (DMU21), and so on.

**Table 1: DMUs to be evaluated by SDEA models**

#	Setting/DMU	INPUT	OUTPUT	
		Hurst	TCP-CI-1	TCP-CI-2
1	Arch12 – VIRTUALBOX – PCNet PCI – Docker	0.6149959	39957.13	40460.15
2	Arch12 – VIRTUALBOX – PCNet Fast – Docker	0.6380813	39073.06	39547.95
3	Arch12 – VIRTUALBOX – PRO 1000 MT Desktop – Docker	0.7139144	50269.86	52329.73
4	Arch12 – VIRTUALBOX – PRO 1000 T Server – Docker	0.6303365	56440.84	58064.13
5	Arch12 – VIRTUALBOX – PCNet PCI – LXC	0.6542005	45482.37	46504.35
6	Arch12 – VIRTUALBOX – PCNet Fast – LXC	0.7290385	44845.22	45956.15
7	Arch12 – VIRTUALBOX – PRO 1000 MT Desktop – LXC	0.5670684	118651	124434.1
8	Arch12 – VIRTUALBOX – PRO 1000 T Server – LXC	0.8320479	93425.87	100499.43
9	Arch12 – VMWARE – Docker	0.7176149	44272.17	45957.72
10	Arch12 – VMWARE – LXC	0.8115613	57585.39	64205.71
11	Fedora24 – VIRTUALBOX – PCNet PCI – Docker	0.5340552	55072.61	56100.76
12	Fedora24 – VIRTUALBOX – PCNet Fast – Docker	0.5435163	55372.15	56390.03
13	Fedora24 – VIRTUALBOX – PRO 1000 MT Desktop – Docker	0.522739	139738	140595.4
14	Fedora24 – VIRTUALBOX – PRO 1000 T Server – Docker	0.5856474	117021	124173.4
15	Fedora24 – VIRTUALBOX – PCNet PCI – LXC	0.7378332	49399.3	50474.68
16	Fedora24 – VIRTUALBOX – PCNetFast – LXC	0.6086283	53662.17	54736.49
17	Fedora24 – VIRTUALBOX – PRO 1000 T Server – LXC	0.607753	112054.4	118863.2
18	Fedora24 – VMWARE – Docker	0.6737299	210654.2	215748.2
19	Fedora24 – VMWARE – LXC	0.8155753	216234.9	225723
20	OpenSuse42.2 – VIRTUALBOX – PCNet PCI – Docker	0.7755444	43965.42	44949.2
21	OpenSuse42.2 – VIRTUALBOX – PCNet Fast – Docker	0.6776561	38840.21	39616.7
22	OpenSuse42.2 – VIRTUALBOX – PRO 1000 MT Desktop – Docker	0.6322082	142966.8	145050.6
23	OpenSuse42.2 – VIRTUALBOX – PRO 1000 T Server – Docker	0.5502623	109558.4	117163.7
24	OpenSuse42.2 – VIRTUALBOX – PCNet PCI – LXC	0.6908904	42968.26	43593.76
25	OpenSuse42.2 – VIRTUALBOX – PCNet Fast – LXC	0.6469991	47821.65	48783.42
26	OpenSuse42.2 – VIRTUALBOX – PRO 1000 MT Desktop – LXC	0.6177616	136018.3	137573.2
27	OpenSuse42.2 – VIRTUALBOX – PRO 1000 T Server – LXC	0.5861757	109950	117765.1
28	OpenSuse42.2 – VMWARE – Docker	0.7692495	79959.19	81449.1
29	OpenSuse42.2 – VMWARE – LXC	0.6591165	211346.7	215868
30	Ubuntu14 – VIRTUALBOX – PCNet PCI – Docker	0.6374863	40411.54	40800.91
31	Ubuntu14 – VIRTUALBOX – PCNet Fast – Docker	0.6240196	40860.83	41254.45
32	Ubuntu14 – VIRTUALBOX – PRO 1000 MT Desktop – Docker	0.8451504	146021.3	147540.3
33	Ubuntu14 – VIRTUALBOX – PRO 1000 T Server – Docker	0.6515093	94393.76	96690.69



34	Ubuntu14 – VIRTUALBOX – PCNet PCI – LXC	0.7819212	41395.06	41968.03
35	Ubuntu14 – VIRTUALBOX – PCNetFast – LXC	0.7150646	40860.83	41254.45
36	Ubuntu14 – VIRTUALBOX – PRO 1000 MT Desktop – LXC	0.7451546	140121.8	141303.4
37	Ubuntu14 – VIRTUALBOX – PRO 1000 T Server – LXC	0.7007306	93457.18	96677.18
38	Ubuntu14 – VMWARE – Docker	0.6205952	72877.09	74536.27
39	Ubuntu14 – VMWARE – LXC	0.7187318	104526.2	106928
40	Ubuntu16 – VIRTUALBOX – PCNet PCI – Docker	0.5748441	44336.33	45014.47
41	Ubuntu16 – VIRTUALBOX – PCNet Fast – Docker	0.7135971	42825.88	43589.12
42	Ubuntu16 – VIRTUALBOX – PRO 1000 T Server – Docker	0.5855856	119909.3	126752.4
43	Ubuntu16 – VIRTUALBOX – PCNet PCI – LXC	0.6616379	60512.06	62106.1
44	Ubuntu16 – VIRTUALBOX – PCNetFast – LXC	0.8082247	56227.09	58150.26
45	Ubuntu16 – VIRTUALBOX – PRO 1000 MT Desktop – LXC	0.5152529	130299.5	132285.3
46	Ubuntu16 – VIRTUALBOX – PRO 1000 T Server – LXC	0.5759278	107895.7	115581.9
47	Ubuntu16 – VMWARE – Docker	0.8549348	179721.8	186928.8
48	Ubuntu16 – VMWARE – LXC	0.6895153	213735.9	220903.7

Table 2 presents all super-efficient (SE) rankings with highlight to DMU29 as the most efficient setting in entire SDEA with CRS. DMU29 also make part of half of VRS DEA frontiers as the most efficient DMU to offer VN services. DMU29 has an intermediary value of Hurst Parameter ( $H=0.6591165$ ), and the third highest TCP bandwidth mean. Other highlights are respectively the DMUs 48, 13, 19 and 45.

The blue colored DMUs in Table 2 should be disregarded on DM because they have or an infeasible solution when their scores are equal to unity or have scores less than unity. So only SE scores should be considered.

**Table 2: Summary of all SDEA rankings**

Ranking	CRS					VRS				
	Super CCR-I	Super CCR-O	Super SBM-I-C	Super SBM-O-C	Super SBM-C	Super BCC-I	Super BCC-O	Super SBM-I-V	Super SBM-O-V	Super SBM-V
1	DMU29	DMU29	DMU29	DMU29	DMU29	DMU48	DMU13	DMU48	DMU29	DMU29
2	DMU48	DMU48	DMU48	DMU18	DMU18	DMU29	DMU29	DMU29	DMU13	DMU13
3	DMU18	DMU18	DMU18	DMU48	DMU48	DMU13	DMU19	DMU13	DMU19	DMU19
4						DMU45	DMU48	DMU45	DMU48	DMU45
5						DMU19	DMU45	DMU19	DMU45	DMU48

Note in Table 3 that all SDEA models with CRS have the same score, where only DMU29 is considered SE. Both SDEA models with VRS as Super BCC-I and Super SBM-I-V have the same ranking and scores. On CRS, Super BCC-I, Super SBM-O-V, and Super SBM-V models have DMUs with some similar value of scores and different ranking.

**Table 3: Scores of the most efficient DMUs by SDEA models of stochastic process**

	Model/DMU	DMU13	DMU18	DMU19	DMU29	DMU45	DMU48
C R S	Super-CCR-I		0.9777668		1.0255315		0.978212
	Super-CCR-O		0.9777668		1.0255315		0.978212
	Super SBM-I-C		0.9777668		1.0255315		0.978212
	Super SBM-O-C		0.9764337		1.0241332		0.9724316
	Super SBM-C		0.9764337		1.0241332		0.9724316
V R S	Super-BCC-I	1.0177239		1	1.0275529	1.0145188	1.0718597
	Super-BCC-O	1.0388015		1.0218163	1.0370719	1	1.0143305
	Super SBM-I-V	1.0177239		1	1.0275529	1.0145188	1.0718597
	Super SBM-O-V	1.0338652		1.0167289	1.0362643	1	1.0105414
	Super SBM-V	1.0177239		1.0167289	1.0241332	1.0145188	1.0105414

For DM the best SDEA model is Super SBM-V because neither DMU of its SE frontier was dropped, reflecting the best settings to offer more stable VN services. Thus, Super SBM-V seeks to decrease  $H$  and increase the TCP transfer rate mean simultaneously.

## CONCLUSIONS

This work evaluates stochastic process referred for VN traffic, showing that each DMU/setting has a different but persistent TCP traffic performance behavior due to the SS with LRD. To this end, this study executed a series of measurements based on RFCs 2544/6815 guidelines to mount and compute statistics. These RFCs indicate only 60 seconds to evaluate devices, but this work evaluates each DMU using 600 seconds to increase accuracy. A SS' Evaluation Framework was also proposed to DM. After a complete SDEA analyses, the highlight is Super SBM-V model, that at the same time seek to minimize  $H$  and maximize TCP bandwidth mean, better-reflecting TCP traffic to offer better services on VN. In summary, SDEA analyses indicate OpenSuse 42.2 as the most efficient Linux distro, followed by Fedora 24, and Ubuntu 16-04. As type II hypervisor, the highlight was VMWARE, and LXC as the container tool.

On future works, one must show other SDEA frontiers of this research related to UDP jitter and delay on VNs which also had SS with LRD. Moreover, it is crucial ever scale up the number of DMUs, mainly because these softwares are always in constant updates and should be evaluated frequently.

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# A TRIPLE BOTTOM LINE APPROACH FOR MEASURING SUPPLY CHAINS SUSTAINABILITY USING DATA ENVELOPMENT ANALYSIS

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## ABSTRACT

*This paper deals with the issues faced by those who endeavor in measuring sustainability in supply chains (SC) by using a comprehensive approach. Elkington's Triple Bottom Line (TBL) divide sustainability in three aspects: environmental (E), economic and social (E2S). Firms publish their business (including SC) sustainability impacts through Corporate Social Responsibility reports (CSR). According to Global Reporting Initiative (GRI), a CSR framework, reported information should be sufficiently accurate/detailed towards performance, but reports are rather qualitative. Fast fashion (FF) is a recent phenomenon of production/promotion of cheap/readily disposable clothes. Data Envelopment Analysis (DEA) is the adequate tool to identify best practices regarding sustainability (multidimensional) and supply chains in FF. To allow comparability and tackle lack of quantitative data, TBL clusters (output) are proposed: for each Disclosure, a three layers scoring scale: absence (1), qualitative only (2), 2nd layer plus quantitative (4). This work's main contribution is the use of DEA as a powerful tool to measure sustainability in SC and the TBL clusters link all dimensions in an innovative way.*

**Keywords:** Data Envelopment Analysis; Sustainability; Supply Chains; Triple Bottom Line; Fast Fashion

## INTRODUCTION

Supply chains (SC) are the main gateway to Sustainability. The article focus will be on measuring the performance of supply chains against sustainability disclosures in Corporate Social Responsibility (CSR) reports with Global Reporting Initiative (GRI) framework. The GRI framework uses the Triple Bottom Line (TBL) approach for addressing Sustainability. TBL is a multidimensional form to structure environmental, economic and social concerns.

The paper will analyze the textile and apparel sector (fast fashion), comparing performance using Data Envelopment Analysis (DEA). Comparability (benchmarking) between reports will be allowed by using a scale for qualitative and quantitative data to address the TBL. Therefore, the benchmarking process will be undertaken by using inputs/outputs out of economic data and by the scores given by CSR reports analysis.

One definition for Sustainable Supply Chain Management (SSCM) is, according to Seuring & Müller (2008), the management of material, information and capital flows as well as cooperation among companies along the supply chain while taking goals from all three dimensions of sustainable development (triple bottom line – TBL). Considering integration of sustainability into supply chain management (SCM) an ever increasing matter, “how to measure supply chain wide sustainability performance is paramount” (Seuring & Gold, 2013).

Multi-criteria decision making (MCDM) techniques, in the analytical models, and mathematical programming models (e.g. multi objective technique) are the most often chosen model types for SSCM assessment (Brandenburg, Govindan, Sarkis, & Seuring, 2014). DEA is a technique located in the

confluence of analytical and mathematical programming models. So far, there is no research that uses DEA for measuring sustainability in supply chains, accounting for the three dimensions simultaneously, with data out of CSR reports, using the GRI G4 framework.

Comparability between reports is not straightforward. To overcome uneven reports' constraint, a scale was built considering a layer-scoring system for quantitative and qualitative data. Therefore, the sum of scores for socio-economic and environmental dimensions will be used, along with financial data, as DEA input/outputs.

The main contributions of this article are: delving into the GRI G4 disclosures and extracting their essence; considering all the TBL dimensions at the same time for measurement; assessing supply chain for sustainability matters using real world data (or empirically); somehow bridging the gap between CSR reports to allow comparability; although in an indirect fashion (by proxy) and with limited boundaries (tier 1), supply chains are being measured; comparability provided by the scale and the sum of scores of socio-economic and environmental dimensions; the insight of disclosures from a layer standpoint (layer 1 through 3); applying DEA for measuring sustainability (via TBL) in a comprehensive way, by including all dimensions in the analysis (non-financial) along with financial data in the model.

The remainder of this article is structured as follows: Section 2 (Literature Review), Section 3 (Methods), Section 4 (Results), Section 5 (Discussion), and Section 6 (Conclusion).

## **METHODS**

Reporting practices in CSR are deemed as a proxy for sustainability behavior (Nielsen & Thomsen, 2007). Global Reporting Initiative (GRI) has long been widely recognized as the most reliable framework for disclosing information regarding sustainability (Nikolaeva & Bicho, 2011). Information for each identified material Aspect can be reported as Disclosures on Management Approach (DMA) and/or Indicator (GRI, 2013). Indicators yield information on the economic, environmental and social performance or impacts of the relationship organization – material Aspects. The DMA gives an opportunity to explain how economic, environmental and social impacts (in relation to material Aspects) are managed and delivers narrative information on how an organization identifies, assess, and responds (proactive vs. reactive) to its actual and potential material TBL.

A model was devised for building/selecting outputs and inputs that would take into account the sector characteristics, DEA assumptions and G4 Disclosures (General, Specific, and DMA). DMA is frequently overlooked in the literature. There is a complimentary condition between the Indicators and DMAs. For management use, when firms intend to disclose their sustainability accomplishments, a mandatory condition is to couple Indicators with DMAs at some level.

Two output TBL clusters were conceived as a means of dealing with the great number of Disclosures vs. number of DMUs, and to permit comparability among quantitative data achieved with different methodologies/assumptions. Besides the quantitative data, there is quite a deal of qualitative data that necessarily would need some kind of transformation into quantitative to fit in the model. The cluster is a technique to normalize the data to allow comparability and overcome the high number of disclosures

issue, to minimally capture all dimensions of sustainability in a firm's supply chain. The TBL clusters grouped G4 Disclosures in Socio-economic (E2S) and Environmental (E). The distribution of Disclosures inside the clusters are shown in Table 1.

**Table 1: Disclosures' Distribution (Input, TBL Clusters Output (E2S,E), Output)**

Factor	Input (Fin)	TBL Output Cluster (E2S)	TBL Output Cluster (E)	Output (Fin)
Scale (firm)	G4-9 (cost of sales)			G4-9 (net sales)
Training		G4-(LA9, HR2, SO4, 56)		
Water			G4-(EN8,EN9,EN10)	
Operations / Supplier		G4-(LA14, HR10, SO9, SO1, SO2, EC9, HR4, HR5, HR6, 6, 12, LA15 (DMA), HR11 (DMA), SO10 (DMA))	G4-(EN32, EN33, EN33 (DMA))	
Governance		G4-(37, 42, 45)		
Energy			G4-(EN3, EN4)	
Water/Waste			G4-(EN22, EN23, EN27 (u), EN27 (d))	
Transportation			G4-(EN30 (u), EN30 (d))	
Environmental Protection			G4-(EN31 (u), EN31 (d))	
Customer		G4-(PR1, PR1 (DMA), PR2, PR3, PR4)		
Material Aspects / Boundaries		G4-(19, 21)		
Community Investments		G4-(EC1, EC7, EC8)		
Greenhouse Gases			G4-(EN15, EN16, EN17)	
Stakeholder Engagement		G4-(24, 25, 27)		
Financial (Fin); upstream (u), downstream (d)				

## RESULTS AND DISCUSSIONS

Performing a Data Envelopment Analysis (DEA), under the technology assumption of Variable Returns to Scale (VRS), with an output based efficiency measure (orientation), yields the results given in Table 2 (efficiency scores and weights) and Table 3 (peers).

**Table 2: Efficiency scores and weights (VRS, output oriented)**

Firms	Efficiency	Cost (Sales) (v1)	TBL E2S Cluster (u1)	TBL E Cluster (u2)	Net Sales (u3)
<b>Inditex*</b>	1	1.13493E-10	0	0.00019111	0
<b>Esprit</b>	1.134892999	1.03876E-09	0	0	4.34998E-10
<b>GAP</b>	1.235244068	0	0.001160358	0.021092224	0
<b>H&amp;M</b>	1	0	0	0.018181818	0
<b>MANGO</b>	1	0	0.014084507	0	0
<b>Uniqlo</b>	1.105429723	0	0.001065516	0.019368249	0
<b>Nike</b>	1.020293636	0	0	0	0
<b>PUMA</b>	1	0	0	0.019607843	0
<b>Target</b>	1	0	0	0	0
<b>M&amp;S</b>	1.064937474	0	0.001001959	0.018212959	0

\*Inditex group (Bershka, Pull & Bear and Zara)

The results presented by Table 3 reveals that under VRS (output-oriented), which is the most generally assumption related to scale, five firms and their supply chains were qualified as efficient (Inditex, H&M, MANGO, PUMA and Target). The zero weights (dual form) presented by the inefficient DMUs suggests

the presence of slacks. From a managerial perspective and for the sake of sowing the seeds of sustainability (including threshold and context related issues), it is more practical to reach for the TBL clusters scores of peers used more than once for comparisons (envelopment map), which in this technology are Inditex (90,54), MANGO (71,37) and PUMA (85,51).

**Table 3: Peers and lambdas (VRS, output oriented)**

Firms	Peer1	Peer2	Peer3	Inditex ( $\lambda_1$ )	MANGO ( $\lambda_2$ )	PUMA ( $\lambda_3$ )	Target ( $\lambda_4$ )
Inditex	Inditex	-	-	1	0	0	0
Esprit	Inditex	MANGO	-	0.015179	0.984821	0	0
GAP	Inditex	PUMA	-	0.924738	0	0.075262	0
H&M	Inditex	MANGO	-	0.934689	0.065311	0	0
MANGO	MANGO	-	-	0	1	0	0
Uniqlo	Inditex	MANGO	PUMA	0.669353	0.291866	0.038781	0
Nike	Inditex	Target	-	0.839679	0	0	0.160321
PUMA	PUMA	-	-	0	0	1	0
Target	Target	-	-	0	0	0	1
M&S	Inditex	PUMA	-	0.906753	0	0.093247	0

Performing a Data Envelopment Analysis (DEA), under the technology assumption of Constant Returns to Scale (CRS), with an output based efficiency measure (orientation), yields the results given in Table 4 (efficiency scores and weights) and Table 5 (peers).

**Table 4: Efficiency scores and weights (CRS, output oriented)**

Firms	Efficiency	Cost (Sales) (v1)	TBL E2S Cluster (u1)	TBL E Cluster (u2)	Net Sales (u3)
Inditex*	1	1.13493E-10	0	0	0
Esprit	1.135673823	9.86456E-10	0.000844107	0	4.12232E-10
GAP	1.450511206	1.75034E-10	0	0.000294739	0
H&M	1.017915969	1.22594E-10	0	0.000206435	0
MANGO	1	9.67859E-10	0.014084507	0	0
Uniqlo	1.168538136	1.86341E-10	0	0.000313779	0
Nike	1.281638869	0	0	0	0
PUMA	1.232131792	6.67027E-10	0	0.001123202	2.78301E-10
Target	1.671264858	0	0	0	0
M&S	1.458435606	1.78691E-10	0	0.000300897	0

**Table 5: Peers and lambdas (CRS, output oriented)**

Firms	Peer1	Peer2	Inditex ( $\lambda_1$ )	MANGO ( $\lambda_2$ )
Inditex	Inditex	-	1	0
Esprit	Inditex	MANGO	0.031382	0.846638
GAP	Inditex	MANGO	0.916663	0.203425
H&M	Inditex	-	0.942348	0
MANGO	MANGO	-	0	1
Uniqlo	Inditex	MANGO	0.671825	0.340121
Nike	Inditex	-	1.705597	0
PUMA	Inditex	MANGO	0.031794	1.516689
Target	Inditex	-	5.401147	0
M&S	Inditex	MANGO	0.868534	0.49264

The results displayed in Table 4 reveals that under CRS, a less generally assumption related to scale, just two firms and their supply chains were qualified as efficient (Inditex and MANGO). With eight DMUs classified as inefficient (and zero weights spotted), from a triple bottom line approach as whole, these

firms and supply chains will be better off if they consider Inditex as their benchmarking and role model, in terms of enhancing sustainability practices and reporting.

The Table 6 provides the outlook on scale efficiency (SE) of the firm's supply chains and in which part of the production function they fall.

**Table 6: Scale Efficiency and Frontier Position (IRS, MPSS, DRS)**

Inditex	Esprit	GAP	H&M	MANGO	Uniqlo	Nike	PUMA	Target	M&S
1	0.981	0.999	0.982	1	0.992	0.805	0.812	0.598	0.981
MPSS	IRS	IRS	DRS	MPSS	DRS	DRS	DRS	DRS	DRS
IRS (Increasing Returns to Scale); MPSS (Most Productive Scale Size); DRS (Decreasing Returns to Scale)									

IRS and DRS are flip sides of the same coin, in terms of calling for some kind of high level approach to tackle the issue, leading to an invaluable occasion to analyzing impacts, risks and opportunities regarding sustainability and its inexorably increasing role in the future of any entrepreneurship.

In essence, the measures are not just serving as a “thermometer” of sustainable practices. Instead, they are in charge of inducing leadership, instigating benchmarking, making firms want to take the edge (frontier vs. average methods) on TBL solutions and at the same time instilling innovation. Each Disclosure has an importance of its own and must make room for capturing the “effects” on TBL implementation/operation, revealing some kind of TBL trend or lack of it.

Notwithstanding, the CSR as the synergy of Disclosures (Indicators + DMA) is becoming a feasible and reasonable proxy for measuring sustainability performance and efficiency on supply chains. Although indirectly, the model can be classified as useful tool for analysis (“rear mirror”). The results rendered by DEA processing can confirm that companies (and supply chains) considered efficient (or almost) are committed to sustainability, according to their CSR reports, like Inditex, MANGO, PUMA and H&M. Nevertheless, it can't be seen as a “carte blanche” for ignoring the challenges of high resource demands, difficult multiple labor issues and heavy waste (production and disposing).

## CONCLUSIONS

The analysis conducted herein made use of performance indicators set by GRI G4 Guidelines. Therefore, standard metrics was not a problem. Three issues can be interpreted as the mainstream difficulties in performance indicators. First, whether the Indicator is present or not in the report. Second, the metrics required by the Indicator were presented to the full or halfway (leading to the three layers for Disclosures). Last, different standards, methodologies and assumptions used by reporters to obtain quantitative data to fulfill Disclosure requirements. The workaround to equalize uneven data and blend quantitative and qualitative data together was the idea of the TBL Clusters, clearly including the social dimension into the analysis.

An important feature of the scale for computing scores for TBL clusters, is allowing mixing qualitative and quantitative data. In consequence, it enabled the combination of financial and non-financial (sustainability) for application in a DEA structure. When it comes to sustainability, the lack of a consistent threshold and context is an ongoing problem. Therefore, DEA with its piecewise comparison and data-driven non-parametric technique overcomes this limitation, at the same time providing

meaningful readings of the practices in the real world. This approach has the limitations of not applying the actual data reported, instead using a score as a proxy. However, it was possible to account for any Disclosure, including the ones which clearly permit to identify the laggards in sustainability measurement and reporting (e.g. failing to do any reporting on G4-EN17 – Other indirect greenhouse gas (GHG) emissions (Scope 3), a typical supply chain Disclosure).

One of the main topics that gives consistency to the present work is also an issue for future research to overcome – the restriction to CSR report analysis to GRI G4 reports. The desired state is developing a methodology for sustainability performance measurement independently of the framework adopted to structure the disclosing of firms (including their supply chains) TBL practices. Thus, there is a need for devising a methodology (measures and how to apply) whose main strength is being “frameworkless” providing researchers and practitioners a useful tool or “sustainability compass”.

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# ADDING A DIFFERENT MEANING TO FDEA BY TRIANGULAR NORMS

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## ABSTRACT

*The Fuzzy Data Envelopment Analysis (FDEA) pioneered by Sengupta and branched out as can be seen in the survey of Hatami-Marbini et al. and later Emrouznejad et al. This field has not yet reached the desired level in its entirety. Current studies of FDEA are still under the framework of fuzzy mathematical programming and away from the representing the full power of DEA. The triangle membership functions and the triangular norms (t-norm) in fuzzy logic constitute an important place, while the simplex concept based on a triangular surfaced polyhedron is the basic stones of linear programming and DEA. Therefore, our scope of the study is the mathematical demonstration of the relationship between t-norms and DEA. This will add a different meaning to the FDEA..*

**Keywords:** Fuzzy Data Envelopment Analysis; DEA; t-norm; triangular norms

## INTRODUCTION

It was aimed to develop a new fuzzy DEA model. During the researches carried out for this purpose, it was seen that both the simplex and triangular norm concepts defines a polyhedron, which has triangular faces. (Schweizer, Sklar 1961, Dantzig 1963, Klement et al. 2000a, Klement, Mesiar 2005) In multivairate case, one of the instruments which is used to aggregate fuzzy data or fuzzy memberships are triangular norms (t-norms) based on triangular inequality (Heath 1908, Klement et al. 2000b, Khamsi, Kirk 2001, Alsina, Nelsen 2009). At this point, in the sense that these two methods have a common ground, the relation between the DEA, which transforms the data into a range of [0,1] in its model equation and produces a solution by linear programming techniques (Charnes et al. 1978, Charnes et al. 1994) and t-norms has been investigated (Menger 1942). In general, spline approximation techniques are used for this purpose (De Boor 1978, Dierckx 1995, Beliakov 2000, Beliakov 2004). As an evaluation of the research, It was foreseen that the results given by different methods which are difficult to comprehend in the literature (Beliakov 2005, Beliakov 2009, James 2016) can be obtained more easily by using DEA.

When one of the inputs or outputs is odd, DEA provided t-norm conditions. In fact, t-norm applications also achieved the result that the proof of single output multiple input usage is valid. As a plus, it was shown that the DEA can also be used for multiple output single input situations. It is also emphasized that the proposed method is more practical than the existing methods (Gölcükcü 2017). In response to the question of where the proposed method is fuzzy, the t-norms correspond to the fuzzy intersection, which is presented as an alternative to the existing situations within the scope of the study. The proposed model is applied to a sample set of data on education and to the tertiary statistics of countries compiled by UNESCO as an application. It was seen that the results approved the proposed model..

## METHODS

### Key Studies

- Karl Menger (1942)-Triangular Norms
- Dantzig (1947)-Simplex Method (Dantzig 1982)
- Farell (1957) -Efficiency Measurement
- Zadeh (1965) -Fuzzy Sets
- Charnes, Cooper and Rhodes (1978)-Data Envelopment Analysis

are the pioneering works. As an addition; Tanaka et al. (1973) proposed fuzzy mathematical programming and Zimmermann contributed the literature with his various works (Zimmermann 1975, Zimmermann 1978, Zimmermann, Zysno 1980, Zimmermann 1983, Zimmermann 1985, Zimmermann 1986, Zimmermann 1987b, Zimmermann 1987a). Sengupta (1992a, 1992b) is the beginning of Fuzzy Data Envelopment Analysis.

Verbally, t-norm is expressed as  $T$ , defined as a binary operation on the unit interval  $[0,1]$  with an neutral element 1, and having the properties of commutativity, associativity and monotonicity . In fact, it is a function  $T: [0,1]^2 \rightarrow [0,1]$  such that for all  $x, y, z \in [0,1]$  which satisfies (Klement et al. 2000a, Klement, Mesiar 2005);

- $T(x, 1) = x$
- $T(x, b) = T(b, x)$
- $x \leq c \text{ and } b \leq d \Rightarrow T(a, b) \leq T(c, d)$
- $T(T(a, b), c) = T(a, T(b, c))$

T-norm conditions are often taken as descriptors of intersection concepts in fuzzy logic; That is, any fuzzy intersection must be t-norm (Bergmann 2008). Hence, nowadays t-norms are assumed to correspond to fuzzy intersection and are now used instead of each other (Gupta, Qi 1991). Similarly, t-conorms, which are dual of t-norms, are used instead of fuzzy union.

As known DEA aggregates the set of variables to a single DEA score in its formulation. With this point of view DEA is a nonparametric aggregator similar with t-norm concept. Thus, Gölcükcü (2017) aproved that, when one of the inputs or outputs is odd, DEA provided t-norm conditions. In fact, t-norm applications also achieved the result that the proof of single output multiple input usage is valid. As a plus, it was shown that the DEA can also be used for multiple output single input situations. It is also emphasized that the proposed method is more practical than the existing methods. As a matter of fact, the proposed models and classical CCR models are generators of each other. Consequently, the DEA could be added as a t-norm under the following list.

### Family of t-Norms

1. Basic t-norm and t-conorm
2. Schweizer-Sklar t-norm and t-conorm
3. Hamacher t-norm and t-conorm
4. Frank t-norm and t-conorm
5. Yager t-norm and t-conorm
6. Dombi t-norm and t-conorm
7. Sugeno-Weber t-norm and t-conorm
8. Aczél-Alsina t-norm and t-conorm
9. Mayor-Torrens t-norm and t-conorm

### The purposal of the Study

Type : Data Envelopment Analysis

t-norm :

$$\text{Max } z_o = (1 - \mu) y_o$$

s.t.

$$(1 - \mu) y_j - \sum_{i=1}^m v_i x_{ij} \leq 0 \quad (1)$$

$$\sum_{i=1}^m v_i x_{io} = 1$$

$$j = 1, \dots, n, \mu, v_i \geq 0$$

## RESULTS AND DISCUSSIONS

In order to see the effect of the proposed model on the real world data, OECD data of 29 countries with respect to education is analysed by model (1). As given in Table 1., the number of academic publications (Y) is taken as output and the total number of researchers in higher education (X1) and the expenditure per researcher (X2) are inputs.

**Table 1: OECD Data and Results**

No	Country	Data			Efficiency Score	
		Y1	X1	X2	Classical CCR	Proposed Model
1	Germany	161.86	335.928	151.594	0.801	0.801
2	Austria	22.776	44.601	131.235	0.356	0.356
3	Belgium	30.851	42.981	137.747	0.488	0.488
4	United Kingdom	187.005	355.06	80.164	1	1
5	Czech Republic	20.123	32.173	95.724	0.433	0.433
6	Denmark	23.216	37.539	99.041	0.464	0.464
7	Estonia	2.698	6.247	57.771	0.292	0.292
8	Finland	18.376	29.157	101.806	0.428	0.428
9	France	117.72	172.7	125.714	0.982	0.982

10	Netherlands	55.346	37.629	114.993	1	1
11	Ireland	12.724	15.281	110.309	0.564	0.564
12	Spain	84.72	150.582	73.472	0.916	0.916
13	Swedish	36.057	55.365	112.857	0.576	0.576
14	Italy	100.627	141.58	129.555	0.939	0.939
15	Iceland	1.374	2.477	65.792	0.368	0.368
16	Japan	130.49	388.831	158.463	0.571	0.571
17	Korea	74.105	181.284	157.36	0.552	0.552
18	Latvia	1.649	6.929	28.937	0.162	0.162
19	Luxembourg	1.636	0.935	179.438	1	1
20	Hungary	10.089	23.112	65.117	0.314	0.314
21	Norway	18.736	30.583	84.652	0.446	0.446
22	Poland	37.39	80.223	56.927	0.677	0.677
23	Portugal	21.813	55.707	37.726	0.578	0.578
24	Slovakia	6.589	18.465	40.156	0.303	0.303
25	Slovenia	5.921	5.494	108.445	0.721	0.721
26	Chile	9.132	11.058	115.049	0.557	0.557
27	Turkey	39.327	113.409	60.178	0.552	0.552
28	New Zealand	14.163	25	47.993	0.519	0.519
29	Greece	18.473	54.602	33.836	0.514	0.514

As is clear from the results, the proposed and classic model results are the same and can be used instead of each other. Apart from the theoretical suggestion, If the results are evaluated; United Kingdom, Netherlands and Luxembourg are the efficient countries, whereas Slovakia, Estonia and Latvia are the most inefficient countries with efficiency score of 0.303, 0.292 and 0.162 respectively according to used variables. In inefficient countries which have efficiency score below 1, regardless of whether they are developed or not, it can be mentioned that there is a wastefulness in the human capital and financial resources.

## CONCLUSIONS

Although the scope of the work is theoretical and OECD data is used to support this theoretical extension, it is worth evaluating the obtained results. In particular, the fact that the developed countries of the present day do not become efficient as a result of the analysis is also a matter of research and is revealed by the results of the DEA which shows that there is waste of resources arising from high welfare.

As mentioned above t-norms and t-conorms are used to define fuzzy intersection and fuzzy union respectively. Eventually, while the proposed model provides the form of t-norms, the classical model will lead the researchers to result practically. This is also a new opening for DEA, which is a method that transforms the data to the range of  $[0,1]$ . The proposed model gave solution, when the set of outputs have single elements as the applications of t-norm or stochastic DEA. Or, on the contrary, when input set have single element, it produces a solution. Whereas, DEA gave solution in multi output-multi input case. Adapting this feature of DEA to proposed t-norm DEA model is a future improvement area.

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# BANK EFFICIENCY AND GROWTH IN OECD COUNTRIES

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## ABSTRACT

*This study analyses the relationship between economic growth and bank efficiency in OECD countries. In the first stage, banks' efficiencies over time are measured by Data Envelopment Analysis based Window analysis. In the second stage, bidirectional relationship between banks' efficiencies and growth are investigated with a dynamic panel data analysis model. The findings show that banks' efficiencies have been decreasing across developed countries that may have resulted from structural changes in banking and new macroeconomic environment after global financial crisis. The results also suggest that efficiency is not effective on growth for all groups of countries, and vice versa. The condition of being uncorrelated may be called as "efficiency neutrality". This may resemble the Keynes' liquidity trap where monetary expansion is ineffective on growth and efficiency of banks. Efficiency decrease and efficiency neutrality may indicate that monetary expansion have not boosted consumption and/or investment, and consequently banking loans.*

**Keywords:** Bank efficiency; economic growth; Data Envelopment Analysis; window analysis; system GMM

## INTRODUCTION

Financial sector has been always in the focus of researchers. Because it is believed that financialization and/or monetary expansion initially boost consumption and investment, then real sector. However, the new normal of global financial crisis and post crisis period, which is characterized by the extraordinary monetary expansion (about three to five times compared to pre-crisis level in US, UK, EU and Japan) and negative interest rates, raises doubt about relationship between financial development and economic growth.

This study aims to investigate bidirectional relationship between efficiency of banks and economic growth in OECD countries. Total gross domestic product (GDP) of OECD countries constitutes about 63% of world economy. The results will reveal tendency in the efficiency of financial sectors, and then relationship between efficiency and growth investigating a way from microeconomics to macroeconomics. There is a vast amount of literature that target to analyse relationship between financial development, efficiency and economic growth. Financial development and financial efficiency are two different concepts. Financial development is generally associated with either quantity of credit volume, private sector loans or private sector loans per employee, ratio of credit to GDP etc. Financial efficiency, on the other hand, is generally associated with quality of financial activities (Berger, Hasan, & Klapper, 2004; Hasan, Koetter, & Wedow, 2009; Koetter & Wedow, 2010).

Studies associating bank efficiency to economic growth are comparatively new and limited (See Table 1).

**Table 1: Studies that analyse the relationship between efficiency of financial sector and economic growth.**

Year Author	Sample Period	1st Stage	Second Stage		Conclusion
		Mtd/Eff.	2nd Stage	Independent	
2017 Diallo	38 2009	DEA IE	Regression • GDP growth	<ul style="list-style-type: none"> <li>• Bank efficiency</li> <li>• Market capitalization</li> <li>• Tot capitalization</li> <li>• Concentration</li> </ul>	<b>Bank efficiency</b> relaxed credit constraints and <b>increased the growth rate for financially dependent industries</b> during the crisis.
2017 Mirzaei & Moore	49 2001- 2010	SFA CE, PE	Regression • Ind. growth	<ul style="list-style-type: none"> <li>• Bank CE, PE</li> <li>• Average firm size • Share in VA</li> <li>• Fin. dependence</li> <li>• Prop. rights • Stock turn. ratio</li> </ul>	<b>Industries that rely on external finance grow faster</b> in countries with <b>efficient banking</b> . However, eff. effect is mainly derived from the cost side during the crisis period.
2016 Belke et al	12 EU 2000- 2013	SFA CE, PE	System GMM • GDP p.worker	<ul style="list-style-type: none"> <li>• Fin Q (Efficiency)</li> <li>• Labor Force Growth • Education</li> <li>• HHI, Heritage, Lerner Index</li> <li>• Income/branch</li> <li>• Bank income pc • Fin. volume</li> </ul>	Results show that relatively more <b>profit efficient banks foster the economic growth</b> . The link between financial quality and growth is valid in normal" times as well as in bad" ones.
2016 Ferreira	EU 1999- 2013	DEA CE	Diff GMM • GNP	<ul style="list-style-type: none"> <li>• Bank efficiency</li> <li>• Int. rate • Gov. net lending-borr.</li> <li>• Bank concentration • Equity/TA</li> </ul>	<b>Bank efficiency contributes positively to economic growth</b> , confirming that well-functioning banks are at least a necessity to the increase of the gnp.
2013 Ferreira	27 EU 1999- 2013	DEA CE	Gran. Caus. • GDP per c. • Gr.cap. Grwth	<ul style="list-style-type: none"> <li>• Cost efficiency</li> <li>• ROE</li> <li>• ROA</li> </ul>	<b>Positive causality from bank performance to economic growth</b> . However, econ. growth positively contributes to the bank ROA and ROE ratios but not certainly in case of CE.
2012 Mensah et al	AFR 1999- 2008	SFA CE	Diff GMM • GDP p.c.	<ul style="list-style-type: none"> <li>• Cost efficiency</li> <li>• Priv. loans • Pop. growth rate</li> <li>• Investments/gdp • Govt spend.</li> <li>• Econ. Freedom • Corrup. Ind.</li> <li>• Infl. rate • Bank concentration</li> </ul>	There is a <b>positive relationship between banking sector efficiency and economic growth</b> , confirming the critical role banks play in the economy.
2010 Koetter & Wedow	DEU 1995- 2005	SFA CE	System GMM • GDP p.worker	<ul style="list-style-type: none"> <li>• Cost efficiency</li> <li>• Bank loans and sec./GDP</li> <li>• Growth rate employed</li> <li>• Tertiary ed./total workers</li> <li>• HHI bank assets • Lerner index</li> </ul>	<b>Quality measure has a significantly positive effect on growth</b> .
2009 Hasan et al	11 EU 1996- 2004	SFA CE, PE	System GMM • GDP	<ul style="list-style-type: none"> <li>• Efficiency</li> <li>• Financial volume</li> </ul>	<b>Regional economic growth in mature economies and recent periods benefits significantly from banks that are more efficient</b> .
2001 Lucchetti et al	ITA 1982- 1994	SFA CE	System GMM • GDP p.c.	<ul style="list-style-type: none"> <li>• Inefficiency</li> <li>• Loan/gdp • Human capital</li> <li>• Number of bankruptcies</li> <li>• Share of loans by coop. banks</li> <li>• Share of priv. sector loans</li> </ul>	The empirical results show the existence of an independent effect exerted by <b>the efficiency of banks on regional growth</b> .
2016 Hasan et al	60 1960- 2011		BMA	<ul style="list-style-type: none"> <li>• <b>Net interest margin</b></li> <li>• Bank Z-score • Private credit</li> <li>• Market cap. • Market turnover</li> </ul>	Common indicators of financial development are not robustly related to long-term growth. However, a new indicator—the <b>efficiency of financial intermediaries—is robustly related to long-term growth</b> .
2016 Mirzaei & Moore	QAT 2000- 2006		Panel FE • Growth of VA	<ul style="list-style-type: none"> <li>• <b>Cost to income ratio</b></li> <li>• <b>Overheads to total assets</b></li> <li>• <b>Interest rate spread</b></li> <li>• Credits • GDPgrowth</li> </ul>	<b>A competitive, efficient and stable banking system is indeed a source of enhancing financially dependent industries to grow faster</b> .
2015 Yusifzada & Mammadova	118 2004- 2011		System GMM • GDP growth p.c.	<ul style="list-style-type: none"> <li>• <b>NIM and ROA</b></li> <li>• Gov. expenditure</li> <li>• Trade • Secondary educ.</li> </ul>	The impact of efficiency differs depending on the level of financial development and has an inverted S-shape function
2015 Capelleblancard & Labonne	24 OECD 1970- 2008		System GMM • GDP growth p.c.	<ul style="list-style-type: none"> <li>• <b>Priv. credit/employees</b></li> <li>• Private credit/GDP</li> <li>• Employees, • Initial GDP</li> <li>• Second. School • Govern. Expen.</li> <li>• Inflation • Openness ratio</li> </ul>	<b>Fail to find a positive relationship between financial deepening and economic growth in OECD countries</b> over the last 40 years
2013 Saqip	50 2005- 2009		Panel • GDP growth p.c.	<ul style="list-style-type: none"> <li>• <b>Net interest margin</b></li> <li>• Investment to GDP • Enrollment</li> <li>• M2/GDP • Priv. Credit to GDP</li> </ul>	<b>Development and efficiency of financial sector stimulates economic growth</b> .
2013 Ayadi et al	11 SEMC 1984- 2010		Panel • GDP growth p.c.	<ul style="list-style-type: none"> <li>• <b>Cost efficiency</b></li> <li>• Financial dev. • Opnennes • FDI</li> </ul>	An improvement in <b>banking efficiency is not sufficient to improve growth</b> , additional conditions must be met, such as better quality institutions, regulations and supervision.

PE: Profit efficiency, CE: Cost efficiency, IE: Intermediation efficiency, p.c.=Per capita

Koetter and Wedow (2010), Hasan et al. (2009), Hasan et al. (2016) refer to efficiency as quality of financial sector. Some studies argue that bank efficiency can be related to economic growth much more



than traditional quantity measures (Hasan et al., 2016; Hasan et al., 2009). We found 16 studies integrating efficiency. Six of them implement model in one step, and they commonly use net interest margin as a proxy for efficiency. Almost all employ a panel data analysis except (Hasan et al., 2016) that uses Bayesian model averaging. Compared to financial development studies, the results of these studies are miscellaneous. Saqib (2013), Mirzaei and Moore (2016) and Hasan et al. (2016) indicate a positive effect of efficiency on growth, while Ayadi, Arbak, Naceur, and De Groen (2015) and Capelle-Blancard and Labonne (2016) fail to find a relationship. Yusifzada and Mammadova (2015) found impact of efficiency to differ depending on the developed, developing and emerging countries.

Nine studies researched effects of efficiency on economic growth in two consecutive stages. In the first stage, they employed a frontier based efficiency measurement tool, either Stochastic Frontier Analysis (SFA) or DEA. In second stage, they used an econometric model of time series or panel data. In all studies, except for (Diallo, 2017), cost efficiency is measured. Ferreira (2013), Ferreira (2016) and Diallo (2017) used DEA, while all others utilize SFA to measure bank efficiency (Belke, Haskamp, & Setzer, 2016; Hasan et al., 2009; Koetter & Wedow, 2010; Lucchetti, Papi, & Zazzaro, 2001; Mensah, Abor, Aboagye, & Adjasi, 2012; Mirzaei & Moore, 2017). Therefore, instead of using a proxy for efficiency in an econometric model, they used efficiency measured in first stage. Majority of the papers use a dynamic model of panel data analysis (Difference GMM or System GMM) in the second stage. All models identified a positive effect of bank efficiency on economic growth. One of the drawbacks of studies, which handle relationship between financial development and economic growth, is that although effects of efficiency on growth is studied to some extent, opposite direction, i.e. effects of growth on efficiency still seems to be untouched and needs further study.

## METHODS

In the first stage of the study, efficiency scores of banks in OECD countries are measured, and then in the second stage, effects banks' efficiencies on economic growth is analysed. To see whether growth is effective on bank efficiency, opposite possibility is also investigated.

With Window Analysis (WA) perspective we employed a Slack Based Model (SBM) of Data Envelopment Analysis (DEA) that evaluates minimum distance to Constant Returns to Scale (CRS) frontier (SBM-Max) (Tone, 2015). SBM is a non-radial model that considers input or output slacks simultaneously and proposes a non-proportional rate of decrease/increase for inputs/outputs of inefficient units. Efficiency in financial sector has many aspects such as profit, cost, intermediation, production, operation. In our study, we measured the efficiency of banks considering intermediation role.

To measure the relationship between bank efficiency and economic growth in the second stage, following equation is formulated:

$$gdp_{it} = \beta_0 + \beta_1 gdp_{it-1} + \beta_2 eff_{it} + \eta_i + \lambda_t + \varepsilon_{it}$$

where  $gdp_{it}$  is gross domestic product (volume, 2010=100) and  $eff_{it}$  is intermediation efficiency measured by DEA-SBM in the first stage.  $\eta_i$  is unit effect,  $\lambda_t$  is time specific effect, and  $\varepsilon_{it}$  is error term.

Including lagged value of *gdp* to the model makes it dynamic. Since causality may exist in both directions, influence of economic growth on efficiency is also investigated. To estimate equations, system Generalized Methods of Moments (GMM) estimator suggested by Arellano and Bover (1995) and Blundell and Bond (1998) is used. System GMM estimator is designed for situations where cross-section dimension is greater than time dimensions in data. System GMM has some advantages over difference GMM, because it combines regressions expressed in first-differences and levels in a system of equations, thereby correcting unobserved heterogeneity, time-invariant components of measurement error, omitted variables bias and potential endogeneity bias.

2970 banks' data of 31 OECD countries for the period of 2011-2016 are used in this study. For efficiency measurement with DEA, equity, deposits and other interest bearing liabilities are used as inputs while loans and other earning assets are used as outputs.

## RESULTS AND DISCUSSIONS

In the first stage, efficiency of each individual bank is measured within the country they belong to, and then, based on asset size, weighted average efficiency is calculated for each country to represent banking sector. The efficiencies of banking sectors for OECD countries are presented in Table 2.

**Table 2. Efficiency scores of banking sectors in OECD countries.**

	AUS	AUT	BEL	CAN	CHE	CHL	CZE	DEU	DNK	ESP	EST	FIN	FRA	GBR	GRC	HUN	IRL	ISR	ITA	JPN	KOR	MEX	NLD	NOR	POL	PRT	SVK	SVN	SWE	TUR	USA
2011	95	97		91					93	93			82	96	99		94	91	95	85	97		82	85	96				88	98	80
2012	94	98		90					92	90			81	96	98		95	92	94	82	99		80	83	94				83	98	80
2013	94	97	93	87	98	90	94	91	91	89	99	95	79	93	99	99	97	93	92	77	97	91	82	88	97	95	99	94	78	98	77
2014	93	99	92	87	98	94	91	91	96	90	78	95	79	91	98	89	94	91	92	74	99	91	79	90	97	91	100	94	81	98	77
2015	92	96	92	88	97	96	88	88	95	87	86	96	78	90	97	90	94	89	91	73	98	89	80	88	97	90	98	92	78	98	78
2016	91	95		85	95	93			95	86		92	79	89			87					93	76	84	98				78	98	79

Note: Green color indicates more, red indicates less efficient.

A decisive decrease is observed in banks' efficiencies globally. This decrease is even sharper in developed countries compared to developing ones, and for European Union member countries compared to non-members. There may be two main reasons for this trend: structural changes in (1) banking and (2) macroeconomic environment. The first reason may be related to structural changes in banking industries around the globe. After the global financial crisis, international community focused on strengthening banking sectors through introduction of new regulations like Basel III that required banks to attain higher level of capital requirements. These new regulations subjected banks to more strengthened supervision, tighter risk management criteria and more transparent trading activities of derivatives. Some banks have recapitalized in some countries like Italy and Portugal, and there have been consolidation within banking sector in some countries. The second reason is related to macroeconomical environment. Because of lower interest rates and lower growth rates after the global financial crisis, smaller banks using deposit as a main source of funding and banks with less diversified sources have become more vulnerable to fluctuations. When interest rates, and consequently deposit rates, tend to approach zero lower bound,

banks could no longer maintain spreads between loans and deposits. Net interest margin squeezes and thus profitability decreases (IMF, 2017b). This jeopardizes intermediation function of banking and forces banks to find new profitable areas especially in emerging countries.

The correlation between banks' efficiencies and GDP is about 4%, which indicates almost no relationship. Table 2 illustrates effect of efficiency on GDP. The Hensen test and AR(2) tests indicate consistent results. All insignificant efficiency coefficients show that efficiency is not effective on economic growth. Although this finding contradicts with majority of the studies indicating positive effect of efficiency on growth, it confirms findings of Capelle-Blancard and Labonne (2016). Capelle-Blancard and Labonne (2016) also fail to find a positive relationship between economic growth and financial deepening in OECD countries over last 40 years. Ayadi et al. (2015) indicated that an improvement in banking efficiency is not sufficient for economic growth, some better additional environmental conditions are required. Moreover, Hasan et al. (2016) state that it becomes more common in literature to conclude about diminishing and eventually negative results of financial development. Rousseau and Wachtel (2011) also specify that positive correlation between economic growth and financial sector is typical before 1990; however, it decreases in subsequent years. There are some studies arguing that financial development lead to growth only up to a certain threshold (Hasan et al., 2016).

Table 2a. Effect of efficiency on GDP				Table 2b. Effect of GDP on efficiency			
	OECD	Developed	Developing		OECD	Developed	Developing
L.gdp	1.219*** (0.036)	1.358*** (0.157)	1.106*** (0.028)	L.eff	0.811*** (0.152)	1.014*** (0.052)	0.478*** (0.092)
eff	5.341 (5.156)	14.052 (16.622)	-1.336 (13.967)	gdp	0.000 (0.000)	-0.000 (0.000)	0.002 (0.002)
Const	-25.834*** (6.186)	-47.860 (29.769)	-7.195 (12.181)	Const	0.160 (0.133)	-0.002 (0.054)	0.302* (0.137)
Obs #	130	97	33	Obs #	118	92	26
Country #	31	22	9	Country #	31	22	9
Instr #	18	18	17	Instr #	18	18	5
AR1 p / R <sup>2</sup>	0.704	0.311	0.254	AR1 p / R <sup>2</sup>	0.009	0.003	0.516
AR2 p	0.307	0.742	0.325	AR2 p	0.395	0.348	0.437
Hansen p	0.139	0.451	0.911	Hansen p	0.572	0.570	0.425

*Notes: Robust standard errors in parentheses. \*\*\*, \*\* and \* indicates significance level of 0.01, 0.05 and 0.10. P is for probability value.*

Searching for opposite relationship, results do not indicate effect of GDP on efficiency of banks also (Table 3). Findings are in parallel with Ferreira (2013)'s conclusions stating that contribution of economic to cost efficiency is not certain. The condition of irrelevancy between efficiency and economic growth may be called as "efficiency neutrality". Both efficiency decrease and "efficiency neutrality" may indicate that monetary expansion has not boosted consumption or investment, and then consequently banking loans. Effects of monetary expansion on economic growth were not witnessed after global crisis. This resembles the Keynes' liquidity trap where monetary expansion is ineffective.

## CONCLUSIONS

This study, first, estimates intermediation efficiency of 2970 commercial banks in OECD countries for the period of 2011-2016 as employing a SBM model of DEA. Then relationship between banks' efficiency and economic growth is investigated using System GMM estimator.

Generally a decrease in efficiency is observed, especially in developed countries. This may results from structural changes in banking and new macroeconomic environment after global financial crisis. Dynamic panel data analysis shows no relationship between efficiency and economic growth in both directions which may be called as “efficiency neutrality”. As some researchers claim, relationship between financial efficiency and economic growth may have been weakened in post financial crisis period. Efficiency decrease and efficiency neutrality may indicate that monetary expansion have not boosted consumption or investment, and then consequently banking loans. It seems that monetary expansion did not push up economic growth after crisis. This resembles the Keynes’ liquidity trap where monetary expansion is ineffective.

The findings are important as they may change our thoughts about interaction between financial development and economic growth. It reveals the link between microeconomics and macroeconomics in banking sector. However, more studies need to be conducted with data of longer period to confirm the changing relationship that believed to exists before 90s. Country based examinations should also be experimented to see whether this relationship differs from country to country.

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# BENCHMARKING OF INDIAN ELECTRICITY DISTRIBUTION UTILITIES USING DATA ENVELOPMENT ANALYSIS

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## ABSTRACT

*Before undertaking innovation in a reform there is a need to undertake prior appraisal of existing utilities by analyzing their performance, efficiency and effectiveness. This paper presents a framework for evaluating the cost-efficiency of Indian distribution Utilities (DISCOMs). Majority of the DISCOMs in India are government undertakings. The Efficiency evaluation is based on Data Envelopment Analysis (DEA) and the analysis promises to provide a useful tool to the regulating commission for tariff setting and also in future for calculating X-factor for Performance based regulation. benchmarks were derived for network cost as inputs, while estimated average Energy sold (KWh), Energy consumed per square kms (KWh), Energy consumed per thousand populations (KWh), Customers (Million) and Distribution line length were chosen as the output variables. The results obtained indicate widespread inefficiencies in the Indian Electric distribution utilities.*

**Keywords:** Benchmarking, Data envelopment analysis, Discom

## INTRODUCTION

Recent years have witnessed wide and rapid acceptance of Power Industry restructuring in several developing countries. This has been necessitated by the pressing need for improvements in the existing power services in most of these countries, many of which are plagued by sub-optimal sector performances. Huge demand-supply gap is often a universal problem in developing countries and the distribution sectors are frequently financially crippled (Scott and Seth, 2013). High distribution losses, poor management, low market densities, poor metering and billing practices and weak institutions are some of the common problems besieging the developing nations (Jamash et al., 2015). Under these circumstances, a performance enhancing strategy to initiate improvements may incorporate wide sector reforms as initiated in a number of developing countries. The improvement of efficiency of the utilities would provide efficient and economic power supply to the consumers i.e., providing a defined output for minimal inputs.

The Indian Power sector has been undergone wide reforms with the implementation of Electricity Act 2003 (Thakur et al, 2005), but the reforms have not yet been able to check deteriorating financial and operating conditions of utilities. One of the major reasons behind this has been the existence of a financially weak distribution sector, which has been the monopoly of the government owned State Electricity Boards (SEBs) that face huge revenue loss and are unable to make payments to the generating units (Pargal and Banerjee, 2014). Thus reform has not succeeded in improving technical efficiency or in improving financial position of the sector. Also it could not reduce the losses or improve customer satisfaction. The objective of this paper is to develop a DEA based model for assessing the cost efficiency of Indian distribution utilities. Based on the efficiency analysis, benchmarks can be set, and utility efficiency scores can be obtained based on the set benchmarks. These scores can help develop a strategic plan for mitigating the factors that contribute to the system inefficiencies. Such studies can greatly help in

avoiding structural and contractual inefficiencies in the emerging design of the power sector. This is especially crucial and significant currently as the important features of the sector (like tariff policy) are being redesigned in the ongoing reform process.

## **METHODS**

This section details an empirical study of applying DEA to evaluate operating efficiencies of Utilities. The models have been employed for benchmarking with respect to the cost named as Distribution-cost-DEA (DC-DEA), based on the operating expenditure (OPEX), which can be controlled by adoption of better management practices. An input-oriented approach was chosen because the very first objective of the analysis was to suggest benchmark for cost efficiency in order to produce a given output at minimal cost. In this study, two DEA models were applied: CCR model developed by Charnes et al., 1985 and BCC model developed by Banker et al, 1984. CCR model produces constant return to scale (CRS) efficiency frontier and evaluates overall (or aggregate) efficiency score. BCC model produces variable return to scale (VRS) efficiency frontier. Thus, the overall efficiency can be decomposed into the technical efficiency and the scale efficiency.

## **INPUTS AND OUTPUTS OF THE MODEL**

The input/output selection for the present study consists of those parameters, which directly affect the distribution of electricity and working culture of the utility. Input variable is operational expenditure (OPEX) of distribution, which constitutes the administrative and general cost (A&G), operation and maintenance cost (O&M), interest/depreciation and miscellaneous cost. The analysis precluded the power purchase cost in the network cost (input) because this cost is essentially of non-controllable nature. The OPEX based model used in this study for analyzing cost efficiency of utilities is based on the accounting information reported by the companies and not on the traditional physical parameters and standard prices, it suits with the Indian conditions, because the price of labor and material costs tend to vary according to locality. Output variables are chosen in such a way that they reflect the degree to which the distributor utility is meeting its objective of facilitating the flow of energy to consumers. Thus, energy sold forms an important output. Energy consumed per square kms (KWh) incorporates the comparison of utilities based on the area that they are serving. The network lengths were used as an indicator of the geographical dispersion of the customer base. India is the second largest country in the world in terms of population and customer numbers for various utilities constitutes an important output variable acting as a proxy for the number of connection points for each distributor.

## **EMPIRICAL RESULTS AND DISCUSSIONS**

CCR model is applied with constant returns to scale, to evaluate the overall efficiency of each utility and, BCC model, with variable returns to scale to calculate technical efficiency and scale efficiency of all 27 utilities for DC-DEA model. The results are shown in Table 1, it is evident from Table 1 that only 3 utilities are overall efficient with an efficiency score of 100%. These utilities belong to the states of Andhra Pradesh, Tamil Nadu and Pondicherry. It is interesting to note that all 3 states are neighbors and

belongs to the southern part of India. The remaining 24 utilities are relatively input inefficient to varying degrees. The mean total efficiency score of the 27 utilities is 36.53, when the best practices are excluded the mean inefficiency score is 28.6. The low efficiency scores clearly demonstrate the poor performance trends of the SEBs. Furthermore, we also found that most of the medium and small sized Utilities are relatively inefficient. It is also evident from the analysis that four out of the worst five Utilities belongs to north east region and this region is also political not stable. By using BCC model, with variable returns to scale the technical efficiency scores are evaluated. Also, the scale efficiency can be derived by the ration of overall efficiency to technical efficiency. When Technical efficiency scores are evaluated, two additional Utilities of Delhi and Maharashtra also become efficient with technical efficiency scores of 100%, but they are not scale efficient. This implied that they should adjust their scales of operation to improve their scale efficiencies as well as overall efficiencies . In the present analysis 21 inefficient utilities exhibited increasing returns to scale, implying that they can increase the scales to effectively improve their efficiencies. Only 3 utilities show decreasing returns to scale, implying that they must decrease their scales to possibly improve their efficiencies. A very interesting fact of the above analysis is all the 22 inefficient utilities are inefficient mainly due to the technical inefficiency because their technical inefficiency scores are lower than scale efficiency scores. This implied that these utilities are urgently required to improve their technical efficiencies so that they can improve productivity and make better use of their resources.

**Table 1: Various Efficiencies Scores**

S. No.	SEB	Total Efficiency	Technical Efficiency	Scale Efficiency	Returns to Scale
1	Andhra Pradesh	100	100	100	Constant
2	Assam	2.5	7.26	34.43	Increasing
3	Bihar	17.9	25.96	68.95	Increasing
4	Delhi	54.9	100	54.9	Decreasing
5	Gujarat	89.4	89.83	99.52	Increasing
6	Haryana	30.6	36.63	83.53	Increasing
7	Himachal Pradesh	10.8	17.1	63.15	Increasing
8	Jammu & Kashmir	10.5	20.73	50.65	Increasing
9	Karnataka	75.8	79.89	94.88	Increasing
10	Kerala	29.2	34.41	84.85	Increasing
11	Madhya Pradesh	52.1	54.42	95.73	Increasing
12	Maharashtra	90.6	100	90.6	Decreasing
13	Meghalaya	1.8	8.04	22.38	Increasing
14	Orissa	5.8	7.45	77.85	Increasing
15	Punjab	43.8	46.43	94.33	Decreasing
16	Rajasthan	55.8	62.56	89.19	Increasing
17	Tamil Nadu	100	100	100	Constant
18	Uttar Pradesh	54.1	56.61	95.56	Increasing
19	West Bengal	21.8	29.31	74.37	Increasing
20	Arunachal Pradesh	0.4	2.86	13.98	Increasing
21	Goa	31.9	48.62	65.61	Increasing
22	Manipur	0.9	6.54	13.76	Increasing
23	Mizoram	1.4	9.1	15.38	Increasing
24	Nagaland	1.4	13.31	10.51	Increasing
25	Pondicherry	100	100	100	Constant
26	Sikkim	0.9	8.78	10.25	Increasing
27	Tripura	2	12.24	16.33	Increasing
	<b>Average</b>	<b>36.52</b>	<b>43.63</b>	<b>63.72</b>	



**Table 2: Probable Cost Saving (Millions of Rupees (Note: Rupee 64.318=1US\$))**

SEB	Actual Cost	Target Cost	Savings	% Savings
Assam	3290.71	230.95	3050.75	92.7
Bihar	1340.27	340.85	990.41	74
Gujarat	970.39	870.49	90.89	10.1
Haryana	1110.03	400.67	700.35	63.3
Himachal Pradesh	1490.24	250.53	1230.70	82.8
Jammu & Kashmir	1070.51	220.29	850.21	79.2
Karnataka	970.33	770.76	190.56	20.1
Kerala	1540.22	530.07	1010.14	65.5
Madhya Pradesh	1430.44	780.07	650.36	45.5
Meghalaya	2310.8	180.64	2130.15	91.9
Orissa	5360.71	390.99	4960.71	92.5
Punjab	1510.71	700.44	801.26	53.5
Rajasthan	960.4	600.31	360.08	37.4
Uttar Pradesh	1370.68	770.95	590.72	43.3
West Bengal	1310.16	380.45	920.70	70.6
Arunachal Pradesh	6560.3	180.78	6370.51	97.1
Goa	390.12	190.02	200.09	51.3
Manipur	2830.64	180.56	2650.07	93.4
Mizoram	2000.58	180.25	1820.32	90.8
Nagaland	138.7	180.46	1200.23	86.6
Sikkim	205.46	180.04	1870.42	91.2
Tripura	1510.39	180.549	1320.84	87.7
Kerala	1540.22	530.07	1010.14	65.5
Madhya Pradesh	1430.44	780.07	650.36	45.5
Meghalaya	2310.8	180.64	2130.15	91.9
Orissa	5360.71	390.99	4960.71	92.5
Punjab	1510.71	700.44	801.26	53.5
Average of inefficient SEBs	1880.63	460.73	1410.89	66.1

Table 2 clearly demonstrates the necessity for induction of efficiency in the distribution operation services in India. As per the model DC-DEA, it is theoretically possible to save Rs. 1410 Million per annum by induction of efficiency in the services as per the current best practices in utilities. Given the model inadequacies and the field constraints and requirements, this amount may actually be less in reality. But one must consider the fact that these improvements are with respect to the Indian best practices and may still be lagging behind the best practices and efficiencies in the distribution services in developed parts of the world. What is obvious is the fact that, there is a definite indication of the existence of possibility of making huge savings through efficiency improvements.

## CONCLUSIONS

The results of the study indicate the existence of cost inefficiency; majority of the utilities is not producing at the minimum level of the cost. To improve the performance of utilities, a number of measures might be taken, as this is the first attempt for accessing the efficiency of distribution utilities in India based on DEA thus, there is further scope for model improvement by incorporating power quality measurement. All the stakeholders can benefit from above study. Such a study can also be used for the following:

- The model will provide the efficiency scores of the utilities so they can identify their shortcoming, can set targets and try to achieve these targets

- The model can be used by regulating commission for tariff setting and as a tool for developing a monitoring system.
- The analysis can have future application in the form of X-factor calculations under the incentive based regulation
- Preferential allocation of the Government funds.
- Such a study can help create awareness and competition amongst the utilities, for sustained improvements in the distribution sector.

## ACKNOWLEDGEMENT

The author is grateful for the financial support from Ministry of Human Resource Development, Government of India and MANIT Bhopal for providing financial support to attend DEA 2017 Conference at Prague.

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# **BUSINESS INTELLIGENCE SOLUTION FOR EVALUATING THE MATCHING EFFICIENCY OF PUBLIC EMPLOYMENT SERVICES IN FINLAND**

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## **ABSTRACT**

*We study the performance of public employment services in Finland. In this project, a business intelligence application is developed in close collaboration with the Ministry of Economic Affairs and Employment. This paper focuses on the matching of vacancies and jobseekers. We utilise one-stage data envelopment analysis and panel data to determine the shape of a matching function, and analyse the effects of operational conditions. The performance of the regional public employment service offices is assessed and the required efficiency measures and performance indicators are regularly reported to the decision-makers in the ministry and the offices using a state-of-the-art data visualization software.*

**Keywords:** *public employment services, matching, data envelopment analysis, business intelligence*

## **INTRODUCTION**

Public service providers often have rich data in their transactional systems and customer databases. These data ought to be utilised in decision-making and should also be available for public use to enhance the transparency and accountability of the government. By utilising the data sources, productive efficiency evaluation methods, and advanced visualisation software, this paper develops a framework for a business intelligence (BI) solution to assess the performance of regional public employment services (PES).

This project was initiated by the Ministry of Economic Affairs and Employment (MEAE). The main goal was to design indicators to evaluate the efficiency of regional PES offices and implement these indicators in a BI solution for PES. We closely collaborated with MEAE and the software vendor. The results of this study are made visible to PES decision-makers on a continuous basis. Moreover, it is intended that the results should be communicated to political decision-makers and the general public to increase accountability and transparency.

In Finland, PES are a part of central government. They are organised in 15 regional offices, providing placement and recruitment services, and labour market training programmes for jobseekers. They also serve employers searching for employees and entrepreneurs starting new businesses. Improving employment is one of the key initiatives of the government. Finland had been in recession practically from the financial crisis of 2008 until 2016. The current employment rate is 69%, which is a challenge for a country with a rapidly ageing population. The country has been undergoing a strong structural change in which the employees of traditional industries like manufacturing have declined and the emerging services have not been sufficient to compensate. At the same time, with a relatively high unemployment rate,

10.7% in May 2017 (“Statistics Finland - Labour force survey,” 2017), employers have difficulties in finding employees. The average filling time of vacancies announced in PES doubled from 20 days in 2008 to 40 days in 2016 (“MEAE Employment service statistics”, 2017). Thus, efficient PES have a critical role for the economy.

In the rest of the paper, key principles in the development of a data-driven performance evaluation BI solution are explained. In this paper, we specifically focus on the efficiency in matching unemployed jobseekers and employers searching for a workforce. First, we depict our approach founded on the concepts of labour and production economics. Then, we utilise a non-parametric technique to estimate the efficiency of PES offices. The indicators are made available for recurring use and they are presented to more than 500 decision-makers using state-of-the-art visual analytics software. Some of the findings of the research study are presented in this paper.

## **THEORY**

The phenomenon of concurrent high unemployment and recruiting difficulties is called the matching problem. In such a situation, one of the main sources of persistent unemployment is a mismatch between the skills of employees and the needs of employers.

In labour economics, the performance of regions is often studied by estimating the matching function (Blanchard & Diamond, 1989; Pissarides, 1990). Such a function maps the stock of jobseekers and the stock of vacancies to the maximum achievable number of job matches. This approach is employed in numerous efficiency studies (see, e.g., Fahr & Sunde, 2002; Ibourk et al. 2004; Ilmakunnas & Pesola, 2003; Sheldon, 2003; Hynninen, Kangasharju & Pehkonen, 2009; Tomic, 2014).

In this paper, our data-driven approach to the productivity of PES offices benefits the decision-makers in various ways: (1) it provides a framework for constant monitoring of productivity and efficiency, (2) the decision-making process is transparent and the results are reproducible, and (3) it takes into account nationwide changes in the labour market.

## **METHODS**

Even though the majority of matching efficiency studies have been based on econometric techniques utilising Cobb-Douglas-type matching functions, a non-parametric approach has also been used. Sheldon (2003) evaluated the matching efficiency of Swiss PES offices using data envelopment analysis. We also preferred the non-parametric approach, as we cannot be certain of the exact functional form of the matching function. Explaining the results of a non-parametric approach to decision-makers is also usually easy and straightforward. Following the matching theory, we assume the matching function is concave and monotonically increasing.

### Model and data

The output is the number of *job matches* that are registered as unemployed people who are recruited on the open job market during the past 12 months. The first input is the *stock of unemployed*, which includes customers who are registered as unemployed or laid off. It also includes customers who are participating

in the employment programmes of PES. The second input is the *stock of vacancies*, which includes the vacancies that are open in the PES database. This is a proxy of all vacancies in the region, because employers are not obligated to register vacancies. Both stocks are 12-month averages.

There are only fifteen regional PES offices in Finland. In order to get a sufficient data window to estimate the matching function, we utilise the annual panel data from 2006 to 2016. As the offices are present 11 times in the data window, we take a conservative approach and assume constant returns to scale, to minimise the risk of overestimating the performance of the smallest and largest PES offices.

## ESTIMATING THE MATCHING FUNCTION

In this paper, and unlike similar studies, in order to estimate a more robust matching function against outliers and annual fluctuations, the method of one-stage data envelopment analysis (DEA-1) is employed (Johnson & Kuosmanen, 2012). The estimated matching function reflects the central tendency of PES offices rather than the best practices.

A matching function  $f: \mathbb{R}_+^2 \rightarrow \mathbb{R}$  models the relationship between jobseekers ( $U_{it}$ ) and job vacancies ( $V_{it}$ ) and the number of job matches ( $M_{it}$ ) in the period of study ( $t$ ) for PES offices ( $i$ ). The nationwide environmental variables ( $z_t$ ) represent labour market conditions that affect the productivity of PES offices. The matching function is defined as  $M_{it} = f(U_{it}, V_{it}, z_t)$ . Following Banker and Natarajan (2008) and Johnson and Kuosmanen (2012), function  $f$  is estimated via a non-parametric partial log-linear concave least squares regression problem that models a stochastic intertemporal matching function:

$$\begin{aligned} \min \sum_i \varepsilon_i^2 \\ \text{s. t.} \end{aligned} \tag{1}$$

$$\ln(M_{it}) = \ln(\varphi_{it}) + z_t \delta_t + \varepsilon_{it} \quad \forall i, t$$

$$\varphi_{it} = \alpha_{it} U_{it} + \beta_{it} V_{it} \quad \forall i, t$$

$$\varphi_{it} \leq \alpha_{ht} U_{it} + \beta_{ht} V_{it} \quad \forall i, h, t, w$$

$$\alpha_{ht}, \beta_{ht} \geq 0 \quad \forall h, t$$

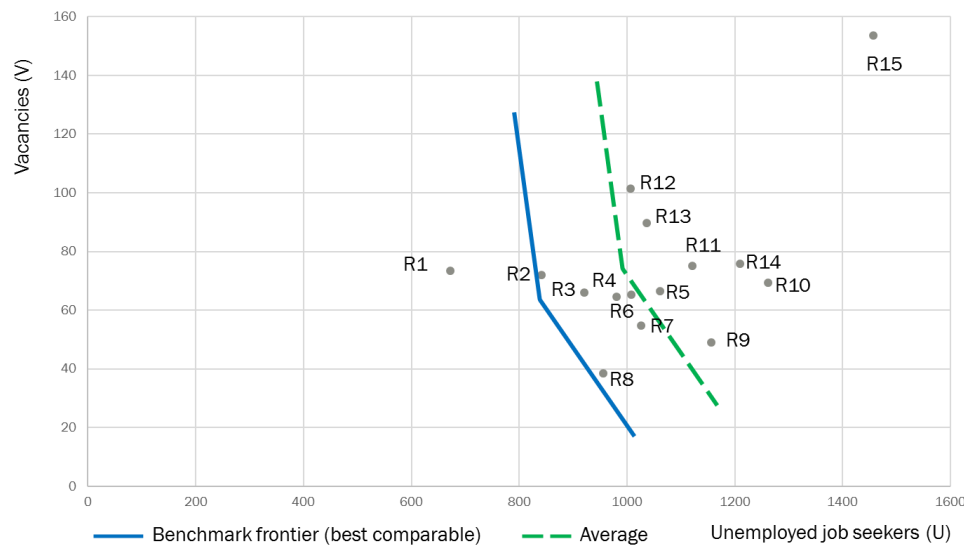
where  $\delta_t$  represents the marginal effect of annual variable  $z_t$ , and  $\alpha_{it}$  and  $\beta_{it}$  are the marginal effects of stocks of jobseekers and job vacancies, respectively.

### Finding the benchmark level

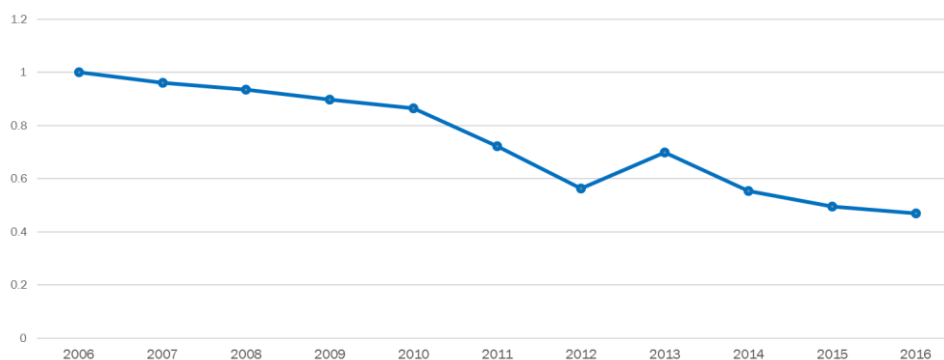
Using Problem (1), an average curve that passes through the cloud of data is estimated. This curve is represented in Figure 1. To obtain the matching function, the average curve is corrected by shifting to the best comparable office. By using the performance of a real unit as the benchmark level, we follow the benchmarking traditions of the previous PES evaluations and make the results understandable for users.

The matching function is represented in Figure 1, which demonstrates the average curve and shift for one year. During that year, the matching function consists of two effective linear segments, which jointly make an efficient frontier. In this setting, office R1 is considered to be a super-efficient unit and it is not used as a reference point for other units.

The nationwide environmental factors in Problem (1) are in the form of dummy variables capturing the annual effects of the labour market conditions affecting all offices. The trajectory of the dummy variable is presented in Figure 2.



**Figure 1. The intertemporal average curve and the estimated matching function in one year.**



**Figure 2. Annual development of labour market matching in Finland 2006-2016.**

As Figure 2 shows, the annual development of labour market matching has declined in Finland in the period of 2006 to 2012. The jump in 2013 is at least partly a result of changes in registration practices, and again the matching has decreased since then.

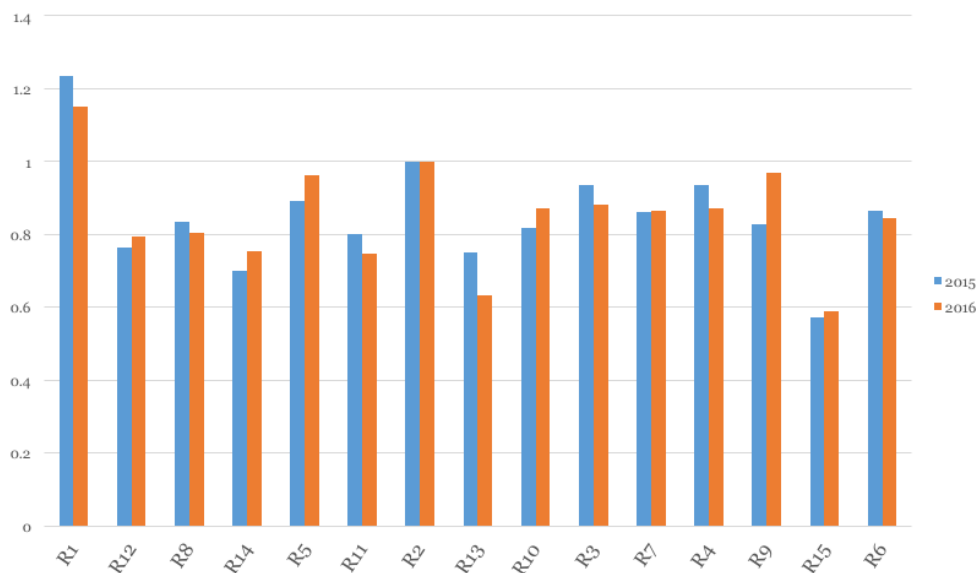
Taking the regional differences into account

Besides the nationwide development of the job market, the labour market of every region is affected by specific regional environmental factors. The traditional approach is to regress the residuals from Problem (1) on these variables and use them to explain the inefficiency (Coelli, Rao, O'Donnell, & Battese, 2005). By using DEA-1, the environmental factors are incorporated in Problem (1) similarly to the annual variables.

Our analysis shows that shares in the flow of new jobseekers that contain gender (male), immigrants, and a low level of education are associated with lower matching efficiency both regionally and temporarily. However, using these environmental variables to correct the inefficiency estimation requires further consideration. Since there are only 15 PES offices, it is challenging to separate the effect of regional exogenous characteristics from persistent inefficient local practices in PES offices. As a symptom, the magnitudes of coefficients exceed realistic levels. From the policy-makers' point of view, it is recognised that correcting the performance indicators may have questionable behavioural consequences, and it may be interpreted as tolerating the discrimination in labour market matching based upon ethnic origin and gender. For these reasons, it was decided not to correct the impact of the environmental variables on the matching efficiency indicators presented in the BI solution.

## RESULTS AND DISCUSSIONS

Figure 3 illustrates the matching efficiency of the regions in 2015 and 2016. The benchmark unit is office R2 for both years. As described above, R1 is super-efficient. The average inefficiency compared to the benchmark level is around 15% and the maximum is around 40%. This shows that there is a realistic improvement potential in matching efficiency, which significantly affects the length of unemployment and recruitment times of new employees. These insights help the decision-makers to set realistic targets for PES offices and to search for good practices to catch up with the best-performing PES offices.



**Figure 3. Relative matching efficiency of the regions in 2015 and 2016.**

The main focus of this paper is matching efficiency. Another popular framework to evaluate PES performance is to consider PES offices as production units utilising the workforce and other resources, like commercial agencies and occupational education institutions, to provide recruitment and training services (see EU Commission 2013 for a review of this approach). Both perspectives are important in PES evaluations. The application of this paper in evaluating Finnish PES offices includes modules on cost and operational workforce efficiency that are based on DEA. Overall efficiency also combines the matching and cost-efficiency perspectives.

The regional efficiency indicators are updated in a Qlikview-based BI system on a monthly basis. The BI system automatically retrieves data from the operational databases of PES offices. The data retrieval and visualization were realised by a software expert. The efficiency calculations are done in an R-based programme that is interfaced with Qlikview. These PES efficiency indicators are available to more than 500 decision-makers across the country. The users are directors, managers, and specialists in PES offices, regional economic development, and the MEAE.

## CONCLUSIONS

In this paper, we presented a BI solution to evaluate the efficiency of regional PES, focusing on matching efficiency. To the best of our knowledge, this is one of the very few BI solutions in government utilising DEA. This was made possible by using solid theoretical foundations from economics, advanced analytics (in our case DEA-1), and a state-of-the-art visualisation solution. This required close cooperation with the decision-makers, software vendor, and specialists in efficiency evaluation.

## ACKNOWLEDGEMENTS

We would like to offer our special thanks to Hanna Hämäläinen and Mika Tuomaala from the Ministry of Economic Affairs and Employment; Olli Vihanta and Jorma Anttila from the Development and Administrative Services Centre; and Jari Autio from the software vendor Pengon Oy for their good cooperation. We also thank the HSE Foundation for the financial support that enabled the presentation of our work at DEA2017 in Prague.

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# CHANGE IN TECHNICAL EFFICIENCY IN CASE OF SLOVAK BANKS

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## ABSTRACT

*The objective of the paper is to measure the technical efficiency of Slovak domestic commercial banks using the non-parametric Data Envelopment Analysis (DEA). An input oriented window DEA model with constant and variable return to scale was applied, to investigate the efficiency of commercial banks' deposits to loan transformation process. The analysis was focused on the 2005-2015 time period, as during this period has the banking sector gone through a massive structural changes, changes in regulation, and was affected by the financial crisis. To capture the differences in the development three sub-periods (2005-2008; 2009-2012; 2013-2015) were considered. The development of the banking market as well as the development of economy has led to changes in efficiency. Therefore, the last part of the paper was focused on the efficiency changes determinants between individual sub-periods using the Malmquist index.*

**Keywords:** Window Data Envelopment Analysis, intermediation approach, commercial banks, Slovak republic, Malmquist index

## INTRODUCTION

A critical part of financial transactions is carried out through commercial banks. That's why it is necessary to evaluate the efficiency of the process where funds transform from creditors to debtors. Conservative measures as financial ratios are nowadays still present when evaluating the banks' efficiency. More sophisticated methods are trying to measure the overall efficiency, taking multiple inputs and multiple outputs into account. Data Envelopment Analysis (DEA) is one of them and allows to create a so called reference group of units with the best practice in terms of efficiency and to determine which of the units are inefficient compared to those in reference group, as well as to provide a measurable scale of the inefficiencies present.

Sherman and Gold (1985) were the first who have applied DEA in banking sector. They have used DEA analysis to evaluate operating efficiency of 14 saving bank branches. In their study they have not only measured the level of efficiency, but also defined how to eliminate inefficiency by adjusting inputs and outputs of inefficient bank branches. Pastor et al. (1997) has analysed the efficiency of several US and European banks using the value added approach for comparability and to define the inputs and outputs. Casu and Molyneux (2003) in their study used the intermediation approach to evaluate efficiency of 750 selected European banks. The results showed relatively low average efficiency scores, nevertheless, it was possible to detect a slight improvement in the efficiency levels through time.

In the Slovak Republic as well as in the Czech Republic DEA models have been used to measure efficiency of financial institutions in last years for example in works of Zimková (2014), Bod'a and

Zimková (2015), Palečková (2015) and others. Zimková (2014) estimated the technical efficiency and the super-efficiency on the sample of 16 banking institutions in Slovakia in 2012. She found out that more than half of the institutions were found technically efficient by applying basic input-oriented DEA model under the assumption of a variable return to scale. The technical efficiency was also analysed in the work of Palečková (2015), who found the increase in the average efficiency of the Slovak commercial banks during the period 2004-2013. Nevertheless, only Boďa and Zimková (2015) have quantified the productivity change in sub periods using Malmquist index, therefore one of this papers objective was the decomposition of total productivity change between three sub-periods into catch-up and frontier-shift components using the Malmquist index in the Slovak banking sector.

In this paper a DEA window analysis is used in order to evaluate the efficiency in a small set of units (nine banks) within three sub-periods that are aggregating years with similar characteristics. An input oriented model with constant and variable return to scale is applied, to investigate the efficiency of deposits to loan transformation process. The last part of the paper is focused on the efficiency changes determinants between individual sub-periods using the Malmquist index.

## METHODS

DEA is one of the methods for the measurement of the relative efficiency of production units (DMU) that are using the same multiple inputs to produce the same multiple outputs. By DEA models application on the available dataset the efficient frontier and the efficiency score of each DMU can be identified. Each DMU unit can be using the DEA method easily visualised on the efficiency frontier and assessed in terms of its efficiency.

As the DMU units in this study are individual banks operating in the banking sector in Slovakia selected. In general let's consider  $n$  banks ( $DMU_j, j=1,2,...,n$ ), each of them consumes  $m$  different inputs ( $x_{ij}, i=1,2,...,m$ ) to produce  $s$  different outputs ( $y_{rj}, r=1,2,...,s$ ). Let mark the matrix of inputs as follows  $X = \{x_{ij}, i=1,2,...,m; j=1,2,...,n\}$  and the matrix of outputs  $Y = \{y_{rj}, r=1,2,...,s; j=1,2,...,n\}$ . Since each used inputs and produced outputs have for each individual bank different level of significance, they have also different weights that are computed using optimisation methods.

Having these conditions and available dataset of inputs and outputs can be the efficiency of a particular DMU ( $x_o, y_o$ ) obtained as the result of linear programming problem. According to Cooper, Seiford, and Tone (2007) can be the input-oriented model with slack variables that assume a variable return to scale (BCC model) used. DMU ( $x_o, y_o$ ) is considered fully technical efficient if and only if the optimal  $\theta$  value is equal to 1 and all the slack variables are equal to zero. In this case, we can talk about the Pareto-Koopmans efficiency. If  $\theta$  is equal to 1, but slack variables aren't equal to zero we can talk about the "pseudo-efficiency". If the slack variables are equal to zero but  $\theta$  is below 1 then the value  $\theta$  signals a technical inefficiency. This inefficiency can be eliminated by radial reduction in all of the DMU's ( $x_o, y_o$ ) inputs of by  $(1 - \theta)$  and thus move onto the efficiency frontier. If the slack variables aren't equal to zero and  $\theta$  is below 1, then to achieve a technical efficiency of a unit a non-radial shift expressed by slack variables is necessary.

The BCC model differs from the CCR model only in the adjunction of the condition where  $\sum_{j=1}^n \lambda_j = 1$ . The

efficiency calculated by the BCC model is often called as the pure technical efficiency and is a component of the overall technical efficiency calculated by the CCR model. The second efficiency component is the scale efficiency (SE). This variable controls for the constant or variable return to scale condition. If the scale efficiency is equal to 1, the DMU (bank) is operating under conditions of constant return to scale, indicating, that the bank operates at the most efficient scale size. If SE is less than 1, the bank operates under conditions of variable return to scale, either increasing or decreasing.

After the efficiency estimation, an analysis of factors determining the efficiency change over time was performed. For this purpose the Malmquist index was used with its decomposition into the driving forces of productivity change. The Malmquist productivity index evaluates a productivity change of a DMU between two time periods as the product of “catch-up” and “frontier shift” terms. The catch-up (or recovery) term reflects the degree that a DMU attains for improving its efficiency, while the frontier shift (or innovation) term reflects the change in the efficient frontier surrounding the DMU between the two time periods. The catch-up effect from the period 1 to period 2 can be measured by the following formula:

$$Catch - up = \frac{\text{Efficiency of } (x_o, y_o) \text{ in period 2 with respect to period 2 frontier}}{\text{Efficiency of } (x_o, y_o) \text{ in period 1 with respect to period 1 frontier}} \quad (1)$$

If the “catch-up” effect value is greater than 1, it can be interpreted as a progress in the relative efficiency from period 1 to period 2. The “catch-up” effect value equal to 1 indicates no changes in the relative efficiency, and a value below 1 indicates a regress in relative efficiency

In addition to “catch-up” effect, the “frontier-shift” effect must be taken into account in order to fully evaluate the productivity change since the “catch-up” effect is determined by the efficiencies being measured as the distances from the respective frontiers. The frontier-shift is defined as follows:

$$Frontier - shift = \sqrt{\phi_1 \phi_2} \quad (2)$$

Where  $\phi_1$  and  $\phi_2$  are defined by following formulas:

$$\begin{aligned} \phi_1 &= \frac{\text{Efficiency of } (x_o, y_o) \text{ in period 1 with respect to period 1 frontier}}{\text{Efficiency of } (x_o, y_o) \text{ in period 1 with respect to period 2 frontier}} \\ \phi_2 &= \frac{\text{Efficiency of } (x_o, y_o) \text{ in period 2 with respect to period 1 frontier}}{\text{Efficiency of } (x_o, y_o) \text{ in period 2 with respect to period 2 frontier}} \end{aligned} \quad (3)$$

Frontier-shift higher than 1 indicates progress in the frontier technology around the evaluated production unit DMU  $(x_o, y_o)$ , from period 1 to period 2, while frontier-shift equal to 1 and frontier-shift lower than one indicate the status quo and regress in the frontier technology.

The Malmquist index is computed as the product of catch-up effect and frontier-shift effect. The Malmquist index higher than 1 indicates a progress in the total factor productivity change of the evaluated production unit DMU  $(x_o, y_o)$ , from period 1 to period 2. The Malmquist index equal to 1 indicates a status quo, and the Malmquist index lower than 1 indicates deterioration in the total factor productivity.

## RESULTS AND DISCUSSIONS

This research, is focused on the efficiency evaluation of domestic (not foreign controlled) commercial banks. The analysis is based on domestic banks' data that cover more than 75% of the total banking assets in 2015. A panel of 9 universal only commercial banks operating in Slovak banking market during the analysed period of 2005 to 2015 was created. The reason for older data exclusion was that in 2004 the new Basel II rules were implemented.

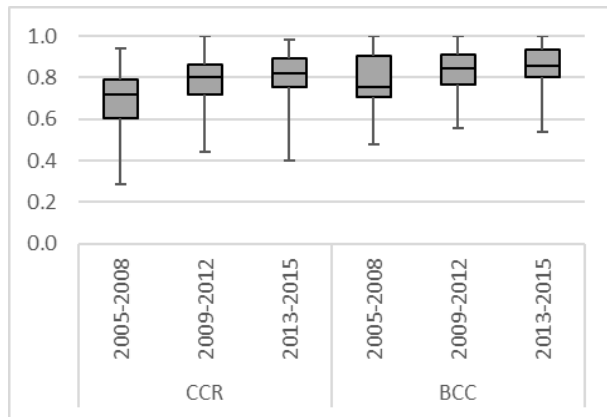
Due to small sample a three sub-periods (windows) DEA analysis was used. A DEA window analysis works on the moving averages principle and so can be used when dealing with small sample size. The first sub-period (2005 – 2008) was characterised by accession of Slovak Republic into the European Union and significant changes in the commercial banks' operations, in the structure of their services and in the orientation on mortgage banking and asset management. The second sub-period (2009-2012) was characterised by the adoption of Euro as the national currency. During this period was the development of banking market influenced also by the financial crisis that has hit banking sectors all around the world since 2008 gradually (Pitoňáková, 2015). Last sub-period (2013 – 2015) was characterised by non-standard operations of European Central Bank (ECB), by the policy of negative interest rates, and by the tightening regulation of National Bank of Slovakia (NBS) in the retail lending. This period is characterised also by implementation of capital buffers.

For the “relative” efficiency evaluation, the intermediation approach was used, as the main role of commercial banks is the realisation of financial intermediation. As the clients' deposits are the main source of funds and loans are main part on the assets side of balance sheet, the use of deposit as the input and loans as the output variable was decided. Personnel expenses as an input variable were selected due to the fact that most of the bank services are provided by their employees. To avoid the unreasonably high efficiency scores in the estimation, a limitation of the variables was decided due to the rule that number of DMUs should be three times the number of variables. The data were extracted from banks' end-of-year unconsolidated balance sheets based on international accounting standards. All data were reported in thousands of EUR as the reference currency and where the national currency was present (Slovenská koruna – SKK), conversion rate of the National Bank of Slovakia for the selected year was used.

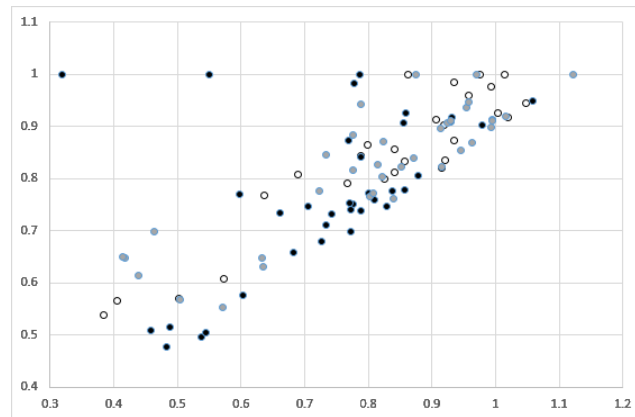
The development of input and output average values during the analysed period shows that the volume of loans, deposit and personnel expenses rose significantly till 2008. During the first sub-period the volume of deposit increased by more than 93%, the volume of loans more than 125%, and the value of personnel expenses more than 57%. The financial crisis in 2009 has temporary stopped this growth, therefore, during the second sub-period (2009-2012) the deposit increased only by 8%, loans only by 13%, and personnel expenses by 7.6%. The renewal of growth from 2013 (third sub-period), increased the deposits

by 13% till the end of analysed period, growth of loans exceeded 19% but only a 5% increase in personnel expenses.

Following the described methodology, the efficiency of all banks in three sub-periods (2005-2008; 2009-2012; 2013-2015) using CCR and BCC input oriented model was evaluated. In the Figure 1 the distributions of efficiencies in each sub-period are presented using the CCR or BCC.



**Figure 1: Results of DEA models in three sub-periods**



**Figure 2: BCC efficiency versus Loan to Deposit ratio**

The CCR model reported an increase of the technical efficiency medians from the first to second period (by 11.67%) with a slowdown (2.62%) towards the third one (71.88%, 80.05%, 82.15%). The slowdown in the second period could be also supported by the financial crisis, that had hit the banking sectors all around the world since 2008. The adoption of Euro currency in 2009 could have also its effect on the personnel expenses. The BCC model was applied due to the other than optimal size operation of the banks. The efficiencies of the BCC model, were higher during all the analysed periods (75.61%, 84.19%, 85.57%). Based on the results presented in Figure 1 a higher variability can be seen under the CCR model. The CCR efficiency can be decomposed into BCC efficiency and scale efficiency. In our sample can be said, that most banks operated under the conditions of variable return to scale, mostly decreasing return to scale.

One of the DEA models advantage compared to isolated financial ratio methods is, that DEA can take into account more than two variables at time and the efficiency of evaluated banks is considered within a group. This fact could be illustrated in Figure 2 by comparison of DEA efficiency score (BCC y axis) with financial ratio in the form of Loan to Deposit ratio (LTD x axis). LTD is the ratio when all the granted loans would come from deposits or other sources (e.g. interbank loans, issuing debt securities, etc.). LTD value above 1 indicates that the bank has lent more that received on deposits, so the additional fund had to borrowed from secondary often riskier and costly sources. Low values of LTD indicate either their income inefficiency on loans or inability to place their deposits. A LTD value in the range of 0.8-0.9 can be considered as fair and signalling an appropriate mix of prudence and regulatory requirements compliance. In the Figure 2 most of the banks in the first sub-period had the LTD in a lower range of 0.4 - 0.8 (black dots). On the other hand, in the third sub-period most of the banks were clustered in a range of 0.8 - 1 (white dots). A clear difference in the BCC efficiency and efficiency calculated through the LTD is

visible in the Figure 2. BCC efficient banks have the LTD value below 0.8 and vice versa. This confirms the authors' assumption, that in the efficiency analysis it is necessary to use more methods simultaneously.

In the last part of the paper changes between individual sub-periods using Malmquist index were examined. The results of this analysis is recorded in Table 1.

**Table 1: Productivity change indexes**

Years	Number of banks	Catch-up effect	Frontier-shift effect	Malmquist index
2005-2009	9	1.0952	1.0250	1.1394
2009-2013	9	0.9490	1.0290	0.9716
2013-2015	9	1.0405	1.0517	1.0928

An average progress of 13.94% in the total factor productivity is recorded in the first time window. This productivity growth of the analysed banks was positively determined by the technical efficiency increase by 9.52% and progress in the frontier technology by 2.5%. During the second time window was the Malmquist index (MI) below 1 indicating a deterioration in the total factor productivity with an annual decrease rate of 2.84%. This deterioration was supported by the decline in the relative efficiency by 5.1% and even a positive innovation effect (2.9%) could not tip the scales in the total factor productivity. During the third time window the MI index recorded a 9.28% increase in total factor productivity with increase in both, the relative technical efficiency (4.05%) and innovation progress (5.17%). Frontier shift effect representing the effect of innovation was always positive and therefore played in the Slovak environment a crucial role. However, the source for this effect can be latent and of any type like technological change or progress, macroeconomic development central bank policies or even government regulations. All these issues together with information technology uptake influenced the banks' ability to offer more sophisticated products and services, enabling them to take their products closer to clients and so increase their efficiency.

## CONCLUSIONS

The aim of the paper was to measure the efficiency in the Slovak banking sector using the non-parametric input-oriented DEA model with constant and variable return to scale. To define the input and output variables the intermediation approach was used. The results showed that the Slovak banking sector's productivity increased during the analysed period with the exception of the 2009 where a slowdown was recorded due to the financial crisis and subsequent changes in banks regulatory requirements or their loan assessment behaviour.

The relative technical efficiency was investigated within three sub-periods. The CCR model recorded increasing median values of the technical efficiency during the three periods (71.68%, 80.05% and 82.15%). The BCC model showed more clustered results with the median increase of 11.35% between the first and second period and one of 1.64% between the last two periods. The slowdown of the average efficiency in the second sub-period could be the result the financial crisis or the adoption of Euro currency in 2009. A comparison of the DEA model results with the traditional LTD value approach was

carried out. Differences in the findings support the authors suggestion for application a combination of methods when dealing with effectivity analysis or assessment.

Based on the MI index can be generally concluded that during the analysed period 2005 - 2015, have the Slovak banks increased their productivity mainly due to technological progress. The banking sector took advantages mainly of information technologies and reached a higher production frontier. The development of the innovation effect was a bit slower than the relative efficiency increase which could be caused by negative impacts like restrictive regulatory requirements, financial crisis, slower economic development, etc.

## ACKNOWLEDGEMENT

Research behind this paper was supported by research project VEGA 1/0446/15.

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# COMPARING R&D EFFICIENCY OF EUROPEAN COUNTRIES USING ROBUST DEA

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## ABSTRACT

*We evaluate the efficiency of research and development (R&D) of 28 European union countries in the year 2014. For this purpose we construct a robust data envelopment analysis (Robust DEA) model. R&D expenditure and number of scientist and engineers are considered as inputs in the model while h-index, high-tech share and number of patent applications are considered as outputs. We also analyze relationships between GDP per capita and input/output variables and their specialized efficiencies.*

**Keywords:** *R&D efficiency, robust data envelopment analysis*

## INTRODUCTION

The goal of this paper is to evaluate the efficiency of inputs and outputs in research and development (R&D) of 28 European union (EU) countries in the year 2014. We focus on macro-trends because they can easily be compared between states of the EU. On the other hand, the micro-trends are influenced by specific characteristics of the countries which are impossible to cover in this paper.

Let us begin with the concept of efficiency. Generally, the efficiency refers to the optimal use of resources or inputs in production of outputs. In our case we define an efficiency in the non-parametric sense of data envelopment analysis (DEA) introduced by Charnes et al. (1978). We apply a more modern robust approach of Hladík (2017). This robust DEA have many advantages over the classical DEA such as robustness to changes of input/output variables, ranking of efficient units and natural normalization while retaining the same order of ranking as classical DEA.

The key aspect of any DEA analysis is the identification of input and output variables. This is a non-trivial task because inputs and outputs can be characterized and measured in several ways, some outputs can also be considered as inputs (e.g. institutional quality or number of Ph.D. students) and many results of R&D only appear after many years or decades (Cincera et al., 2009). Ekinici and Ön (2015) review different approaches for input/output indicator selection by many authors. Among the most used indicators are R&D expenditure, number of researchers, number of scientific publications, number of patents and institutional quality. We respect this characteristics of R&D and use this indicators (or its proxy variables) in our DEA model for efficiency evaluation.

There are several studies related to R&D in EU. Goolsbee (1998) proved that at macro-level (the level of states) the labor market have strong relationship with the R&D funding. Lee and Park (2005) measure and cluster R&D productivity of 27 Asian countries. Wang and Huang (2007) use Tobit regression where

manpower and R&D stockpile act as inputs and patents and publications as outputs. Cincera et al. (2009) investigate the relationship between public R&D spending and the additional R&D in the business sector among OECD countries. Johansson et al. (2015) analyze capacity to produce new knowledge proxied by patents granted in 18 industries in 11 European economies.

## METHODOLOGY

We use a robust approach to DEA introduced by Hladík (2017). In this model, the ranking  $r = 1 + \delta$  of analyzed decision making unit (denoted as  $DMU_0$ ) is obtained by solving non-linear optimization problem

$\max \delta$

$$\text{s.t. } (1 - \delta)y_0^T u \geq 1,$$

$$(1 + \delta)x_0^T v \geq 1,$$

$$(1 + \delta)Yu - (1 - \delta)Xv \leq 0,$$

$$u, v \geq 0$$

where  $x_0, y_0$  are input/output vectors for  $DMU_0$ ,  $X, Y$  are input/output matrices for other DMUs, and  $v, u$  are input/output weights. This generalized linear fractional programming problem can be effectively approximated by linear programming (Hladík, 2017).

The ranking of units is based on robustness to input/output data changes. For efficient units, the method measures the largest allowable variation of input/output data such that the analyzed unit remains efficient. For inefficient units, the method measures the smallest possible variation of input/output data such that the analyzed unit becomes efficient. The robust DEA ranks both inefficient and efficient units. This is an advantage over classical DEA which ranks only inefficient units. The classification to inefficient and efficient units as well as order of inefficient rankings is exactly the same as in classical DEA. The difference between methods is in values of rankings. As in classical DEA, the rankings are invariant to scaling of variables. The rankings given by robust DEA are also naturally normalized which allows for comparison of efficiencies between different models.

## DATA

We use data from two sources. The first data source is Eurostat which provides a comprehensive information about a lot of areas of R&D and education (European Commission, 2017). The other data source is Scimago Journal & Country Rank which covers numbers of articles in Scopus Preview by specific criteria (Scimago, 2017). We analyze data of 28 EU countries for the year 2014. As inputs we consider R&D expenditure and number of scientist and engineers while as outputs we consider h-index of country, high-tech share and number of patent applications. We also compare all variables with annual GDP per capita.

R&D Expenditure covers a total intramural R&D expenditure per states. This data are compiled by the national statistical authorities. The data are collected by various methods – samples, census or surveys and using the administrative data sources. The unit is Euro per inhabitant across all sectors.

Number of scientist and engineers refers to a number of persons who have scientific or technological training and are working in areas related to science and technology.

For approximation of “quality” of research we use a proxy variable h-index which is a country's number of articles ( $h$ ) that have received at least  $h$  citations. Data are provided by Scimago Journal & Country Rank, a data collection from Scopus Preview.

High-tech share refers to share of high-technology products from a country on total export of that country. In our case the total trade includes only extra-EU trade (the intra-EU trade is excluded).

Number of patent applications refers to patent applications/granted to the European Patent Office. This variable includes three main categories: high-tech patent applications/granted, ICT patent applications/granted, biotechnology patent applications/granted.

## RESULTS

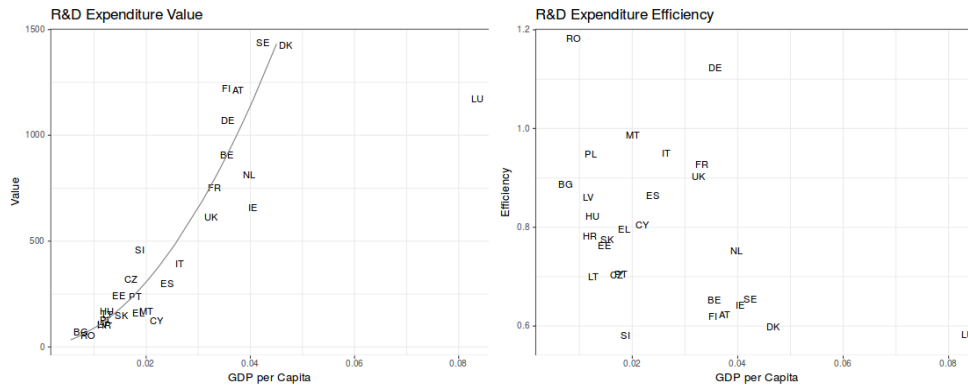
Firstly, we evaluate efficiencies in the model with all 2 inputs and 3 outputs. There are 9 efficient and 19 inefficient countries out of 28 EU countries. The most efficient countries are Germany, Luxembourg, Malta and Romania. The efficiency of Romania may be surprising but it is because of its very low R&D expenditure. On the other hand, the least efficient countries are Czech Republic, Estonia, Lithuania and Portugal. Efficiency of each EU country can be found in Figure 1. Our results are similar to related studies such as Cincera et al. (2009).



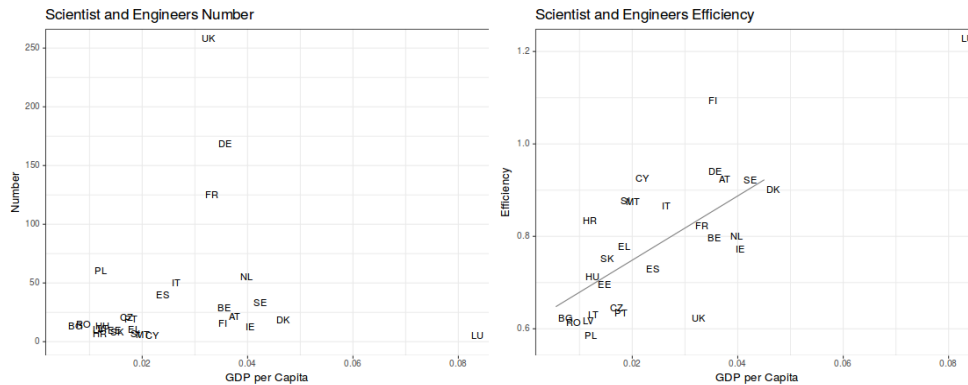
**Figure 1: Estimated efficiency for 28 EU countries**

Secondly, we analyze efficiencies of models with a single input and all the outputs or a single output with all the inputs. As in Lee and Park (2005) we refer to these efficiencies as specialized R&D efficiencies. We investigate relationships between GDP per capita and input/output variables values and their specialized efficiencies. Figures 2-6 show that for some variables there indeed exist a relationships to GDP per capita. R&D expenditure value increases with GDP per capita while for its specialized

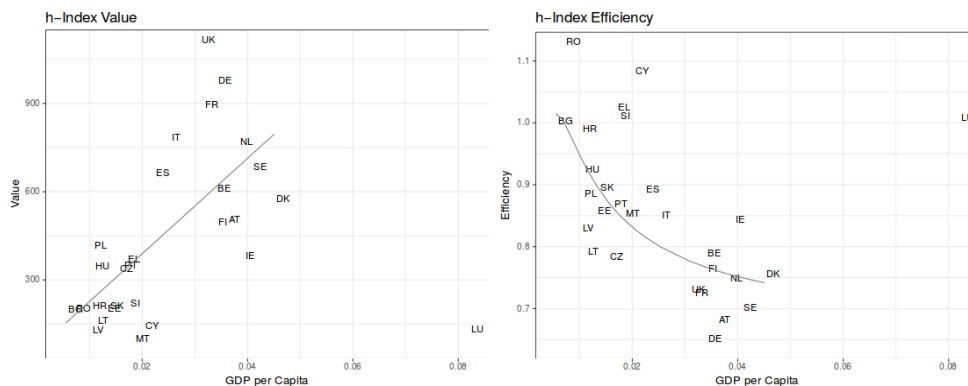
efficiency there is no clear relationship. On the other hand, the number of scientists and engineers have no clear relationship to GDP per capita while scientists and engineers specialized efficiency increases with GDP per capita. H-index increases with GDP per capita but its specialized efficiency decreases. The same relationships can be found for high-tech share. Finally, the number of patent applications have no clear relationship to GDP per capita while patent applications specialized efficiency increases with GDP per capita.



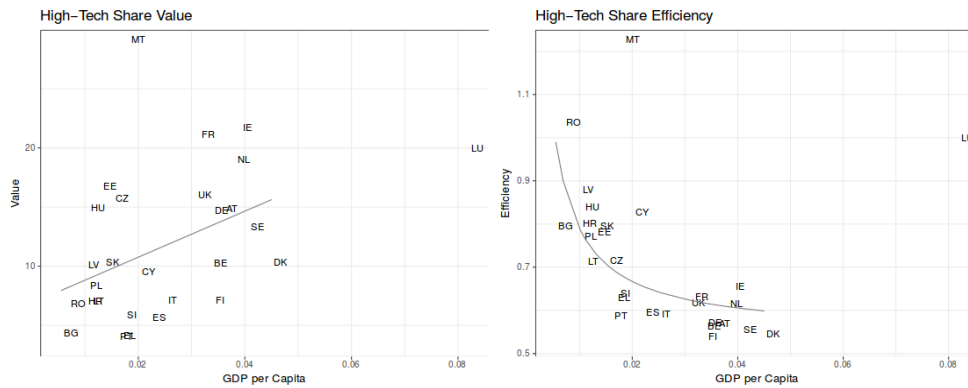
**Figure 2: Dependency of R&D expenditure value and specialized efficiency on GDP per capita**



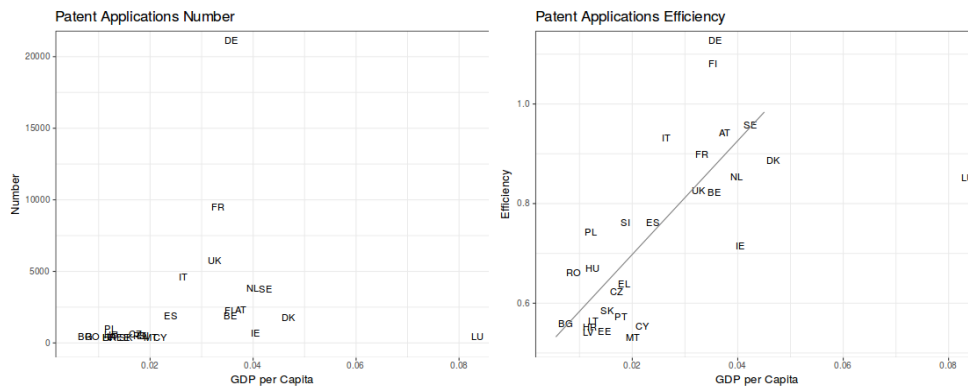
**Figure 3: Dependency of scientist and engineers number and specialized efficiency on GDP per capita**



**Figure 4: Dependency of h-index value and specialized efficiency on GDP per capita**



**Figure 5: Dependency of high-tech share value and specialized efficiency on GDP per capita**



**Figure 6: Dependency of patent applications number and specialized efficiency on GDP per capita**

## DISCUSSION

Our results correspond with the results of previous studies on different years. There are some countries which are generally ineffective (e.g. Czech Republic) in the context of our analysis of full and specialized models as well as studies in other papers. We also find some interesting relationships between input/output variables and GDP per capita. The causality of these relationships should be inspected more precisely in future research. An other open question is the time-series analysis and the stability of efficiency.

## ACKNOWLEDGEMENTS

The work on this paper was supported by the grant IGS F4/93/2017 of University of Economics, Prague.

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# COMPARISON OF $\alpha$ -LEVELS IN FUZZY DATA ENVELOPMENT ANALYSIS

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## ABSTRACT

*Data Envelopment Analysis (DEA) is a noticeable technique which proved itself in performance measurement. It is combined with many techniques and applied in a wide range of areas. In real world, in general, the real data processed in DEA analysis are not crisp and have some problem which can be considered as fuzzy. Most commonly used efficiency evaluation method in Fuzzy DEA (FDEA) application is  $\alpha$ -cut approach which transforms fuzzy data to crisp. Our proposed method is an attempt to compare these levels with each other and can also be used when there exist missing values which can be accepted as fuzzy and if some  $\square$  levels attained from a possibility distribution to these missing values. Furthermore, it can also be used to measure the effects of different input or output levels of performance targets.*

**Keywords:**  $\alpha$ -level, efficiency comparison, Fuzzy DEA

## INTRODUCTION

Within the wide range and applications of Data Envelopment Analysis (DEA), Fuzzy DEA context is extensively discussed by Hatami-Marbini et al. (2011) and Emrouznejad, Tavana (2014); They reviewed literature and classified the Fuzzy DEA (FDEA) application into six catagories. Recent studies can also be assessed within this context. As a different point of view, Gölcükcü et al. (2016) showed in his thesis that DEA is a t-norm that forms the basis of fuzzy sets.

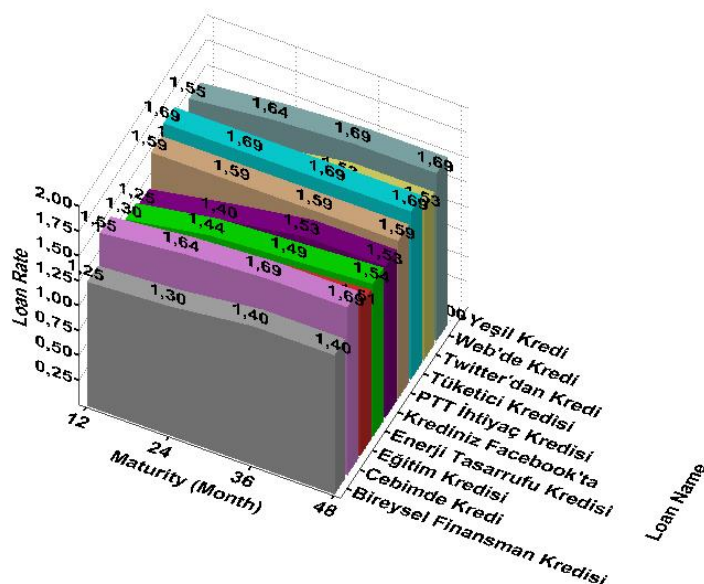
The subject of Banking under DEA context is reviewed by Paradi, Zhu (2013). Some recent literature was given by Kevork et al. (2017) about DEA and banking application. One of which is Curi et al. (2015) and they also have given a short review. In consort with these studies, various authors focused on banking. (Afsharian, Ahn 2017, Afsharian et al. 2017, Eskelinen 2017, Hatami-Marbini, Toloo 2017, Li et al. 2017, Palečková 2017) Malmquist analysis is another concept which was propsed by Caves et al. (1982) dedicated to the early work of Sten Malmquist (1953) and extensively studied by Rolf Färe (Färe, Hunsaker 1986, Färe, Grosskopf 1992, Färe et al. 1992, Färe et al. 1994, Färe et al. 1994, Färe et al. 1998, Färe et al. 2011, Färe et al. 2012).

Furtermore, there are also some studies concerning group comparison with Malmquist (Camanho, Dyson 2006, Valami 2009, Thanassoulis et al. 2015), there are also other studies using rather than malmquist index (Naito et al. 2009, Kukozumi et al. 2012) for group comparison.

Besides all, Gölcükcü et al. (2016) proposed a matrix model for Malmquist index and make group comparison more practical. In this study, the work of Gölcükcü et al. (2016) is aimed to extend by adding another dimension. This dimension could be taken in any way; sub grouping, term/period comparision or

## METHODS

On the other hand, the prevention of fluctuations in inputs allows us to examine all aspects of performance changes in output. So, all sub categories of output variables included in a bank. Figure 1. depicts the sub categories of consumer loan for one bank. The figure is just like a wave on the sea. All of them are consumer loan given by a selected sample bank, but each of them has different name and loan rate.



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As could be seen in Figure 1., each sub category has trapezoidal shape and can be assumed to be a membership function in fuzzy sets. They could be decided as fuzzy because the interest rates depends on the amount that customer demands and maturity period. These are determined by customer. Loan rate are determined by the bank according to customers ability to bargain and credibility.

Consequently, it could be represented as;

$$A = \frac{1.25}{12} + \frac{1.30}{24} + \frac{1.40}{36} + \frac{1.40}{48} \quad (1)$$

for “Bireysel finansman kredisi (Individual financing loan)” of selected bank in Figure 1. Within above assumptions, the input data set is given in Table 1. and some selected sample of consumer loans (outputs) in Table 2.

**Table 1: Deposit Rates (Inputs)**

No	Bank Name	TL	USD	EURO
1	ABank	10.7	2.6	1.35
2	Akbank	9.25	1.25	0.7
3	Albaraka Türk	9.59	2.09	1.57
4	Anadolubank	12.5	3	1.5
5	Burgan Bank	14	3.75	1.75
6	Denizbank	13.5	4	1.15
7	Fibabanka	14.25	3.75	1.8
8	Garanti Bankası	8.5	1.4	0.5
9	HSBC	10.15	1.4	1
10	Kuveyt Türk	7.89	1.7	1.41
11	Odea Bank	10	2	1.5
12	QNB Finansbank	9	1.5	0.5
13	Şekerbank	10	2.35	1.1
14	TEB	8	1.65	1.1
15	Türkiye Finans	8.62088	2.0598	1.83592
16	Türkiye Halk Bankası	10.25	1.75	1.75
17	Türkiye İş Bankası	9.25	1.95	1.15
18	Vakıfbank	11.5	2.25	1.5
19	Yapı Kredi Bankası	10.65	1.95	1.5
20	Ziraat Bankası	10	1.9	1.1

**Table 2: Some selected sample of consumer loans (selected outputs)**

Bank No	Bank Name	Loan Code	Name of Consumers Loan	Maturity Period (Month)			
				12	24	36	48
1	ABank	1	İhtiyaç Kredisi	1.41	1.41	1.42	1.44
		2	Evime Nakit Kredi	1.27	1.24	1.24	1.24
6	Burgan Bank	1	Finansal Destek Kredisi / Bireysel İhtiyaç Kredisi	1.99	1.99	1.99	0
7	Denizbank	1	Bireysel Finansman Kredisi	1.25	1.30	1.40	1.40
		2	Tüketici Kredisi	1.69	1.69	1.69	1.69
		3	Eğitim Kredisi	1.25	1.35	1.46	1.51
		4	Cebimde Kredi / Web'de Kredi	1.55	1.64	1.69	1.69
		6	Krediniz Facebook'ta / Twitter'dan Kredi	1.25	1.4	1.53	1.53
		9	Enerji Tasarrufu Kredisi	1.30	1.44	1.49	1.54
		10	PTT İhtiyaç Kredisi	1.59	1.59	1.59	1.59
		11	İhtiyaç Kredisi	1.29	1.35	1.41	1.51

As could be seen from Figure 1 and Table 2 the services of banks are fuzzified by diversification. In this case, in order to calculate the efficiency of any bank in the sector according to other banks, it is proposed a malquist based approach. With this object in mind, all combinations of loans are calculated for each bank and decided as an individual sub-DMU. From this point of view, the banks were accepted as a group. Only, the loans which have same loan rate for all periods are merged. Hence, maturity periods represents periods. Table 3. shows these combinations of loans for a sample bank.

## MODEL

Malmquist Productivity index introduced by Caves et al. dedicated to the idea of index of Stern Malmquist concerns with efficiency change of DMUs over time. Malmquist model for group and period comparison could be represented as follows;

$$M^{t_{g,p}, t_{g,p}+k_{g,p}} = \underbrace{\frac{D^{t_{g,p}+k_{g,p}}(Y^{t_{g,p}+k_{g,p}}, X^{t_{g,p}+k_{g,p}})}{D^{t_{g,p}}(Y^{t_{g,p}}, X^{t_{g,p}})}}_{\substack{\text{Efficiency change} \\ \text{or} \\ \text{catch-up effect} \quad (C)}} \sqrt{\underbrace{\frac{D^{t_{g,p}}(Y^{t_{g,p}+k_{g,p}}, X^{t_{g,p}+k_{g,p}})}{D^{t_{g,p}+k_{g,p}}(Y^{t_{g,p}+k_{g,p}}, X^{t_{g,p}+k_{g,p}})} \frac{D^{t_{g,p}}(Y^{t_{g,p}}, X^{t_{g,p}})}{D^{t_{g,p}+k_{g,p}}(Y^{t_{g,p}}, X^{t_{g,p}})}}_{\substack{\text{Technplogial change} \\ \text{or} \\ \text{frontier-shift effect} \quad (T)}}} \quad (2)$$

In above model  $t$ ; is the recetor beginning period and  $t+k$  is  $k$  period later than  $k$ th period.  $D(Y,X)$  is the distance which is the inverse of CCR model given in (3).  $g$  represent groups,  $p$  represent periods in this study. After obtaining results of model given (3) , the matrix model given in (4) (Gölcükcü et al. 2016), is used for the cross comparison of groups namely banks which is the first stage of our analyze. Then, as a second step, period comparisons were made using the group comparison Malmquist index results obtained in the first stage.

$$\begin{aligned} D_o^{t_{g,p}}(x_o^{t_{g,p}}, y_o^{t_{g,p}})^{-1} &= \min \theta \\ \text{s.t.} \\ \sum_{i=1}^n \lambda_i x_{ji}^{t_{g,p}} &\geq x_o^{t_{g,p}} \\ \sum_{i=1}^n \lambda_i y_{jr}^{t_{g,p}} &\leq \theta y_o^{t_{g,p}} \\ \lambda_i &\geq 0 \\ i &= 1, \dots, n; j = 1, \dots, p; r = 1, \dots, s \\ g &= 1, \dots, G; p = 1, \dots, P \end{aligned} \quad (3)$$

Equivalent of the above model and an application scheme was proposed by Gölcükcü et al. (2016) as matix form as fallows:

$$MI^{BA} = \underbrace{(K^{-1}1K)}_{IE^{BA}} \boxtimes \underbrace{[(GK^{-1}) \boxtimes \{(K1) \boxdiv G^T\}]^{\frac{1}{2}}}_{IF^{BA}} \quad (4)$$

G is the geometric mean matrix of inverse of group efficiency scores, K is the diagonal of G,  $\boxtimes$  is the elementwise multiplication of matrices,  $\boxdiv$  is the elementwise division of matrices.  $\frac{1}{2}$  is the elementwise square root of the selected matrix.

## RESULTS AND DISCUSSIONS

Table 4. represents the first stage results which is the comparison of maturity periods for only one selected bank. The highlighted value of 0,934 indicates that the performance of 24 months loans of selected banks are %93,4 of 12 month loans or on contrary the value 1,074 indicates that the performance of 12 month loans are %107,4 of 24 month loans for selected bank. If you have any table use the following formatting.

Table 5. represents the second stage results which is the comparison of groups namely banks over maturity period comparison. The highlighted value of 0,934 states that performance of 24 month loans of selected bank is %93,4 of its 12 month loans. The highlighted value 0,873 (just at the left of 0,934) states that the performance of 24 month loans of selected bank (Fibabanka) is %87,3 of Denizbank 12 month loans.

## CONCLUSIONS

This study is an extension of Gölcükcü et al. (2016) and take it one step forward by double layer malmquist analysis. In the first layer maturity periods are compared and in the second layer groups (banks) are compared with each other. The results is hypermatrix whose elements are matrices. This study is aimed to simplify the complex calculation of this hypermatrix, thus, study facilitates interpretation. Although the results obtained are kept in pages and only a portion is given here, it provides the desired upshot. These results are also important because they allow cross-comparison of the upper and lower boundaries of different  $\alpha$ - levels.

## APPENDICES

**Table 3: Output combinations of a selected bank**

Input 1 (Consumer Loan)					Input 2 (Housing loan)					Input 3 (Vehicle loan)				
sub-category name	12	24	36	48	sub-category name	12	24	36	48	sub-category name	12	24	36	48
Bireysel Finansman	1.25	1.3	1.4	1.4	Mortgage	0.91	0.91	0.97	1.01	Taşıt Kredisi	1.31	1.31	1.31	1.31
Tüketici	1.69	1.69	1.69	1.69										
Eğitim	1.25	1.35	1.46	1.51										
Cebimde /	1.55	1.64	1.69	1.69										

[illegible]

**Table 4. Period Comparison Results for a sample bank (Stage 1)**

Periods	P_12 vs 12	P_24 vs 12	P_36 vs 12	P_48 vs 12	P_12 vs 24	P_24 vs 24	P_36 vs 24	P_48 vs 24
P_12 vs 12	1	1.071	1.143	1.266	1	1	1.067	1.183
P_24 vs 12	0.934	1	1.067	1.183	0.934	0.934	0.997	1.105
P_36 vs 12	0.875	0.937	1	1.108	0.875	0.875	0.934	1.035
P_48 vs 12	0.79	0.845	0.902	1	0.79	0.79	0.843	0.934
P_12 vs 24	1	1.071	1.143	1.266	1	1	1.067	1.183
P_24 vs 24	1	1.071	1.143	1.266	1	1	1.067	1.183
P_36 vs 24	0.937	1.003	1.071	1.186	0.937	0.937	1	1.108
P_48 vs 24	0.845	0.905	0.966	1.071	0.845	0.845	0.902	1
P_12 vs 36	1	1.071	1.143	1.266	1	1	1.067	1.183
P_24 vs 36	1	1.071	1.143	1.266	1	1	1.067	1.183
P_36 vs 36	1	1.071	1.143	1.266	1	1	1.067	1.183
P_48 vs 36	0.902	0.966	1.031	1.143	0.902	0.902	0.963	1.067
P_12 vs 48	1	1.071	1.143	1.266	1	1	1.067	1.183
P_24 vs 48	1	1.071	1.143	1.266	1	1	1.067	1.183

P_36 vs 48	1	1.071	1.143	1.266	1	1	1.067	1.183
P_48 vs 48	1	1.071	1.143	1.266	1	1	1.067	1.183

**Table 4. Period Comparison Results for a sample bank (Stage 1)-(Continued)**

Periods	P_12 vs 36	P_24 vs 36	P_36 vs 36	P_48 vs 36	P_12 vs 48	P_24 vs 48	P_36 vs 48	P_48 vs 48
P_12 vs 12	1	1	1	1.108	1	1	1	1
P_24 vs 12	0.934	0.934	0.934	1.035	0.934	0.934	0.934	0.934
P_36 vs 12	0.875	0.875	0.875	0.97	0.875	0.875	0.875	0.875
P_48 vs 12	0.79	0.79	0.79	0.875	0.79	0.79	0.79	0.79
P_12 vs 24	1	1	1	1.108	1	1	1	1
P_24 vs 24	1	1	1	1.108	1	1	1	1
P_36 vs 24	0.937	0.937	0.937	1.038	0.937	0.937	0.937	0.937
P_48 vs 24	0.845	0.845	0.845	0.937	0.845	0.845	0.845	0.845
P_12 vs 36	1	1	1	1.108	1	1	1	1
P_24 vs 36	1	1	1	1.108	1	1	1	1
P_36 vs 36	1	1	1	1.108	1	1	1	1
P_48 vs 36	0.902	0.902	0.902	1	0.902	0.902	0.902	0.902
P_12 vs 48	1	1	1	1.108	1	1	1	1
P_24 vs 48	1	1	1	1.108	1	1	1	1
P_36 vs 48	1	1	1	1.108	1	1	1	1
P_48 vs 48	1	1	1	1.108	1	1	1	1

**Table 5. Bank (Group) Comparison Results over a sample period for comparison (24 Month vs 12 Month) (Stage 2)**

Bank Name	ABank	Akbank	Albaraka Türk	Anadolubank	Burgan Bank	Denizbank	Fibabanka	Garanti Bankası	HSBC	Kuveyt Türk
ABank	1	1	1	1	1	0.987	1.047	1	1	0.997
Akbank	1	1	1	1	1	0.992	1.015	1	1.039	1.001
Albaraka Türk	1	1	1	1	1	0.965	1.008	1	1.039	1.001
Anadolubank	1	1	1	1	1	0.985	1.042	1	1	0.997
Burgan Bank	1	1	1	1	1	0.965	1.008	1	1.039	1
Denizbank	1.007	1.004	1.018	1.007	1.018	1	1.059	1.004	1.016	1.002
Fibabanka	0.913	0.927	0.93	0.915	0.93	0.873	<b>0.934</b>	0.927	0.93	0.925
Garanti Bankası	1	1	1	1	1	0.992	1.015	1	1	0.997
HSBC	0.944	0.926	0.926	0.944	0.926	0.914	0.952	0.944	<b>0.944</b>	0.923
Kuveyt Türk	0.999	0.997	0.997	0.999	0.998	0.991	1.015	0.999	1.039	<b>0.998</b>
Odea Bank	1	1	1	1	1	1	1.031	1	1	0.997
QNB Finansbank	1	1	1	1	1	0.997	1.015	1	1.039	0.998
Şekerbank	1	1	1	1	1	0.965	1.008	1	1.039	1
TEB	1	1	1	1	1	0.997	1.015	1	1.039	1.001
Türkiye Finans	1.008	1	1	1.006	1	0.969	1.008	1	1.039	1.001
Türkiye Halk B.	1	1	1	1	1	0.98	1.012	1	1.039	1
Türkiye İş B.	1	1	1	1	1	0.992	1.015	1	1.039	1.001
Vakıfbank	1	1	1	1	1	0.992	1.015	1	1	0.997
Yapı Kredi B.	1	1	1	1	1	0.99	1.015	1	1	0.997

Ziraat B.	1	1	1	1	1	0.981	1.013	1	1.033	0.999
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**Table 5. Bank (Group) Comparison Results over a sample period for comparison (24 Month vs 12 Month) (Stage 2)-(Continued)**

Bank Name	Odea Bank	QNB Finansbank	Şekerbank	TEB	Türkiye Finans	Türkiye Halk Bankası	Türkiye İş Bankası	Vakıfbank	Yapı Kredi Bankası	Ziraat Bankası
ABank	1	1	1	1	0.984	1	1	1	1	1
Akbank	1	1	1	1	1	1	1	1	1	1
Albaraka Türk	1	1	1	1	1	1	1	1	1	1
Anadolubank	1	1	1	1	0.988	1	1	1	1	1
Burgan Bank	1	1	1	1	1	1	1	1	1	1
Denizbank	1	1.002	1.018	1.002	1.01	1.01	1.004	1.004	1.005	1.01
Fibabanka	0.92	0.927	0.93	0.927	0.93	0.929	0.927	0.927	0.927	0.928
Garanti Bankası	1	1	1	1	1	1	1	1	1	1
HSBC	0.944	0.926	0.926	0.926	0.926	0.926	0.926	0.944	0.944	0.929
Kuveyt Türk	0.999	0.999	0.998	0.997	0.998	0.998	0.997	0.999	0.999	0.999
Odea Bank	1	1	1	1	0.991	1	1	1	1	1
QNB Finansbank	1	1	1	1	1	1	1	1	1	1
Şekerbank	1	1	1	1	1	1	1	1	1	1
TEB	1	1	1	1	1	1	1	1	1	1
Türkiye Finans	1.004	1	1	1	1	1	1	1	1	1
Türkiye Halk B.	1	1	1	1	1	1	1	1	1	1
Türkiye İş B.	1	1	1	1	1	1	1	1	1	1
Vakıfbank	1	1	1	1	1	1	1	1	1	1
Yapı Kredi B.	1	1	1	1	1	1	1	1	1	1
Ziraat B.	1	1	1	1	1	1	1	1	1	1

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# DEA BASED PRICING FOR MAJOR PORTS IN INDIA

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## ABSTRACT

*Ports are economic entities that provide cargo loading and unloading facility to and from ships. The major ports in India are facing stiff competition from other ports and from other modes of transportation as well. These ports are now in the process of transiting from service ports to landlord ports. In this transition process ports are continuing to provide certain services while outsourcing the rest. Under these changed circumstances traditional pricing approaches are no more applicable. The port cost depends not only on cost of inputs used by ports but also cost of outsourcing of various activities. The cost of outsourcing in turn does not depend only on the marginal cost of the service provider but also on their market power. In this paper a DEA based pricing approach considering the market power of the service providers, measured using Lerner's index, and setting up a ceiling price for outsourced services is proposed.*

**Keywords:** Port pricing; DEA; Outsourcing; Lerner's Index; Ceiling price

## INTRODUCTION

### Port Pricing: Literature Survey

Ports are points of cargo handling from and to ships. Ships' cost at port has a significant bearing on total landed cost of goods traded across countries. Hummels (2007) finds that sea cargo has not experienced the strong reduction in transport costs that is usually assumed to have occurred as a result of containerisation. Until the mid-1980s, higher fuel costs and port charges wiped out any efficiency gains due to containerisation. Today, the decline in transport costs in international shipping since the 1980s is largely or even completely cancelled out by rising port congestion costs following the strong growth that has occurred recently in transport volumes (Meersman et al, 2012). Clark, Dollar and Micco (2002) refer to data from the World Bank indicating that transport costs are on the rise in many areas of trade, especially in the context of developing countries. Apart from various reasons such as cost of fuel, port cost contributes significantly to the total transport cost. Clark, Dollar and Micco (2002), who analysed the importance of distance and transport costs for the international flow of goods out of and into Latin American countries, find that trade flows are especially sensitive to port efficiency. They opined that if better structured port charges are introduced, the incentives to increase port efficiency may also be enhanced which may in turn remove important obstacles to trade.

Pricing is an important aspect in determining port cost. Haralambides et al., (2001) in their report on ATENCO project to the European Commission, concluded that pricing in ports can and should be based on costs. The determination of which costs should be reflected in prices largely depends on the type of port organisation. Prices in service or comprehensive ports reflect a multitude of different costs - many of them joint costs, difficult to allocate in a way that is not largely arbitrary – compared to prices in landlord



ports where more clear lines of responsibility and accountability exist. There exists an extensive body of papers discussing and suggesting various port pricing structures. Several of these studies are reviewed by Strandenes and Marlow (2000), who categorise the approaches taken as (1) cost-based pricing (e.g. Button, 1979); (2) methods for cost recovery (e.g. Gilman, 1978, Meyerick, 1989, Talley, 1994, Bergantino, 1977 and 2000); (3) congestion pricing (e.g. Jansson and Rydén, 1979, Vanags, A. H., 1977,); (4) strategic port pricing (Arnold, 1985, UNCTAD 1995); or (5) quality pricing (Strandenes and Marlow, 2000).

There are studies concentrating on the general theory (see Haralambides *et al*, 2001, Haralambides, 2002, Meersman *et al*, 2002, Abbes, 2007, Meersman *et al*, 2007, De Borger *et al*, 2008), and based on empirical studies as well (Acciaro, 2013). These studies includes cost axioms port pricing based on US public ports by Talley (1994), efficient port pricing by Perez- Labajos and Esteban Garcia (2000), strategic pricing of privatized ports by Ashar (2001) and a more recent work on determinants of port infrastructure pricing by Bandara *et al* (2013).

Bennathan and Walters (1979) concluded that strictly setting price equal to marginal cost is best only in a perfectly competitive free economy or in an efficient socialist economy. In practice, the port is confronted with organised and largely foreign-owned shipping cartels. Haralambides *et al*. (2001, p. 939), opined that from a theoretical perspective, and assuming that a number of conditions are fulfilled, long-run marginal costs represent the most appropriate basis for efficient pricing. The authors go on to say that irrespective of the cost basis chosen, the principle that prices should accurately reflect (not to say recover) social opportunity costs is crucial (Haralambides *et al*, 2001, p. 939; see also Haralambides and Veenstra, 2003).

Only the short *Port Pricing* term marginal cost indicates precisely the difference in costs between acceptance and refusal of an additional user (Blauwens *et al*, 2002, p. 427). There has been always debate between choices of short term marginal cost against long term marginal cost in port pricing. This is so because short term marginal cost tend to ignore fluctuations arising out of peak and off peak loads. They suggested that price should be a kind of average of the short-term marginal cost at different moments so that this average can, under certain conditions, be approximated by the long-term marginal cost. Meersman *et al* (2004) make a distinction between four elements of marginal costs in port operations, being costs for provision of infrastructure, costs associated with the use of the transport mode, costs for supplying port services, and external costs. This distinction may constitute the basis for implementation of a pricing approach that is grounded on the marginal cost principle.

The governance structure of seaports has radically changed over the last decades. It now includes private sector participation in seaport operation. Ports have embarked a restructuring process such that the operational aspects of the seaports have been transferred to private sector operators whose financial and operational capacity have also been much more internationalized. This has also resulted in a new port management decision making protocol as well as pricing strategies and practice, from a more centralized public sector affair to a more corporatized business process. Bergantino (2002) analyses EU policy papers on transport infrastructure and points out that the EU is changing its port charging policy with the aim of facilitating shipping. Reeve (2010) analyses differences between port organizations models in terms of the price level implemented. The author finds that a landlord or vertically separate dport yields the highest

profit for the port authorities and the highest prices for customers. Strandenes and Marlow (2000) concluded that on the basis of the potential conflict situations that may arise from different objectives of the various stakeholders, there is no single solution to the problem which is port pricing.

Thus it can be inferred that there is a need for a detailed study of port pricing. The best approach is to start from the heterogeneous nature of ports, taking into account the different market players, with different – possibly conflicting- interests.

#### Application of DEA in Pricing

Agrell (2000) extends earlier results joining Data Envelopment Analysis (DEA) cost norms with the modern approach to regulation, based on agency theory. A comparison to the popular CPI-X scheme and to a more advanced DEA-based cost scheme implemented in Norway has been made in this paper, the paper provides a fragment to a regulatory manual. Giannoccaro (2008) applied modified DEA technique, proposed by Korhonen and Luptacik (2004) in order to separate the technical and the ecological efficiency of different water pricing policies.

Sueyoshi(1995) proposed a profit based pricing approach. He extended the same to include a budget constraint Sueyoshi (1997).

#### Governance of Major Ports in India

The ports in India are in transition phase. Prior to 1991, the year India introduced liberalisation in its public governance, the ports served as service ports. Under this model the ports provided all kinds of services required for vessel operation and cargo handling. Under the liberalization policy ports intend to transgress to landlord model. At this stage, however, the ports are outsourcing some of its services while itself carrying out other components of operations. The vessel and cargo handling operations can be broadly categorized under three heads, namely, on-board vessel operation (stevedoring), on shore cargo handling (quay operation) and at yard operations (terminal management). Some ports have outsourced some of these services. The port retains the pricing decision of its services, based on which the customers pay the port charges. There are around 200 ports across the 7000 Km coastline in India. The major ports run by the government are facing competition with the other ports.

Traditional approaches suggest that ports priced their services based on cost of operations and the port margins dictated by the market forces. The rationality of the port charges were governed by the Tariff authority for Major Ports (TAMP) in India.

In view of partial outsourcing of its operations, the cost of outsourcing however is not dependent on the cost of inputs alone. It depends on the cost quoted by the third party who would carry out the job on behalf of the port. The cost would also vary with the number of service providers available in the market. The port has to resort to open bidding to get the best competitive rate. However, ports need to ascertain the ceiling price beyond which the cost of outsourcing would be acceptable. In other words, the ceiling price restricts the bidder to quote below the ceiling price and ensure viability of port charges that the port levies from its customers.

In this paper Lerner's index is proposed to be used to arrive at the cost of outsourced services. Lerner's index is expressed as:

$L = 1 / e$ , where  $e$  is the price elasticity of demand.

That is,  $L = (P-MC)/P = 1 / e$ , (1)

where  $P$  is the price of the product or service and  $MC$  is the marginal cost of the product.

The Lerner index measures a firm's level of market power by relating price to *marginal cost*. When either exact prices or information on the *cost* structure of the firm are hard to get, the Lerner index uses price elasticity of demand in order to measure market power: Lerner's index provide better option to compute price as it considers price not only as function of demand but also availability of service providers i.e., less the number of service provider higher the Lerner's index .

From equation (1) we get:

$P_K (1-L) = MC$  or

$P_K = MC / (1-L)$  (2)

Where  $P_K$  is the price to be paid to the outsourced agency for the  $K^{th}$  service.

The number of service provider for three different components, namely, stevedoring, quay and yard operation may differ. For example, it is observed that the number of stevedores is less than the quay operators. Hence, cost of services will vary with market power as well. Moreover, the number of service provider will differ from port to port.

## METHODS

In this paper a pricing model for ports in India is suggested based on DEA technique proposed by Sueyoshi (1997). The proposed model factors the application of Lerner's Index and imposes a constraint associated with ceiling price of outsourcing.

### Port Pricing Model

Sueyoshi. T., (1997) proposed the following model

$$\begin{aligned}
 &\text{Maximize} && P_i(Y) Y - c \\
 &\text{Subject to} && \sum c_j \lambda_j - c \leq 0 \\
 &&& \sum Y_j \lambda_j + Y \leq 0, \\
 &&& c_i \leq \text{Budget (of } i^{\text{th}} \text{ DMU)} \\
 &&& lb \leq \sum \lambda_j \leq ub
 \end{aligned}$$

$c \geq 0$ ,  $Y \geq 0$ , and  $\lambda_j \geq 0$ ,  $c$  is the marginal cost of operation

Sueyoshi (1997) assumed that magnitude of price  $P_i(y)$  depends on output quantity  $Y$  and  $P_i(y)$ , can be expressed by a linear inverse demand curve i.e

$$P_i(y) = a_r - b_r y_r$$

The total revenue

$$P_i(Y) Y = \sum (a_r - b_r y_r) y_r = \sum (a_r y_r - b_r y_r^2)$$

$a_r \geq 0$ , and  $b_r \geq 0$

Where  $P_i(Y)$  is a row vector representing services prices.  $P_i(Y)$  has  $s$  components ( $r=1, \dots, s$ ) and magnitude of each components depends on output quantity.

Reformulating the above formulation for arriving at prices for port the following mathematical model is suggested:

$$\begin{aligned} &\text{Maximize} && P_i(Y) Y - c \\ &\text{Subject to} && \sum c_j \lambda_j - c \leq 0 \\ &&& \sum Y_j \lambda_j + Y \leq 0, \end{aligned}$$

$$c_i \leq CP_i + PB_i$$

$$CP = \sum P_K$$

$$lb \leq \sum \lambda_j \leq ub$$

$$c \geq 0, Y \geq 0, \text{ and } \lambda_j \geq 0$$

Where  $P$  is price charged by the port to the customer for the comprehensive operation, say cost of one container handling.  $CP_i$  is the ceiling price paid to the outsourced agencies and is derived as given in equation (2) by the  $i^{\text{th}}$  port. The Port Budget is based on cost of services provided by the  $i^{\text{th}}$  port.

The input variables include the total number of quay and yard cranes, other equipment, locomotives and berths. The annual throughput is considered as the output variable, denoted as  $Y$ . The cost associated with these inputs can be derived using the concept of total annual cost of equipment. For example, for a equipment with a purchase price of 200USD, the annual cost can be computed as:

**Table 1: Annual Costs**

1	Year	1	2	3	4	5	6	7	8
2	Hours worked in year	2.000	2.000	2.000	1.900	1.900	1.800	1.800	1.700
3	Purchase Price \$200,000								
4	RMV % if sold at end of year	65%	46%	34%	26%	20%	16%	13%	11%
5	RMV \$ if sold at end of year	\$130.000	\$92.000	\$68.000	\$52.000	\$40.000	\$32.000	\$26.000	\$22.000
6	Loss in value in year	\$70.000	\$38.000	\$24.000	\$16.000	\$12.000	\$8.000	\$6.000	\$4.000
7	Cost to keep & run for the year	\$11.000	\$16.500	\$22.000	\$27.500	\$33.000	\$38.500	\$44.000	\$49.500
8	Total cost for the year	\$81.000	\$54.500	\$46.000	\$43.500	\$45.000	\$46.500	\$50.000	\$53.500
9	Cost per hr. for the year	\$40.50	\$27.25	\$23.00	\$22.89	\$23.68	\$25.83	\$27.78	\$31.47

Source: <https://www.constructionequipment.com/annual-and-life-date-costs>

## RESULTS AND DISCUSSIONS

The above method reveals that the profit of an Indian port would not only depend on the input cost but also on the market power of the agencies providing services. Ports can pre-empt their profitability given their knowledge on efficient use of resources and the availability of service providers. In the present context, an input oriented DEA of major container handling ports in India (table 2) reveal that ports are on varied state of efficiency.

Firm	1	2	3	4	5	6	7	8	9	10	11	12
CRSTE	1.000	0.588	1.000	0.687	1.000	0.976	1.000	1.000	0.713	0.471	1.000	0.210
VRSTE	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
SCALE Efficiency	1.000	0.588	1.000	0.687	1.000	0.976	1.000	1.000	0.713	0.471	1.000	0.210
	-	Irs	-	drs	-	Irs	-	-	irs	irs	-	Irs

Any one port subscribing to outsourcing needs to assess its scale efficiency and accordingly adjust surplus use of inputs to match the efficient port. This is so required to arrive at the right input quantity followed by input cost to determine the right pricing strategy. The thumb rule on use of Lerner's index could be;  $L \approx 1/n$  where  $n$  is the number of available firms. This approximation indicates that availability of ten or more firms will lead to get a competitive cost of outsourcing by the ports. Else the ports have to operate at lower margin of profit in order to remain competitive.

## CONCLUSIONS

The traditional approach to pricing of port services is no more suitable for ports, especially in India, as the ports are in transition phase of governance. The port cost depends not only on cost of inputs used by ports but also cost of outsourcing of various activities. The cost of outsourcing in turn does not depend only on the marginal cost of the service provider but also on their market power. Hence a DEA based pricing approach considering the market power of the service providers, measured using Lerner's index, and setting up a ceiling price for outsourced services is proposed in this paper for the decision makers.

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# DESIGNING A NETWORK DATA ENVELOPMENT ANALYSIS (NDEA) MODEL TO EVALUATE RESILIENCE OF SUPPLY CHAIN WITH STOCHASTIC DATA

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## ABSTRACT

*In today's global economy, are affected by a wide range of disruption in conditions of uncertainty, thus they faced up with high risk. Risks have negative effects on the supply chain and leading to lower profitability and competitive advantage. In this study, the data envelopment analysis model for designing a resilience supply chain network under conditions of uncertainty is taking into consideration, resilience performance indicators with stochastic data in the automotive industry. The purpose of this study is to measure performance a decision-making by combining two approaches: Resilience and chance constrained network DEA and analysing of resilience supply chain with using data envelopment analysis in conditions of uncertainty. A number of major indicators of resilience industry include: flexibility, responsiveness, accessibility, agility, diversity and adaptability which will be measured by using several operational level. The first step is to identify the key supply chain processes in the industrial environment and establish relationships between organizational units and identify inputs and outputs of organization are obtained randomly. After the formation of the main DMUs in order to measure the efficiency of processes, explain of the network DEA model in conceptual framework, the objectives, constraints, variables and then apply linearization techniques have implemented.*

**Keywords:** *Chance- Constrained Data Envelopment Analysis, Supply chain Resilience, Resilience Indicators, Three-level Supply Chain*

## INTRODUCTION

SC is defined as ‘the network of organizations that are involved, through upstream and downstream linkages, in the different processes and activities that produce value in the form of products and services in the hands of the ultimate consumers (Oliver and Webber,1992). Since today’s SCs are more vulnerable to disasters and the business environment is varying continuously, risks happen. For managing these risks, we require considering resilience(Carvalho et al.,2012). Recently companies are affected by a wider range of disruptions than before. It is quite difficult to forecast how SC would behave when different disruptions happen, while we live in an unpredictable world and serious disruptions to supply chain (SC) activities may occur. The concept of resilience is multidimensional and multidisciplinary. Resilience helps to recover system states after incidents take place rather than prevent incidents from occurring (Dinh et al., 2012). Moreover, resilience is one of the ways to fight disruptions in the SC. In the literature, resilience means ‘the ability to react to an unforeseen disturbance and to return quickly to their original

state or move to a new, more advantageous one after suffering the disturbance (Peck ,2005). Disruptions could occur at any section of the network, in any of the processes, for a wide variety of reasons such as transportation delays, power outages, or natural or man-made disasters. A resilient supply chain is a system that has the ability to recover quickly from disruptions and ensure customers are minimally affected.

The resilience of the firm can be identified and evaluated using resilience performance indicators (RPI) They can be measured on a regular basis by using several operational level measures. Six major indicators of resilience were taken into consideration for this study and the data give a reading on the indicators of flexibility, responsiveness, accessibility, agility, diversity and adaptability. Our problem employs a representative a real-world case study of Iran Saipa manufacturing. The level of a firm's supply chain resilience can be evaluated by certain performance measures. Some of the resilience indicators of the firm or its supply chain are described below:

#### *Flexibility indicators*

Flexibility in a supply chain is the ability to manage changes quickly without undue effort and loss. It can be related to a system, product or process. Flexibility needs to be designed into the SC and is reflected in its structure, its inter-organisational processes as well as its strategies (Tang and Tomlin 2008). Process flexibility is related to flexible production and warehousing systems (Ivanov et al., 2014). Several determinants measure the flexibility of a system (Rajesh, 2016):

Stock out rate, Inventory accuracy rate, Number of small disruptions managed through flexibility, Percentage increase in sales from design flexibility.

#### *Responsiveness indicators*

For managing risk, it is important for a firm's supply chain to be coordinated with its partners to improve information sharing and increase responsiveness. Visibility and velocity are the two major indicators of supply chain resilience that contributes to the agility of the supply network (Cantor et al., 2014). Several measures can be used for the responsiveness of the supply chain system: On-time delivery ratio, Contract issue time, Contract approval time, Put-away time ratio.

#### *Accessibility indicators*

This indicates the visibility of supply networks and increased accessibilities reduces the vulnerabilities and increases trust in supply chains. Supply chain risk increases due to the lack of accessibility and high supply accessibility guarantee resilience. From a complexity theory perspective, accessibility increases the interactive complexities of networks, but increases the trust and transparency of operations (Brandon-Jones et al., 2014). The major indicators of accessibility in supply chains are: Dealer accessibility, Retailer accessibility, Customer accessibility, Network intensity.

#### *Agility indicators:*

The ability of an SC to respond quickly to unpredictable changes in demand or supply is



its agility (Christopher and Peck 2004). If the response times to demand changes or supply disruption are too long, the organisation and supply networks are exposed to risk. To prevent that risk, the SC must be designed agile. Agility has two main factors: velocity and visibility:

1. Velocity refers to the rate by which the SC recovers from the disturbance.
2. Visibility implies a clear view of upstream and downstream inventories, demand and supply

conditions, and production and purchasing schedules. It also implies internal visibility with clear lines of communications and agreement on 'one set of numbers' (Christopher and Peck 2004).

#### Diversity indicators:

The variety of products has got the ability to meet customer's needs, respond to market fluctuations and enhance the flexibility.

#### Adaptability indicators:

It is defined in three different structures: readiness, recovery and responsibility. These three structures show that the resilience of an SC can effect on three distinct phases: prior to an event (event readiness), throughout the event (response) and after the event (recovery). In other words, an adaptable SC should act properly in all periods (Azadeh et al., 2014).

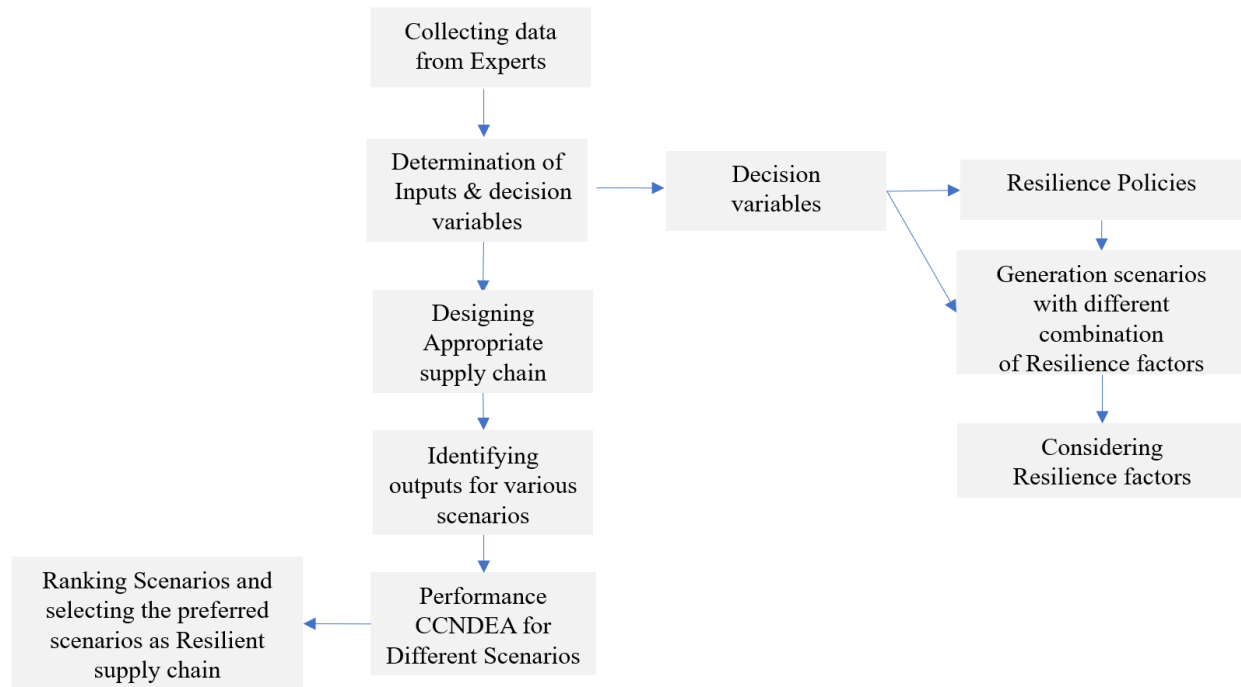
By increasing complexity and vulnerability of the supply chains, a growing number of studies in the operations and supply chain literature have stressed the importance of conducting resilience assessments as part of routine supply chain risk management practices (Chopra and Sodhi, 2004).

In the supply chain resilience literature, the view of the supply chain as complex adaptive system has been adopted by (Surana et al., 2005) who studied a variety of complex network topologies emphasising the importance of resilience as a critical factor for assuring supply chain performance.

(Klibi, et al., 2010) defined an SCN design problem under uncertainty and presented a review of the optimisation models proposed in the literature. To address the aforementioned research need, a model built using the network data envelopment analysis (DEA) modeling approach (Fare and Grosskopf, 2000), DEA has been adopted to evaluate and compare risk exposures of individual tiers in supply chains (see among Azadeh and Alem, 2010; Olson and Wu, 2011; Talluri et al., 2013). Talluri et al. (2006) proposed a chance-constrained DEA (CCDEA) based on uncertain inputs and outputs to assess variations in vendor attributes in presence of supply risks. Azadeh and Alem (2010) developed, fuzzy DEA models for supplier risk evaluation under uncertainty. The suggested chance-constrained network DEA model allows to: (i) account for uncertainties associated with risk and resilience levels in the given supply chains, (ii) assess supply chain resilience to the identified risks at the level of the overall supply chain and its individual tiers, and (iii) rank supply chains based on their level of resilience to risk. To the best of our knowledge, this study is the first to introduce the application of network DEA modelling to facilitate the assessment of supply chain resilience to risks. Due to the complex nature of the various parameters affecting some systems such as big companies and Unpredictable issues, an exact mathematical model for the systems does not exist, thus uncertainty models is used for these systems.

## METHODS

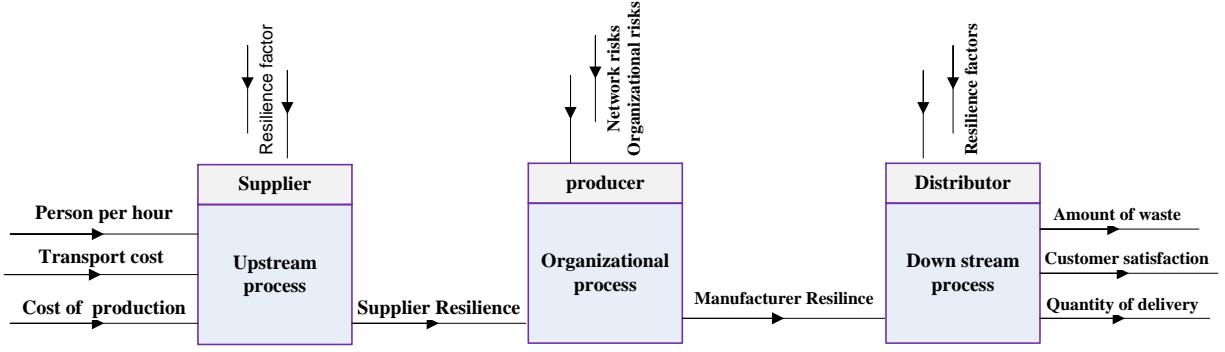
In this paper, a three-echelon SC is assumed which assembles the parts gradually and finally hands them over to the final plant. To address the purpose of this study, a multimethod research approach is adopted as follows: first, data envelopment analysis (DEA) modeling and probability theory are used to build a chance-constrained network DEA model to assess risk resilience of the overall supply chains and their individual tiers; next, the proposed model is tested using a survey of middle-level and top-level managers representing Auto industry sectors in SIPA corporation. And then, Different scenarios with different resilience policies in Auto production are considered. the scenarios are assumed as a DMU in CCNDEA<sup>3</sup> and the finally, best scenario is selected in various conditions. Finally, the resilience policies are debated and Choosing the best strategy in Automotive Manufacturing between different stages in an SC is an important issue especially when we have uncertainty parameters. In order to do so, a process along with the CCNDEA approach is proposed (Figure1).



**Figure1. The resilient SC with ambiguous inputs and outputs**

As demonstrated in figure2, in the context of this study, DEA is capable to incorporate multiple risk and resilience measures as inputs and outputs to a supply chain system and enables the comparison between the current level of resilience to various supply chain risks with the desirable levels that supply chain decision makers aim to achieve. Additionally, chance-constrained network DEA makes these comparisons possible at both tier-specific (i.e. individual companies forming supply chains) and system-wide (supply chains as entities) levels of analysis.

<sup>3</sup> Chance- Constrained Network Data Envelopment Analysis



**Figure 2. Proposed model for Three level supply chain with resilience factors**

This section introduces the CCNDEA model with deterministic input and stochastic output factors to evaluate the performance of DMUs (suppliers) under varying levels of supplier performance risk. CCDEA is a popular technique for supplier evaluation and ranking has been used by Land et al. (1993), Land provides the first established CCDEA model with the following standard notation and formulation:

Notation used:

$i, j = 1, \dots, I$  : supplier sample index (the set of decision-making units),

$m = 1, \dots, M$ : input factors,

$n = 1, \dots, N$  :output factors,

$X = [x_{mi}]$  : sample input matrix, with dimension  $M * I$ ,

$Y = [y_{ni}]$  : sample output matrix, with dimension  $N * I$ ,

$X^m$  : row vector of  $X$ ,

$Y^n$  : row vector of  $Y$ ,

$y_{n0}$  : one output factor of the supplier under evaluation,

$x_{m0}$  : one input factor of the supplier under evaluation,

$X_0 = [x_{m0}]$ : column vector of inputs of the particular DMU investigated,

$Y_0 = [y_{n0}]$ : column vector of outputs of the particular DMU investigated,

$\theta$  : (radial input) contraction factor, DMU's efficiency score,

$i=1, \dots, I$ , weights vector, for all DMUs (column vector of DMU loadings, determining the “best practice” for the DMU being evaluated).

## MODEL

$$\min z = \theta - \varepsilon(e \cdot s^+ + e \cdot s^-)$$

$$\begin{aligned}
&\text{subject to } E(Y^n \lambda - y_{no}) - F^{-1}(1-p) \delta_n - s_n^+ = 0, \quad n=1, \dots, N \\
&\theta x_{m0} - X^m \lambda - s_m^- = 0, \quad m=1, \dots, M \\
&\theta \text{ unrestricted}, \lambda, s_n^+, s_m^- \geq 0, n=1, \dots, N, m=1, \dots, M
\end{aligned}$$

In Model,  $s^+ = [s_n^+]$  and  $s^- = [s_m^-]$  are the column vectors of the output and input slack variables,  $e$  is the row vector of suitable dimension with unity in all positions, and  $\varepsilon > 0$  is a non-Archimedean infinitesimal. Again, a DMU is CCDEA-efficient if and only if  $\theta^* = 1$  and  $s^{+*} = s^{-*} = 0$ .

In the CCDEA case, a series of computations are presented based upon some guiding assumptions on the individual and the joint probability distributions of the output and input factors. First, the output factors for each supplier are regarded as random variables following a normal distribution.

## RESULTS AND DISCUSSIONS

To address the research aim above, our study adopts a multimethod research approach which allows to design and test an analytical model for assessment of resilience to supply chain risk. First, DEA modelling and stochastic factors are adopted to develop an analytical model for risk and resilience assessment in a three-tier supply chain. This section begins by describing the dataset that consists only of quantitative data due to our focus on uncertainty performance. Following the dataset description, the methodology discussion covers the traditional CCDEA models and our incorporation of varying levels of performance risk into the CCDEA models of efficiency evaluation.

**Table 1: The results of CCDEA approach: technical efficiency (T. E.) and ranking (R.) of scenarios for each power.**

P	0.1						0.01					
	T.E	R.	Min	Mean	Median	Stdev	T.E	R.	Min	Mean	Median	Stdev
Scenario 1	0.68	5	0.8321	0.9440	0.9467	0.0480	0.71	6	0.8374	0.9550	0.9643	0.0449
Scenario 2	0.91	1	0.8323	0.9443	0.9469	0.0479	0.90	2	0.8375	0.9555	0.9653	0.0448
Scenario 3	0.83	4	0.8324	0.9448	0.9471	0.0470	0.91	1	0.8378	0.9555	0.9649	0.0445
Scenario 4	0.90	2	0.8325	0.9449	0.9473	0.0479	0.78	5	0.8380	0.9556	0.9965	0.0443
Scenario 5	0.54	7	0.8337	0.9451	0.9475	0.0477	0.87	3	0.8383	0.9559	0.9653	0.0442
Scenario 6	0.87	3	0.8348	0.9461	0.9468	0.0476	0.53	9	0.8385	0.9561	0.9653	0.0440
Scenario 7	0.51	10	0.8375	0.9478	0.9506	0.0460	0.85	4	0.8427	0.9570	0.9662	0.0433
Scenario 8	0.53	8	0.8391	0.9476	0.9513	0.0466	0.71	6	0.8472	0.9583	0.9677	0.0424
Scenario 9	0.63	6	0.8376	0.9456	0.9546	0.0472	0.54	8	0.8514	0.9599	0.9702	0.0417
Scenario 10	0.52	9	0.8321	0.9432	0.9483	0.0494	0.67	7	0.8540	0.9599	0.9713	0.0444

In this study, the CCDEA approach is used as an effective method to rank the scenarios and analyse the data. All the performance indicators are imported to the CCDEA model in order to determine the efficiency score and rank of scenarios. Table 1 shows the results of using the CCDEA approach for three levels are computed for ten CCDEA scenarios.

## CONCLUSIONS

SCs are facing many unexpected situations that increase their vulnerability to disturbances. So SCs must be resilient to survive. In this paper, we assumed that disruptions may occur in a production system. We considered a 3 echelon SC with 10 different scenarios; each one reflects a policy against disruption. The

factors of resilience assumed in this paper are responsiveness, accessibility, agility, diversity and flexibility. In order to rank the scenarios, the CCNDEA approach has been used. We can conclude that applying the factors of resilience leads to better outputs and among these factors responsiveness and diversity are more important to be considered. The difference in stochastic outputs after applying the flexibility policy is significant, and it is recommended that the SCs be re-engineered with the visibility characteristic. Also redundancy comes to our attention as another important resilient factor, especially because the disturbance considered in this paper is in a transportation system. Future studies may further explore the applied aspects of supply chain performance and resilience assessment. They may adopt different methodological approaches to improve the assessment of supply chain resilience at various levels—moving from a system-wide perspective of the supply chain toward an organization-specific perspective of individual supply chain tiers.

## ACKNOWLEDGEMENT

The authors are grateful for the valuable comments and suggestions from the respected reviewers, which have enhanced the strength and significance of our paper. The data of this study is supported by Sipa Corporation.

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# **DEVELOPMENT OF NETWORK DYNAMIC DATA ENVELOPMENT ANALYSIS FOR INDUSTRIAL MANAGEMENT INSTITUTE PERFORMANCE EVALUATION SYSTEM USING GAMES THEORY**

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## **ABSTRACT**

*The main objective of this study is to measure performance cause and effective processes in Industrial Management Institute uses data envelopment analysis for Dynamic Network and approach to game theory. According to this study, the performance evaluation with the approach of the Data Envelopment Analysis using operations research techniques to obtain the solution of the model will be addressed in the Organization of industrial management. Therefore, the first step is to identify the main industrial management institute and determine the relationships between the levels and units of each process and identify all the inputs and outputs of system which have been achieved through communication between processes. After the formation of the original DMUs in order to measure the efficiency of operational processes, the definition of a Network of DEA model within the framework of the conceptual model, objectives, constraints, variables and parameters and then applying linear construction techniques. Next step, Development of Network Dynamic Data Envelopment Analysis will be achieved using Games Theory, and finally solve the provided model by the software LINGO.*

**Keywords:** Network DEA, Dynamic, Game Theory, Performance Evaluation, Business

## **INTRODUCTION**

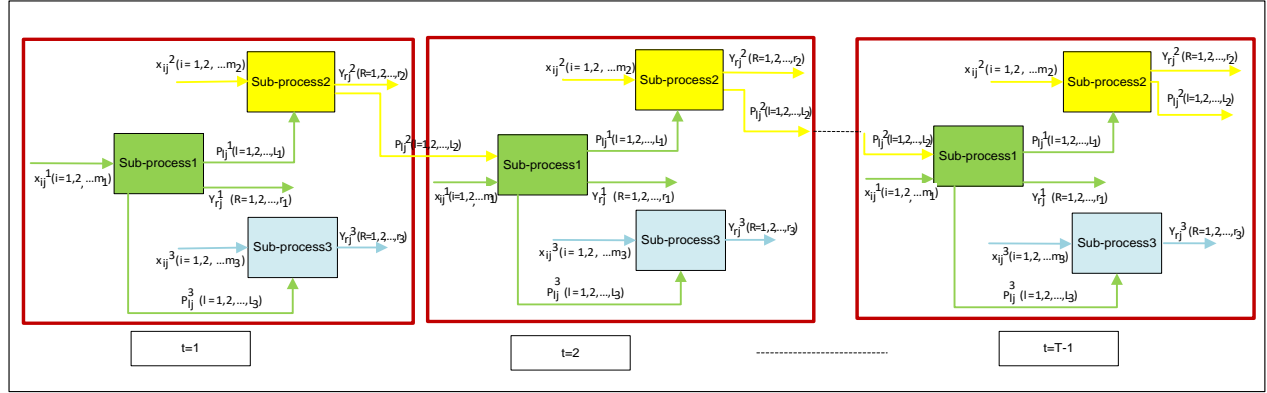
Data envelopment analysis (DEA), introduced by Charnes, Cooper and Rhodes (1978), is an effective tool for measuring the relative efficiency of peer decision making units (DMUs) that have multiple inputs and multiple outputs. In the standard DEA approach, internal structures of DMUs are ignored and the units are treated as black boxes. Lewis, Sexton (2004), Single-stage DEA models are appropriate for measuring the relative efficiency of simple processes and cannot represent the internal operations of a DMU. These internal operations may consist of several sub-DMUs. The NDEA models are developed to effectively handle the internal operations of the DMUs. The NDEA essentially considers the internal operations of the DMUs. The NDEA models are developed to effectively handle the internal operations of the DMUs. The NDEA essentially considers the internal operations of a DMU by modeling the complicated interactions among multiple sub-DMUs in the efficiency evaluation problems. By focusing on the efficiency score of a sub-DMU, managers can improve the performance of the sub-DMU which in turn improves the overall efficiency of the DMU. Recently, researchers have started to look into two-stage network structures or processes where outputs from the first stage become inputs to the second stage. The outputs from the first stage are referred to as intermediate measures. Seiford and Zhu (1999), use a two-stage process to measure

the profitability and marketability of US commercial banks. Hwang (1977), expressed two stage processes and be implemented in the banking industry. Chilingirian and Sherman (2004), describe a two-stage process in measuring physician care. Kao and Hung (2008), assessed the relative efficiency of 6 scientific departments including 41 educational departments affiliated with Taiwan Cheng Chung National University using output oriented BCC-DEA model. Kao and Hwang (2008), offered a new method of measuring the overall efficiency of such a process. Chen et al. (2009), use a weighted Additive model to summation the two stages and decompose the efficiency of the overall process. Moreover Liang et al. (2008), develop a number of DEA models that use the concept of game theory. Specifically, Liang et al. (2008) develop a leader–follower model borrowed from the notion of Stackelberg games, and a centralized or cooperative game model where the combined stage is of interest. The purpose of this study can be performance measurement and determination of cause and effect process. The network of businesses, industrial management institute with process approach adopted with the latest organizational chart to determine the cause and effect process. This processes to be able to determine the relationship between different organizational units.

## METHODS

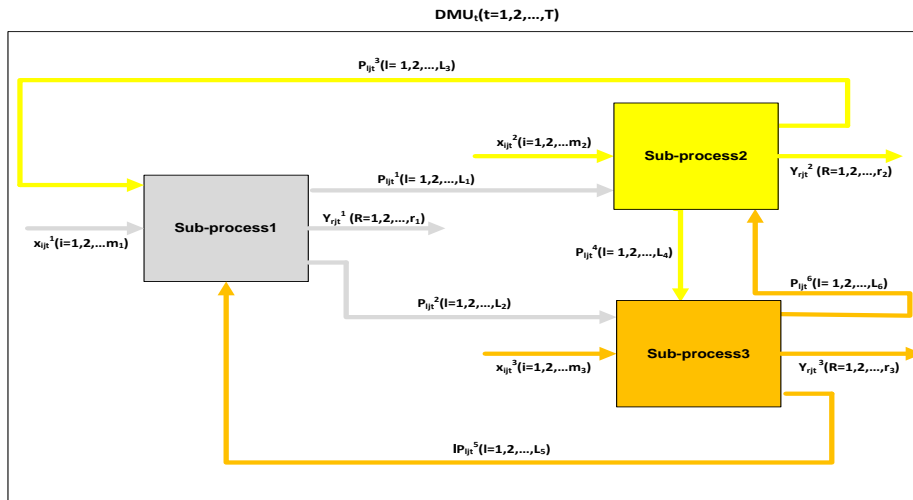
In this article, a network structure has been proposed to assess the cause and effect processes in the Industrial Management Institute. The effect activity consists of three sub-processes of education, communication and customer affairs and IT, and the cause activity consists of three sub-processes of education, consulting and publishing. The complicated process among the units of the headquarters, line and high amount of inputs and outputs of each process has formed a complicated network for the Industrial Management Institute processes. The current paper applies directly the Nash bargaining game theory to the efficiency of DMUs that have processes. three individuals bargaining with each other for a better payoff, which is characterized by the DEA ratio efficiency of each individual stage. In general, the resulting Nash bargaining game model is highly non-linear, given the nature of ratio forms of DEA efficiency. In the present study due to the nature of the processes of industrial management institute for many years, a process that will create more profits for IMI is necessary to be effective that the rest of the processes under the process to achieve their maximum. organizational units depending on your bargaining power and status can have the leading role In the bargaining game. Players are the same units introduced in cause and effect processes that they are working in a competitive environment to achieve greater profits. So the process as a player and the whole process as a centralized model is considered with the purpose of optimizing the efficiency in the bargaining game theory. A Network DEA (NDEA) model is being developed in order to measure the performance in the cause and effect Processes. the network DMU presented in Fig 1 and 2 contain three sub-processes which are associated with publishing, education and consulting in the cause process and communication and customer service, education, information technology in the effect process. consider the following assumptions and extend the usual DEA modeling notations to proposed network format. A schematic view of the DMU and its input, intermediates, and outputs are presented in Figure 1 and 2.





**Figure1: Conceptual model of the cause processes network considering the inputs, outputs and feedback for multi period**

In Figure 1, a model of the cause processes network considering with inputs, outputs and intermediate products is presented for multi periods. The index  $t$  is the same as the DMUs being evaluated in the context of time. each  $DMU_t$  ( $t=1,2,\dots,T$ ) consists of sub-process1(Publication), subprocess2(Education) and sub-process3(Consulting). Sub-process1 consumes,  $m_1$  inputs  $x_{ijt}^1$  ( $i=1,2,\dots,m_1$ ) and  $L_2$  intermediate measures  $P_{ijt}^2$  ( $l=1,2,\dots,L_2$ ) to produce  $r_1$  outputs  $Y_{ijt}^1$  ( $R=1,2,\dots,r_1$ ) and  $L_1$  intermediate measures  $P_{ijt}^1$  ( $l=1,2,\dots,L_1$ ) and  $L_3$  intermediate measures  $P_{ijt}^3$  ( $l=1,2,\dots,L_3$ ). Sub-process2 consumes  $L_1$  intermediate measures  $P_{ijt}^1$  ( $l=1,2,\dots,L_1$ ) and  $m_2$  inputs  $x_{ijt}^2$  ( $i=1,2,\dots,m_2$ ) to produce  $R_2$  outputs  $Y_{ijt}^2$  ( $R=1,2,\dots,r_2$ ). Sub-process3 consumes  $L_3$  intermediate measures  $P_{ijt}^3$  ( $l=1,2,\dots,L_3$ ) and  $m_3$  inputs  $x_{ijt}^3$  ( $i=1,2,\dots,m_3$ ) to produce  $R_3$  outputs  $Y_{ijt}^3$  ( $R=1,2,\dots,r_3$ ). The efficiency of total process, sub-process1, sub-process2 and sub-process3 in period  $t$  are parameterized using  $e_t$ ,  $e_t^1$ ,  $e_t^2$ ,  $e_t^3$ , respectively.



**Figure3: Conceptual model of the effect processes network considering the inputs, outputs and feedback for one period**

In Figure 2, a model of the effect processes network considering with inputs, outputs and intermediate products is presented for one periods. The index  $t$  is the same as the DMUs being evaluated in the context of time.

Game-Theoretic approach model for cause processes: The (CRS) efficiency scores for each DMU in the first-second and third stages can be defined by  $e_j^1, e_j^{1.1}, e_j^2, e_j^3$  respectively,

$$e_j^{(1)} = \frac{\sum_{r_1=1}^{s_1} u_{r_1}^1 y_{r_1 j} + \sum_{l=1}^{l_1} w_l^1 p_{l j} + \sum_{l=1}^{l_2} w_l^2 p_{l j}^2}{\sum_{i=1}^{m_1} v_i^1 x_{ij}^1} \quad j=1, e_j^{(1.1)} = \frac{\sum_{r_1=1}^{s_1} u_{r_1}^1 y_{r_1 j} + \sum_{l=1}^{l_1} w_l^1 p_{l j} + \sum_{l=1}^{l_2} w_l^2 p_{l j}^2}{\sum_{i=1}^{m_1} v_i^1 x_{ij}^1 + \sum_{l=1}^{l_3} w_l^3 p_{l j-1}^3} \quad j \neq 1, j=2, \dots, J \quad (1)$$

$$e_j^{(2)} = \frac{\sum_{r_2=1}^{s_2} u_{r_2}^2 y_{r_2 j}^2 + \sum_{l=1}^{l_3} w_l^3 p_{l j}^3}{\sum_{i=1}^{m_2} v_i^2 x_{ij}^2 + \sum_{l=1}^{l_1} w_l^1 p_{l j}^1} \quad j=1, \dots, J, e_j^{(3)} = \frac{\sum_{r_3=1}^{s_3} u_{r_3}^3 y_{r_3 j}^3}{\sum_{i=1}^{m_3} v_i^3 x_{ij}^3 + \sum_{l=1}^{l_2} w_l^2 p_{l j}^2} \quad j=1, \dots, J$$

The individual efficiency scores  $e_j^1, e_j^{1.1}, e_j^2, e_j^3$ , it is reasonable to define the overall efficiency of the entire three stage process for DMU as  $e_j = e_j^1 \cdot e_j^{1.1} \cdot e_j^2 \cdot e_j^3$  since

$$e_j = \frac{\sum_{r_1=1}^{s_1} u_{r_1}^1 y_{r_1 j}^1 + \sum_{r_2=1}^{s_2} u_{r_2}^2 y_{r_2 j}^2 + \sum_{r_3=1}^{s_3} u_{r_3}^3 y_{r_3 j}^3}{\sum_{i=1}^{m_1} v_i^1 x_{ij}^1 + \sum_{i=1}^{m_2} v_i^2 x_{ij}^2 + \sum_{i=1}^{m_3} v_i^3 x_{ij}^3} = \frac{\sum_{r_1=1}^{s_1} u_{r_1}^1 y_{r_1 j}^1 + \sum_{l=1}^{l_1} w_l^1 p_{l j}^1 + \sum_{l=1}^{l_2} w_l^2 p_{l j}^2}{\sum_{i=1}^{m_1} v_i^1 x_{ij}^1} \cdot \frac{\sum_{r_1=1}^{s_1} u_{r_1}^1 y_{r_1 j}^1 + \sum_{l=1}^{l_1} w_l^1 p_{l j}^1 + \sum_{l=1}^{l_2} w_l^2 p_{l j}^2}{\sum_{i=1}^{m_1} v_i^1 x_{ij}^1 + \sum_{l=1}^{l_3} w_l^3 p_{l j-1}^3}$$

$$\cdot \frac{\sum_{r_2=1}^{s_2} u_{r_2}^2 y_{r_2 j}^2 + \sum_{l=1}^{l_3} w_l^3 p_{l j}^3}{\sum_{i=1}^{m_2} v_i^2 x_{ij}^2 + \sum_{l=1}^{l_1} w_l^1 p_{l j}^1} \cdot \frac{\sum_{r_3=1}^{s_3} u_{r_3}^3 y_{r_3 j}^3}{\sum_{i=1}^{m_3} v_i^3 x_{ij}^3 + \sum_{l=1}^{l_2} w_l^2 p_{l j}^2} = e_j^1 \cdot e_j^{1.1} \cdot e_j^2 \cdot e_j^3$$

The above overall efficiency definitions ensures that  $e_j \leq 1$  from  $e_j^1 \leq 1$  and  $e_j^{1.1} \leq 1$  and  $e_j^2 \leq 1$  and  $e_j^3 \leq 1$  the overall process is efficient if and only if In fact three stages can be regarded as three players in Nash bargaining game theory. We can approach the efficiency evaluation issue of three stage processes by using Nash bargaining game theory. A bargaining problem can then be specified as the triple (N, S, b) consisting of participating individuals, feasible set and breakdown point. The solution is a function F that is associated with each bargaining problem (N, S, b), expressed as F(N, S, b). We here first construct the most ideal DMU and use its DEA efficiency scores as the breakdown point. To do that we assume that if three stages negotiate, their efficiency scores will be the best. (in the first stage, which consumes the minimum amount of input values, while producing the most amount of output or intermediate measure and other stages are the same). The efficiency for above three most ideal DMUs is the best among the existing DMUs. Efficiency scores of the three most ideal DMUs in the first; second and third stage as and  $\theta_{\min}^1, \theta_{\min}^{1.1}, \theta_{\min}^2, \theta_{\min}^3$  as our breakdown point. Then DEA bargaining model for a specific DMU<sub>0</sub> to(2) can be express as:

$$\begin{aligned}
& \text{Max} \left( \frac{\sum_{r_1=1}^{s_1} u_{r_1}^1 y_{r_1 o}^1 + \sum_{l=1}^{l_1} w_l^1 p_{l o}^1 + \sum_{l=1}^{l_2} w_l^2 p_{l o}^2}{\sum_{i=1}^{m_1} v_i^1 x_{i o}^1} - \theta_{\min}^1 \right), \left( \frac{\sum_{r_1=1}^{s_1} u_{r_1}^1 y_{r_1 o}^1 + \sum_{l=1}^{l_1} w_l^1 p_{l o}^1 + \sum_{l=1}^{l_2} w_l^2 p_{l o}^2}{\sum_{i=1}^{m_1} v_i^1 x_{i o}^1 + \sum_{l=1}^{l_3} w_l^3 p_{l o}^3} - \theta_{\min}^{1.1} \right). \\
& \left( \frac{\sum_{r_2=1}^{s_2} u_{r_2}^2 y_{r_2 o}^2 + \sum_{l=1}^{l_3} w_l^3 p_{l o}^3}{\sum_{i=1}^{m_2} v_i^2 x_{i o}^2 + \sum_{l=1}^{l_1} w_l^1 p_{l o}^1} - \theta_{\min}^2 \right), \left( \frac{\sum_{r_3=1}^{s_3} u_{r_3}^3 y_{r_3 o}^3}{\sum_{i=1}^{m_3} v_i^3 x_{i o}^3 + \sum_{l=1}^{l_2} w_l^2 p_{l o}^2} - \theta_{\min}^3 \right) \\
& s.t. \\
& \frac{\sum_{r_1=1}^{s_1} u_{r_1}^1 y_{r_1 o}^1 + \sum_{l=1}^{l_1} w_l^1 p_{l o}^1 + \sum_{l=1}^{l_2} w_l^2 p_{l o}^2}{\sum_{i=1}^{m_1} v_i^1 x_{i o}^1} \geq \theta_{\min}^1, \frac{\sum_{r_1=1}^{s_1} u_{r_1}^1 y_{r_1 o}^1 + \sum_{l=1}^{l_1} w_l^1 p_{l o}^1 + \sum_{l=1}^{l_2} w_l^2 p_{l o}^2}{\sum_{i=1}^{m_1} v_i^1 x_{i o}^1 + \sum_{l=1}^{l_3} w_l^3 p_{l o}^3} \geq \theta_{\min}^{1.1} \\
& \frac{\sum_{r_2=1}^{s_2} u_{r_2}^2 y_{r_2 o}^2 + \sum_{l=1}^{l_3} w_l^3 p_{l o}^3}{\sum_{i=1}^{m_2} v_i^2 x_{i o}^2 + \sum_{l=1}^{l_1} w_l^1 p_{l o}^1} \geq \theta_{\min}^2, \frac{\sum_{r_3=1}^{s_3} u_{r_3}^3 y_{r_3 o}^3}{\sum_{i=1}^{m_3} v_i^3 x_{i o}^3 + \sum_{l=1}^{l_2} w_l^2 p_{l o}^2} \geq \theta_{\min}^3 \\
& \frac{\sum_{r_1=1}^{s_1} u_{r_1}^1 y_{r_1 j}^1 + \sum_{l=1}^{l_1} w_l^1 p_{l j}^1 + \sum_{l=1}^{l_2} w_l^2 p_{l j}^2}{\sum_{i=1}^{m_1} v_i^1 x_{i j}^1} \leq 1, j=1, \frac{\sum_{r_1=1}^{s_1} u_{r_1}^1 y_{r_1 j}^1 + \sum_{l=1}^{l_1} w_l^1 p_{l j}^1 + \sum_{l=1}^{l_2} w_l^2 p_{l j}^2}{\sum_{i=1}^{m_1} v_i^1 x_{i j}^1 + \sum_{l=1}^{l_3} w_l^3 p_{l j-1}^3} \leq 1, j \neq 1, j=2, \dots, n \\
& \frac{\sum_{r_2=1}^{s_2} u_{r_2}^2 y_{r_2 j}^2 + \sum_{l=1}^{l_3} w_l^3 p_{l j}^3}{\sum_{i=1}^{m_2} v_i^2 x_{i j}^2 + \sum_{l=1}^{l_1} w_l^1 p_{l j}^1} \leq 1, j=1, \dots, n, \frac{\sum_{r_3=1}^{s_3} u_{r_3}^3 y_{r_3 j}^3}{\sum_{i=1}^{m_3} v_i^3 x_{i j}^3 + \sum_{l=1}^{l_2} w_l^2 p_{l j}^2} \leq 1, j=1, \dots, n, v_i, u_r, w_d \succ 0
\end{aligned}$$

next we will prove that the feasible set S is both compact and convex. Linear model can be solved. Game-Theoretic approach model for effect processes: Modeling is an effected processes and it is made nonlinear according to the cause processes; therefore, the common nonlinear programming model shows the relative efficiency of the whole network of cause processes like what has been shown in Figure 2 for muti periods and named it DMUo.

## RESULTS AND DISCUSSIONS

In this section, we apply the new Nash bargaining game on set of real data from Industrial Management Institue.the inputs to the cause process for sub-process1 are good sent and performance of annual plans.the outputs are the requests sent,conveying knowledge and information. the intermediate measures are books and journals sent to education department, New edition announced to the education unit and

consulting. The inputs for sub-process2 are New courses and Specific educational contracts. The outputs are The graduates of the courses and Assessment of courses. The intermediate measure is Request for books and journals for the publication unit. The inputs for sub-process3 are Consulting unit experts and Consulting agreements. The outputs are The sent requests to receive financial resources and The requests sent to attract man force, change the personnel status and training. The CRS efficiency scores for the least ideal DMUS in the first and second and thired stages are calculated as  $\theta_{\min}^1=0.0455$ ,  $\theta_{\min}^{1.1}=0.0432$ ,  $\theta_{\min}^2=0.05128$ ,  $\theta_{\min}^3=0.0342$  respectively.

**Table 1: Result of Industrial Managemet Institute with breakdown point for cause process**

	$e_0^{1*}$	$e_0^{1.1*}$	$e_0^{2*}$	$e_0^3$	Overall
DMU 1	0.2510	0.2500	0.6279	0.3278	0.0129
DMU 2	0.7500	0.7321	0.8560	0.0771	0.0362
DMU 3	0.9500	0.7689	0.9312	0.2121	0.1442
DMU 4	1.0000	1.0000	1.0000	1.0000	1.0000
DMU 5	0.9584	0.8726	0.3478	0.7321	0.2129
DMU 6	.....	.....	.....	.....	.....
DMU 7	0.9000	0.1000	0.8661	0.9281	0.0481
DMU 8	.....	.....	.....	.....	.....
DMU 9	1.0000	1.0000	1.0000	1.0000	1.0000
DMU 10	0.6500	0.7200	0.6900	0.9700	0.3132

In this table we show impossible examples with the “...”.

## CONCLUSIONS

A discussed in DEA literature, DMUs can take a three-stage structure with intermediate measures connecting the stages, and acting as outputs from the first stage and inputs to second or third stage. The common practice of separately apply the standard DEA approach to each individual stage cannot address the conflict in such three-stage processes caused by the intermediate measures. Some existing studies address such conflict from different points of view, including Kao and Huang (2008), Chen et al. (2009), and Liang et al.(2008). This paper introduces the Nash bargaining game model as a way of dealing with the conflict arising from the intermediate measures, and presents an alternative approach to evaluate the efficiency scores for three- stage and the overall process. The goal is not the best system performance during the negotiation but the goal is finding the most efficiency during the negotiation.

## ACKNOWLEDGEMENT

The authors thank the Industrial Management Institute for their data and valuable suggestions and comments.

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# ENVIRONMENTAL EFFICIENCY OF MARINE CAGE LOBSTER AQUACULTURE IN VIETNAM

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## ABSTRACT

*Marine cage lobster cultivation in Vietnam has been facing a negative feedback on productivity in recent years of the overuse of nutrient inputs. This study employed a bootstrap DEA to measure to which extent the nutrient release can be reduced while producing the current level of output and a truncated regression to identify the determinants of the environmental efficiency. Cross sectional data of 353 marine cage lobster farms in Vietnam was used. Farms were grouped into spiny lobster, green lobster and mixed cultivation (both type of lobster). The mean of the environmental efficiency built on the materials balance principle and DEA cost minimization, for those three groups was 0.383, 0.379 and 0.636, respectively. The result of bootstrap truncated regression showed the statistical significant effect of age, volume of cultivating area and distance from the nearest farm on environmental efficiency for the green and mixed cultivation groups. Education and movement have effects on spiny and mixed cultivation but in opposite direction. The cage cleaning frequency during the cultivating period has a positive effect on the spiny group but a negative effect on the green lobster farms.*

**Keywords:** lobster aquaculture, materials balance model, bootstrap DEA, environmental efficiency

## INTRODUCTION

Aquaculture has been one of the fastest-growing sectors over the world in the past decades. However, it also has been criticized in causing adverse environmental impacts. Since the 1990s the typical examples of environmental harmful aquaculture are salmon farming (Asche & Tveteras, 2005) and shrimp farming (Boyd & Clay, 1998 ; Martinez-cordero & Leung, 2004 ; Naylor et al., 2000). Marine cage lobster aquaculture in Vietnam has also faced a similar problem since 2000.

Because in Vietnam cage lobster is exclusively fed on fish by-catch, the cultivation has been confronted with a number of environmental problems in the form of nutrient surpluses (Hung, Khuong, Phuoc, & Thao, 2010 ; Lee, Hartstein, & Jeffs, 2015a, 2015b ; Wu, 1995). The organic waste from faeces and uneaten feed, which accounted one third of the total amount feeding (Tuan, 2011), can degrade marine water quality, damage the local fauna and increase nutrient concentrations in the water (Asche & Tveteras, 2005 ; Minh et al., 2015). Apart from the effect on the environment, the organic waste sedimentation also causes negative feedback effects on lobster productivity due to lowering the resistance

to diseases (Asche & Tveteras, 2005 ; Minh et al., 2015). As a consequence there are frequent lobster disease outbreaks and decreases in production (Minh et al., 2015).

To pursue sustainable development in marine cage lobster aquaculture in Vietnam a lot of attention went to finding methods to reduce nitrogen surplus by for instance poly-cultivating lobster with other species of aquatic animal (Chien, 2005 ; Du, Hoang, Du, & Thi, 2004 ; Vinh & Huong, 2009) or by using manufactured feed pellets instead of fish by-catch (Irvin & Williams, 2009 ; Smith, Irvin, & Mann, 2009 ; Tuan, 2012). Other studies simply tried estimating the nitrogen loading to the marine environment (An & Tuan, 2012 ; Chien, 2005 ; Thao, 2012). However this seems to be meaningless because the environmental consequences of the inputs were still unspecified (Brooks, 2007). Ly (2009) used not only the mass balance model to measure the nitrogen release but also the treatment cost, productivity change and a cost benefit analysis to assess environmental impacts of growing out lobster in this country. However, it seems not meet the need of accurate information of those methods and be difficult to find the relevant benchmarks in comparative farm analysis, thereby identifying room for improvement. No study focused on how environmentally efficient the lobster farms in Vietnam are or which factors affect their environmental efficiency and to which extent the nutrient losses can be reduced while maintaining the current level of output. This study, therefore, aims to estimate the environmental efficiency of marine cage lobster aquaculture in Vietnam and to identify its determinants using bootstrap data envelopment analysis (DEA) and truncated regression.

## METHODS

### Materials Balance Principle (MBP) and Data envelopment analysis (DEA)

The Material Balance Principle is the rule of “what goes in must come out” of mass conservation. It regulates the transformation of materials in closed systems such as agricultural production. The MBP implies that the balance of nutrient equals the nutrient in inputs minus the nutrient in output.

Consider the case of  $n$  farms or decision making units (DMUs). Each farm uses  $K$  inputs ( $x$ ) to produce  $M$  conventional outputs ( $y$ ). Moreover, this production also produces an emission of polluting substance ( $z$ ).

The amount of emission is defined by the balance of nutrients:  $Z = a'x - b'y$

Where  $a$  and  $b$  are nutrient content in inputs and outputs. It is possible that some inputs could almost have zero amounts of nutrients. And the vectors  $a$  of those inputs may include zero values.

The nutrient from producing lobster released to marine environment, which is calculated by MBP, was considered as potential damage or pollution in this study. This pollution will be least with the minimum nutrient balances. When the output  $y$  is constant, the nutrient balances will be minimized if the nutrient content in input  $x$  is minimized. Therefore, the pollution minimization is defined in the same manner as cost minimization using input-oriented data envelopment analysis model, where the nutrient content is treated as an input price.  $N(y, a) = \min\{a'x(x, y) \in T\}$

The input vector that gives the minimum nutrient content is called  $x_{EE}$ . The minimum nutrient content can be equal to  $a'x_{EE}$ . The nutrient content of the observed input vector is  $a'x$ . Then the input-orientated

environmental efficiency (EE) is defined as the ratio of minimum nutrient content over observed nutrient content.  $EE = \alpha' x_{EE} / \alpha' x$

Input-orientated technical efficiency (TE) is defined as:  $TE(y, x) = \min_{\theta} \{ \theta | (\theta x, y) \in T \}$

Where  $\theta$  is the technical efficiency (TE) score having values ranging from zero to one ( $0 \leq \theta \leq 1$ ). If the input vector at which the farm is technically efficient is  $x_{TE}$ , the technical efficiency can be re-written as:  $TE = \theta = \frac{x_{TE}}{x} = \alpha' x_{TE} / \alpha' x$

The input orientated environmental efficiency can be decomposed into technical efficiency and environmentally allocative efficiency as follow:

$$EE = \frac{\alpha' x_{EE}}{\alpha' x} = \frac{\alpha' x_{EE}}{\alpha' x_{TE}} \times \frac{\alpha' x_{TE}}{\alpha' x} = EAE \times TE$$

Where environmentally allocative efficiency is:  $EAE = \alpha' x_{EE} / \alpha' x_{TE}$

#### Determinants of environmental efficiency

The determinants of the environmental efficiency of marine cage lobster farms in Vietnam were identified using bootstrapped truncated regression (Algorithm 2) (Simar & Wilson, 2007):

$$\bar{\delta}_i = \alpha + Z_i \beta + \varepsilon_i, i = 1, \dots, n$$

Where  $\bar{\delta}_i$  is the reciprocal of the bias-corrected DEA environmental efficiency scores,  $\alpha$  is constant term,  $\beta$  is a vector of parameters,  $Z_i$  is a vector of specific variables and  $\varepsilon_i$  is the statistical noise,  $\varepsilon_i \sim N(0, \sigma_\varepsilon^2)$  with left truncation at  $1 - Z_i \beta$ .

#### Data and variables

Data was collected from marine cage lobster farms in Phu Yen and Khanh Hoa provinces from August to November 2016. Those two provinces were selected as primary sampling units because they account for more than 94% of the total lobster cages in Vietnam (Minh et al., 2015 ; Petersen & Phuong, 2010). In total 361 farmers were interviewed using a structured questionnaire, which was designed based on the result of expert interviews in July 2016. This covers about 4% of the estimated marine cage lobster farms in the study area. 8 farms were found to be outliers and removed to avoid sensitivity of the DEA approach. The final sample of 353 farms was used in this study. Based on the type of lobster cultivated, the samples were found to be grouped into 150 spiny lobster farms, 166 green lobster farms and 37 mixed cultivation farms. Mixed cultivation means that both types of lobster (spiny and green lobster) are cultivated in the farm, but in different cages. The information of nutrient content in the inputs was based on the study of Chien (2005).

In order to identify the determinants of the environmental efficiency the reciprocal of the bias-corrected DEA environmental efficiency scores were regressed on a set of variables in a truncated regression



model. The description of the variables used in DEA and in the bootstrap Truncated regression model are shown in Table 1.

**Table 1: Description of the variables in DEA and bootstrap Truncated regression model**

Variables	Description	Unit
<b>DEA model</b>		
<i>Outputs</i>	Total quantity of spiny (green) lobster produced	Kilogram
<i>Inputs</i>		
Fingerling	Spiny (green) fingerling cultivated per production cycle	Unit
Feed	Total quantity of trash fish for feed per production cycle	Kilogram
Labor	Total working hours used per production cycle	Man-hours
<b>Truncated regression</b>		
<i>Dependent variable</i>	Reciprocal of bias corrected DEA environmental efficiency scores	
<i>Independent variables</i>		
<i>Farmer's characteristics</i>		
Age	Represents age of lobster farmer	Year
Education	Level of education of lobster farmer	Grade
Household size	Number of members in the household	Number
<i>Production characteristics</i>		
Cultivating period	Time period the farmer had cultured lobster at the current location up to present	Year
Movement	Dummy taking 1 if the farmer used to shift their lobster cultivation to another area and 0 if they did not	Dummy
Volume	Total volume of cultivating area	M <sup>3</sup>
<i>Production environment</i>		
Cleaning	Cage cleaning times during the cultivating period	Times
From the nearest	The distance from the current farm to the nearest farm	Meter
From the coast	The distance from the current farm to the coast	Kilometer
Other discharge	existence of other production which close to the current farm discharge to marine environment (1 = there was, and 0 = otherwise)	Dummy

## RESULTS AND DISCUSSIONS

The results in Table 2 shows large ranges in technical, environmentally allocative and environmental efficiency of all three groups. The mean environmental efficiency of spiny lobster and green lobster groups were almost the same with 0.383, 0.379, respectively while it was 0.636 for mixed cultivation group. This implies that these farms, especially spiny and green lobster groups, were producing substantially environmentally inefficient. Compared to the best practice, these farms should be able to produce as much lobster with an input bundle that contains 61.7%, 62.1% less nutrient respectively. For mixed cultivation farms this is 36.4%. This reduction would mean that less pollution is released or less potential damage is caused to the marine environment. The Kruskal-Wallis test in Table 2 for the difference in efficiency measures also shows statistical significant differences in all technical, environmentally allocative and environmental efficiency among three groups.

**Table 2: Efficiency scores under VRS with DEA bootstrap**

	Spiny lobster			Green lobster			Mixed cultivation			P-value of Kruskal-Wallis
	Mean	Min	Max	Mean	Min	Max	Mean	Min	Max	
Bias corrected										
TE	0.7564	0.4355	0.9186	0.6661	0.2679	0.9154	0.8961	0.6276	0.9472	< 2.2e-16***
EAE	0.5093	0.1421	1.0110	0.5698	0.2122	1.0230	0.7040	0.2739	1.0020	1.087e-06***
EE	0.3831	0.0919	0.8855	0.3794	0.0757	0.8210	0.6361	0.2016	0.9181	2.836e-09***
Lower bound										
TE	0.7064	0.4057	0.8832	0.5989	0.2536	0.8882	0.8463	0.6103	0.9098	
EAE	0.4936	0.1370	1.0730	0.4898	0.1547	1.0600	0.6976	0.2657	1.0060	

EE	0.3436	0.0858	0.8136	0.2863	0.0522	0.6958	0.5940	0.1890	0.8528
Upper bound									
TE	0.8256	0.4879	1.1850	0.7521	0.2861	1.1490	0.9986	0.6549	1.1660
EAE	0.5255	0.1487	1.0330	0.6363	0.2489	1.0540	0.7153	0.3002	1.0000
EE	0.4358	0.1001	1.1900	0.4839	0.0954	1.1640	0.7266	0.2303	1.1430

The results in Table 3 show that there was a statistically significant positive effects of age and volume of cultivating area but a negative effect of distance from the nearest farm on the environmental efficiency of green and mixed cultivation farms. The farmers with higher level of education and those who used to shift their farms to another area were more environmentally efficient for mixed cultivation but less efficient for the spiny lobster group. An increase in cage cleaning frequency during the cultivating period improves the environmental efficiency of spiny lobster farms but reduces it for green lobster farms. Moreover, the spiny lobster farms which weres far from the coast and not close to other discharges to the marine environment (excluding lobster production) were more efficient.

**Table 3: Bootstrap Truncated regression with  $\alpha=0.05$**

	Spiny lobster	Green lobster	Mixed cultivation
Intercept	3.475605e+00*** (1.3688045531 5.698621053)	1.649250939 (-0.401937794 3.5945038990)	10.469988900*** (6.23787823 15.855628228)
<b>Farmer's characteristics</b>			
Age	3.145844e-02 (-0.0031351893 0.065711622)	-0.044546912** (-0.085166615 -0.0065622810)	-0.063701128* (-0.16148744 0.002391057)
Education	8.122287e-02* (-0.0212257647 0.184301725)	0.006908474 (-0.096917842 0.1146060145)	-0.484427939*** (-0.80035518 -0.261986961)
Household size	-2.000559e-01 (-0.4810763016 0.086724269)	0.082863584 (-0.158110403 0.3251277663)	-0.457163307* (-0.96810533 0.038310780)
<b>Production characteristics</b>			
Cultivating period	2.849439e-03 (-0.0835108263 0.088191248)	0.030553485 (-0.032785051 0.0986035590)	0.126434423 (-0.05325549 0.323117585)
Movement	7.520733e-01*** (0.2015415498 1.332660231)	-0.589526381 (-1.684484596 0.3810496008)	-2.122990734* (-5.86312999 0.202767824)
Volume	8.363636e-06 (-0.0002679257 0.000286717)	-0.001632062* (-0.003784246 0.0001861504)	-0.012023244*** (-0.02084506 -0.006428133)
<b>Production environment</b>			
Cleaning	-6.748289e-03*** (-0.0104306246 -0.03346568)	0.005837206 *** (0.002009499 0.0102984394)	-0.009503529 (-0.02338617 0.003185516)
From the nearest	-2.880053e-03 (-0.0098723634 0.002966766)	0.036446978** (0.009197293 0.0665204826)	0.122993209*** (0.05162746 0.221413441)
From the coast	-5.804806e-01*** (-0.7707863551 -0.415247739)	0.051547634 (-0.414800913 0.4746297514)	-0.433705411 (-1.50605110 0.600739658)
Other discharge	8.589097e-01** (0.0000645983 1.742553545)	-0.517009414 (-1.992638467 0.6072494740)	-1.741870900 (-5.10255048 0.333468701)

## CONCLUSIONS

This study shows that the marine cage lobster farms in the study area are substantially environmental inefficient. Moreover, the results of the bootstrap truncated regression in the second stage might be useful for local authorities and extension services to improve the environmental efficiency and thereby improve the marine water quality. This study was also useful for further study in identifying the trade-off between being environmentally efficient and cost efficient.

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# ESTIMATING RETURNS TO SCALE OF TWO-STAGE NETWORK PROCESSES

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## ABSTRACT

*Network data envelopment analysis (NDEA) deals with evaluating the performance of a set of homogeneous decision-making units (DMUs) taking into account the internal structure of DMUs. A large number of studies in NDEA are based on two-stage structures. In this context, the methodology was offered by Fukuyama and Mirdehghan (2012) [Fukuyama, H., & Mirdehghan, S.M. (2012). Identifying the efficiency status in network DEA. European Journal of Operational Research, 220, 58–92] for Identifying the efficiency status of network efficient DMUs. The current research with the aim of proposing a DEA approach to determine right- and left-hand RTS (returns to scale) of the identified network efficient DMUs. Finally, a numerical example is provided to illustrate the proposed approach.*

**Keywords:** Data envelopment analysis (DEA); Network DEA; Returns to scale (RTS); Efficiency

## INTRODUCTION

Returns to scale (RTS) is one of the important issues in data envelopment analysis (DEA). Because, it provides useful information about the optimal size of decision making units (DMUs) (Banker et al., 2004). Hitherto, there are many attempts on RTS of DMUs but all of them consider the production process as a “black box” and ignore the sub-processes or the intermediate products. However, in the real world, the production processes (or DMUs) include the intermediate products which they should be considered for determining RTS. Hence, in this study, we propose a DEA approach to estimate right- and left-hand RTS of the network efficient DMUs (Fukuyama & Mirdehghan, 2012) with two sub-processes. Also, a numerical example is presented to illustrate our proposed approach.

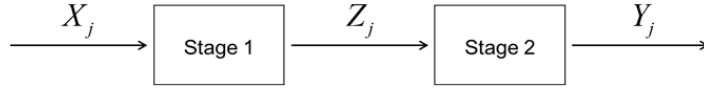
### The proposed approach

Suppose that there is a set of  $n$  DMUs ( $DMU_j; j = 1, \dots, n$ ) which each one includes two-stage structure as represented in Figure 1. This figure indicates  $DMU_j$  with input vector  $X_j = (x_{1j}, \dots, x_{mj})^t \in \mathbb{R}_+^m$ , intermediate products  $Z_j = (z_{1j}, \dots, z_{qj})^t \in \mathbb{R}_+^q$ , and output vector  $Y_j = (y_{1j}, \dots, y_{sj})^t \in \mathbb{R}_+^s$ . Note that the superscript “ $t$ ” represents a vector transport.

Assume that  $T_1$  and  $T_2$  denote the production possibility set (PPS) of Stages 1 and 2, respectively, which they are defined as below:

$$T_1 = \left\{ \begin{pmatrix} \mathbf{x} \\ \mathbf{z} \end{pmatrix} \mid \sum_{j=1}^n \lambda_j^1 \mathbf{x}_j \leq \mathbf{x}, \sum_{j=1}^n \lambda_j^1 \mathbf{z}_j \geq \mathbf{z}, \sum_{j=1}^n \lambda_j^1 = 1, \lambda_j^1 \geq 0; j = 1, \dots, n \right\},$$

$$T_2 = \left\{ \begin{pmatrix} \mathbf{z} \\ \mathbf{y} \end{pmatrix} \mid \sum_{j=1}^n \lambda_j^2 \mathbf{z}_j \leq \mathbf{z}, \sum_{j=1}^n \lambda_j^2 \mathbf{y}_j \geq \mathbf{y}, \sum_{j=1}^n \lambda_j^2 = 1, \lambda_j^2 \geq 0; j = 1, \dots, n \right\}.$$



**Figure 1: Two-stage production process**

First, we specify the network efficient DMUs by using Fukuyama and Mirdehghan's approach (Fukuyama & Mirdehghan, 2012). Then, right- and left-hand RTS of these DMUs are determined as will be presented in Sub-sections 2.1 and 2.2.

#### Right-hand RTS

In this section, we introduce the following DEA model to estimate right-hand RTS of the network efficient  $DMU_o$  ( $o \in \{1, \dots, n\}$ ) and its Stages 1 and 2.

$$\begin{aligned} \max \quad & \beta_o^1 + \beta_o^2 \\ \text{s. t.} \quad & \sum_{j=1}^n \lambda_j^1 x_{ij} \leq (1 + \delta)x_{io}, \quad i = 1, \dots, m, \\ & \sum_{j=1}^n \lambda_j^1 z_{hj} \geq \beta_o^1 z_{ho}, \quad h = 1, \dots, q, \\ & \sum_{j=1}^n \lambda_j^2 z_{hj} \leq \beta_o^1 z_{ho}, \quad h = 1, \dots, q, \\ & \sum_{j=1}^n \lambda_j^2 y_{rj} \geq \beta_o^2 y_{ro}, \quad r = 1, \dots, s, \\ & \sum_{j=1}^n \lambda_j^1 = 1, \\ & \sum_{j=1}^n \lambda_j^2 = 1, \\ & \lambda_j^1, \lambda_j^2 \geq 0, \quad j = 1, \dots, n, \end{aligned} \tag{1}$$

where  $\delta$  is a small positive parameter. It is obvious that model (1) is feasible. Assume that  $\beta_o^{1*}$  and  $\beta_o^{2*}$  are the obtained optimal values from solving model (1). Then, we have:

**Table 1: The results of right-hand RTS**

Right-hand RTS of Stage 1 of $DMU_o$	Right-hand RTS of Stage 2 of $DMU_o$	Right-hand RTS of $DMU_o$
--------------------------------------	--------------------------------------	---------------------------

If $(1 + \delta) < \beta_o^{1*}$ then IRS <sup>1</sup> .	If $\beta_o^{1*} < \beta_o^{2*}$ then IRS.	IRS
	If $\beta_o^{1*} = \beta_o^{2*}$ then CRS.	IRS
	If $\beta_o^{1*} > \beta_o^{2*}$ then DRS.	IRS or CRS or DRS
If $(1 + \delta) = \beta_o^{1*}$ then CRS <sup>2</sup> .	If $\beta_o^{1*} < \beta_o^{2*}$ then IRS.	IRS
	If $\beta_o^{1*} = \beta_o^{2*}$ then CRS.	CRS
	If $\beta_o^{1*} > \beta_o^{2*}$ then DRS.	DRS
If $(1 + \delta) > \beta_o^{1*}$ then DRS <sup>3</sup> .	If $\beta_o^{1*} < \beta_o^{2*}$ then IRS.	IRS or CRS or DRS
	If $\beta_o^{1*} = \beta_o^{2*}$ then CRS.	DRS
	If $\beta_o^{1*} > \beta_o^{2*}$ then DRS.	DRS

<sup>1</sup> Increasing returns to scale. <sup>2</sup> Constant returns to scale. <sup>3</sup> Decreasing returns to scale.

**Remark 1.** Since  $DMU_o$  is network efficient, so  $\beta_o^{1*} \geq 1$  and  $\beta_o^{2*} \geq 1$ .

#### Left-hand RTS

To estimate left-hand RTS of the network efficient  $DMU_o$  and its Stages 1 and 2, we present the following DEA model:

$$\begin{aligned}
\min \quad & \alpha_o^1 + \alpha_o^2 \\
\text{s. t.} \quad & \sum_{j=1}^n \lambda_j^1 x_{ij} \leq \alpha_o^1 x_{io}, \quad i = 1, \dots, m, \\
& \sum_{j=1}^n \lambda_j^1 z_{hj} \geq \alpha_o^2 z_{ho}, \quad h = 1, \dots, q, \\
& \sum_{j=1}^n \lambda_j^2 z_{hj} \leq \alpha_o^2 z_{ho}, \quad h = 1, \dots, q, \\
& \sum_{j=1}^n \lambda_j^2 y_{rj} \geq (1 - \eta) y_{ro}, \quad r = 1, \dots, s, \\
& \sum_{j=1}^n \lambda_j^1 = 1, \\
& \sum_{j=1}^n \lambda_j^2 = 1, \\
& \lambda_j^1, \lambda_j^2 \geq 0, \quad j = 1, \dots, n,
\end{aligned} \tag{2}$$

where  $\eta$  is a small positive parameter. It is clear that model (2) is feasible model. Suppose that  $\alpha_o^{1*}$  and  $\alpha_o^{2*}$  are the obtained optimal values from solving model (2). Then, we have:

**Table 2: The results of left-hand RTS**

Left-hand RTS of Stage 1 of $DMU_o$	Left-hand RTS of Stage 2 of $DMU_o$	Left-hand RTS of $DMU_o$
If $\alpha_o^{1*} > \alpha_o^{2*}$ then IRS.	If $\alpha_o^{2*} < (1 - \eta)$ then DRS.	IRS or CRS or DRS
	If $\alpha_o^{2*} = (1 - \eta)$ then CRS.	IRS

	If $\alpha_o^{2*} > (1 - \eta)$ then IRS.	IRS
If $\alpha_o^{1*} = \alpha_o^{2*}$ then CRS.	If $\alpha_o^{2*} < (1 - \eta)$ then DRS.	DRS
	If $\alpha_o^{2*} = (1 - \eta)$ then CRS.	CRS
	If $\alpha_o^{2*} > (1 - \eta)$ then IRS.	IRS
If $\alpha_o^{1*} < \alpha_o^{2*}$ then DRS.	If $\alpha_o^{2*} < (1 - \eta)$ then DRS.	DRS
	If $\alpha_o^{2*} = (1 - \eta)$ then CRS.	DRS
	If $\alpha_o^{2*} > (1 - \eta)$ then IRS.	IRS or CRS or DRS

**Remark 2.** Due to  $DMU_o$  is network efficient, so  $\alpha_o^{1*} \leq 1$  and  $\alpha_o^{2*} \leq 1$ .

### NUMERICAL EXAMPLE

Consider five hypothetical production processes (or DMUs) with one input ( $x$ ), one intermediate product ( $z$ ), and one output ( $y$ ) as shown in Table 3. By using the Fukuyama and Mirdehghan's approach (Fukuyama & Mirdehghan, 2012), it is deduced that DMUs 1 and 2 are network efficient and DMUs 3, 4, and 5 are not network efficient.

**Table 3: DMUs' data**

	DMU <sub>1</sub>	DMU <sub>2</sub>	DMU <sub>3</sub>	DMU <sub>4</sub>	DMU <sub>5</sub>
$x$	2	2.5	3	5	5
$y$	2.25	3.125	4	5	3
$z$	2.25	3.125	3.125	3.125	1

Now, we apply models (1) and (2) to determine right- and left-hand RTS of DMUs 1 and 2 and their Stages 1 and 2. Tables 4 and 5 show the obtained results from solving models (1) and (2).

**Table 4: The results of right-hand RTS, ( $\delta = 0.001$ )**

DMU	$\beta_o^{1*}$	$\beta_o^{2*}$	Stage 1	Stage 2	The entire system
1	1.0016	1.0016	IRS	CRS	IRS
2	1.0014	1	IRS	DRS	DRS

**Table 5: The results of left-hand RTS, ( $\eta = 0.001$ )**

DMU	$\alpha_o^{1*}$	$\alpha_o^{2*}$	Stage 1	Stage 2	The entire system
1	1	1	CRS	IRS	IRS
2	0.9993	0.9990	IRS	CRS	IRS

As presented in Table 4, since  $1 + \delta = 1.001 < \beta_o^{1*} = \beta_o^{2*} = 1.0016$ , so Stages 1 and 2 of  $DMU_1$  have increasing and constant right-hand RTS, respectively. Hence, the entire system (or  $DMU_1$ ) has increasing right-hand RTS. Moreover, left-hand RTS of Stages 1 and 2 of  $DMU_1$  are respectively constant and increasing because, as seen in Table 5,  $1 - \eta = 0.999 < \alpha_o^{1*} = \alpha_o^{2*} = 1$ . Therefore,  $DMU_1$  has increasing left-hand RTS. Right- and left-hand RTS of Stages 1 and 2 of  $DMU_2$  can be analyzed, similarly.

## CONCLUSIONS

In this research, we propose a DEA approach to estimate right- and left-hand RTS of the network efficient DMUs with two-stage structures. It is necessary to mention that this work can be extended to the production processes (or DMUs) with more than two-stage structures.

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# EVALUATING THE PERFORMANCE OF COMPANIES LISTED IN TEHRAN STOCK EXCHANGE BASED ON FINANCIAL RATIOS USING DEA (THE CASE OF CHEMICAL & MEDICAL COMPANY)

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## ABSTRACT

*Many Chemical & Medical companies embark on process improvement initiatives, but lack evidence that these programs result in high efficiency. Therefore, the purpose of this study is to examine the performance of chemical companies listed in Tehran Stock. The study uses Data Envelopment Analysis (DEA) to understand Chemical co. efficiency and effectiveness a survey to understand the Chemical process improvement implementation. To measure the performance of companies, the total assets, equity, and ADS was used as inputs and outputs consist of NPM, ROE, and ROA. In the course of this research, formulation of hypotheses and systematic removal of 32 companies by stock exchange, the 5-year period 2011- 2015 were selected with reference to their financial statements, the information necessary to measure the variables, extraction and statistical tests on they were carried out. The results of the DEA indicate the existence of different degrees of financial ratios in Medical and Chemical Companies. When by Anderson Peterson model, companies can be ranked and categorized into three groups: strangely super-efficient, supper-efficient and efficient.*

**Keywords:** Performance Evaluation, Financial Ratios, Tehran Stock Exchange, DEA Technique, Chemical & Medical Companies,

## INTRODUCTION

Many studies have been conducted to achieve a proper criterion to assess the performance of companies and managers in order to ensure the consistency of a company with the interests of the actual investors and make a basis to make economic decisions of potential investors and creditors. The results obtained from these studies have provided four approaches in relation to the performance criteria (Abzari, et. Al, 2013). These consist of the following approaches:

1. The accounting approach: In this approach, the figures contained in the financial statements are used, including profit, earnings per share, operating cash flow, return on asset and return on equity to assess the performance (Yahyazadeh, et. Al., 2014).
2. The economic approach: According to this approach in which economic concepts are used, the performance of the business unit is assessed with a focus on the profitability power of the company's assets and according to the rate of return and the rate of cost of capital used, the economic value added, adjusted EVA and MVA are in this group.
3. The integrated approach: In this approach, a combination of accounting and market information is used to assess the performance such as Tobin's Q ratio and price to earnings ratio.
- 4- The financial management approach: In accordance with this approach, most of the theories of financial management are used such as capital asset pricing model and risk and return concepts. The main emphasis of this approach is to determine the excess return per share (Noravesh & Hidari, 2015).

In a general classification, the criteria of measuring performance can be divided into two groups of financial and non-financial criteria. Non-financial criteria include the criteria for the production, marketing, administration and social criteria and financial ratios are the techniques that have been suggested as financial criteria. Some finance researchers have suggested that composite indices (financial and non-financial) should be used. However, this is difficult because determining the type of the criteria and their correlation as well as determining the value and weight of each of them as a total is not a simple task (Yang & Huang, 2015).

The financial ratios indicate the strength or weakness of the companies compared with other companies in the same industry, leading companies and last year's performance of the same company (Malhotra. & Malhotra R. 2008). Financial ratios are calculated simply, while their interpretation is often difficult and controversial, especially when two or more ratios show the conflicting signs to each. The main problem of the financial statements' ratio analysis is that each of financial ratios assess one aspect of the financial performance of an organization so that a group assesses the ability of liquidity, a group assesses profitability, another part assesses the ability to grow and finally the last group assesses on the practice of organization's operation (Athanassopoulos & Ballantine 1995) .

Although the importance of organizational performance is widely known, but there is a significant discussion about technical and conceptual issues of the performance measurement. Measure the performance individually and as is clear from all directions is not possible. The organization's performance is a wide combination of intangible earnings, such as increasing the organizational knowledge and objective and tangible receipts, such as the economic and financial results. Among objective indices of organizational performance, profitability indices such as return on asset, return on equity, return on investment, return on equity and earnings per share can be noted. The indices are also traditional indices of the performance. Subjective criteria of organizational performance include the indices that are formed based on the judgment of the beneficiary groups (Moradzadehfard, et. Al, 2012).

The present study proposes the data envelopment analysis technique method to solve this problem is. This method with ratios' aggregation allocates a single score named "efficiency" to each studied unit. In fact, this technique by entering ratios as the model input and output turns them to a single criterion that increases the ability to assess and compare the performance. Tehran Stock Exchange is one of the most influential organizations on the development of decision-making, policy-making, decision-making, production, and support of manufacturers. Therefore, a study for the assessment of the listed companies in it is very important. Hence, the question rises that how is the efficiency of pharmaceutical and chemical companies listed in the Tehran Stock Exchange in terms of DEA technique?

## **THEORETICAL FRAMEWORK**

The main key to achieve the goal is to assess the performance. An efficient assessment system to identify the executive strengths and weaknesses is the current status of the organizations and the realization of programs and achieving goals required (Khorasani, 2011). Control and performance measurement systems are official information-based procedures and matters that are used by managers to protect and

reform the organization's activities. One of the reasons of the failure of managers' efforts who are interested in improving their organization's financial performance is the lack of an appropriate tool to assess the performance of organizations. A set of new economic conditions, existing changes in the new patterns of management and administration of the organizations and the inefficiency of traditional methods of assessment have created the need for the change and development of the criteria for measuring the financial performance, the profitability of a unit is closely related to the amount of investment and none of the traditional methods pay attention to the amount invested. A summary of views of some researchers in the field of the limitations of traditional profit-based criteria, including Stewart (1991), Chen and Dodd (2001), Beak and Kmi (2002), Hendriksen and Berda (1992), and Biddle et al. (2010) has been classified as below:

1. The ability to manipulate a lot due to the commitment of using different accounting methods and different use of standards
2. Relying on the limiting principles and methods such as the principle of conservatism and accrual method
3. The lack of a prospective vision and ignoring the factors such as the advances in the technology and new production technologies, the innovation of new products, the time value of money and etc.
4. Inattention to the value creation factors such as the intellectual capital and intangible asset
5. Inattention to the actions of the cost of financing through the equity (Cheng et al ,2017) .

Melnyk et al. (2004) shows that the criteria and measurements of the performance are obtained from the high attention to the last years, but according to Evans' (2004) opinion the analysts for the analysis of the performance results in terms of competitive comparison and criteria among companies active at the industry level in the capital market need a more efficient method. On the other hand, it has been reported that the criteria based on financial indices when used in a dynamic environment to assess the business and industry performance have disadvantages as well (Atkinson et al. 1997). Financial experts over many years have developed various models for the valuation of the companies, these models can be very simple or very complex and advanced. Among the models, several model have been tested that according to the research conducted in other world stock markets have the most efficiency. These models include: Gordon, two-stage dividend discount, the discounted free cash flow payable to shareholders, adjusted present value, the price to earnings ratio and the discount model of Residual Income (Sorensen and Williamson, 2015). Also, Melnyk et al. (2004) in another classification performance assessment criteria are divided into five different approaches applied as follows: DEA, hierarchical process, gray relationships' analysis, balanced scorecard and the analysis of financial statements (Abzari, et al. 2013 & Saiedi 2015).To measure the efficiency of decision-making units (DMUs), Data Envelopment Analysis (DEA) is often used. Charnes, Cooper, and Rhodes (1978) introduced it more than 40 years ago when they developed their CCR model, which remodeled the fractional linear measure of efficiency into linear programming.

DEA is a mathematical programming model to assess the efficiency of decision-making units that have multiple inputs and outputs. Karaman et al. (2016) pointed out, using data envelopment analysis model to assess the units relatively needs to determine two basic characteristics, the nature of the pattern and

returns to the pattern scale. (A) The nature of the input, if in the assessment process with constant output level trying to minimize the inputs is the used input pattern nature. (B) The nature of the output, if in the assessment process with constant input level, output level, trying to increase the output level, is used output pattern nature.

## **LITERATURE REVIEW**

Several research have been conducted on the function of data envelopment analysis technique based on the assessment of the efficiency of the economic units, but the relationship between this technique and financial variables of the performance assessment or in other words the performance assessment based on the financial statements and the financial performance has not been examined by the technique independently. Foreign research in connection with the use of DEA in education, different industries, banks, hospitals, military and etc. have been conducted with various approaches yet in connection with the Finance recent research including Ehsan et al. (2003) entitled the analysis of financial statements have been conducted on active companies in oil and gas industry exchange using DEA approach during 1982-1992 and the results of using DEA in this study have indicated a reliable basis of practical and managerial efficiency of economic firms. The null hypothesis indicating no relationship between DEA and financial ratios was tested as a traditional performance assessment index, and the results of the null hypothesis were not confirmed i.e. DEA shows information for decision-making.

George & Tzeremes (2012) in a study entitled "Assessment of the efficiency of the industry using financial ratios: the use of automated DEA" examined the overall analysis of ratio data / financial data in order to make a way for the units' measurement. Using automated techniques, this paper has examined the use of 23 production factors in Greece using financial data. The results showed that at the first stage of our critical analysis, we had the applications obtained on the basis of the baseline. After using automated techniques the critical analysis showed that the efficiency components have been significantly improved.

Mahmoudi and Metan (2011) using the technique of DEA and financial variables of the performance assessment such as return on investment and corporate risk grade in metal industries' group companies listed in the Tehran Stock Exchange, between 2003 and 2008 assessed the performance and then using multi-variable regression test attempted to examine the relationship. According to the results obtained the variable of the corporate risk grade with values resulting from DEA technique a positive relationship is shown so that the corporate risk grade is a proper criterion to predict the efficiency of the economic units and introduced as an alternative criterion.

Haritha Saranga & Phani (2004) assessed the internal efficiency pharmaceutical industry companies using DEA in 44 companies that have good financial conditions in order to assess the efficiency and performance with internal growth rate and the results obtained indicate the variables' relationship.

Susumu et al. (2008) in a study assessed investment contrasting strategies using DEA technique and in addition to introducing traditional assessment criteria such as the book value to market price, considered using two traditional indices and DEA useful for the assessment.

Nikomaram et al. (2005). assessed the efficiency of investment companies listed in Tehran Stock Exchange with the help of models of mathematical benchmarking of data envelopment analysis that the researchers in this paper designed the performance assessment system to assess the efficiency of these companies, and with its help measured nineteen investor companies existing in the stock securities' exchange using mathematical models of DEA.

Khajavi et al. (2005) examined the use of DEA in determining the portfolio of the most efficient companies listed in Tehran Stock Exchange that its main purpose has been using data envelopment analysis method to determine the optimum portfolio of the most efficient companies listed in the Tehran Stock Exchange. In this study, the input nature and form envelopment have been used. The study results showed that among 90 companies studied (CCR) model, 29 companies that in fact consisted 32% of all companies were known efficient, and 61 companies were known inefficient.

Azar et al. (2007) in this study has used DEA models in order to measure the effects of the investment on IT on the efficiency of companies listed in the Tehran Stock Exchange. The results obtained suggest that DEA models are appropriate models for rating and assessing the efficiency of decision making units as well as the models of Charnes and Cooper and Rhodes (CCR) have been technically more efficient than the models of Banker and Charnes and Cooper (BCC).

Safai et al. (2007) in this study using a nonparametric approach first classified investment companies into two efficient and inefficient groups, and then using (DEA / AHP) and (CEM) methods ranked (A & P) of efficient companies.

Khajavi et al. (2010) conducted a study entitled "Data envelopment analysis technique complement to the traditional analysis of financial ratios". In this regard, the financial statements of 267 companies listed in Tehran Stock Exchange for the period 2005 to 2007 were analyzed. Financial ratios and data, 4 inputs and 7 outputs of BCC envelopment model consisted the mentioned technique input. The implementation of the mentioned model showed that among 267 surveyed companies, 32 companies had relative efficiency and 235 companies are inefficient. In fact this technique turned different financial ratios and data to a single and comparable criterion named "efficiency" and showed that DEA technique can be a good complement for the traditional analysis of the financial statements using the financial ratios.

## **METHODS**

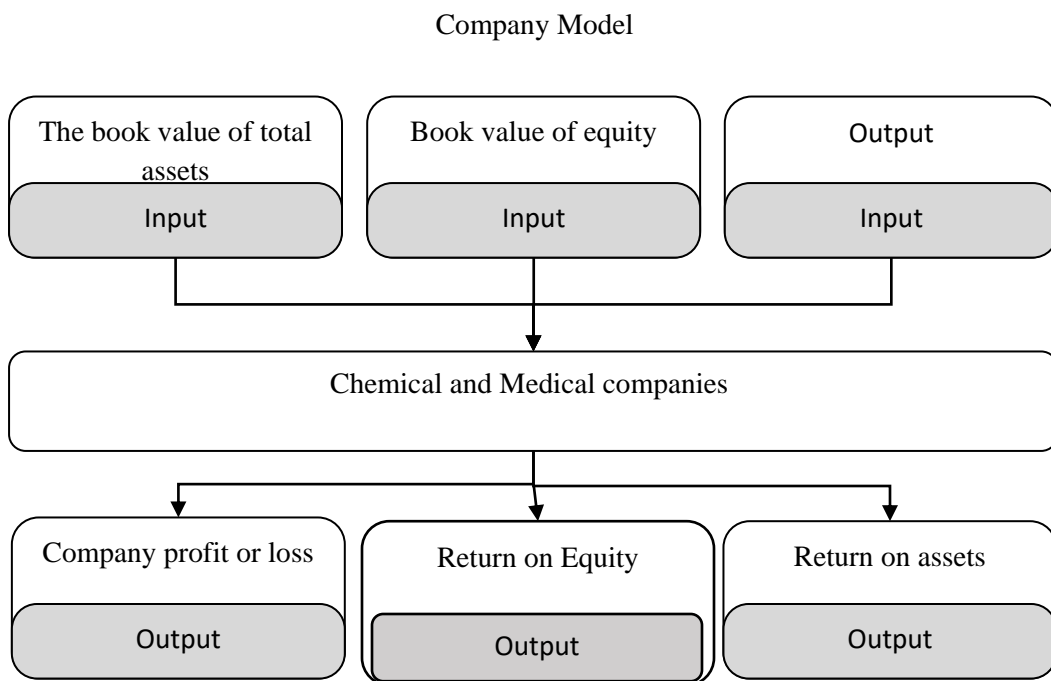
The present study in terms of the purpose is applied and in terms of the method is a library and analytical-casual study based on DEA. The study population includes all companies listed on Tehran Stock Exchange during 2011 to 2016. Due to the widespread population size and the presence of some inconsistencies between the population members, companies with the following characteristics and conditions are considered as available population. Due to limited the population of the manufacturing companies in Tehran Stock Exchange and regarding the applied conditions, the available population is selected fully as a sample in order to get the most observations for statistical analysis.

1. The study period from 2011 to 2016, trading symbol is not out of the Stock Exchange Market board
2. Companies, during the course of the financial year should not be changed its activities.

3. The required financial information, especially the notes to the financial statements is available.
4. In all the years, fiscal year has not changed
5. Every six months at least have a deal and be active and profitable

Therefore, the number of chemical and pharmaceutical companies that have these characteristics are 32 companies.

In order to identify suitable input output combination, it is crucially essential to have a clear understanding of the process being evaluated. In addition, the purpose of the performance measurement effects not only the input output selection but also the model orientation as well. The proposed study of this paper uses three inputs and three outputs for measuring the relative efficiencies of various units. Fig. 1 shows details of the proposed study.



**Figure 1: The structure of the proposed study**

## MODEL AND DATA

Afterwards, to specify overall efficient units CCR model of DEA is used (Charnes et al., 1978). Further, to identify best performing units on their scale BCC model of DEA is used (Banker et al., 1984). The used models are given below as (1) and (2) respectively.

Stage (1)

$$\max : \phi$$

$$s.t : \sum_{j=1}^n \mu_j x_{ij} \leq x_{i0} \quad , \quad i = 1, \dots, m$$

$$\sum_{j=1}^n \mu_j y_{rj} \geq \phi y_{r0} \quad , \quad r = 1, \dots, s$$

$$\sum_{j=1}^n \mu_j = 1 \quad , \quad j = 1, \dots, n$$

$$\mu_j \geq 0 \quad , \quad j = 1, \dots, n$$

$$\phi \text{ free.}$$

Stage (2)

$$\max : w = \sum_{i=1}^m S_i^- + \sum_{r=1}^s S_r^+$$

$$s.t : \sum_{j=1}^n \mu_j x_{ij} + S_i^- = x_{i0} \quad , \quad i = 1, \dots, m$$

$$\sum_{j=1}^n \mu_j y_{rj} - S_r^+ = \phi^* y_{r0} \quad , \quad r = 1, \dots, s$$

$$\sum_{j=1}^n \mu_j = 1 \quad , \quad j = 1, \dots, n$$

$$\mu_j \geq 0 \quad , \quad j = 1, \dots, n$$

$$S_i^- \geq 0 \quad , \quad i = 1, \dots, m$$

$$S_r^+ \geq 0 \quad , \quad r = 1, \dots, s.$$

#### ANDERSON-PETERSON MODEL FOR RANKING DMUS

Anderson, and Peterson (1993) proposed a method for ranking efficient units that can determine the most efficient unit. With the technique, the advantage of efficient units can be more than 1, thus efficient units also like inefficient units can be ranked. This method included 2 stages: at the 1<sup>st</sup> stage as before the efficiency is determined and after identifying the efficient units the constraint related to the efficient unit is removed from the model constraints' set until at this stage the efficiency can also be estimated more than 1.

$$\begin{aligned}
& \text{Max} \quad z = \sum_{r=1}^s y_{rj} \cdot u_r \\
& \text{s.t:} \quad \sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} \leq 0 \\
& \sum_{i=1}^n v_i x_{ij} = 1 \\
& u_r, v_i \geq 0
\end{aligned}$$

$$f(x) = a_0 + \sum_{n=1}^{\infty} \left( a_n \cos \frac{n\pi x}{L} + b_n \sin \frac{n\pi x}{L} \right)$$

## EMPIRICAL RESULTS

### The statistics and information collected from the decision maker units

In this study, 32 decision-making units were examined according to 3 inputs and 3 outputs based on data envelopment analysis collectively. The information on the indices for 32 units are shown in Table 1. In the study, a variety of descriptive statistics' parameters are expressed.

In addition, the range of change based on quartiles are shown in Table 1.

**Table 1: The range of change based on quartiles**

	Input 1	Input 2	Input 3	Output 1	Output 2	Output 3
<b>No.</b>	32	32	32	32	32	32
<b>Missed</b>	0	0	0	0	0	0
<b>Mean</b>	7429128.1	3372603.9	253295.75	1782964.1	0.6006	0.2159
<b>Variation</b>	1.091E+14	3.875E+13	1.69E+11	8.038E+12	0.0787	0.0223
<b>S.D</b>	10443492	6224754.6	411100.76	2835100.5	0.2805	0.1495
<b>Median</b>	2748953.5	890789.5	108644	541576	0.6261	0.2049
<b>Min</b>	253636	102891	16878	19051	0.0354	0.0104
<b>Max</b>	40045220	26688683	1791283	11789545	1.3574	0.7841
<b>Rang</b>	39791584	26585792	1774405	11770494	1.322	0.7737
<b>Total</b>	237732099	107923326	8105464	57054851	19.2208	6.9081
<b>skewness</b>	1.9527	2.7547	2.9032	2.1553	0.2262	1.7002
<b>Kurtosis</b>	2.8151	7.5733	8.431	4.315	0.3795	5.6324

Quarters						
<b>0%</b>	253636	102891	16878	19051	0.035447642	0.01041113
<b>25%</b>	1786163.5	440587.5	54288.75	189939.5	0.426738837	0.10903204275
<b>50%</b>	2748953.5	890789.5	108644	541576	0.6261065525	0.204893105
<b>75%</b>	4622991.5	1612655	211988.75	1228909.25	0.801923855	0.28990244575
<b>100%</b>	40045220	26688683	1791283	11789545	1.357433974	0.784066549



DEA using linear programming and optimization techniques to determine the efficiency of each unit and target enhanced efficiency for each of the units determined a reference set for the inefficient unit and compare the efficiency of different units to the efficiency border.

In input-based models, while the output is maintained at a given level, the input is reduced appropriately and possibly and on the contrary, in output-based models maintaining the appropriate input level the output is increased. "Additive model" is a model that simultaneously considers reduced input and increased output. Primary and secondary problems of additive model are as follows:

Primary Model:

$$\text{Min } Z_0 = -\sum_{r=1}^s s_r^+ - \sum_{i=1}^m s_i^-$$

St:

$$\sum_{j=1}^n \lambda_j x_{rj} - s_r^+ = y_{r0} \quad (r=1,2,\dots,s)$$

$$\sum_{j=1}^n \lambda_j x_{ij} + s_i^- = x_{i0} \quad (i=1,2,\dots,m)$$

$$\sum_{j=1}^n \lambda_j = 1 \quad (j=1,2,\dots,n)$$

$$\lambda_j, s_r^+, s_i^- \geq 0$$

Secondary Model:

$$\text{Max } Y_0 = \sum_{r=1}^s y_{r0} u_r - \sum_{i=1}^m x_{i0} v_i + w$$

St:

$$\sum_{r=1}^s y_{rj} u_r - \sum_{i=1}^m x_{ij} v_i + w \quad (j=1,2,\dots,n)$$

$$\sum_{r=1}^s u_r \geq 1$$

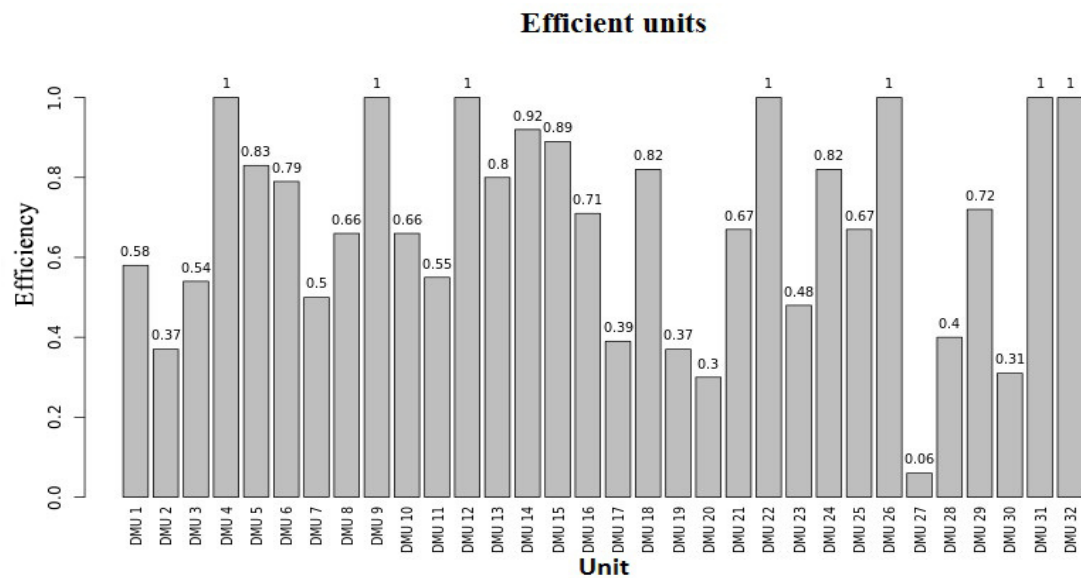
$$\sum_{i=1}^m v_i \geq 1$$

$$u_r, v_i \geq 0 \quad w \text{ Free Mark}$$

The primary model is called BCC model and the secondary model is called CCR model. Note that the value 1 indicates the efficient units. If a unit efficiency is less than 1 it is inefficient. If a unit efficiency is equal to 1 and there is no output shortage and input surplus the efficiency is Pareto. If a unit efficiency is equal to 1 and there is output shortage or input surplus the efficiency is poor. The efficiency or inefficiency of the units are given in Table 2. Figure 1 also shows the efficient and inefficient units.

**Table 2: Efficient and inefficient units.**

Unit	Score	Performance	Unit	Score	Performance
DMU 1	0.577	Inefficient	DMU 17	0.388	Inefficient
DMU 2	0.37	Inefficient	DMU 18	0.817	Inefficient
DMU 3	0.54	Inefficient	DMU 19	0.367	Inefficient
DMU 4	1	Efficient	DMU 20	0.305	Inefficient
DMU 5	0.832	Inefficient	DMU 21	0.668	Efficient
DMU 6	0.791	Inefficient	DMU 22	1	Inefficient
DMU 7	0.499	Inefficient	DMU 23	0.481	Inefficient
DMU 8	0.662	Inefficient	DMU 24	0.815	Inefficient
DMU 9	1	Efficient	DMU 25	0.673	Inefficient
DMU 10	0.665	Inefficient	DMU 26	1	Efficient
DMU 11	0.547	Inefficient	DMU 27	0.057	Inefficient
DMU 12	1	Efficient	DMU 28	0.398	Inefficient
DMU 13	0.802	Inefficient	DMU 29	0.715	Inefficient
DMU 14	0.922	Inefficient	DMU 30	0.307	Inefficient
DMU 15	0.894	Inefficient	DMU 31	1	Efficient
DMU 16	0.714	Inefficient	DMU 32	1	Efficient



**Figure 1: efficient and inefficient units.**

Rank an efficient unit based on Anderson-Peterson technique

Envelopment analysis basic models due to the lack of full ranks among the efficient units do not allow the comparison between the mentioned units easily. Because in these models the efficiency score 1 is allocated to all efficient decision maker units, the need for ranking the efficient units and maintain the inefficiency of the inefficient units is inevitable. In the assessment by AP method (Anderson-Peterson) the unit examined is removed from the assessment, and in this way the number allocated to the efficient units in AP full ranking model (Anderson-Peterson) is greater than or equal 1 as well as the efficient units are ranked. Anderson-Peterson model's primary and secondary problems are as follows:

$$\text{Min } Y_0 = \theta - \left( \sum_{r=1}^s \varepsilon s_r^+ + \sum_{i=1}^m \varepsilon s_i^- \right)$$

St:

$$\sum_{j=1}^n \lambda_j x_{rj} + s_i^- = \theta x_{ik} \quad (i=1,2,\dots,m)$$

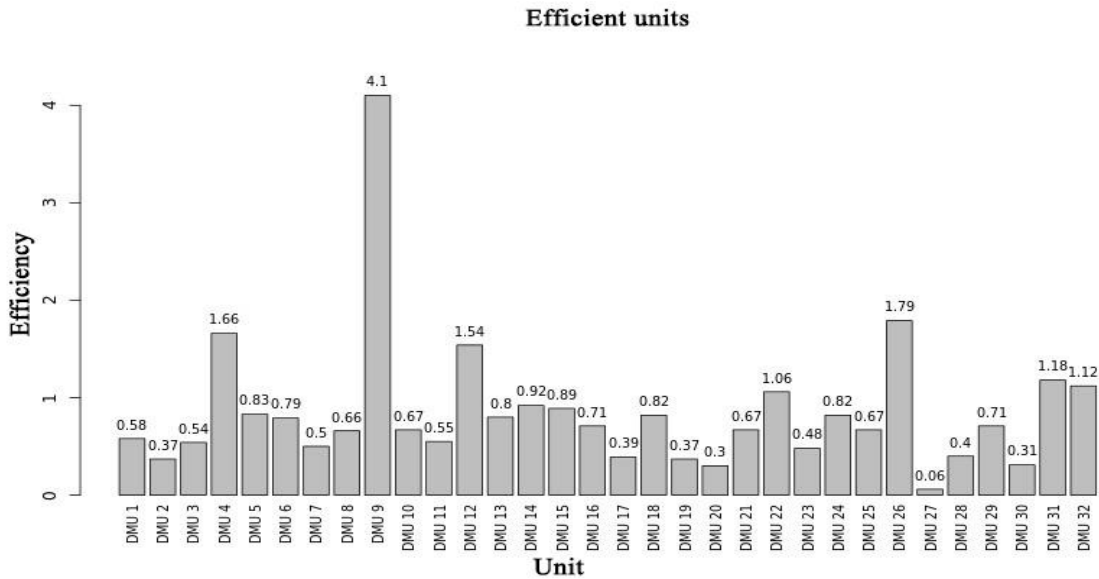
$$\sum_{j=1}^n \lambda_j x_{rj} - s_r^+ = y_{rk} \quad (r=1,2,\dots,s)$$

$$\lambda_j, s_r^+, s_i^- \geq 0 \quad \text{Free on Sign } \theta$$

The efficiency level according to the defined model is shown in Table 3 and Fig. 2. The units with the efficiency 1 and more are super-efficient.

**Table 3: Table of efficiency scores**

Unit	Efficiency	Unit	Efficiency	Unit	Efficiency	Unit	Efficiency
DMU 1	0.577	DMU 9	4.097	DMU 17	0.388	DMU 25	0.673
DMU 2	0.37	DMU 10	0.665	DMU 18	0.817	DMU 26	1.794
DMU 3	0.54	DMU 11	0.547	DMU 19	0.367	DMU 27	0.057
DMU 4	1.664	DMU 12	1.536	DMU 20	0.305	DMU 28	0.398
DMU 5	0.832	DMU 13	0.802	DMU 21	0.668	DMU 29	0.715
DMU 6	0.791	DMU 14	0.922	DMU 22	1.057	DMU 30	0.307
DMU 7	0.499	DMU 15	0.894	DMU 23	0.481	DMU 31	1.181
DMU 8	0.662	DMU 16	0.714	DMU 24	0.815	DMU 32	1.121



**Figure 2: Efficiency Units based on Anderson-Peterson technique**

As seen, like the 1st method the units 31, 26, 22, 12, 9, 4 and 32 are super-efficient but in this method the units can be ranked based on the efficiency. Also, the minimum efficiency of DMU27 is 0.06 and then it

is related to units of 20 and 30 as 0.30 and 0.31. So, the ranking of the super-efficient units based on efficiency units is shown in Table 4

**Table 4: ranking of the super-efficient units**

Unit	Efficiency	Rank
DMU09	4.1	01
DMU26	1.79	02
DMU04	1.66	03
DMU12	1.54	04
DMU31	1.18	05
DMU32	1.12	06
DMU22	1.06	07

## CONCLUSIONS

In this paper, we have presented an empirical investigation to measure the relative performance of 32 municipality units listed in Tehran Stock Exchange. The study has considered three inputs and three outputs for performance measurement and using constant return to scale data envelopment method, the study has determined the relative efficiency of all units and also performed super efficiency among efficient units and provided appropriate ranking for these units. The review of methods and materials revealed that in the standard DEA seven DMUs are rated as efficient and tie for the top position in the ranking. On the other hand, as the results of this article show, the super efficiency score enables us to distinguish between the efficient observations. In addition, based on the results of our survey, we can conclude that most units where either efficient or close to their efficient utilization of their resources. Furthermore, the result showed that DEA technique by taking; the book value of total assets, Book value of equity, and General and administrative expenses as inputs and Company profit or loss, Return on Equity, and Return on assets as outputs the ability to integrate these ratios and turning them into a single criterion that is called efficiency.

## ACKNOWLEDGEMENTS

I acknowledge the invaluable support by the **Tehran Stock Exchange** where data for this study were collected. The clearance of the study by the Director **General of Stock Exchange** is also gratefully acknowledged.

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# EXAMINATION OF THE EFFECT OF PATIENTS' FUNCTIONAL STATUS ON THE OPERATION OF NEURO-MUSCULOSKELETAL REHABILITATION UNITS IN HUNGARY USING DATA ENVELOPMENT ANALYSIS

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## ABSTRACT

*The efficient operation of public healthcare systems is important in every country as a consequence of the enormous amount of resources spent on serving an aging population. This paper analyses the efficiency of neuro-musculoskeletal rehabilitation units in Hungary using data envelopment analysis (DEA). Data pertaining to all rehabilitation units in Hungary are taken from a country-wide data collection. The operation of neuro-musculoskeletal rehabilitation units is strongly influenced by the patients' functional ability. Rehabilitation of patients recovering after a stroke requires significantly more resources, than the treatment of patients suffering degenerative and inflammatory joint disease or soft tissue rheumatism. Consequently, it is important to consider in the analysis, the aim of rehabilitation and the functional status of patients. The paper shows, how different functional status of patients can be incorporated into an output oriented slack based DEA model. A two stage DEA approach is used to analyze the effect of contextual variables related to the functional status of patients on the efficiency scores. The results show that the effect of patients' functional ability on the efficiency of operation depends strongly on the objective of the analysis and on the model applied.*

**Keywords:** *Data Envelopment Analysis, healthcare, system efficiency, musculoskeletal diseases, stroke*

## INTRODUCTION

Efficiency improvement is a major objective of the management of production and service systems. Data envelopment analysis (DEA) is a popular method to evaluate efficiency, particularly in service systems. DEA is used to compare the performance of service (or production) systems using linear programming. The compared systems are called decision-making units (DMU) and various models, inputs and outputs are used depending on the application environment and on the objectives of management (Cooper et al., 2007). In the past decades, the application area of DEA has rapidly expanded (See for example Emrouznejad et al., 2008 or Koltai et al., 2017).

A very important and rapidly growing application area of DEA is health care. Assessing and evaluating the efficiency of healthcare systems are difficult, because several conflictive evaluation criteria must be considered at the same time and the value of inputs and outputs are generally measured on different scales. Several studies show the benefits of health care applications of DEA. (See for example, Kooreman, 1994 or Asandului, 2014)

Hospital departments show differences in organizational structure, in financial conditions, and in the condition and treatment of patients, therefore, it is frequently reasonable to analyze hospital units separately. This paper focuses on the performance evaluation of musculoskeletal rehabilitation units.

The term musculoskeletal disorders (MSD) refers to conditions that involve the nerves, tendons, muscles and supporting structures of the patient (Bruce, 1997). Musculoskeletal conditions comprise over 150 illnesses and syndromes, which are usually progressive and associated with pain. Musculoskeletal conditions are prevalent and their impact is pervasive. They are the most common cause of severe long-term pain and physical disability, and they affect hundreds of millions of people around the world. The treatment of musculoskeletal disorders imposes heavy financial burden on the institutions responsible for treatment and on the whole society; consequently, efficiency improvement in this area is very important.

One of the most complicated and resource consuming rehabilitation task is the treatment of patients recovering from stroke. A stroke is caused by the interruption of the blood supply to the brain, leading to a necrosis (i.e. the cells that die) of the affected part. Stroke often involves functional degradation, for example upper/lower-limb paralysis, speech disorder, cognitive problems, and depression. After a stroke, the patient usually cannot move, walk, bathe, eat, or dress without help. Musculoskeletal rehabilitation activities and methods try to develop these abilities and skills.

In rehabilitation, the functional status of patients is frequently characterized with the Barthel Index (Houlden et al., 2006). This index provides an aggregate measure of every basic ability and life skill (toilet use, mobility, dressing, grooming, feeding, stairs, bathing, bowels, transfer, bladder) needed to be independent. The maximum value of the Barthel Index is 100, which indicates that the patient is able to live without any help. In the following part of the paper, the Barthel Index is used to characterize the patient's functional ability and to evaluate the change of health status as a consequence of medical treatment.

In this paper musculoskeletal rehabilitation departments in the field of in-patient care in Hungary are examined. The objective of the presented research is to analyze, how patient mix influences efficiency. More precisely, the patients treated at the rehabilitation units can be considered homogeneous in DEA, or as a consequence of the differences in the health status of patients and of the differences of the treatment process, homogeneity assumption is violated (Dyson et al., 2001).

In the following part of this paper, first, two DEA models are presented for efficiency analysis. The first model ignores the health status information and only concentrates on the number of patients treated. The second model considers health status information, and concentrates on the effectiveness of the treatment process. Next, the results of the two models and the effect of some health status related contextual variables are examined with correlation analysis. Finally, some important conclusions are drawn and further research possibilities are outlined.

## **THE METHOD APPLIED**

In a two stage DEA approach, first, two output oriented SBM DEA models were solved to get efficiency scores for the quantity oriented, and for the medical result oriented cases. Next, correlation analysis was



performed to analyze the relation of the two results and to analyze the effect of several contextual variables on the efficiency scores.

The inputs and outputs of the two DEA models are summarized in Table 1. The four inputs are defined as follows,

- number of hospital beds: all hospital beds which are regularly maintained and staffed and immediately available for the care,
- number of physicians employed: full time equivalent (FTE) of the number of doctors at the rehabilitation unit (part time doctors with joint affiliation are considered with a 0.3 weight),
- number of nurses employed: full time equivalent (FTE) of the number of nurses at the rehabilitation unit,
- number of professional healthcare workers and other non-physician specialist (psychologists, speech therapists, physiotherapists, qualified masseurs, conductive teachers, physiotherapist assistants, occupational therapists, orthopaedic technician, social assistant, medical physical education, special education teachers, dieticians, other therapists, others).

The outputs defined as follows,

- number of patients discharged: total number of musculoskeletal patients who leave the unit because the rehabilitation treatment is finished or because of transfer to other unit,
- the average change of Barthel Index of patients: the average difference of the Barthel Index of the patients at admission (to the rehabilitation center) and at discharge,
- number of stroke patients: total number of musculoskeletal patients who has any post stroke consequence (non-discretionary output),
- theoretical Barthel improvement possibility, which is defined as 100 minus the entering Barthel Index (non-discretionary output).

**Table 1: Inputs and outputs of the two DEA models**

	Model I	Model II
Objective	Quantity	Medical result
Inputs:	number of beds	number of beds
	number of physicians	number of physicians
	number on nurses	number on nurses
	number of professional healthcare workers and non-physician specialists	number of professional healthcare workers and non-physician specialists
Outputs:	number of patients discharged	the average change of Barthel Index of patients
Non-discretionary outputs:	-	number of stroke patients
	-	theoretical Barthel improvement possibility
	-	(entering 100-Barthel Index)

The classic formulation of the output oriented slack based DEA models, found in Cooper et al (2007) is used, and the models are solved with the PIM-DEA software. The applied data pertaining to all the 76 rehabilitation units operating in Hungary are taken from a country-wide data collection.

## DISCUSSION OF THE RESULTS

Table 2 contains the data of those rehabilitation units, which were found efficient by the applied two DEA models. The first rehabilitation unit (S6) in the table is the only DMU which was found efficient by both models. Based on the data in Table 2, S6 is a large unit with many patients discharged. Small Barthel Index change indicates poor effectiveness, and stroke ratio is relatively large. Based on the data, this unit is a mixture of a general and a stroke patient oriented rehabilitation unit. It is not surprising, that both Model I and Model II indicated efficiency.

The next three rehabilitation units (S44, S61, S68) were found efficient only by the quantity oriented DEA model (Model 1). Based on the data of Table 2, these units are relatively large, and the ratio of stroke patients is very low. The last four rehabilitation units (S2, S4, S14, S24) were found efficient only by the medical result oriented model (Model II). These units are relatively small, they have a higher than average stroke ratio, and average change of Barthel Index is relatively high.

**Table 2: Efficient DMUs based on the two DEA model**

DMU	No. of patients discharged	Change of Barthel Index	Ratio of patients with stroke	Efficiency Model I	Efficiency Model II
S6	1570	4.0	0.52	100.00	100.00
S44	1009	20.0	0.18	100.00	56.41
S61	1000	17.2	0.11	100.00	53.18
S68	845	11.0	0.08	100.00	51.75
S2	102	26.4	0.82	22.74	100.00
S4	401	30.0	0.68	30.47	100.00
S14	524	28.5	0.42	34.30	100.00
S24	176	50.0	0.28	25.65	100.00

The results of rank correlation analysis are listed in Table 3 and Table 4. Table 3 contains the Spearman rank correlation coefficients and Table 4 shows the corresponding *p*-values. Although each number in the tables have interesting practical implication, as a consequence of the limited space available, only the bold face numbers are explained in details in the following.

– The rank correlation coefficient of the efficiency scores provided by the two models for the same DMU shows, that there is no any association between the two efficiency scores ( $\rho=0,0262$ ;  $p\text{-value}=0,8220$ ). This result indicates, that a rehabilitation unit (DMU) efficient according to the quantity based approach (Model 1) is not necessarily inefficient according to the medical result based approach (Model 2). One would expect a negative correlation between the two efficiency score, but this is not justified by the analysis.

– The analysis of the effect of the ratio of stroke patients on the efficiency scores shows ambiguous results. The rank correlation coefficient shows, that there is no any association between the stroke ratio and the volume based efficiency score ( $\rho=-0.1674$ ;  $p\text{-value}=0.1480$ ). On the other hand, a strong association can be assumed between the medical result based efficiency score (Model 2) and the stroke ratio ( $\rho=0,5418$ ;  $p\text{-value}=0,0000$ ). This result indicates, that the ratio of stroke patients influences efficiency if efficiency calculation is based on Barthel improvement, and on the health status information of patients.

– The analysis of the effect of the number of stroke patients on the efficiency scores, however is similar in both cases. The rank correlation coefficient shows, that there is no any association between the number of stroke patients and the volume based efficiency scores ( $\rho=0.2235$ ;  $p\text{-value}=0.0520$ ), and between the medical result based efficiency score (Model 2) ( $\rho=0.1846$ ;  $p\text{-value}=0.1100$ ).

It is interesting to observe, that while the number of stroke patients doesn't correlate with the efficiency scores of Model 2, the ratio of stroke patients correlates with this efficiency score. The important is not the number of the complicated cases, but their ratio.

– The analysis of the effect of the average change of the Barthel Index shows, that a week negative association can be assumed between the volume based efficiency score (Model 1) and the average change of Barthel Index ( $\rho=-0.2681$ ;  $p\text{-value}=0.0190$ ). This result indicates, that if effective medical service is provided, it has an adverse effect on quantity. The average change of Barthel Index is not an output of Model 1, but the more intensive use of resources in order to better improve the health status of patients influences quantity based efficiency.

– The analysis of the effect of the average 100-Barthel value has no association with the volume based efficiency score (Model 1) ( $\rho=-0.2131$ ;  $p\text{-value}=0.0650$ ). This result shows, that the health improvement possibility has no effect on the quantity oriented evaluation.

– The effect of the ratio of patients with planned rehabilitation shows, that the ratio of scheduled patients has a week association with the volume based efficiency score ( $\rho=0.2475$ ;  $p\text{-value}=0.0310$ ). This result shows, that scheduling patients if possible, has a favorable effect on the number of patients treated. When, however, effectiveness is in the focus of efficiency, then scheduling is not relevant since the ratio of scheduled patients has no association with the medical result based efficiency score ( $\rho=-0.1244$ ;  $p\text{-value}=0.02840$ ).

**Table 3: Rank correlation coefficients (Spearman rho)**

<b>Spearman rho</b>	Efficiency Modell 1	Efficiency Modell 2	Stroke ratio	No. of stroke	Barthel change	100-Barthel	Scheduled ratio
Modell 1		<b>0.0262</b>	<b>-0.1674</b>	<b>0.2236</b>	<b>-0.2681</b>	<b>-0.2131</b>	<b>0.2475</b>
Modell 2			<b>0.5418</b>	<b>0.1846</b>	0.4176	0.3484	<b>-0.1244</b>
Stroke ratio				0.7657	0.0291	0.0186	-0.0839
No. of stroke					-0.0872	-0.0167	0.0412
Barthel change						0.6635	-0.3222
100-Barthel							-0.2349
Scheduled ratio							

**Table 4: p-values of the rank correlation analysis**

<b>p-value</b>	Efficiency Modell 1	Efficiency Modell 2	Stroke ratio	No. of stroke	Barthel change	100-Barthel	Scheduled ratio
Modell 1		<b>0.8220</b>	<b>0.1480</b>	<b>0.0520</b>	<b>0.0190</b>	<b>0.0650</b>	<b>0.0310</b>
Modell 2			<b>0.0000</b>	<b>0.1100</b>	0.0000	0.0020	<b>0.2840</b>
Stroke ratio				0.0000	0.8030	0.8730	0.4710
No. of stroke					0.4540	0.8860	0.7240
Barthel change						0.0000	0.0050
100-Barthel							0.0410
Scheduled ratio							

## CONCLUSIONS

The major question raised in this paper is, whether patient mix influences efficiency. A two stage DEA approach was applied to answer this question. First DEA models were applied to calculate efficiency scores. Next, correlation analysis was used to analyze the effect of contextual variables on efficiency.

The first model (Model I) in this paper was quantity oriented. The calculation was based only on the major resources used by the rehabilitation units and on the patients attended. No any characteristics of the functional status of the patients was involved in the calculation. If, however, functional status influences the treatment process and the intensity of resource usage, then that must be reflected in the efficiency score. The analysis of correlation between the volume based efficiency score and the ratio or the number of stroke patients, did not show any evidence that patient mix influences efficiency scores.

In case of the second model (Model II), characteristics of the functional status of patients was involved in the calculation. In this case the ratio of stroke patients had a positive correlation with efficiency, but the number of stroke patients didn't influence the efficiency score.

We may conclude, that if the DEA model does not use any special characteristics of patients, then the activity of the rehabilitation units can be considered homogenous, consequently, patient mix has no effect on efficiency.

The analysis also confirmed, that better improvement of the health status of patients (higher average change of Barthel Index) has an adverse effect on the quantity of patients attended. Consequently, a strictly quantity oriented performance evaluation does not serve the interest of patients.

Finally, we may also conclude, that careful planning in rehabilitation, whenever is possible, has a favorable effect on efficiency.

Health status of patients in this paper was characterized by the Barthel Index. This index has a long history in rehabilitation, it is methodologically well based, but its application contains several subjective elements. Furthermore, several other indicators (eg. FIM) exist in practice (Houlden et al., 2006). Consequently, it would be important to analyze the sensitivity of these results to the applied values of the Barthel Index, or to the type of health status indicators applied. This is a topic of our future research.

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# **A FUZZY DATA ENVELOPMENT ANALYSIS FOR THE SUPPLY CHAIN RESILIENCE ASSESSMENT: AN IRANIAN CAR MANUFACTURER**

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## **ABSTRACT**

*Today's complex supply chains are increasingly susceptible to the turbulent and fast-changing business environment and their economic implications. Resilience as effective strategic planning during disturbances is a way to mitigate supply chain vulnerabilities. After reviewing the literature on a resilient supply chain, this paper extracts a complete series of 16 resilience enablers identified in a questionnaire, which is distributed among 150 experts and staffs of a real case associated with the Iranian automotive supply chain. The reliability of this questionnaire is evaluated by statistical tests and Cronbach's alpha. Then, a Fuzzy Data Envelopment Analysis (FDEA) approach is employed to evaluate the performance of the resilience enablers in the context of supply chain disruptions. Finally, the validation and verification of the obtained results are performed using statistical tests. The case study findings indicate that by improving the resilience enablers, especially ones with the greatest influence on the supply chain performance, firms can be less vulnerable in times of supply chain disruptions. The framework proposed in this study may find a broad practical application in all types of supply chains.*

**Keywords:** Fuzzy data envelopment analysis; Supply chain resilience; Vulnerability; Automotive supply chain.

## **INTRODUCTION**

Today's Supply Chains (SCs) are a complicated network of enterprises and much more susceptible to unpredictable disruptions than in the past. Nowadays, numerous changes in the business model contribute to increasing the risk to which firms are exposed and their complexity. The globalization of businesses, the widespread adaptation of lean manufacturing philosophy in a majority of firms, the reduction of the supplier base, and the developing companies work on the basis of outsourcing are some of these changes that may be resulted in SCs becoming very unable to react to potential disruptions.

Supply chain disruptions can result from internal sources, such as loss of an extremely crucial supplier, unsuccessful integration of SC functions and fire at a manufacturing plant. Disruptions can also be triggered by external sources, such as natural hazards and terrorist attacks. They can lead to undesirable operational and financial impact, such as lost sales and even market share (Ponomarov and Holcomb 2009). As SC risks go up, the importance of developing capabilities that can enable firms to provide an effective and efficient response to the unforeseen events also goes up. This makes clear the essence of Supply Chain Resilience (SCR), which is defined as the ability to return to its original state and even a

more desirable one after being disrupted. A SC can be resilient if it is capable of absorbing the negative effects from unexpected disturbances and recovering from them.

Answering the question of how to build a resilient SC is well supplied in the literature. Based on experiences from a number of specific SC cases from industries, such as electronics and automotive. Christopher and Peck (2004) proposed an initial empirical framework for a resilient SC and claimed that the resiliency can be formed into a SC system structure through four essential elements, including re-engineering, collaboration, agility, and risk management culture. In their proposed framework, the resilience enablers (e.g., availability, flexibility and redundancy) were seen as secondary elements. Sheffi (2005) identified operational flexibility, redundancy, collaboration, and cultural change as the ways in which organizations can respond to unpredictable and high-impact disruptions. According to Töyli et al. (2013), the best level of resilience is achieved in two dimensions, which include agility and robustness. While agility needs visibility and quick response to rapidly depart from an unstable situation, anticipation and preparedness are needed for the robust SC. In another study, among different enablers of the SC, dynamic assortment planning is identified as the decisive causal factor that initiated the effects of many other resilience enablers (Rajesh and Ravi, 2015).

Incorporation of the resilience concept into the SC design process is needed to improve resilience against environmental abrupt changes. The geographical spacing between SC nodes, network complexity, and a number of critical nodes within a SC are the most important measurable design parameters for modeling the entire SC (Falasca et al., 2008). Carvalho (2011) developed a SCR assessment model based on two resilience indices of on-time delivery to capacity shortage and on-time delivery to the materials shortage. The work of Soni et al. (2014) is an extensive study to support organizations in developing a framework for measuring SCR considering the major enablers

After a detailed review of the SCR-focused studies, we provide a complete portfolio of SC enablers, including flexibility, redundancy, decentralization, reengineering/redesign, risk management culture, agility, robustness, corporate social responsibility, market position, organization, trust among members, security, dynamic assortment planning, sustainability, information sharing, and risk and revenue sharing which contribute to enhance the automotive industry SCR.

The related literature states that limited research concentrates on SCR and all the major resilience enablers that affect it. To our knowledge, this is the first attempt that examines a comprehensive portfolio of the resilience enablers in the automotive industry by the FDEA.

## **METHODS**

Due to the nature of the automotive industry, effective performance in the SC section of this industry has a big influence on the performance of the whole manufacturing system. In this regard, this study presents an overall approach for assessing the performance of a real car manufacturer's SC network in Iran by considering the resilience enablers. The required steps are as follows:

- Step 1) Literature survey and identification of potential SCR enablers: According to the literature survey in the previous section, it is understood that the performance of an automotive plant SC under disruption situations can be influenced by the 16 resilience enablers.
- Step 2) Data collection: With respect to the qualitative nature of the majority of the identified factors, a standard questionnaire contained two questions about each factor is designed for this. A total number of 150 Decision-Making Units (DMUs) (i.e., a subset of experts and staff of the considered plant forms the statistical sample of this study). Correspondents answered questions in the range of 1 to 20. After collecting the questionnaires from respondents, the obtained results are analyzed with the Cronbach's alpha to make sure about the reliability of the survey data.
- Step 3) Apply FDEA model to solve the problem: all the 16 identified resilience enablers are taken into account as outputs, and one dummy input is proposed as an input of the conceptual model of this study. Since our focus is placed on the outputs, output-oriented FDEA model is employed to analyze the data. By implementation of various  $\alpha$ -cuts of FDEA model, efficiency score of each DMU is achieved in this step. It should be noted that the preferred  $\alpha$ -cut is selected based on normality test on efficiency residuals.
- Step 4) Verification and validation: The obtained results of the considered FDEA model is validated using conventional deterministic DEA models. For this purpose, the obtained ranks of the DMUs by the FDEA model are compared to the obtained ranks of conventional deterministic DEA models using the Pearson correlation test.
- Step 5) Sensitivity analysis: Identification of the most influential resilience enablers is performed in this step. For this purpose, the preferred FDEA model is run by omitting each output variable from the model separately. After calculating the efficiency scores for the non-existence of each resilience enabler, it is possible to evaluate its weight and performance.

### Data Envelopment Analysis

The Data Envelopment Analysis (DEA) is a quantitative decision support tool, which has been increasingly employed for assessing the performance of systems (Charnes et al., 1978). The main advantage of this approach in comparison with the other evaluation techniques is its ability in evaluating complicated systems with numerous inputs and outputs and often the unclear relationship between them (Cooper et al., 2004). In this section, we intend to briefly explain the mathematical model of output-oriented CCR, BCC and FDEA models.

- Output-oriented CCR model (Charnes et al., 1978): This basic DEA model tries to maximize the output variables so that the input variables are considered to be constant.
- Output-oriented BCC model (Banker et al., 1984): This model is derived from the CCR model. The main difference between these versions of the DEA is related to their envelopment surface. In other words, the condition of output to the fixed scale in the CCR model is substituted by the condition of output to the variable scale in the BCC model.
- Output-oriented FDEA model: The aforementioned DEA models are applicable only in the presence of deterministic data. However, decision making in reality is tainted with uncertainty and is essential to apply an appropriate approach to handle imprecise input and output variables. Accordingly, we apply the FDEA model with triangular fuzzy numbers proposed by Azadeh et al. (2017). After transforming the FDEA model into interval programming by using the  $\alpha$ -cut method, Equations (1)-(6) give an upper bound of efficiencies and the next three ones give a lower bound of efficiencies.



$$Max \sum_{r=1}^{16} (\alpha y_{rj}^m + (1-\alpha) y_{rj}^u) \quad (1)$$

$$s.t. \sum_{i=1}^1 (\alpha x_{ij}^m + (1-\alpha) x_{ij}^l) \geq \sum_{r=1}^{16} (\alpha y_{rj}^m + (1-\alpha) y_{rj}^u) ; \quad \forall j = 1, 2, \dots, 150 \quad (2)$$

$$\sum_{i=1}^1 (\alpha x_{ij}^m + (1-\alpha) x_{ij}^u) = I ; \quad \forall j = 1, 2, \dots, 150 \quad (3)$$

$$Max \sum_{r=1}^{16} (\alpha y_{rj}^m + (1-\alpha) y_{rj}^l) \quad (4)$$

$$s.t. \sum_{i=1}^1 (\alpha x_{ij}^m + (1-\alpha) x_{ij}^u) \geq \sum_{r=1}^{16} (\alpha y_{rj}^m + (1-\alpha) y_{rj}^l) ; \quad \forall j = 1, 2, \dots, 150 \quad (5)$$

$$\sum_{i=1}^1 (\alpha x_{ij}^m + (1-\alpha) x_{ij}^l) = I ; \quad \forall j = 1, 2, \dots, 150 \quad (6)$$

where inputs and outputs related to the  $j$ -th DMU are represented by triangular fuzzy numbers  $\tilde{x}_{ij} = (x_{ij}^l, x_{ij}^m, x_{ij}^u)$  and  $\tilde{y}_{rj} = (y_{rj}^l, y_{rj}^m, y_{rj}^u)$ , respectively. Furthermore,  $\alpha$  is a parameter in the range of (0, 1] and provides different degrees of uncertainties. Different values for this parameter in this study are 0.01, 0.05, 0.1, 0.2, ..., 0.9, 0.95, 0.99 and 1.

## RESULTS AND DISCUSSIONS

The personal judgement of the chosen statistical sample about the performance of the system under SC disruptions is collected through a structured questionnaire. Given the judgement nature of the collected data in this study, effective decision-making must be able to consider the uncertainties of the data. Accordingly, to use the FDEA approach, we need to transform the collected data to triangular fuzzy numbers. To achieve this goal, three points associated with each triangular fuzzy number are estimated with the minimum, total mean, and maximum values of all DMUs related to each resilience enabler.

In order to make certain about the internal consistency or reliability of the collected data, the survey data are assessed using the Cronbach's alpha test in statistical package SPSS version 23. The value of the Cronbach's alpha for all of the considered factors is greater than 0.7. The total value of the Cronbach's alpha is also equal to %72 in this study, which verifies the collected data's reliability.

### FDEA results

According to the obtained results of normality test on efficiency residuals, among different  $\alpha$ -cuts,  $\alpha=0.3$  is the preferred  $\alpha$ -cut value. Therefore, the FDEA model is applied to the collected data related to the 150 DMUs under the  $\alpha$ -cut level of 0.3 by using Auto Assess software (Azadeh et al., 2016). The efficiency score and rank of each DMU are computed by importing all the 16 resilience enablers to the output-oriented model as outputs and considering a single dummy input in the model.

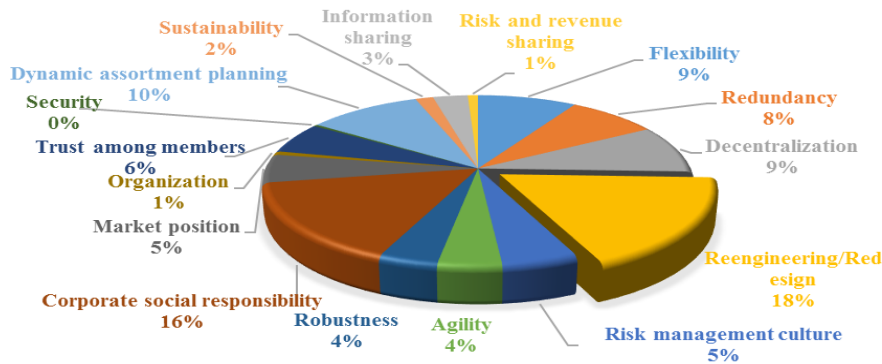
### Validation and verification

To verify and validate the obtained results using the FDEA, the conventional deterministic BCC DEA model is employed. After calculating the efficiency scores using the deterministic DEA model in the

presence of all factors, the Pearson correlation test is used. Since the Pearson correlation coefficient between efficiency scores of the FDEA and deterministic DEA models is 0.823, the obtained results are validated.

### Sensitivity analysis

To determine the relative importance of the resilience enablers in this study, we perform a sensitivity analysis by running the FDEA model 16 times, each time in the absence of one output factor. In addition to identifying the most important factors, the sensitivity analysis also makes it possible to determine how a resilience enabler influence the average efficiency of DMUs. In other words, we are capable of identifying which factors influenced the efficiency positively, which negatively, or not at all. For this purpose, at first, we calculate the difference between average efficiency before and after elimination of each factor. Regardless of positivity or negativity of these values, factors with higher values are selected as the most important factors and suitable planning for improvement of these factors can enhance the performance of the system. Among these factors, organization, security, sustainability, and risk and revenue sharing are selected as the most trivial factors for this particular case study. The weight of each factor is illustrated in Figure 1. The weight of each resilience enabler is calculated based on the percentage of change in the efficiency score after its elimination.



**Figure 1: Weight of each resilience factor in forming performance efficiency**

### **CONCLUSIONS**

The results stated that the reengineering/redesign, corporate social responsibility, dynamic assortment planning, flexibility, and decentralization are the most important enablers in improving the resiliency of the considered SC, respectively. Therefore, enhancing the total performance of this system is achievable through more focusing on these items. Given the current state of the automotive industry in Iran, the obtained results appear reasonable. The difference between the total average efficiency and average efficiency is a negative value when the decentralization factor is removed. This means that this factor has a negative impact on efficiency. A lasting monopoly in the automotive industry SC is a major reason for the negative influence of this factor on the performance efficiency. Increasing the cost of quality, inability in providing high quality after-sales service, and even shutting down of production lines are also often

caused by this reason. This sector possesses a very low public satisfaction concerning quality and price of goods and services. Therefore, it is reasonable that the social responsibility factor is assessed negatively in the face of unforeseen situations. The very negative influence of reengineering/redesign factor on total efficiency can have a direct relationship with the structural weakness of the Iranian industries in the design processes as well as focusing on assembly-based productions. Due to lack of market competition and a ban on entry of foreign car manufacturers at the same level of domestic ones, there is a constant demand for domestic production in general. The result of such policies has caused that the first two largest vehicle manufacturing companies, known as Iran Khodro and Saipa, have more than 94% of the Iran's total market. Therefore, the positive impact of market position factor on the total efficiency seems to be justified. The framework of this study can be applied in other sectors to have a comprehensive insight into their performance under disruptions. This paper suggests some future research as follows: (1) gathering the quantitative data for SCR enablers and (2) considering the financial records of car manufacturing companies along with conventional SCR enablers to assess economic resiliency of the company.

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# FINDING THE STABILITY REGION OF EFFICIENCY OF EFFICIENT FIRMS FOR UPWARD VARIATIONS OF INPUTS AND DOWNWARD VARIATIONS OF OUTPUTS

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## Abstract

*Finding stability region of efficiency for an efficient firm (or generally decision-making unit (DMU)) is one of the important issues from the economic point of view, which the DMU remains efficient within this region. This paper proposes a data envelopment analysis (DEA) approach to find the stability region of efficiency for upward variations of inputs and/or for downward variations of outputs of an (extremely) efficient DMU. Boljućić (2006) [Boljućić, V. (2006). Sensitivity analysis of an efficient DMU in DEA model with variable returns to scale (VRS). Journal of Productivity Analysis, 25 (1-2), 173–192] suggested a DEA procedure to identify the stability region of efficiency. As a matter of fact, the computational complexity of the Boljućić's procedure is high, however, the proposed DEA approach has an acceptable computational complexity. To demonstrate the applicability of the proposed approach in economics, a numerical example is provided.*

**Keywords:** Data envelopment analysis (DEA); Efficiency; Production possibility set (PPS); Stability region (SR); Sensitivity analysis

## INTRODUCTION

Boljućić (2006) introduced a DEA procedure to obtain the SR (stability region) of efficiency for e-efficient (extremely efficient) DMUs. The author used the super-efficiency model and then applied the optimal simplex tableau and parametric programming to obtain all possible variations of inputs and outputs. The suggested procedure is computationally inefficient, especially when it deals with problems with enormous dimensions. To tackle this drawback, this study proposes a DEA approach with an acceptable computational complexity for determining the SRs of efficiency of e-efficient DMUs when their inputs increase and/or their outputs decrease.

## PRELIMINARIES AND THE NEW APPROACH

Consider  $n$  DMUs, i.e.  $\{DMU_j | j = 1, 2, \dots, n\}$ , where each  $DMU_j$  produces  $s$  different semi-positive outputs  $\mathbf{y}_j = (y_{1j}, \dots, y_{rj}, \dots, y_{sj})^T \geq \mathbf{0}_s$  by using  $m$  different semi-positive inputs  $\mathbf{x}_j = (x_{1j}, \dots, x_{ij}, \dots, x_{mj})^T \geq \mathbf{0}_m$ .

The input BCC-efficiency score of a target DMU ( $DMU_o ; o \in \{1, \dots, n\}$ ) can be obtained by using the following envelopment form of the input-oriented BCC model (Banker et al., 1984):

$$\begin{aligned}
& \min \theta \\
& \text{s. t.} \\
& \sum_{j=1}^n \lambda_j x_{ij} + s_i^- = \theta x_{io} \quad i = 1, \dots, m \\
& \sum_{j=1}^n \lambda_j y_{rj} - s_r^+ = y_{ro} \quad r = 1, \dots, s \\
& \sum_{j=1}^n \lambda_j = 1 \\
& \lambda_j \geq 0 \quad j = 1, \dots, n \\
& s_i^- \geq 0 \quad i = 1, \dots, m \\
& s_r^+ \geq 0 \quad r = 1, \dots, s
\end{aligned} \tag{1}$$

Suppose that the optimal solution  $(\theta^*, \lambda_1^*, \dots, \lambda_n^*, s_1^{-*}, \dots, s_m^{-*}, s_1^{+*}, \dots, s_s^{+*})$  for model (1) is obtained:

**Definition 1 (E-efficient).**  $DMU_o$  is called e-efficient if and only if  $E_o = \{j | \lambda_j^* > 0, j = 1, \dots, n\} = \{o\}$  where  $E_o$  is called *reference set* of  $DMU_o$ .

The dual problem of model (1) is expressed as below:

$$\begin{aligned}
& \theta_o^* = \max \sum_{r=1}^s u_r y_{ro} + u_0 \\
& \text{s. t.} \\
& \sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} + u_0 \leq 0 \quad j = 1, \dots, n \\
& \sum_{i=1}^m v_i x_{io} = 1 \\
& v_i \geq 0 \quad i = 1, \dots, m \\
& u_r \geq 0 \quad r = 1, \dots, s
\end{aligned} \tag{2}$$

The following model measures the efficiency score of  $DMU_o$  with more discriminating power:

$$\begin{aligned}
& z^* = \max \sum_{r=1}^s u_r y_{ro} + u_0 \\
& \text{s. t.} \\
& \sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} + u_0 \leq 0 \quad j = 1, \dots, n \\
& \sum_{i=1}^m v_i x_{io} = 1 \\
& v_i \geq \varepsilon^* \quad i = 1, \dots, m \\
& u_r \geq \varepsilon^* \quad r = 1, \dots, s
\end{aligned} \tag{3}$$

where  $\varepsilon^*$  is the optimal objective value of the proposed model by Cook, Kress, and Seiford (1996).

**Definition 2 (S-efficient).**  $DMU_o$  is called s-efficient (strong efficient) if and only if  $z^* = 1$ ; otherwise it is inefficient.

Let  $DMU_o$  be an e-efficient DMU. After eliminating  $DMU_o$  from the reference set, a new PPS is obtained which is called  $T'_V$ . Then, we determine all s-efficient DMUs of  $T'_V$  by using the multiplier form of the BCC model (3). Let  $\Psi = \{DMU_1, \dots, DMU_k\}$  be the obtained set of s-efficient DMUs. We solve the

following two DEA models, which were presented by Huang et al. (1997), for each  $DMU_p \in \Psi$  ( $p = 1, \dots, k$ ):

$$\begin{aligned}
& \max \quad u_0 \\
& \text{s. t.} \\
& \sum_{r=1}^s u_r y_{rp} - \sum_{i=1}^m v_i x_{ip} = u_0 \\
& \sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} \leq u_0 \quad j = 1, \dots, n; j \notin \{p, o\} \\
& \sum_{r=1}^s u_r + \sum_{i=1}^m v_i = 1 \\
& u_r \geq 0 \quad r = 1, \dots, s \\
& v_i \geq 0 \quad i = 1, \dots, m
\end{aligned} \tag{4}$$

$$\begin{aligned}
& \min \quad u_0 \\
& \text{s. t.} \\
& \sum_{r=1}^s u_r y_{rp} - \sum_{i=1}^m v_i x_{ip} = u_0 \\
& \sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} \leq u_0 \quad j = 1, \dots, n; j \notin \{p, o\} \\
& \sum_{r=1}^s u_r + \sum_{i=1}^m v_i = 1 \\
& u_r \geq 0 \quad r = 1, \dots, s \\
& v_i \geq 0 \quad i = 1, \dots, m
\end{aligned} \tag{5}$$

By solving models (4) and (5) for each  $DMU_p \in \Psi$ , we attain all of the defining hyperplanes of  $T'_V$ . Let  $H_1, H_2, \dots$ , and  $H_l$  be the obtained hyperplanes which are as below:

$$H_q = \left\{ \begin{pmatrix} \mathbf{x} \\ \mathbf{y} \end{pmatrix} \mid (\mathbf{u}_q^*)^T \mathbf{y} - (\mathbf{v}_q^*)^T \mathbf{x} = u_{0q}^* \right\}, \quad (q = 1, \dots, l).$$

Thus,

$$T'_V = \bigcap_{q=1}^l H_q^- = \bigcap_{q=1}^l \left\{ \begin{pmatrix} \mathbf{x} \\ \mathbf{y} \end{pmatrix} \mid (\mathbf{u}_q^*)^T \mathbf{y} - (\mathbf{v}_q^*)^T \mathbf{x} \leq u_{0q}^* \right\} \tag{6}$$

**Theorem 1.** Suppose that  $DMU_o$  is an e-efficient DMU. Then, the SR of efficiency for  $DMU_o$  is as

$$(SR)_o = \Gamma_o - T''_V \tag{7}$$

where

$$\begin{aligned}
\Gamma_o &= \left\{ \begin{pmatrix} \mathbf{x} \\ \mathbf{y} \end{pmatrix} \mid \mathbf{x}_o \leq \mathbf{x}, \mathbf{y}_o \geq \mathbf{y} \not\leq \mathbf{0} \right\} \\
T''_V &= T'_V - \cup_q \left\{ \begin{pmatrix} \mathbf{x} \\ \mathbf{y} \end{pmatrix} \mid (\mathbf{u}_q^*)^T \mathbf{y} - (\mathbf{v}_q^*)^T \mathbf{x} = u_{0q}^*, \left( (\mathbf{u}_q^*)^T, -(\mathbf{v}_q^*)^T \right) > \mathbf{0}^T \right\} = T'_V - \Lambda_o.
\end{aligned}$$

**Proof.** See Appendix.

**Q.E.D.**

**Theorem 2.** Suppose that  $DMU_o$  is s-efficient, but not e-efficient. Then,  $(SR)_o = \{DMU_o\}$ .

**Proof.** See Appendix.

**Q.E.D.**

## NUMERICAL EXAMPLE

We consider five DMUs ( $J = \{A, B, C, D, E\}$ ) with single input and single output. Table 1 exhibits the data set of these DMUs.

**Table 1: An illustrative dataset**

DMU	A	B	C	D	E
Input	2	4	6	10	8
Output	10	5	9	14	7

By using the proposed approach to find the SR of efficiency of  $DMU_A$ , we obtain the following SR:

$$(SR)_A = I_A - T_V'' = \{(x, y)^T | x \geq 2, 0 < y \leq 10\} - \{(x, y)^T | x \geq 4, 0.33333y - 0.66667x + 1 < 0, 0.44444y - 0.55556x - 0.66667 < 0, y \leq 14\}.$$

To validate our approach, we change the input-output vector of  $DMU_A$  within  $(SR)_A$  from  $(2, 10)^T$  to  $(3, 2)^T$ ,  $(4, 7)^T$ ,  $(2, 8)^T$ ,  $(5, 9)^T$ , and  $(6, 10)^T$ , respectively. As seen in Table 2, in all of the cases, the efficiency score of  $DMU_A$  is one. So,  $DMU_A$  remains s-efficient within  $(SR)_A$ . Note that the last column of Table 2 depicts the value of  $\varepsilon^*$  which has been calculated in each case, separately.

**Table 2: Sensitivity analysis on  $DMU_A$**

DMU	$(3, 2)^T$	$(4, 7)^T$	$(2, 8)^T$	$(5, 9)^T$	$(6, 10)^T$	$DMU_B$	$DMU_C$	$DMU_D$	$DMU_E$	$\varepsilon^*$
Eff.	<b>1.000</b>	----	----	----	----	1.000	1.000	1.000	0.625	0.100
Eff.	----	<b>1.000</b>	----	----	----	0.999	0.952	1.000	0.500	0.100
Eff.	----	----	<b>1.000</b>	----	----	0.499	0.556	1.000	0.249	0.100
Eff.	----	----	----	<b>1.000</b>	----	1.000	0.833	1.000	0.562	0.100
Eff.	----	----	----	----	<b>1.000</b>	1.000	0.933	1.000	0.600	0.100

## CONCLUSIONS

Boljuncić (2006) introduced a DEA procedure to determine the SR of efficiency for e-efficient DMUs by applying the super-efficiency model, the optimal simplex tableau, and parametric programming to get all possible variations of inputs and outputs. However, his proposed procedure has a high computational complexity, especially when it is used for problems with enormous dimensions, which is not acceptable from a computational point of view. Hence, the current study, with the aim of overcoming this drawback, proposes a DEA approach with an acceptable computational complexity for determining the SRs of

efficiency of e-efficient DMUs for upward variations of their inputs and/or for downward variations of their outputs.

#### APPENDIX. PROOFS OF THE THEOREMS

**Proof of Theorem 1.** Let  $DMU_{\bar{o}} = (x_{1o} + \alpha_1, x_{2o} + \alpha_2, \dots, x_{mo} + \alpha_m, y_{1o} + \beta_1, y_{2o} + \beta_2, \dots, y_{so} + \beta_s)^T$  such that  $\alpha_i \geq 0$  ( $i = 1, \dots, m$ ) and  $\beta_r \geq 0$  ( $r = 1, \dots, s$ ). Assume that  $DMU_{\bar{o}}$  belongs to the SR of efficiency of  $DMU_o$ , i.e.  $DMU_{\bar{o}} \in (SR)_o$ . From (7) we obtain  $DMU_{\bar{o}} \in \Gamma_o$  and  $DMU_{\bar{o}} \notin T_V''$ . It is sufficient to prove that  $DMU_{\bar{o}}$  remains s-efficient when  $DMU_o$  is removed from the reference set. We have  $DMU_{\bar{o}} \notin T_V''$ , hence either

$$DMU_{\bar{o}} \notin T_V', \quad (1)$$

or

$$DMU_{\bar{o}} \in \Lambda_o. \quad (2)$$

We consider the following two cases:

*Case (A).*  $DMU_{\bar{o}} \notin T_V'$ .

On the contrary, suppose that  $DMU_{\bar{o}}$  is inefficient. Then, there exists  $DMU_{\hat{o}} \in \Gamma_o$  such that  $DMU_{\bar{o}}$  is dominated by  $DMU_{\hat{o}}$ . Assume that  $\bar{T}_V$  is the PPS which is made by  $DMU_{\hat{o}}$  and  $DMU_j$  ( $j = 1, \dots, n; j \neq o$ ), therefore

$$DMU_{\bar{o}} \in \bar{T}_V. \quad (3)$$

In this case, by eliminating  $DMU_o$  from the reference set, it is deduced that

$$T_V' = \bar{T}_V. \quad (4)$$

Thereupon, as per Eqs. (3) and (4), we have  $DMU_{\bar{o}} \in T_V'$  and, since  $DMU_{\bar{o}}$  is dominated by  $DMU_{\hat{o}}$ , thus  $DMU_{\bar{o}} \in T_V'$ , which is contradictory to Eq. (1).

*Case (B).*  $DMU_{\bar{o}} \in \Lambda_o$ .

Since  $DMU_{\bar{o}} \in \Lambda_o$ , thus there exists at least a strong defining hyperplane of  $T_V'$ ,  $H_k = \left\{ \begin{pmatrix} \mathbf{x} \\ \mathbf{y} \end{pmatrix} \mid (\mathbf{u}_k^*)^T \mathbf{y} - (\mathbf{v}_k^*)^T \mathbf{x} = u_{ok}^*, ((\mathbf{u}_k^*)^T, -(\mathbf{v}_k^*)^T) > \mathbf{0}^T \right\}$ , which is binding at  $DMU_{\bar{o}}$  that is,  $(\mathbf{u}_k^*)^T \mathbf{y}_{\bar{o}} - (\mathbf{v}_k^*)^T \mathbf{x}_{\bar{o}} = u_{ok}^*$ . Therefore, this issue indicates that  $DMU_{\bar{o}}$  remains s-efficient in  $T_V'$ . It is deduced that  $DMU_o$  remains s-efficient if, after increasing its inputs and decreasing its outputs,  $DMU_o$  stays in  $(SR)_o$ , which completes the proof. **Q.E.D.**



**Proof of Theorem 2.** Since  $DMU_o$  is s-efficient, but not e-efficient, after eliminating  $DMU_o$  from the reference set, it can be represented as a convex combination of the remaining DMUs as below (Charnes et al., 1991):

$$DMU_o = (\mathbf{x}_o, \mathbf{y}_o)^T = \left( \sum_{j \neq o}^n \lambda_j \mathbf{x}_j, \sum_{j \neq o}^n \lambda_j \mathbf{y}_j \right)^T,$$

where  $\sum_{j \neq o}^n \lambda_j = 1$  and  $\lambda_j \geq 0$  for  $j = 1, \dots, n; j \neq o$ .

It is easy to verify that  $DMU_o$  dominates the new DMU obtained by increasing at least one of the inputs of  $DMU_o$  and/or decreasing at least one of its outputs. Therefore, it is not efficient, and hence it does not belong to  $(SR)_o$ . Consequently,  $(SR)_o = \{DMU_o\}$  and the proof is complete. **Q.E.D.**

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# IDENTIFYING TYPE OF RIGHT AND LEFT RETURNS TO SCALES: HYPERPLANE APPROACH

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## ABSTRACT

*According to the local nature of the RTS, two directions have been proposed, including right- and left-side directions, which are attributed to the expansion and contraction of the active DMUs. Almost all of the used methods developed for the directional RTS are limited to the predefined parameters. In light of this, even a small alteration in parameter magnitude can lead to unexpected changes in the results. To avoid such a problem, in this work, an approach for right and left RTS classification has been introduced. In contrast with the ordinary methods, this method is fully free-parameter, supporting advantages such as simplicity, linear programming basis, and accuracy. We compare our approach with an ordinary RTS determination technique. The results confirmed that the method route led to a desirable stability and accuracy.*

**Keywords:** Data envelopment analysis; Efficiency; Right and left returns to scale; mixed integer programming.

## INTRODUCTION

Data Envelopment Analysis (DEA) is a technique based on mathematical programming for the performance assessment and evaluation of the efficiency of a set of homogeneous Decision Making Units (DMUs), each of which consumes multiple inputs to produce multiple outputs. The CCR [1] and the BCC [2] models are two Basic DEA models. The latter is established by developing a variable RTS version of the first. One of the important subjects in DEA is the concept of returns to scale (RTS), which is defined as the ratio of the proportional changes in outputs to the proportional changes in inputs. Nowadays, RTS has allocated a wide contribution of DEA literature to itself. First, Banker [3] introduced RTS estimation of the BCC model and also presented an approach that used the sign of the slope parameter of the BCC model. Several studies have been performed in regard to the RTS status identification. While reviewing these studies, it was found that: a) input and output oriented models may give different results in their RTS findings; and b) the RTS estimated by means of these methods holds only in the current position of the DMU under-evaluation. Focusing on these two points, Golany and Yu [4] discussed the estimation of RTS to the right and left neighborhood of the given DMU and proposed a method based on solving two LP models to do this task. Hadjicostas and Soteriou [5] have presented a more general definition of these two concepts in RTS from a scale elasticity measure point of view and was based on definition, and few assumptions in order to identify the type of right and left RTS. Jahanshahloo et al. [6] and Allahyar and Rostamy-Malkhalifeh [7] also attempted to not only tackle the problems of the GY method [4] but to also determine the type of right and left RTS for each particular DMU. However, following review of the studies on right and left RTS, we could find that almost all are including parameters similarly and by small changes in the magnitude of a parameter the results would change significantly. Eslami and Khoveyni [8] using a multiplier model, tried to determine the status of the right and left RTS. Their method is distinct from other existing methods, but is nonlinear and, due to strict inequalities, is

parametric. Therefore, the results can be achieved, but with difficulty and with a high dependence on parameters.

The current paper considers the problem of distinguishing the type of right and left RTS via a simple approach, similar to the definition of right and left RTS introduced in [5] with a main difference being that it does not depend any assumptions. This approach is based on finding the constant value of right and left defining hyperplanes of each DMU.

The remainder of this paper is structured as follows: the second section describes, briefly, right and left RTS identification. Third section includes the presented approach and a main theorem as well as a numerical example to illustrate our purposes. Finally, in the last section, concluding comments are made and the overall idea summarized.

## PRELIMINARIES

Consider a set of  $n$  DMUs to be evaluated,  $DMU_j$  ( $j = 1, \dots, n$ ), each one consumes  $m$  semi-positive inputs  $\mathbf{x}_j = (x_{1j}, x_{2j}, \dots, x_{mj}) \geq \mathbf{0}_m$  and  $\mathbf{x}_j \neq \mathbf{0}_m$  to produce  $s$  semi-positive outputs  $\mathbf{y}_j = (y_{1j}, y_{2j}, \dots, y_{sj}) \geq \mathbf{0}_s$  and  $\mathbf{y}_j \neq \mathbf{0}_s$ . The production possibility set (PPS) is the set of all inputs and outputs that inputs can produce output. The PPS under variable returns to scale (VRS) assumption is defined as:

$$T_v = \{(\mathbf{x}, \mathbf{y}) | \mathbf{x} \geq \sum_{j=1}^n \lambda_j \mathbf{x}_j, \mathbf{y} \leq \sum_{j=1}^n \lambda_j \mathbf{y}_j, \sum_{j=1}^n \lambda_j = 1, \lambda_j \geq 0 \forall j\}$$

The outer boundary of PPS is known as the efficient frontier that is constructed by several defining hyperplanes. The following well-known BCC model evaluates the considered unit,  $DMU_o$  ( $o \in \{1, \dots, n\}$ ), under VRS assumption [2]:

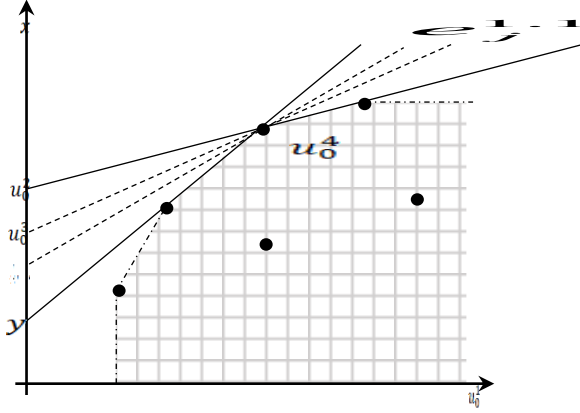
$$\begin{aligned} \theta_o^* = \min \quad & \theta \\ \text{s. t.} \quad & \sum_{j=1}^n \lambda_j x_{ij} \leq \theta x_{io} \quad i = 1, \dots, m \\ & \sum_{j=1}^n \lambda_j y_{rj} \geq y_{ro} \quad r = 1, \dots, s \\ & \sum_{j=1}^n \lambda_j = 1 \\ & \lambda_j \geq 0 \quad j = 1, \dots, n \end{aligned} \tag{1}$$

$DMU_o$  is efficient if  $\theta_o^* = 1$ . Furthermore,  $\theta_o^*$  can be any positive number less than one for inefficient DMUs. The dual of model (1) is as follows:

$$\begin{aligned} \max \quad & \sum_{r=1}^s u_r y_{ro} - u_0 \\ \text{s. t.} \quad & \sum_{i=1}^m v_i x_{io} = 1 \\ & \sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} - u_0 \leq 0 \quad j = 1, \dots, n \\ & v_i \geq 0, u_r \geq 0 \quad i = 1, \dots, m, r = 1, \dots, s \end{aligned} \tag{2}$$

where  $v_i$  and  $u_r$  are  $i^{\text{th}}$  input and  $r^{\text{th}}$  output weights, respectively. On the other hand, the optimal values of these weights, along with  $u_0$ , result a supporting, or defining, hyperplane that the DMU projects (for

more details about hyperplane see [9]). Fig 1 illustrates some possible hyperplanes in a two dimensional example. The dashed lines,  $L_1$  and  $L_4$ , are supporting hyperplanes and solid lines,  $L_2$  and  $L_3$ , are defining ones related to  $DMU_o$ . Furthermore,  $u_0^1, \dots, u_0^4$  are y-intercepts regarding  $L_1, \dots, L_4$ , respectively.



**Fig1. Illustration of supporting and defining hyperplane in a two-dimensional  $T_v$ .**

#### Golany and Yu (GY) method

Focusing on the fact that the RTS is a local phenomenon, [4] proposed the right and left RTS (RRTS and LRTS) concepts and suggested the following two models with the aim of investigating the RTS status in the immediate neighborhood of under evaluated and efficient  $DMU_o$  from the right and left sides, respectively:

$$\begin{aligned}
 & \min \alpha_0 - \varepsilon(\sum_{i=1}^m s_i^- + \sum_{r=1}^s s_r^+) \\
 & \text{s. t. } \sum_{j=1}^n \lambda_j x_{ij} + s_i^- = \alpha_0 x_{io} \quad i = 1, \dots, m \\
 & \quad \sum_{j=1}^n \lambda_j y_{rj} - s_r^+ = (1 + \delta) y_{ro} \quad r = 1, \dots, s \\
 & \quad \sum_{j=1}^n \lambda_j = 1 \\
 & \quad \lambda_j \geq 0, s_i^- \geq 0, s_r^+ \geq 0 \quad j = 1, \dots, n, i = 1, \dots, m, r = 1, \dots, s
 \end{aligned} \tag{3}$$

$$\begin{aligned}
 & \max \beta_0 + \varepsilon(\sum_{i=1}^m s_i^- + \sum_{r=1}^s s_r^+) \\
 & \text{s. t. } \sum_{j=1}^n \lambda_j x_{ij} + s_i^- = (1 - \delta) x_{io} \quad i = 1, \dots, m \\
 & \quad \sum_{j=1}^n \lambda_j y_{rj} - s_r^+ = \beta_0 y_{ro} \quad r = 1, \dots, s \\
 & \quad \sum_{j=1}^n \lambda_j = 1 \\
 & \quad \lambda_j \geq 0, s_i^- \geq 0, s_r^+ \geq 0 \quad j = 1, \dots, n, i = 1, \dots, m, r = 1, \dots, s
 \end{aligned} \tag{4}$$

Where  $\varepsilon$  is a non-Archimedean infinitesimal and the parameter  $\delta$  assumes a positive small arbitrary number to ensure the projection occurs in the immediate neighborhood of  $DMU_o$ . It is noteworthy to mention that the right, or left, side is the side of increasing inputs, or decreasing outputs in the case of left side, proportionally.

**Theorem 1.** Given the optimal solutions  $\alpha_0^*$  and  $\beta_0^*$  of models (3) and (4), the RRTS and LRTS of  $DMU_o$  is determined as follows, respectively:

1) RRTS status:

- 1.i) The RRTS of  $DMU_o$  is increasing if  $1 < \alpha_0^* < 1 + \delta$ .
- 1.ii) The RRTS of  $DMU_o$  is constant if  $\alpha_0^* = 1 + \delta$ .
- 1.iii) The RRTS of  $DMU_o$  is decreasing if  $\alpha_0^* > 1 + \delta$ .
- 1.iv) If model (3) is infeasible, then it can be concluded that there is no data to determine RRTS of  $DMU_o$ .

2) LRTS status:

- 2.i) The LRTS of  $DMU_o$  is decreasing if  $1 - \delta < \beta_0^* < 1$ .
- 2.ii) The LRTS of  $DMU_o$  is constant if  $\beta_0^* = 1 - \delta$ .
- 2.iii) The LRTS of  $DMU_o$  is increasing if  $\beta_0^* < 1 - \delta$ .
- 2.iv) If model (4) is infeasible, then it can be concluded that there is no data to determine LRTS of  $DMU_o$ .

*Proof.* See [5]. ■

#### Presented Method

Giving our concern to eliminate the undesirable effects of parameter dependency in the RTS classification, we introduce an approach to identify the right and left RTS status of each efficient DMU. The method is based on the signs of the lower and upper bounds of  $u_0^*$ . The following theorem expresses the RRTS and LRTS classifications. Although it is similar to the definition presented in [5], it is not based on any extra assumptions or rely on a theorem. For our purposes, we consider two following auxiliary models that Banker and Thrall [10] introduced for RTS identification:

$$\begin{aligned}
 u_0^+ &= \max u_0 \\
 \text{s. t. } &\sum_{i=1}^m v_i x_{io} = 1 \\
 &\sum_{r=1}^s u_r y_{ro} + u_0 = 1 \\
 &\sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} - u_0 \leq 0 \quad j = 1, \dots, n \\
 &v_i \geq 0, u_r \geq 0 \quad i = 1, \dots, m, \quad r = 1, \dots, s
 \end{aligned} \tag{5}$$

$$\begin{aligned}
 u_0^- &= \min u_0 \\
 \text{s. t. } &\sum_{i=1}^m v_i x_{io} = 1 \\
 &\sum_{r=1}^s u_r y_{ro} + u_0 = 1 \\
 &\sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} - u_0 \leq 0 \quad j = 1, \dots, n \\
 &v_i \geq 0, u_r \geq 0 \quad i = 1, \dots, m, \quad r = 1, \dots, s
 \end{aligned} \tag{6}$$

In fact,  $u_0^+$  ( $u_0^-$ ) is the constant component in the equation of a right (left) defining hyperplane. In addition, there are several hyperplanes in each right (left) side and  $u_0^+$  ( $u_0^-$ ) is the maximum (minimum) value between them.

**Theorem 5.** Let  $u_0^+$  and  $u_0^-$  be the optimal objective values of model (5) and model (6) related to an efficient  $DMU_o$ , respectively, then the RRTS and the LRTS are determined as follows:

$u_0^+ > 0 \Rightarrow$  RRTS is decreasing.

$u_0^+ = 0 \Rightarrow$  RRTS is constant.

$u_0^+ < 0 \Rightarrow$  RRTS is increasing.

$u_0^- > 0 \Rightarrow$  LRTS is decreasing.

$u_0^- = 0 \Rightarrow$  LRTS is constant.

$u_0^- < 0 \Rightarrow$  LRTS is increasing.

*Proof.* **Case (i):** Let  $(\mathbf{u}^*, \mathbf{v}^*, u_0^+)$  be the optimal solution of model (5). Now consider a dummy DMU  $\mathbf{z}_\alpha = (\alpha \mathbf{x}_o, \mathbf{y}_o)$  where  $\alpha = 1 + \delta$  and  $\delta$  is a very small positive number. Since  $u_0^+ > 0$ ,  $\mathbf{u}^{*t}(1 + \delta)\mathbf{y}_o - \mathbf{v}^{*t}(1 + \delta)\mathbf{x}_o - u_0^+ = (\mathbf{u}^{*t}\mathbf{y}_o - \mathbf{v}^{*t}\mathbf{x}_o - u_0^+)(1 + \delta) + \delta u_0^+ > 0$ . This inequality reveals that  $\mathbf{z}_\delta$  is an outer point of PPS. This occurs in two situations: 1- there exists a point like  $(\alpha \mathbf{x}_o, (1 + \delta)\mathbf{y}_o) \in \text{PPS}$  such that  $\alpha > (1 + \delta)$ , then according to theorem 1, case (1.iv), the right RTS status of DMU<sub>o</sub> is identified decreasing. 2- there doesn't exist any point like  $(\alpha \mathbf{x}_o, (1 + \delta)\mathbf{y}_o) \in \text{PPS}$  such that  $\alpha > (1 + \delta)$ , then  $\mathbf{z}_\delta$  lies on an inefficient part of frontier from the right side which also shows decreasing RTS status for DMU<sub>o</sub>. **Case (ii),**  $\mathbf{u}^{*t}(1 + \delta)\mathbf{y}_o - \mathbf{v}^{*t}(1 + \delta)\mathbf{x}_o - u_0^+ = 0$ , which implies that right RTS status of DMU<sub>o</sub> is constant. **Case (iii),**  $\mathbf{u}^{*t}(1 + \delta)\mathbf{y}_o - \mathbf{v}^{*t}(1 + \delta)\mathbf{x}_o - u_0^+ < 0$ , thus  $\mathbf{z}_\delta$  is an interior point of PPS and there exists a point  $(\alpha \mathbf{x}_o, (1 + \delta)\mathbf{y}_o) \in \text{PPS}$  such that  $1 < \alpha < (1 + \delta)$ . Then according to theorem 1, case (1.i), the right RTS status of DMU<sub>o</sub> is increasing.

**Cases (iv), (v) and (vi):** Let  $(\mathbf{u}^*, \mathbf{v}^*, u_0^-)$  be the optimal solution of model (6) and consider  $\bar{\mathbf{z}}_\delta = ((1 - \delta)\mathbf{x}_o, (1 - \delta)\mathbf{y}_o)$ , the rest of ratiocination is similar to the above cases. ■

### Numerical example

Here, an example is presented including 12 DMUs with two inputs ( $x_1, x_2$ ) and two outputs ( $y_1, y_2$ ) whose data have been extracted from [6]. The data for these DMUs are listed in Table . Efficient DMUs {1,4,6,7,9,10,11,12}, as well as the results obtained from models (5) and (6) and theorem 2, are indicated in the first five columns of Table 2, while the results of the GY method with two different choices for parameters are reported in the other columns. As indicated in Table 2, the GY method is potentially unable to detect the type of RTS. These findings reveal how sensitive the results are in parameter-based approaches. For instance, DMU<sub>9</sub> and DMU<sub>11</sub> have decreasing LRTS for  $\varepsilon = \delta = 0.0001$  while they have increasing LRTS for  $\varepsilon = \delta = 0.01$ . Comparison of the results of the proposed and the GY methods in Table 2 reveals the similarities of the presented results with the GY method when  $\varepsilon = \delta = 0.01$ , except when their model does not answer. Although the GY method may be supposed to obtain better results by smaller parameter values, it leads to worse results in this comparison.

**Table 1: Input-output data.**

DMU#	1	2	3	4	5	6	7	8	9	10	11	12
$x_1$	6	7	6	7	5	4	5	6	6	8	5	9
$x_2$	4	5	5	4	5	6	8	7	6	8	7	6
$y_1$	6	4	3	5	3	3	5	3	4	6	5	9
$y_2$	2	3	3	4	2	3	6	2	5	6	5	3

**Table 2: The obtained results of the proposed and GY methods for efficient DMUs.**

DMU#	Presented method				GY method			
	$u_0^+$	$u_0^-$	RRTS	LRTS	$\varepsilon = \delta = 0.0001$		$\varepsilon = 0.1, \delta = 0.01$	
					RRTS	LRTS	RRTS	LRTS
1	0	-1	CRS	IRS	CRS	N.F.S*	CRS	N.F.S
4	1.67	-1	DRS	IRS	DRS	N.F.S	DRS	N.F.S
6	-0.63	-1	IRS	IRS	IRS	N.F.S	IRS	N.F.S
7	5.2	-0.6	DRS	IRS	N.F.S	DRS	N.F.S	IRS
9	0.67	-0.58	DRS	IRS	DRS	DRS	DRS	IRS
10	3.25	0.5	DRS	DRS	N.F.S	DRS	N.F.S	IRS
11	0	-0.58	CRS	IRS	CRS	DRS	CRS	IRS
12	2.89	0	DRS	CRS	N.F.S	CRS	N.F.S	CRS

\* No feasible solution

## CONCLUSIONS

The status of RTS is a valuable tool for a manager who wants to know how he/she can improve the operations of a unit. Usually, he/she is interested in knowing how increasing or decreasing inputs can affect the outputs. This paper considered the issue of recognition of the right and left RTS related to efficient DMUs. Correct diagnosis of the right and left RTS of each DMU that is going to make an expansion or contraction plan for their operation is highly important. That said, incorrect detection may lead to a DMU's bankruptcy. All models provided for this purpose are on the basis of parametric models in the DEA field. The aim of this paper is to provide a parameter-free method. The presented approach is based on the defining hyperplanes located on the right side (in the direction of increasing all inputs) and the left side (in the direction of reducing all outputs) of each efficient DMU under evaluation. The results presented in the application section show that for a small change in the magnitude of the parameters using the Golany & Yu method as a parametric method may identify different types of right or left RTS, while the proposed approach always gives a unique identification. Since in practice the existence of some uncontrollable inputs or outputs such that any changes in their magnitude is not possible, the concept of the right and left hyperplane is not found to be useful and should only lead to partial modification in the appropriate directions of the considered hyperplanes. Therefore, developing accurate and parameter-free models to identify directional returns to scale can be suggested for future studies.

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# IMPROVING AN INTEGRATED INTUITIONISTIC FUZZY ELECTRE & DEA METHODOLOGY

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## ABSTRACT

*The purpose of this case is to present a new multi-expert integrating methodology - based Intuitionistic Fuzzy ELECTRE and Data Envelopment Analysis (DEA) to choose the decision making units (DMUs) with most efficiency. Intuitionistic Fuzzy sets (IFS) have been selected since they at the same time ensure; the membership, non-membership, hesitancy function. This methodology do not modify the classical DEA; rather, it improves the analysis by improving full ranking in the classical DEA situation for all DMUs by gather both individual and hesitation opinion for evaluation the importance of criteria and DMUs.*

**Keywords:** multi-criteria decision-making (MCDM), data envelopment analysis (DEA), decision making units (DMU), Elimination Et Choix Traduisant La REalite (ELECTRE), Intuitionistic Fuzzy Set (IFS).

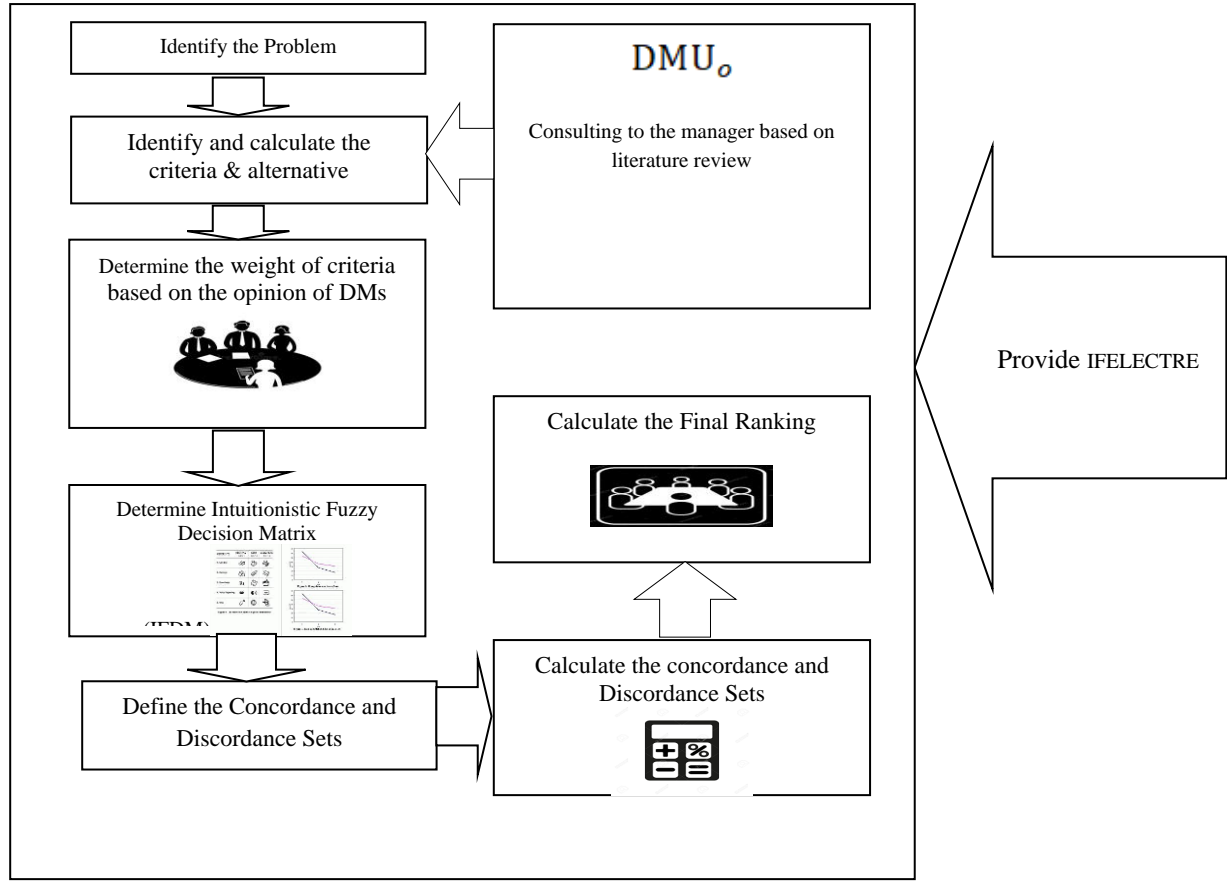
## INTRODUCTION

Multi-Criteria Decision-Making (MCDM) is use to resolve performance evaluation problems with multiple criteria problem, to assist us in detection the best decision making units (DMU). Resolving complicated multi criteria problems with several criteria is one of the best improving areas of research process. (Triantaphyllou, 2000; Yang & Hung, 2007) However, the original MCDM cannot effectively work with vagueness data in MCDM. Zadeh (1965) suggested the fuzzy set theory (FST), which is applied to MCDM problems. It is an effective tool to use for vagueness data and is more natural for humans to express. In the real world, knowledge is often more fuzzy. (Vahdani & Hadipour, 2011) The Fuzzy set (FS) is a suitable tool to deal with imprecise.

DEA has been thoroughly popular for classification DMUs. It applies a DMU to multiple-variate methods with multiple-inputs and multiple-outputs data. The proposed integration methodology has been utilized in different setting to combination them into real world problems. The integrated methodology is basic, easy to utilize, and flexible on data practicable to any number of DMUs. Moreover, it is effective to in confusing problems with a large number of DMUs. An integration of IFELECTRE & DEA is proposed to handle overcome the specific faults of each method with the goal of measuring the performance of DMUs. DEA can only classify DMUs into efficient and less-efficient DMU, and intuitionistic Fuzzy Set (IFS) is described by three types; membership function, non-membership function, and hesitancy function. (Xu, 2014) A combination methodology is proposed to prevent the pitfalls of each model separately. (Sinuany-stern et. al., 2000)

## METHODOLOGY

In this part, we offer an Intuitionistic Fuzzy ELECTRE for the evaluation of the DMUs performance. An algorithm takes into consideration the decisions ensured by multiple DMs. The flowchart of the IFELECTRE steps is shown in Figure 1.



**Figure 1: The flowchart of the IFELECTRE steps**

Step 1: Apply the IFELECTRE method

Step 1.1: At the first step, define the problem and set up the hierarchy, goal, criteria, and DMUs.

Step 1.2: Determine weight of criteria for inputs and outputs, the experts' team using a standard 1–9 scale (Saaty, 1980,1996). Both the distance from each expert with k person can be gathered as the distances of the DMs. Calculate geometric mean of data is as in follows:

$$D_i = \left( \prod_{j=1}^k D_{ij} \right)^{1/k} \quad i = 1, \dots, m$$

Step 1.3: Determine the weight of the group DM's, the multiple inputs given by DM's team within in a DMs do not necessarily carry equal values and are referred to as linguistic terms in IFNs in Table 1.

Next, the aggregated opinions of the experts are collect using the following equations:

$$\lambda_l = \frac{(\mu_l + \pi_l(\frac{\mu_l}{\mu_l + v_l}))}{\sum_{l=1}^k (\mu_l + \pi_l(\frac{\mu_l}{\mu_l + v_l}))} \quad \text{where } \lambda_l \in [0,1] \text{ and } \sum_{l=1}^k \lambda_l = 1 \quad (\text{Rouyendegh, 2011})$$

Where  $\mu_l$ : membership,  $v_l$ : non-membership,  $\pi_l$ : hesitation,

**Table 1. Linguistic Term for Rating DMs (Rouyendegh, 2012)**

Linguistic terms	IFNs
Very serious	(0.80, 0.10)
Serious	(0.50, 0.20)
Average	(0.50, 0.50)
Worse	(0.3, 0.50)
Very worse	(0.20, 0.70)

**Step1.4: Determine an Aggregate Intuitionistic Fuzzy Decision Matrix**

The Aggregate Intuitionistic Fuzzy Decision Matrix is calculated by employing a *Fuzzy Weighted Averaging* operator. (Szmidt 2000; Grzegorzewski, 2004, Rouyendeg, 2011, 2012) In order to reach a precise conclusion each individual opinion obtained from DMs should be merged into a single opinion to build the Aggregate Intuitionistic Fuzzy Decision Matrix model. In this case, construct pairwise comparison for data, and collect each individual opinion utilizing linguistic terms from Table 2.

**Table 2: Linguistic Terms for Rating the DMUs (Rouyendegh,2012)**

Linguistic terms	IFNs
Extremely good (EG)	[1.00; 0.00;0.00]
Very good (VG)	[0.85;0.05; 0.10]
Good (G)	[0.70; 0.20;0.10]
Medium bad (MB)	[0.50; 0.50;0.00]
Bad (B)	[0.40; 0.50;0.10]
Very bad (VB)	[0.25; 0.60;0.15]
Extremely bad (EB)	[0.00, 0.90,0.10]

Let  $R^{(l)} = (r_{ij}^{(l)})_{m \times n}$  be an Intuitionistic Fuzzy Decision Matrix of each DM.  $\lambda = \{\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_k\}$  is the weight of DM.

$R = (r_{ij})_{m \times n}$ , Where

$$r_{ij} = IFWA_{r_{\lambda}}(r_{ij}^{(1)}, r_{ij}^{(2)}, \dots, r_{ij}^{(k)}) = \lambda_1 r_{ij}^{(1)} + \lambda_2 r_{ij}^{(2)} + \dots + \lambda_k r_{ij}^{(k)}$$

$$= \left[ 1 - \prod_{l=1}^k (1 - \mu_{ij}^{(l)})^{\lambda_l}, \prod_{l=1}^k (\nu_{ij}^{(l)})^{\lambda_l}, \prod_{l=1}^k (1 - \mu_{ij}^{(l)})^{\lambda_l} - \prod_{l=1}^k (\nu_{ij}^{(l)})^{\lambda_l} \right]$$

$$R_{ij} = \begin{bmatrix} r_{11} & \dots & r_{1j} \\ \vdots & \ddots & \vdots \\ r_{i1} & \dots & r_{ij} \end{bmatrix} \text{ where } R_{ij} = (\mu_{ij}, \nu_{ij}, \pi_{ij}) \text{ (} i = 1, 2, \dots, m; j = 1, 2, \dots, n \text{) is an element of an Aggregate}$$

Intuitionistic Fuzzy Decision Matrix.

Step 1.5: Calculate an aggregated weighted Intuitionistic Fuzzy Decision Matrix, find the summation of the weight of the criteria and determine the ratings given to the DMUs. The aggregated weighted intuitionistic fuzzy decision matrix is established.  $W$  and  $R$  are two IFSs of the set  $X$ . (Atanassov, 1986):

$R'_{ij} = w * R_{ij}$ ;  $R'_{ij} = (\mu'_{ij}, \nu'_{ij}, \pi'_{ij})$  is an element of an Aggregate Intuitionistic Fuzzy Decision Matrix,  $R$  an Intuitionistic Fuzzy Decision Matrix.

Step 1.6: Calculate the concordance indexes  $C_{xy}$  (confidence in pair-wise comparison of  $x$  and  $y$  DMUs)

$$C_{xy}^1 = \{l | \mu_{xl} \geq \mu_{ly}, v_{xl} \geq v_{ly} \text{ and } \pi_{xl} \geq \pi_{ly} \}$$

$$C_{xy}^2 = \{l | \mu_{xl} \geq \mu_{ly}, v_{xl} < v_{ly} \text{ and } \pi_{xl} \geq \pi_{ly} \} \quad C_{xy}^3 = \{l | \mu_{xl} \geq \mu_{ly}, v_{xl} \geq v_{ly} \}$$

Step 1.7: Calculate the discordance indexes  $D_{xy}$  (degree of disagreement in  $(X_x \rightarrow X_y)$ )

$$D_{xy}^1 = \{l | \mu_{xl} < \mu_{ly}, v_{xl} \geq v_{ly} \text{ and } \pi_{xl} \geq \pi_{ly} \}$$

$$D_{xy}^2 = \{l | \mu_{xl} < \mu_{ly}, v_{xl} \geq v_{ly} \text{ and } \pi_{xl} \geq \pi_{ly} \} \quad D_{xy}^3 = \{l | \mu_{xl} < \mu_{ly}, v_{xl} < v_{ly} \}$$

Step 1.8: Calculate the *Concordance Matrices*

$$C_{xy} = w_c^1 * \sum_{l \in C_{xy}^1} w_l + w_c^2 * \sum_{l \in C_{xy}^2} w_l + w_c^3 * \sum_{l \in C_{xy}^3} w_l$$

Step 1.9: Calculate the *dis-concordance matrices*

$$D_{xy} = \frac{\max_{j \in D_{ij}} w_D * dis(x_{il}, x_{lj})}{\max_{j \in L} dis(x_{il}, x_{lj})}$$

Step 1.10: Calculate the distance among  $X_{il}$  and  $X_{lj}$

$$dis(x_{il}, x_{lj}) = \sqrt{\frac{1}{2}((\mu_{il} - \mu_{lj})^2 + (v_{il} - v_{lj})^2 + (\pi_{il} - \pi_{lj})^2)}$$

Step 1.11: Calculate the  $L_{ij}$  and  $K_{ij}$  matrices

$$C_{xy} \geq \bar{C} \text{ Where, } \bar{C} = \frac{\sum_{x=1, x \neq y}^m \sum_{y=1, y \neq x}^m C_{xy}}{m * (m - 1)}, D_{xy} \geq \bar{D} \text{ Where, } \bar{D} = \frac{\sum_{x=1, x \neq y}^m \sum_{y=1, y \neq x}^m D_{xy}}{m * (m - 1)}$$

The Concordance Dominances Matrices ( $L_{ij}$ ) and Discordance Dominances Matrices ( $K_{ij}$ )

$$L_{ij} = E' - E_{ij} \text{ where } E' = \text{The Maximum value of } E_{ij}$$

$$K_{ij} = F' - F_{ij} \text{ where } F' = \text{The Maximum value of } F_{ij}$$

Step 1.12: Calculate *Aggregate Dominances Matrix (T)* Matrix ( $T$ ) where  $T_{ij} = \frac{K_{ij}}{L_{ij}} + K_{ij}$

Step 1.13: At the last step the whole weight is determined

$$\bar{w} = \sum_{j=1, i=j}^m T_{ij} \quad X^* = \text{Max } \bar{w}$$

## Step 2: DEA

Step 2.1: Use the data from step 1, and determine the decision matrix from the modified DEA algorithm ( $e_{k,k'}$ ). (Rouyendegh, 2010)

$$\begin{aligned}
e_{k,k'} &= \max \sum_{r=1}^t u_r y_{rk} \\
s. t. \\
\sum_{i=1}^m v_i x_{rk} &= 1 \\
\sum_{r=1}^s u_r y_{rk} - \sum_{i=1}^m v_i x_{rk} &\leq 0 \\
\sum_{r=1}^s u_r y_{rk'} - \sum_{i=1}^m v_i x_{rk'} &\leq 0 \\
u_r &\geq 0, v_i \geq 0 \quad r = 1, \dots, t \quad i = 1, \dots, m
\end{aligned}$$

$e_{k,k'}$  matrix include the inputs and outputs modified model.

$$(e_{k,k'})_{m \times n} = \begin{bmatrix} e_{1,1} & e_{1,2} & \dots & e_{1,k'} \\ e_{2,1} & e_{2,2} & \dots & e_{2,k'} \\ e_{k,1} & e_{k,1} & \dots & e_{k,k'} \end{bmatrix}_{m \times n}$$

Where, the pair-wise comparison of e matrix ( $k' = 1, \dots, n$ ,  $k = 1, \dots, n$  and  $k \neq k'$ ).

Step 2.2: The calculation of  $a_{k,k'}$  in DEA modified inputs & outputs matrix.

$$a_{k,k'} = \frac{e_{k,k'}}{e_{k',k}} \quad (A_{k,k'})_{m \times n} = [a_{k,k'}]_{m \times n} = \begin{bmatrix} a_{1,1} & a_{1,2} & \dots & a_{1,k'} \\ a_{2,1} & a_{2,2} & \dots & a_{2,k'} \\ a_{k,1} & a_{k,1} & \dots & a_{k,k'} \end{bmatrix}_{m \times n}$$

Step 2.3: Normalized matrix  $a'_{k,k'}$  in DEA modified inputs & outputs matrix.

$$a'_{k,k'} = \frac{a_{k,k'}}{\sum_{k=1}^n a_{k,k'}} \quad (A'_{k,k'})_{m \times n} = [a'_{k,k'}]_{m \times n} = \begin{bmatrix} \frac{a_{1,1}}{\sum a_{k,1}} & \dots & \frac{a_{1,k'}}{\sum a_{k,k'}} \\ \frac{a_{2,1}}{\sum a_{k,1}} & \dots & \frac{a_{2,k'}}{\sum a_{k,k'}} \\ \frac{a_{3,1}}{\sum a_{k,1}} & \dots & \frac{a_{3,k'}}{\sum a_{k,k'}} \\ \frac{a_{k,1}}{\sum a_{k,1}} & \dots & \frac{a_{k,k'}}{\sum a_{k,k'}} \end{bmatrix}_{m \times n}$$

Where, Each component of the matrix obtained is divided by that column's total score

Step 2.4: The column vector components are calculated by summation over the rows.

$$a''_{k,k'} = \sum_{k=1}^n a'_{k,k'}$$

Step 2.5: Final, normalized matrix in DEA modified inputs & outputs matrix.

$$a'''_{k,k'} = \frac{a''_{k,k'}}{\sum_{k=1}^n a''_{k,k'}}$$

## CONCLUSIONS

The proposed new IFELECTRE & modified DEA integrated methodology could apply the linguistic term measurement of multi DMs teams. This methodology observed that new integrated methodology offer a commitment tool if combination methodology used to show potential result developments, but each method has limitations. Intuitionistic Fuzzy ELECTRE can be used to commit more mix and complex

MCDM problems, where the DMs and experts team have several uncertainty and hesitation in appointing precedence values to the objects noted. Classical DEA needs deterministic or certain information, but new methodology deal data with qualitative and quantitative. However, the integration algorithm is that the Intuitionistic Fuzzy ELECTRE has been accumulating mathematically from multiple data. As a Intuitionistic Fuzzy ELECTRE & DEA methodology included in the data set with qualitative and quantitative, the DMUs are determined as relatively fully-ranked. This algorithm propose the best of both method by avoiding the pitfalls of each method.

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# USING INTERACTIVE MOLP METHODS TO SOLVE IMPRECISE DEA

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## ABSTRACT

The basic data envelopment analysis (DEA) determines efficiency of a set decision making units (DMUs) in exact input-output environments. For imprecise data, some methods have been developed to calculate the efficiency scores. The aim of this paper is showing an equivalence relation between one of the model of data envelopment analysis with imprecise data and multi objective linear programming. Relation between DEA and multiple objective linear programming(MOLP) conducted to using interactive multiple objective models for solving the DEA problem in exact situation and find the most preferred solution. In this paper, the authors will show how an imprecise DEA(IDEA) model can be equivalent by MOLP model and using an interactive method with name PROJECT method for solve the IDEA model. The PROJECT method is able to obtain any efficient solution, contain nonsupported efficient solution and also identify the MPS, even in nonconvex cases. also, we will use the data envelopment scenario analysis (DESA) model since this model focuses on decreases total input consumption and increase total output production which results in solving one mathematical model instead of n models.

**Keywords:** *Data Envelopment Analysis, Imprecise Data, interactive Multiple Objective, PROJECT method*

## INTRODUCTION

The traditional data envelopment analysis (DEA) measures the relative efficiencies of a set of decision making units with exact data of inputs and outputs. DEA model [charnes et al, 1987], originally is a non-linear fractional mathematical programming model, known as the Charnes, Cooper and Rhodes (CCR) model. In many real decision and planning problems, the problem consist in optimizing (maximizing or minimizing) several conflicting objective functions. In these cases, it is impossible to find a solution which optimized all the objective functions simultaneously. In recent decades, many methods have been developed to solve multi objective linear programming (MOLP). Recently, the relation between data envelopment analysis (DEA) and multiple objective linear programming [Ebrahimnejad, 2012], have been studied from several viewpoints by many researchers. It would be worth mentioning that important result obtained by this equivalent relation is using interactive MOLP to solve DEA model and obtain most preferred solution (MPS). Also, Cooper et al (1999) introduced application of DEA whose data was imprecise. In this paper we want using the equivalence relation between MOLP and DEA models in exact environment, and expansion this relation between MOLP and DEA models with imprecise data. Following that, we use the interactive MOLP method with name PROJECT method to solve the imprecise DEA.

### Data Envelopment Scenario Analysis (DESA)

At the first we introduce the model proposed by Thanassoulis and Dyson (1992), the index sets of inputs  $I = \{1...m\}$  and outputs  $O = \{1,...,s\}$  their subsets  $I \equiv I_g \cup \bar{I}_g$  and  $O \equiv O_g \cup \bar{O}_g$  where  $I_g$  and  $O_g$  are used to display inputs and outputs where bounds of target achievements are imposed in the construction of the following target model:

$$\begin{aligned}
 & \underset{\lambda_j, Z_k, \theta_i}{Max} \quad \sum_{k \in O} P_k^+ Z_k - \sum_{i \in I} P_i^- \theta_i \\
 & st \quad \sum_{j=1}^n \lambda_j x_{ij} = \theta_i x_{io} \quad i \in I \\
 & \quad \sum_{j=1}^n \lambda_j y_{kj} = Z_k y_{ko} \quad k \in O \\
 & \quad \theta_i x_{io} \geq G_i \quad i \in I_g \\
 & \quad Z_k y_{ko} \leq G'_k \quad k \in O_g \\
 & \quad A_i \leq \theta_i \leq 1/B_i, \quad A_i, B_i \in [0,1], \quad \forall i \in I \\
 & \quad \Gamma_k \leq 1/Z_k \leq 1/\Delta_k, \quad \Gamma_k, \Delta_k \in [0,1], \quad \forall k \in O \\
 & \quad \lambda_j \geq 0 \quad \forall j
 \end{aligned} \tag{1}$$

Where  $x_{ij}$  and  $y_{kj}$  are the  $i$ th input and  $k$ th output, respectively.  $P_i^-$ ,  $P_k^+$  the user-specified constants reflecting the decision makers' preferences over the improvement of input/output components,  $\theta_i$  is contraction rate of input  $i$  and  $Z_k$  is development rate of output  $k$ .  $G_i$  and  $G'_k$  are bounds for assessment targets of input  $i$  and output  $k$ ,  $A_i, B_i$  are remarked as the lower and upper bounds of  $\theta_i$  also  $\Gamma_k, \Delta_k$  are regarded as the lower and upper bounds of  $Z_k$ .

### DESA Model with Imprecise Data

In this section, we will consider the DESA with imprecise data, such that in this model we can solve one model instead of  $n$  models, this model can be formulated as follows:

$$\begin{aligned}
 & \underset{\lambda_j, Z_k, \theta_i}{Max} \quad \sum_{k \in O} P_k^+ Z_k - \sum_{i \in I} P_i^- \theta_i - M \sum_{i \in I_f} s'_i - M \sum_{k \in O_f} s''_k \\
 & st \quad \sum_{r=1}^n \sum_{j=1}^n \lambda_{jr} x_{ij} = \theta_i \sum_{j=1}^n x_{ij} \quad i \in I \\
 & \quad \sum_{r=1}^n \sum_{j=1}^n \lambda_{jr} y_{kj} = Z_k \sum_{r=1}^n y_{kr} \quad k \in O \\
 & \quad \sum_{j=1}^n \theta_i x_{ij} \geq G_i^L - s'_i \quad i \in I_f \\
 & \quad \sum_{r=1}^n Z_k y_{kr} \leq G_k'^U + s''_k \quad k \in O_f \\
 & \quad \sum_{j=1}^n \lambda_{jr} = 1 \quad \forall r \\
 & \quad \lambda_j \geq 0 \quad \forall j, \quad \theta_i \text{ free}, \quad Z_k \text{ free}, \quad k \in O \\
 & \quad s'_i \geq 0 \quad \forall i \in I_f, \quad s''_k \geq 0 \quad \forall k \in O_f
 \end{aligned} \tag{2}$$



Where  $[G_i^L, G_i^U]$  and  $[G_k'^L, G_k'^U]$  indicate the interval of existing resource and bounds for total input  $i$  consumption and total output  $k$  production, respectively.  $s_i', s_k''$  are intended for the legal of total input diminution and total output generation, respectively and  $M$  in the objective function is a penalty factor that has to be intended by decision maker.

### *Imprecise DESA Model Based On Interval Arithmetic*

In this section, the authors use Wang model (2005) for transformed above model to two new models with name undesirable model and desirable model. The following models result in the best lower bound efficiency and the best upper bound efficiency for each DMU. These interval DEA models are suitable for interval input and output data rather than for crisp input and output data. Where  $x_{ij} \in [x_{ij}^L, x_{ij}^U]$ ,  $y_{kj} \in [y_{kj}^L, y_{kj}^U]$ ,  $\theta_i \in [\theta_i^L, \theta_i^U]$ ,  $Z_k \in [Z_k^L, Z_k^U]$ .

$$\begin{aligned}
 & \text{Max}_{\lambda_j, Z_k, \theta_i} \quad \sum_{k \in O} P_k^+ Z_k^L - \sum_{i \in I} P_i^- \theta_i^L - M \sum_{i \in I_f} s_i' - M \sum_{k \in O_f} s_k'' \\
 \text{st} \quad & \sum_{r=1}^n \sum_{j=1}^n \lambda_{jr} x_{ij}^L = \theta_i^L \sum_{j=1}^n x_{ij}^U \quad i \in I \\
 & \sum_{r=1}^n \sum_{j=1}^n \lambda_{jr} y_{kj}^U = Z_k^L \sum_{r=1}^n y_{kr}^L \quad k \in O \\
 & \sum_{j=1}^n \theta_i^L x_{ij}^U \geq G_i^U - s_i' \quad i \in I_f \\
 & \sum_{r=1}^n Z_k^L y_{kr}^L \leq G_k'^L + s_k'' \quad k \in O_f \\
 & \sum_{j=1}^n \lambda_{jr} = 1 \quad \forall r \\
 & \lambda_{jr} \geq 0 \quad \forall j, \quad \theta_i^L \text{ free}, \quad Z_k^L \text{ free}, \quad k \in O \\
 & s_i' \geq 0 \quad \forall i \in I_f, \quad s_k'' \geq 0 \quad \forall k \in O_f
 \end{aligned} \tag{3}$$

Where  $G_i^U$  is limit to total consumption of input  $x_{ij}^U$  and  $G_k'^L$  is limit to total production of output  $y_{kr}^L$

$$\begin{aligned}
 & \text{Max}_{\lambda_j, Z_k, \theta_i} \quad \sum_{k \in O} P_k^+ Z_k^U - \sum_{i \in I} P_i^- \theta_i^U - M \sum_{i \in I_f} s_i' - M \sum_{k \in O_f} s_k'' \\
 \text{st} \quad & \sum_{r=1}^n \sum_{j=1}^n \gamma_{jr} x_{ij}^L = \theta_i^U \sum_{j=1}^n x_{ij}^L \quad i \in I \\
 & \sum_{r=1}^n \sum_{j=1}^n \gamma_{jr} y_{kj}^U = Z_k^U \sum_{r=1}^n y_{kr}^U \quad k \in O \\
 & \sum_{j=1}^n \theta_i^U x_{ij}^L \geq G_i^L - s_i' \quad i \in I_f \\
 & \sum_{r=1}^n Z_k^U y_{kr}^U \leq G_k'^U + s_k'' \quad k \in O_f \\
 & \sum_{j=1}^n \lambda_{jr} = 1 \quad \forall r \\
 & \gamma_{jr} \geq 0 \quad \forall j, \quad \theta_i^U \text{ free}, \quad Z_k^U \text{ free}, \quad k \in O \\
 & s_i' \geq 0 \quad \forall i \in I_f, \quad s_k'' \geq 0 \quad \forall k \in O_f
 \end{aligned} \tag{4}$$

Where  $G_i^L$  is limit to total consumption of input  $x_{ij}^L$  and  $G_K^U$  is limit to total production of output  $y_{kr}^U$

### Establish an Equivalence Relationship Between DESA with Imprecise Data and MOLP

In this section, we want introduce one multiobjective linear program (MOLP) and show this model and imprecise model are equivalence. In desirable model, suppose that

$$\bar{f}_k(\lambda) = \sum_{r=1}^n \sum_{j=1}^n \lambda_{jr} y_{kj}^U \quad k = 1 \dots s \quad (5)$$

$$\tilde{f}_i(\lambda) = \sum_{r=1}^n \sum_{j=1}^n \lambda_{jr} x_{ij}^L, \quad i = 1 \dots m \quad \lambda = (\lambda_{11}, \lambda_{jr}, \dots, \lambda_{nm})^T \quad (6)$$

Where  $f_i'' = \tilde{f}_i(\lambda^*)$ ,  $f_k' = \bar{f}_k(\lambda^*)$

And we can obtain  $\lambda^*$  by solved follow model.

$$\begin{aligned} \text{Max} \quad & \sum_{r=1}^n \sum_{j=1}^n \lambda_{jr} y_{kj}^U \\ \text{s.t} \quad & \lambda \in \Omega \end{aligned} \quad (7)$$

$$\begin{aligned} \text{Min} \quad & \sum_{r=1}^n \sum_{j=1}^n \lambda_{jr} x_{ij}^L \\ \text{s.t} \quad & \lambda \in \Omega \end{aligned} \quad (8)$$

Such that  $f' = [f_1', \dots, f_s']$  and  $f'' = [f_1'', \dots, f_m'']$  are ideal points in object space in (5-6).

Suppose that:  $\sum_{j=1}^n x_{ij} > 0 \quad i = 1, \dots, m$   $\sum_{r=1}^n y_{kr} > 0 \quad k = 1, \dots, s$

$$w_k = \frac{1}{\sum_{r=1}^n y_{kr}^U} \quad \forall k \quad v_i = \frac{1}{\sum_{j=1}^n x_{ij}^L} \quad \forall i \quad (9)$$

$$\bar{f}_k^* = \frac{F^{\max}}{w_k} = F^{\max} \sum_{r=1}^n y_{kr}^U, \quad \tilde{f}_i^* = \frac{H^{\min}}{v_i} = H^{\min} \sum_{j=1}^n x_{ij}^L \quad \tau_i = \theta_i^U - H^{\min}, \quad \gamma_k = F^{\max} - Z_k^U \quad (10)$$

$$F^{\max} = \max_{1 \leq k \leq s} \{w_k f_k'\} = \max_{1 \leq k \leq s} \left\{ \frac{f_k'}{\sum_{r=1}^n y_{kr}^U} \right\} \quad (11)$$

$$H^{\min} = \min_{1 \leq i \leq m} \{v_i f_i''\} = \min_{1 \leq i \leq m} \left\{ \frac{f_i''}{\sum_{j=1}^n x_{ij}^L} \right\} \quad (12)$$

We can write equivalence model as follow model:

$$\begin{aligned}
& \underset{\lambda_j, Z_k, \theta_i}{Max} \quad \sum_{k \in O} P_k^+ Z_k^U - \sum_{i \in I} P_i^- \theta_i^U \\
& st \quad \tilde{f}_i(\lambda) - \theta_i^U \frac{1}{v_i} = 0 \quad i = 1, \dots, m \\
& \quad Z_k^U \frac{1}{w_k} - \bar{f}_k(\lambda) = 0 \quad k = 1, \dots, s \\
& \quad g_i v_i \leq \theta_i^U \quad i \in I_f \\
& \quad g'_k w_k \geq Z_k^U \quad k \in O_f, \quad \lambda \in \Omega
\end{aligned} \tag{13}$$

The first  $m$  constraint in (13) can be equivalently transformed as follows:

$$\tilde{f}_i(\lambda) - \theta_i^U \frac{1}{v_i} = 0 \Leftrightarrow v_i \tilde{f}_i(\lambda) = \theta_i^U \Leftrightarrow v_i \tilde{f}_i(\lambda) - H^{\min} = \theta_i^U - H^{\min} \Leftrightarrow v_i (\tilde{f}_i(\lambda) - \tilde{f}_i^*) = \theta_i^U - H^{\min} \tag{14}$$

Similar way the second  $s$  constraints in (13) can be equivalently transformed as follow

$$Z_k^U \frac{1}{w_k} \bar{f}_k(\lambda) = 0 \Leftrightarrow -w_k \bar{f}_k(\lambda) = -Z_k^U \Leftrightarrow F^{\max} - w_k \bar{f}_k(\lambda) = F^{\max} - Z_k^U \Leftrightarrow w_k (\bar{f}_k^* - \bar{f}_k(\lambda)) = F^{\max} - Z_k^U \tag{15}$$

Such that  $P_k^+ = P_i^- = 1, k \in O, i \in I$

Moreover, the objective function of (13) becomes:

$$\underset{\lambda_j, Z_k, \theta_i}{Max} (\sum_{k \in O} Z_k^U - \sum_{i \in I} \theta_i^U) = \underset{\lambda_j, Z_k, \theta_i}{Min} (\sum_{i \in I} \theta_i^U + \sum_{k \in O} (-Z_k^U)) \Rightarrow \underset{\lambda_j, Z_k, \theta_i}{Min} (\sum_{i \in I} (\theta_i^U - H^{\min}) + \sum_{k \in O} (F^{\max} - Z_k^U)) = \underset{\lambda_j, Z_k, \theta_i}{Min} (\sum_{i \in I} \tau_i + \sum_{k \in O} \gamma_k)$$

Also, for any  $\lambda \in \Omega$ , we have

$$F^{\max} - Z_k^U \geq w_k \bar{f}_k^* - Z_k^U \geq w_k \bar{f}_k(\lambda) - Z_k^U = 0 \quad k = 1, \dots, s$$

$$\theta_i^U - H^{\min} \geq \theta_i^U - v_i \tilde{f}_i^* \geq \theta_i^U - v_i \tilde{f}_i(\lambda) = 0 \quad i = 1, \dots, m$$

$$\bar{f}_k^* = \frac{F^{\max}}{w_k} \geq \frac{w_k f'_k}{w_k} = f'_k = \max_{\lambda \in \Omega} \bar{f}_k(\lambda) \quad k = 1, \dots, s$$

$$f_i^* = \frac{H^{\min}}{v_i} \leq \frac{v_i f_i''}{v_i} = f_i'' = \min_{\lambda \in \Omega} \tilde{f}_i(\lambda) \quad i = 1, \dots, m$$

In the end, we can rewrite the imprecise DESA model as a minimax formulation as follows:

$$\begin{aligned}
& \underset{\lambda_j, Z_k, \theta_i}{Min} \quad \sum_{i \in I} \tau_i - \sum_{k \in O} \gamma_k \\
& st \quad v_i (\tilde{f}_i(\lambda) - \tilde{f}_i^*) = \tau_i \quad i = I \\
& \quad w_k (\bar{f}_k^* - \bar{f}_k(\lambda)) = \gamma_k \quad k = O \\
& \quad g_i v_i - H^{\min} \leq \tau_i \quad i \in I_f \\
& \quad g'_k w_k - M \leq \gamma_k \quad k \in O_f
\end{aligned} \tag{16}$$

$$\sum_{j=1}^n \lambda_{jr} = 1, \quad \forall r \quad \lambda_{jr} \geq 0, \quad M \geq 0 \quad j, r = 1, \dots, n$$

Therefore, imprecise DESA model and minimax MOLP formulation are equivalence, this result lead to using interactive methods for solving imprecise DESA models and obtain most preferred solution.

#### PROJECT Algorithm for Solving DESA Model with Imprecise Data

The aim of an interactive method in MOLP is to improve DM's utility in an iterative way, especially if the utility function is not expressed explicitly. PROJECT method is a comprehensive interactive method, this method proposed (Luque, 2009) based on the GRIST method and together with the methodology of reference point techniques. The GRIST method can be used to identify a normal vector at a given efficient solution on the efficient frontier. The PROJECT method, offers a better way to search for a new solution that the GRIST method as local tradeoffs provided by the DM are maintained through the use of reference point approaches. also the PROJECT method to showing the decision maker (DM) preferences in the process of assessing efficiency in the DESA model with any prior judgement. Steps of the algorithm:

Step 1: the ideal values of outputs and inputs are calculated.

Optimize each of the upper bounds of total outputs and the lower bounds of total inputs.

Step 2: Iteration  $t = 0$ . Select an initial point. Set the initial weighting parameters for all DMUs, and reach the initial solution of the decision variables.

Step 3: Calculate the normal vectors and check optimality condition

At interaction  $t$ , estimate the normal vector. Generate reference total output and input, after then, decision maker (DM) select the most preferred solution and the interactive process will be finished. otherwise, the DM is asked to obtain local indifference tradeoffs on objectives

Step 4: determine the trade-off direction.

we obtain the projection of the DM's indifference tradeoffs on to the tangent plane of the efficiency frontier and determines the new tradeoff direction.

Step 5: Calculate the desired step sizes, the step sizes are to be determined by the largest and smallest desired step.

Step 6: Define the new reference points.

Step 7: select the most preferred solutions by the decision maker.

Decision maker select the best solutions, otherwise we go to step 6.

#### **CONCLUSIONS**

In this paper, we obtained an equivalence relation between the imprecise data envelopment scenario analysis and the minimax MOLP formulation. following that, with use interactive PROJECT method for estimating efficiency the imprecise DESA model and obtain most preferred solution. it must be pointed that any efficient solution can be generated with this interactive method, even including nonsupport

efficient solutions for nonconvex problems. Also, this approach results in decreasing total input and increasing total output at the same time. Instead of solving  $n$  independent linear programming models.

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# INNOVATION EFFICIENCY OF THE VISEGRAD GROUP STATES – RECOMMENDED FIELDS FOR IMPROVEMENT

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## ABSTRACT

*Innovation, a term introduced by J. Schumpeter in 1911, has since then been continuously explored at the micro-, meso- and macroeconomic levels. In 2017, Global Innovation Index (GII) Edition 10 was published. Based on the composite indicators, 127 states were ranked in terms of their innovativeness. The GI aims at reflecting the multi-dimensional facets of innovation and providing tools that can facilitate tailoring policies that can promote long-term output growth, improved productivity, and business development. This study focuses on the innovative performance of 28 member states of the EU in order to assess the innovation efficiency of the V4 states and identify the possible areas of their inefficiency. Data Envelopment Analysis has been applied (using GI composite indicators) to determine this. Based on the DEA computation, the Czech Republic, Hungary and Slovakia have been identified as efficient states, while Poland was found to have 9% relative inefficiency. In order to include Poland in the efficiency frontier, the DEA projected the need to raise all the output indicators by 10-25%. The most significant area of innovation inefficiency in Poland arises from inadequate Knowledge Diffusion. To advance its efficiency, Poland must seek solutions inspired principally by Slovakia and Croatia.*

**Keywords:** innovation; efficiency; Visegrad Group; DEA

## INTRODUCTION

*Innovation* was the term that J. A. Schumpeter, an Austrian economist, introduced in 1911, in his seminal work titled *Theorie der wirtschaftlichen Entwicklung*. The first English edition of this influential volume was published in 1934 under the title, *The Theory of Economic Development* (Schumpeter, 1934) and since then Schumpeter's reputation has been firmly established as a "prophet of innovation" (McCraw, 2007). Schumpeter, who recognised innovation as the critical dimension of economic change, sought to prove that innovation-initiated market power could deliver better results than the "invisible hand" and price competition (Pol and Carroll, 2006). It is therefore not surprising that the concept of innovation has become an extensively explored issue at the micro-, meso- and macroeconomic levels, accompanied by an assortment of definitions whose origin (authorship) is difficult to identify. Explanations that best mirror the spirit of this term include: 1) Innovation involves exploiting new ideas that lead to the creation of a new product, process or service, which is introduced into the market, putting it into practice and exploiting it in a manner that results in new products, services or systems which add value or improve the quality. 2) Innovation involves technological transformation and management restructuring. 3) Innovation also implies exploiting new technology and employing "out-of-the-box" thinking to generate new value and induce significant changes in society.

Innovation is treated as the crucial driver of economic growth and prosperity (Acs and Varga 2002; Tang, 2006) and as Gossling and Rutten (2007) have reported, innovation and economic welfare have an interdependent relationship – innovation creates the wealth and wealth is the prerequisite for innovation. Besides, Fritsch and Slavtchev (2011) also indicate that regions differ in terms of the 'quality' or

‘efficiency’ of the regional innovation systems, resulting in various innovative output levels, even assuming that the inputs are quantitatively and qualitatively identical. All of these facts encourage a comparison of firms and branches (Thornhill, 2006; Sieradzka, 2013; Wolak-Tuzimek, 2016), municipalities (Makkonen, 2011; Mikušová Meričková et al., 2015), regions (Buesa et al., 2010; Kozuň-Cešlak, 2016), states (Hasan and Tucci, 2010; Zdrzil et al., 2016) in terms of their innovativeness.

The Global Innovation Index (Cornell University, INSEAD and WIPO 2017), among the most popular innovativeness-focused investigations, is recognised worldwide for its annual ranking of countries based on their capacity for, and success in, innovation. The GII report aims at capturing the multi-dimensional facets of innovation and providing the tools that can facilitate tailoring policies to encourage long-term output growth, improved productivity, and job development. In 2017, Edition 10 of the GII was released, which identified the innovation leaders, as well as those that were not as good from among the 127 states examined.

The GII is computed by calculating the simple average of the scores in the two sub-indices of the Innovation Input Index and Innovation Output Index, which include five and two pillars, respectively. Each pillar is distinguished into sub-pillars, each of which include individual indicators (82 in total).

This study focuses on the innovative performance of the 28 European Union (EU) member states in order to assess their innovation efficiency, and subsequently to identify the possible areas (sources) of inefficiency in the Visegrad Group states. Therefore, the main aim of this research was to determine the success of transforming the innovation inputs into innovation outputs in the Czech Republic, Hungary, Poland and Slovakia, compared with the other EU members. To accomplish this the technique of mathematical programming Data Envelopment Analysis (DEA) was applied.

## **METHODS**

The Data Envelopment Analysis, due to its many benefits and relatively very few limitations, has been extensively explored in the scholarly literature (Emrouznejad and Yang, 2017). The DEA approach is a convenient tool to investigate the technical efficiency, especially of those entities whose performance is difficult to define in monetary terms (Emrouznejad et al., 2014).

The DEA introduced by Charnes, Cooper and Rhodes in 1978 (Charnes et al., 1978), based on the work of Farrell (1957), offered a basic DEA model – the radial CCR model with the assumption of constant returns to scale (an abbreviation that arises from the first letters of the providers’ names). The DEA-CCR model was extended to constitute technologies that reveal variable returns to scale by Banker, Charnes and Cooper in 1984 (Banker et al., 1984), called DEA-BCC, respectively. The benchmarks of the DEA method analysed units only against the best ones that form the frontier of efficiency (productivity frontier). An object (decision making unit, DMU) is recognised as 100% efficient (DEA score = 1) when comparisons with other units in a sample do not offer evidence of inefficiency in the use of any input or output. If any object is not at the frontier, it indicates inefficiency - its distance from the frontier defines the inefficiency level and a DEA score < 1. Over the years, simple DEA models have been developed through several modifications which permit the users to have a better fit of the appropriate DEA variant to

the specific needs of the researchers (for mathematical foundations of the DEA please refer **Charnes et al.**, 1994; Cooper et al., 2007; Ray, 2004; Emrouznejad and Tavana, 2014).

The critical advantages of the DEA method that make it suitable as a quantitative tool to estimate the efficiency of the innovative performance at a country level are as follows (see: Kozuń-Cieślak, 2010):

- DEA enables the evaluation of models with many inputs and outputs
- DEA applies data with heterogeneous names (expressed in different units of measurement)
- DEA does not require assumptions regarding functional dependence between inputs and outputs
- DEA does not call for the inputs and outputs to be assigned weights
- DEA detects extreme values which are invisible in other methods (instead of fitting regression curves to the average values, DEA constructs the frontier based on extreme data)
- DEA delivers input-output projections for inefficient DMUs to improve their performance.

All the attributes mentioned above that fit the requirements of this study include two stages. First, the innovative performance of the 28 EU member states will be examined using one input measure (the GII-initiated Innovation Input Sub-Index) and six output measures (the GII-initiated Innovation Output Sub-Pillars). For full description and scores of all the input-output measures listed in Table 1 see the GII 2017 report (available also on-line: [www.globalinnovationindex.org](http://www.globalinnovationindex.org)).

**Table 1: DEA input-output measures**

Input	Outputs
Innovation Input Sub-Index, ISI	Knowledge creation, KC
	Knowledge impact, KI
	Knowledge diffusion, KD
	Intangible assets, IA
	Creative goods and services, CG
	Online creativity, OC

*Source: Own work based on Global Innovation Index, Cornell University, INSEAD and WIPO 2017.*

Second, the study will focus on the four Visegrad Group states in order to identify the areas (sources) of their inefficiency. At this stage, the DEA-projections of the innovative outputs will be examined and solutions for the inefficient V4 states will be recommended. For these purposes, the DEA-BCC output-oriented model will be applied (computed with *DEA Solver LV 8.0*).

## RESULTS AND DISCUSSIONS

The DEA computations identified sixteen efficient EU states (for details see Appendice Table A1). These six are the “old” EU members (Germany, Ireland, Luxembourg, Netherlands, Sweden and UK) and ten new ones (Bulgaria, Croatia, Cyprus, the Czech Republic, Estonia, Hungary, Latvia, Malta, Romania and Slovakia), including three V4 states. The relative DEA inefficiency scores of twelve other EU states range between 9% and 25%. In the cases of Portugal, Poland, Italy, France, Belgium, Denmark and Finland, the DEA scores bear close similarity (0.912-0.899). Slovenia, Spain and Austria show relative inefficiency from 11 to 18%. The last two places in this innovation efficiency ranking are occupied by Greece (0.775)



and Lithuania (0.751). Generally speaking, all these DEA scores for the inefficient states show very small differences (the coefficient of variation is 6%).

Poland is the only V4 state which has been determined as relatively inefficient with 9% gap of DEA efficiency. This raises the question regarding the key areas of Poland's inefficiency.

In this study, the DEA-BCC output-oriented model presented the projection of increase in outputs that would propel Poland towards the efficiency frontier. According to this projection, Poland requires improvement principally in the area of *Knowledge diffusion* (by about 25%). Other outputs that need to increase are as follows: *Knowledge impact* by 13%, *Knowledge creation*, *Intangible assets*, *Creative goods and services* and *Online creativity* by nearly 10% each.

*Knowledge diffusion*, the output measure that should show the greatest improvement (by 25%), reveals four indicators that have achieved inadequate levels, viz., *Intellectual property receipts*, *High-tech net exports*, *ICT services exports* and *Foreign direct investment net outflows* (for a description of the indicators see Appendice Table A2). These components of *Knowledge diffusion* are significant areas (sources) of the relatively poor efficiency innovation in Poland. Hence, the economic policy in Poland should be directed primarily at these four dimensions of pro-innovative activity. To improve its innovation efficiency, Poland should draw its solutions mainly from Slovakia and Croatia, the countries that the DEA had identified as reference DMUs with the intensity factors ( $\lambda$ ) of 42% and 37%, respectively.

## CONCLUSIONS

The DEA computations identified Poland among the 28 EU member states as the only innovatively inefficient V4 state. The most significant area of Poland's inefficiency was found to be insufficient *Knowledge Diffusion*. The Polish government needs to consider introducing some tools of economic policy that could result in increasing (by about 25%) knowledge-initiated benefits like, intellectual property receipts, high-tech net exports, telecommunications, computer and information services exports, as well as foreign direct investment net outflows. The Slovak and Croatian solutions are strongly recommended as the reference practices for Poland.

Further research must focus on:

- examining the Slovak and Croatian solutions that enabled them to reach the efficiency frontier,
- modifying the DMU group, implying the replacement of the 28 EU members with countries achieving more similar GDP per capita,
- applying the other DEA variants, e.g. window analysis.

## APPENDICE

**Table A1: Innovation efficiency of EU-28 states (DEA BCC-O)**

DMU	Score	Rank	Projection (%)							Reference states (with $\lambda > 10\%$ )
			ISI	KC	KI	KD	IA	CG	OC	

Austria	0.816	26	0	22.5	22.5	22.5	22.5	22.5	36.2	0.39 UK	0.35 LU	0.10 MT
Belgium	0.905	21	0	10.5	10.5	139.9	17.7	10.5	39.8	0.53 MT	0.45 NL	
Bulgaria	1	1										
Croatia	1	1										
Cyprus	1	1										
Czechia	1	1										
Denmark	0.902	22	-3.5	13.5	19.5	44.1	10.9	10.9	10.9	0.61 NL	0.36 UK	
Estonia	1	1										
Finland	0.899	23	-5.8	11.2	11.2	11.2	11.2	24.4	31.5	0.60 DE	0.32 SE	
France	0.906	20	-8.1	14.6	10.4	34.0	10.4	10.4	54.4	0.66 LU	0.21 IR	0.13 MT
Germany	1	1										
Greece	0.775	27	0	39.4	73.6	41.1	29.0	65.2	29.0	0.59 BG	0.32 LV	0.10 LU
Hungary	1	1										
Ireland	1	1										
Italy	0.911	19	0	9.8	9.8	9.8	9.8	51.6	21.1	0.49 MT	0.26 CZ	0.15 BG
Latvia	1	1										
Lithuania	0.751	28	0	118.3	51.2	74.2	33.1	33.1	41.7	0.52 BG	0.41 LU	
Luxembourg	1	1										
Malta	1	1										
Netherlands	1	1										
Poland	0.912	18	0	9.7	13.1	25.4	9.7	9.7	9.7	0.42 SK	0.37 CR	0.16 LU
Portugal	0.912	17	0	56.4	9.7	38.8	9.7	51.0	41.1	0.45 LU	0.30 BG	0.25 MT
Romania	1	1										
Slovakia	1	1										
Slovenia	0.886	24	0	75.3	12.9	55.4	12.9	22.3	64.3	0.41 LU	0.39 MT	0.21 BG
Spain	0.836	25	0	19.6	19.6	19.6	19.6	69.3	51.8	0.42 MT	0.41 LU	0.14 IR
Sweden	1	1										
UK	1	1										

Source: Own computations using DEA Solver LV 8.0

**Table A 2: Knowledge diffusion indicators**

Indicator	Description
<i>Intellectual property receipts</i>	Charges for the use of intellectual property (% of total trade), not included elsewhere receipts for the use of proprietary rights (e.g. patents, trade-marks) and for licenses to reproduce or distribute intellectual property embodied in produced originals or prototypes (e.g. copyrights on books, computer software).
<i>High-tech net exports</i>	High technology exports minus re-exports (% of total trade), commodities with a high intensity of R&D (aerospace, computers and office machines, electronics, telecommunications, pharmacy, scientific instruments, electrical and non-electrical machinery, chemistry).
<i>ICT services exports</i>	Exports of telecommunications, computer and information services (% of total trade).
<i>Foreign direct investment net outflows</i>	Outflows of investment from the reporting economy to the rest of the world (% of GDP). It is the sum of equity capital, reinvestment of earnings, and other capital associated with a resident in one economy having control or a significant degree of influence on the management of an enterprise that is resident in another economy.

Source: *Global Innovation Index*, Cornell University, INSEAD and WIPO 2017.

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# INTRODUCTION A FUZZY EFFICIENCY SCORE IN DATA ENVELOPMENT ANALYSIS WITH FUZZY DATA

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## ABSTRACT

*Data Envelopment Analysis (DEA) is a widely applied approach for measuring the relative efficiencies of a set of Decision Making Units (DMUs), which use multiple inputs to produce multiple outputs. In real world problems, the data available may be imprecise. With fuzzy inputs and fuzzy outputs, the optimality conditions for the crisp DEA Models need to be clarified and generalized. The corresponding fuzzy linear programming problem is usually solved using some ranking methods for fuzzy sets. In this paper, we introduce a fuzzy efficiency score and a numerical method for ranking DMUs with fuzzy data.*

**Keywords:** Data envelopment analysis (DEA), Fuzzy mathematical programming, Ranking

## INTRODUCTION

In recent years, fuzzy set theory has been proposed as a way to quantify imprecise and vague data in DEA models. The DEA models with fuzzy data ("fuzzy DEA" models) can more realistically represent real-world problems than the conventional DEA models. Fuzzy set theory also allows linguistic data to be used directly within the DEA models. Fuzzy DEA models take the form of fuzzy linear programming models. A typical approach to fuzzy linear programming requires a method to rank fuzzy sets, and different fuzzy ranking methods may lead to different results. The problem of ranking fuzzy sets has been addressed by many researchers.

Following Kao's work, this paper introduces a new  $\alpha$ -level based approach to generalize essential concepts of DEA literature such as efficiency, efficient unit and a numerical algorithm for ranking DMUs with fuzzy data that the main idea is taken from Classification of Units based on efficiency classes  $E^{++}$ ,  $E^{+}$  and  $E^{-}$ .

## FUZZY DEA

Let us assume that we have  $n$  DMUs where DMU $_j$  ( $j = 1, \dots, n$ ) consumes input levels  $\tilde{x}_{ij}$  ( $i = 1, \dots, m$ ) to produce output levels  $\tilde{y}_{rj}$  ( $r = 1, \dots, s$ ), where all  $\tilde{x}_{ij}$  and  $\tilde{y}_{rj}$  are convex bounded fuzzy numbers.

Since the input and the output level of DMUs are not assigned exactly and then are uncertain as fuzzy numbers, so we expect the relative efficiency score of DMU $_o$  ( $o \in \{1, \dots, n\}$ ) also to be assigned in an uncertain manner as a fuzzy number.

With the assumption of fuzzy numbers  $\tilde{x}_{ij}$  and  $\tilde{y}_{rj}$  being convex and bounded, from it can be derived that each  $\alpha$ -level of  $\tilde{x}_{ij}$  and  $\tilde{y}_{rj}$  is a bounded interval as follows:

$$\begin{aligned} [\tilde{x}_{ij}]_{\alpha} &= [x_{ij}^{\alpha,L}, x_{ij}^{\alpha,U}] & [\tilde{y}_{rj}]_{\alpha} &= [y_{rj}^{\alpha,L}, y_{rj}^{\alpha,U}], \\ \alpha &\in (0,1], & j &= 1, \dots, n, \quad i = 1, \dots, m, \quad r = 1, \dots, s. \end{aligned}$$

By any choice of  $\alpha$ , a set is obtained that consists of  $n$  DMUs with interval data, and we can compute its efficiency interval by using (1),(2). Therefore, for any  $\alpha$ , an efficiency interval is obtained. As previously mentioned, the efficiency interval of DMU<sub>o</sub> for an  $\alpha$ -level is denoted by  $[h_o^{\alpha,L}, h_o^{\alpha,U}]$ , where

$$\begin{aligned} h_o^{\alpha,L} &= \text{Min} \theta \\ \text{s.t.} \quad & \sum_{j \neq o} \lambda_j x_{ij}^{\alpha,L} + \lambda_o x_{io}^{\alpha,U} \leq \theta x_{io}^{\alpha,U} & i = 1, \dots, m \\ & \sum_{j \neq o} \lambda_j y_{rj}^{\alpha,U} + \lambda_o y_{ro}^{\alpha,L} \geq y_{ro}^{\alpha,L} & r = 1, \dots, s \\ & \lambda_j \geq 0 & j = 1, \dots, n \\ & \theta & \text{free} \end{aligned}$$

and

$$\begin{aligned} h_o^{\alpha,U} &= \text{Min} \theta \\ \text{s.t.} \quad & \sum_{j \neq o} \lambda_j x_{ij}^{\alpha,U} + \lambda_o x_{io}^{\alpha,L} \leq \theta x_{io}^{\alpha,L} & i = 1, \dots, m \\ & \sum_{j \neq o} \lambda_j y_{rj}^{\alpha,L} + \lambda_o y_{ro}^{\alpha,U} \geq y_{ro}^{\alpha,U} & r = 1, \dots, s \\ & \lambda_j \geq 0 & j = 1, \dots, n \\ & \theta & \text{free} \end{aligned}$$

with  $h_o^{\alpha,L}$  and  $h_o^{\alpha,U}$  sometimes being equal.

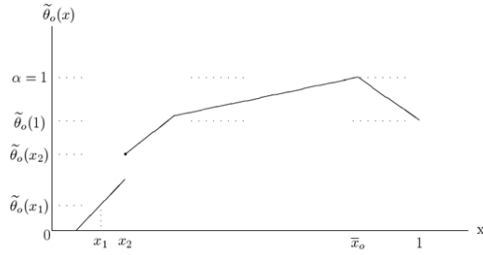
**Theorem 1.** If  $0 < \alpha_1 < \alpha_2 \leq 1$ , then  $[h_o^{\alpha_2,L}, h_o^{\alpha_2,U}] \subseteq [h_o^{\alpha_1,L}, h_o^{\alpha_1,U}]$ .

From the above theorem, the efficiency interval intricately shrinks as  $\alpha$  increases from 0 to 1. From intricate properties, the efficiency score can be introduced by a fuzzy set.

**Definition1.** With the above assumptions, the fuzzy set  $\tilde{\theta}_o$  on interval (0,1] with the membership function is the relative efficiency score of DMUo ,

$$\tilde{\theta}_o(x) = \sup\{t \mid 0 < t \leq 1, \quad x \in [h_o^{t,L}, h_o^{t,U}]\}$$

If there is some  $t$  such that  $x \in [h_o^{t,L}, h_o^{t,U}]$ , otherwise  $\tilde{\theta}_o(x) = 0$ .



**Figure 1:** An example for fuzzy efficiency score  $\tilde{\theta}_o$

In the following part, we show that fuzzy set  $\tilde{\theta}_o$  satisfies fuzzy number conditions. But, first, we present some important results. The following theorem states that the efficiency interval obtained from a special  $\alpha$ -level data is equal to the  $\alpha$ -level  $\tilde{\theta}_o$ , that is denoted by  $[\tilde{\theta}_o]_\alpha$

**Theorem2.** If  $0 < \alpha \leq 1$ , then  $[\tilde{\theta}_o]_\alpha = [h_o^{\alpha,L}, h_o^{\alpha,U}]$

**Conclusion1.**  $\tilde{\theta}_o$  is a convex bounded fuzzy set.

Unimodality is another property of function  $\tilde{\theta}_o(x)$  which is necessary for a fuzzy number.

**Theorem3.** The function  $\tilde{\theta}_o(x)$  is unimodal.

Hereafter, we assume  $[\tilde{\theta}_o]_1 = \{\bar{x}_o\}$ .

**Conclusion 2.** If  $[\tilde{\theta}_o]_\alpha = \{x\}$  for any  $0 < \alpha < 1$ , then  $x = \bar{x}_o$ .

From the above theorems, the function  $\tilde{\theta}_o(x)$  is maximized at point  $\bar{x}_o$ . In the next theorem, we show that this function is increasing and decreasing before and after  $\bar{x}_o$ , respectively.

**Theorem 4.** The function  $\tilde{\theta}_o(x)$  is increasing on interval  $(0, \bar{x}_o]$  and if  $\bar{x}_o < 1$ , then it is decreasing on interval  $[\bar{x}_o, 1]$ .

The following theorem shows that  $\tilde{\theta}_o(x)$  is a piecewise continuous function on the interval (0,1].

Theorem 5. a) The function  $\tilde{\theta}_o(x)$  is either continuous or right continuous on the interval  $(0, \bar{x}_o)$ .

b) If  $\bar{x}_o < 1$ , then  $\tilde{\theta}_o(x)$  is either continuous or left continuous on the interval  $(\bar{x}_o, 1]$ .

From the above theorems, we conclude that if the data for the DMUs are Fuzzy numbers, then the relative efficiency scores of the DMUs are also fuzzy numbers. Below, we define the concept of an efficient DMU in fuzzy DEA.

**Definition 2.** DMU<sub>o</sub> is efficient if and only if  $[\tilde{\theta}_o]_\alpha = \{1\}$  for any  $0 < \alpha \leq 1$ .

The above definition has a very strong condition for a DMU to be efficient. Even it may happen that none of DMUs is efficient. Because it is possible that  $E^{++}$  is empty when all DMUs have interval data. In the next section, we present a method for the ranking of DMUs with fuzzy data.

## RANKING

Definitions 5 and 6 are fundamentally theoretical criteria with respect to the efficiency score  $\tilde{\theta}_o$  and the efficient DMUs, but in practice, it is not possible that  $\tilde{\theta}_o$  or efficient DMUs are exactly determined by these definitions.

Here we present a numerical ranking method to determine the rank order of DMUs approximately. Consider the following  $k$  real numbers

$$0 < \alpha_1 < \dots < \alpha_k = 1$$

Hence, these numbers intricately form  $k$  sets of inputs and outputs as follows:

$$[\tilde{x}_i]_{\alpha_{l+1}} \subseteq [\tilde{x}_i]_{\alpha_l}, \quad [\tilde{y}_r]_{\alpha_{l+1}} \subseteq [\tilde{y}_r]_{\alpha_l} \quad i = 1, \dots, m, \quad r = 1, \dots, s, \quad l = 1, \dots, k-1$$

So from Theorems 3, 4, we have  $l = 1, \dots, k-1$

$[\tilde{\theta}_o]_{\alpha_{l+1}} \subseteq [\tilde{\theta}_o]_{\alpha_l}$  Since each  $\alpha$ -level set of  $\tilde{\theta}_o$  is a closed interval, as mentioned previously, the following statements can be considered. If for some  $l' (l' < k)$ :

$DMU_o \in E^{++}$ , then  $DMU_o \in E^{++}$  for all  $l$  where  $l > l'$ .

$DMU_o \in E^-$ , then  $DMU_o \in E^-$  for all  $l$  where  $l > l'$ .

Moreover, for  $\alpha_k = 1$ , we know the inputs and outputs are crisp data and  $[\tilde{\theta}_o]_1 = \theta_o = \{\bar{x}_o\}$ . At this level, if DMU<sub>o</sub> is efficient ( $\theta_o = 1$ ), then DMU<sub>o</sub> is in  $E^{++}$  or  $E^+$  for all  $\alpha_l < 1$ , and otherwise ( $\theta_o < 1$ ) then DMU<sub>o</sub> is in  $E^+$  or  $E^-$  for all  $\alpha_l < 1$ .

It must be considered that the nearer  $\alpha_1$  is to zero, the more exactly the efficient units are assessed. This does not mean that at least one unit will be certainly efficient. Also, after studying the ranking method below, it will be clear that the higher the number of  $\alpha_1$ , with the distances between them selected equally, the more justly and precisely the ranking of the DMUs is carried out.

Therefore, suppose  $\alpha_1 > 0$  is sufficiently small and the other  $\alpha_l$  are selected as follows:

$$\alpha_l = \alpha_1 + \frac{(1 - \alpha_1)(l - 1)}{k - 1}, \quad l = 2, \dots, k$$

hence  $\alpha_k = 1$ .

**Definition3.** The set of DMUs are in  $E^{++}$  for  $\alpha_l$ -level data are denoted

by  $E_{\alpha_l}^{++}$ ; that is

$$E_{\alpha_l}^{++} = \{DMU_o \mid h_o^{\alpha_l, L} = 1\}, \quad l = 1, \dots, k$$

and also the set of DMUs are in  $E^+$  for  $\alpha_l$ -level data are denoted by  $E_{\alpha_l}^+$ ; that is

$$E_{\alpha_l}^+ = \{DMU_o \mid h_o^{\alpha_l, L} < 1 \text{ \& } h_o^{\alpha_l, U} = 1\}, \quad l = 1, \dots, k$$

Obviously  $E_1^+ = \phi$  and the DMUs in  $E_{\alpha_1}^{++}$  have the highest efficiency score.

If a DMU is efficient as defined in Definition 6, then it must be in  $E_{\alpha_1}^{++}$ , but the reverse does not necessarily hold true.

Suppose  $p = 1$  and  $J = \{DMU_1, \dots, DMU_n\}$ . There are two cases for  $E_{\alpha_1}^{++}$  as:

I.  $E_{\alpha_1}^{++} = \phi$

II.  $E_{\alpha_1}^{++} \neq \phi$

If  $E_{\alpha_1}^{++} = \phi$  then one can ask which DMUs are more efficient, the DMUs in  $E_{\alpha_1}^+$  or in  $E_{\alpha_2}^{++}$ ? We claim the DMUs in  $E_{\alpha_2}^{++}$  are more efficient because these DMUs are the DMUs in  $E_{\alpha_1}^+$  that join  $E_{\alpha_2}^{++}$  for  $\alpha_2$ -level data. Therefore, when  $E_{\alpha_1}^{++} = \phi$ , we turn to  $E_{\alpha_2}^{++}$ .



With the  $E_{\alpha_1}^{++} = \phi$  assumption, there are also two cases for  $E_{\alpha_2}^{++}$ . If  $E_{\alpha_2}^{++} = \phi$ , similarly, we turn to  $E_{\alpha_3}^{++}$  and this continues until  $E_{\alpha_{l'}}^{++} \neq \phi$  for some  $l'$  ( $l' = 1, \dots, k$ ). There is certainly such an  $l'$ , because  $E_{\alpha_k}^{++} = E_{\alpha_1}^{++} \neq \phi$  the end.

In this case, let

$$E_p = E_{\alpha_{l'}}^{++} \quad J = J - E_p \quad p = p + 1.$$

Now again we form  $E_{\alpha_l}^{++}$  and  $E_{\alpha_l}^+$  ( $l = 1, \dots, k$ ) for the reduced set  $J$  and compute  $E_p$  similarly. This process continues until  $J = \phi$ . It can be at most repeated for  $n$  iterations. In the end, we will obtain  $E_1, \dots, E_p$  ( $p \leq n$ ) from the above process. Clearly the DMUs in  $E_1$  have higher efficiency scores than the ones in  $E_2, \dots, E_p$  and also the DMUs in  $E_2$  have the same relation with other DMUs in  $E_3, \dots, E_p$ , and so on. Therefore, by arranging DMUs on the basis of these sets we can rank them.

It is possible that  $E_1, \dots, E_p$  have more than one member, so some DMUs may have the same rank. This problem can be removed by the following change to determine  $E_1, \dots, E_p$ . In each iteration, if  $E_p$  has more than one member, then by going back to one level before  $\alpha_{l'}$ , i.e.,  $\alpha_{l'-1}$ , we choose a DMU in  $E_p$  which has the highest  $h_j^{L, \alpha_K - 1}$  as the member of  $E_p$  and omit the other members. Even, if  $E_p$  still has more than one member, we can go back to  $\alpha_{l'-2}, \dots, \alpha_1$  and choose  $E_p$  as a singleton. In the end, if  $E_p$  is not a singleton yet, then we can conclude that all members of  $E_p$  have the same rank order.

**Definition4.**  $q$  denotes the rank order of a DMU when the  $DMU \in E_q$ ,  $q = 1, \dots, p$ , after using the ranking method above.

## CONCLUSIONS

The purpose of this study was to develop the concepts of efficiency and ranking in DEA with fuzzy data, which was presented by introducing an  $\alpha$ -level approach and a numerical method for ranking DMUs.

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# INVESTIGATING THE SUPER-EFFICIENCY OF THE TOP ICT COMPANIES WORLDWIDE

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## ABSTRACT

*This study investigates the cross-layer efficiency of ICT, where the convergence blurs the boundaries of the industry and transforms the dynamics of the competition. By utilizing super-efficiency model of DEA, efficiency of the layers of the ICT ecosystem is benchmarked during the time period between 2005 and 2015. Our results show that overall efficiency of the ICT industry is increasing throughout the analysis period, in which layer 1 (networked elements) has consistent efficiency increases as the leader and layer 3 (platforms, e-commerce, software and services) emerges as the challenger. On the contrary, the inferior efficiency scores of network operators (layer 2) after 2009 put pressure on the capacity of the operators to invest in the networks on which ICT rises and risk the sustainable development of ICT industry. Our results also exhibits that Japanese ICT companies have higher average efficiency scores, which are closely followed by the US and lately by the Indian firms.*

**Keywords:** ICT industry; super-efficiency; data envelopment analysis; DEA

## INTRODUCTION

Information and communication technologies (ICT) industry is a powerful driver of economic growth in today's digital world. According to the World Bank, the share of global ICT service exports in overall service exports is 31.39% in 2015, whereas this ratio is 11.09% for global ICT goods<sup>4</sup>. In OECD, contribution of ICT to the total labor productivity increase is 1.99% in 2013 (OECD, 2015). As the world economy become more dependent on ICT, firms in this industry need to deal with rapidly changing market and shape shifting competitors. Convergence process blurs the boundaries of the industry and force companies to continuously investigate the dynamics of the competition and value chain, benchmark their efficiency and evolve strategies accordingly.

In line with its economic importance, efficiency analysis within the different segments of ICT industry (e.g. computer manufacturers, telecom operators, IT services etc.) is a popular research area. For example, Sengupta (2005) compared the efficiency of 12 companies from US computer industry for the years 1987 to 1998. Chen and Ali (2004) analyzed the technical efficiency change within the group of 8 computer companies between 1991 and 1997. On the other hand, DEA is widely used for the efficiency analysis of the telecom operators (e.g. Tsai et al. 2006, Hung and Lu 2008, Sadjadi and Omrani 2010, Diskaya et al. 2011 etc.). Apart from the operators, Marthur (2007) applied DEA to assess the export performance of 92 Indian software companies, whereas Chou and Shao (2014) compared the efficiency of the information technology (IT) service industry in 25 OECD countries between 1995 and 2007. The work of Halkos and

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<sup>4</sup> Source: The World Bank (<http://data.worldbank.org>. Accessed on 15.05.2017)

Tzeremes in 2007 is one of the few studies on the whole ICT industry, where they used DEA to evaluate the competitive structure of the top 50 largest ICT companies in 2003.

We believe that, rather than working on isolated layers, a comparative efficiency analysis of the whole ICT would provide important insights on the dynamics of the competition and the sustainability of the industry. Therefore, this study investigates and benchmarks the cross-layer and cross-country efficiency of ICT between 2005 and 2015 with the super-efficiency model of DEA. The model is applied to 68 top ICT companies worldwide, which are grouped into the layers according to Fransman's (2010) multi-layered New ICT Ecosystem Model<sup>5</sup>.

## **METHODS**

DEA is non-parametric mathematical programming technique to generate a relative technical efficiency score for each decision-making units (DMUs) and identify the efficiency frontiers, which utilize multiple input-output combinations (Gökgöz, 2010). Charnes Cooper and Rhodes (CCR) in 1978 developed the original CCR model of DEA, which was based on constant returns to scale (CRS). The underlying assumption is that all DMUs are operating under optimal scale. The CRS assumption was relaxed by Banker, Charnes and Cooper (BCC) in 1984 to variable returns to scale (VRS), which assumes that firms are not operating at an optimal scale. The use of CCR model gives the technical efficiency scores mixed with scale efficiencies, whereas BCC model will allow to calculate the pure technical efficiency and the scale efficiency (SCE) scores separately (Cooper et al., 2000). DEA reports the relative efficiency scores as a number between 0 and 1. The DMUs having the score 1 are regarded as the efficient ones relative to the other units in the sample, whereas below 1 is regarded as inefficiency.

An advantage of DEA is that it does not superimpose a particular structure on the data to identify the efficient units, rather the best-practice is empirically constructed with the inputs and outputs. This significant feature has made DEA be an effective tool in corporate strategy, benchmarking and change implementation programs, and it has been widely applied in many areas since its introduction (Zhu, 2014). However, DEA does not provide a mechanism to benchmark the efficiency within the DMUs which are on the frontier line with the DEA score of 1. Super-efficiency DEA model produces efficiency scores greater than 1, offers a solution for ranking the efficient DMUs. When a DMU under evaluation is not included in the reference set of the original DEA model, the resulting DEA model is called super-efficiency model (Zhu, 2014). Therefore, this study utilizes the super-efficiency model of DEA for cross-layer efficiency benchmarks.

## **DATA AND DESCRIPTIVE STATISTICS**

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<sup>5</sup> Fransman's model horizontally aggregates the ICT industry into four permeable layers where layer 1 is the networked elements which builds the physical infrastructure of the information network (e.g. Apple, Cisco, HP, Samsung etc.). Layer 2 is the network operators which provides services to access the information network (e.g. AT&T, Deutsche Telekom, Vodafone etc.). Layer 3 is the platforms, e-commerce, content and software where information and content is generated and shared within the network (Google, Microsoft, Amazon, Reuters, Timer Warner, Walt Disney etc.). Layer 4 is the consumers where digital content and information is consumed.

The sum of the 2015 revenues of the 68 companies in our sample is 84.02% of the global ICT spending<sup>6</sup> in the same year (please see Table 3 for the full list of the sample). There are 22 companies in layer 1 (networked elements), 23 in layer 2 (network operators) and 23 in layer 3 (platforms, e-commerce, content, software and service). Companies are selected from 2014 and 2015 Financial Times Global 500 (FT500) lists based on the availability of the financial data, which is collected from company annual reports/SEC filings, then deflated to 2010 values. The outliers are removed in iterations.

Input/output variables are selected, which have the same meaning across different tax regimes and reporting standards and are publicly available. Therefore, *revenue* is used as the output (Calabrese et al. 2002, Chen and Ali 2004, Diskaya et al. 2011, etc.), whereas *total assets* (Chen and Ali 2004), *capital expenditure* (Diskaya et al. 2011 etc.), *total equity* (Chen and Ali 2004) and *number of employees* (Calabrese et al. 2002, Chen and Ali 2004, Diskaya et al. 2011 etc.) are the inputs.

As Table 1 shows, network operators (layer 2) gets the highest earnings within the ICT ecosystem with the average of \$45,306 million, followed by layer 1 (\$42,392 million) and layer 3 (\$21,185 million) companies. Layer 2 is also the most investment intensive layer in ICT, where the average asset size is \$93,713 million, which is more than total of average assets of layer 1 and layer 3. Capital expenditure (CAPEX) and equity figures also confirm that network operators play a dominant role in undertaking the investment burden of ICT industry (Fransman, 2010). On the other hand, Table 1 also shows that layer 3 is more labor intensive compared to the other two layers.

**Table 1: Descriptive statistics**

		Revenue <sup>a</sup> (O)	Total Assets <sup>a</sup> (I)	CAPEX <sup>a</sup> (I)	Equity <sup>a</sup> (I)	Employees (I)
Layer 1	Mean	42,392	51,726	2,908	24,992	107,965
	Median	25,132	37,027	1,269	19,847	71,917
	Std. Dev.	42,758	46,059	4,006	23,803	101,506
Layer 2	Mean	45,306	93,713	7,864	35,932	102,009
	Median	36,431	57,438	6,138	19,721	53,523
	Std. Dev.	34,807	80,197	6,134	36,603	92,409
Layer 3	Mean	21,185	37,919	1,100	17,307	83,184
	Median	12,941	26,347	421	9,913	51,000
	Std. Dev.	22,930	36,139	1,567	18,414	94,510

Source: Own calculations of authors. (Notes: (a) Values are in million US dollars. "O": output. "I": input. "Std. Dev.": standard deviation.)

## RESULTS AND DISCUSSIONS

Based on the output oriented VRS model of super-efficiency DEA, Table 2 displays the average super-efficiency scores of the ICT layers between 2005 and 2015. The efficiency of the whole sample has an increasing trend for the complete analysis period, where it started with 0.757 in 2005 and ended-up with 0.903 in 2015.

**Table 2: Average super-efficiency scores of the ICT layers**

	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	Average
Layer 1	0.749	0.767	0.789	0.790	0.831	0.908	0.866	0.940	0.906	0.941	0.950	0.855
Layer 2	0.744	0.760	0.830	0.811	0.881	0.802	0.708	0.729	0.702	0.819	0.830	0.781
Layer 3	0.778	0.744	0.764	0.803	0.775	0.829	0.857	0.810	0.889	1.020	0.930	0.833

<sup>6</sup> Worldwide ICT spending in 2015 is \$3,413,324 Million. (Source: <http://www.gartner.com> Accessed on 01.01.2017)

Whole Sample 0.757 0.757 0.794 0.802 0.829 0.845 0.809 0.825 0.831 0.926 0.903 0.824

Source: Own calculations of authors (Notes: Average is the geometric mean of each row.)

It is notable that, all three layers started almost at the same level of super-efficiency in 2005 and 2006, after when layer 2 took the lead until 2009. However, as seen in Figure 1 (a), layer 2 companies experienced a steep efficiency fall starting from 2010. The average super-efficiency score of layer 2 dropped from 0.881 in 2009 to first 0.802 in 2010 and dipped to 0.702 in 2013. The efficiency gap between layer 2 and other two layers reached up to 20% during this period. Although the network operators started to pick up speed in 2014/2015 period, the efficiency gap is still significant. Figure 1 (a) also shows that layer 1 companies climbed to the super-efficiency leadership after 2010. With the highest average of 0.855 among the ICT layers, layer 1 exhibits a consistent and increasing trend in super-efficiency. On the other hand, layer 3 had an impressive jump of almost 21% from 2012 to 2014, which is a strong signal for this layer to play the leading role in efficiency.

Table 3 displays the average super-efficiency scores of the ICT companies in our sample. Accenture, Apple, Softbank, Hitachi and KDDI are the top 5 performers for the whole analysis period between 2005 and 2015. If we look at the performance of the companies during the last 5 years, we see that with impressive efficiency rise, Applied Materials and Verizon took over the positions of Softbank and KDDI in the top 5 list. Steep efficiency drops are notable for T-Mobile US (33%), Nokia (36%) and Vodafone (24%) in the last five years compared to the whole period.

**Table 3: Average super-efficiency scores of the top ICT companies**

No	Company	A11	A5	No	Company	A11	A5	No	Company	A11	A5
1	Accenture, L3, US	<b>2.37</b>	<b>2.25</b>	24	21st Cen. Fox, L3, US	0.86	<b>1.10</b>	47	Wipro, L3, Ind	0.64	0.78
2	Apple, L1, US	<b>1.65</b>	<b>2.01</b>	25	Reed Elsevier, L3, Neth	0.85	0.98	48	Naspers, L3, S. Afr	0.61	0.71
3	Softbank, L2, Jap	<b>1.48</b>	<b>1.13</b>	26	America Mov., L2, Mex	0.82	0.85	49	Intel, L1, US	0.60	0.61
4	Hitachi, L1, Jap	<b>1.35</b>	<b>1.29</b>	27	NT&T, L2, Jap	0.81	0.76	50	Telenor, L2, Nor	0.59	0.69
5	KDDI, L2, Jap	<b>1.32</b>	<b>1.22</b>	28	Vivendi, L3, Fra	0.80	0.87	51	ADP, L3, US	0.58	0.75
6	HP, L1, US	<b>1.20</b>	<b>1.07</b>	29	AT&T, L2, US	0.80	0.74	52	WPP, L3, UK	0.57	0.65
7	Samsung, L1, Korea	<b>1.19</b>	<b>1.18</b>	30	Ericsson, L1, Swe	0.77	0.86	53	Yahoo!, L3, US	0.57	0.66
8	T-Mobile US, L2, US	<b>1.14</b>	0.81	31	Emerson, L1, US	0.76	0.86	54	Schneider, L1, Fra	0.56	0.64
9	Verizon, L2, US	<b>1.13</b>	<b>1.41</b>	32	Texas Inst., L1, US	0.74	0.75	55	Tencent, L3, Chn	0.55	0.50
10	NTT Docomo, L2, Jap	<b>1.13</b>	<b>1.03</b>	33	Adobe, L3, US	0.74	0.80	56	China Mob., L2, Cn	0.54	0.55
11	IBM, L3, US	<b>1.09</b>	<b>1.08</b>	34	Telefonica, L2, Spa	0.73	0.68	57	EMC, L1, US	0.53	0.60
12	Applied Mat, L1, US	<b>1.09</b>	<b>1.53</b>	35	Google, L3, US	0.72	0.64	58	Walt Disney, L3, US	0.53	0.55
13	Nokia, L1, Fin	<b>1.05</b>	0.69	36	Qualcomm, L1, US	0.70	0.87	59	Murata, L1, Jap	0.52	0.66
14	Oracle, L3, US	<b>1.04</b>	<b>1.02</b>	37	Deutsche Tel., L2, Ger	0.70	0.63	60	Comcast, L2, US	0.49	0.58
15	Panasonic, L1, Jap	<b>1.03</b>	<b>1.19</b>	38	Bharti Airtel, L2, Ind	0.70	0.82	61	Singtel, L2, Sing	0.47	0.53
16	Salesforce.com, L3, US	<b>1.00</b>	<b>1.00</b>	39	Swisscom, L2, Swi	0.69	0.78	62	BCE, L2, Can	0.46	0.52
17	Sony, L1, Jap	0.98	<b>1.14</b>	40	Canon, L1, Jap	0.68	0.70	63	Telia, L2, Swe	0.46	0.48
18	CBS, L3, US	0.96	<b>1.22</b>	41	Vodafone, L2, UK	0.67	0.43	64	TSMC, L1, Chn	0.46	0.48
19	Time Warner, L3, US	0.90	1.05	42	SAP, L3, Ger	0.67	0.68	65	Micron Tech., L1, US	0.45	0.54
20	ASML, L1, Neth	0.89	0.97	43	Telstra, L2, Aust	0.66	0.78	66	China Uni, L2, Chn	0.38	0.43
21	Tata Cons., L3, Ind	0.89	0.98	44	Infosys, L3, Ind	0.65	0.77	67	Reuters, L3, Cana	0.36	0.44
22	Cisco, L1, US	0.87	1.00	45	Orange, L2, Fra	0.65	0.58	68	Corning, L1, US	0.31	0.31
23	Microsoft, L3, US	0.86	0.86	46	MTN Group, L2, S. Afr	0.64	0.68				

Source: Own calculations of authors. (Notes: A11 is the geometric mean of the 11 years between 2005 and 2015. A5 is the geometric mean of the last 5 years between 2011 and 2015. L1: layer 1-networked elements. L2: layer 2-network operators. L3: layer 3-platforms, e-commerce, content, software and service. US: United States, Jap: Japan, Korea: South Korea, Fin: Finland, Neth: Netherlands, Ind: India, Mex: Mexico, Fra: France, Swe: Sweden, Spa: Spain, Ger: Germany, Swi: Switzerland, UK: United Kingdom, Aust: Australia, S. Afr: South Africa, Nor: Norway, Chn: China, Sing: Singapore and Can: Canada. Bold indicate the efficiency scores  $\geq 1.00$ .)

From the perspective of country/region of origin, dominance of US in our sample is remarkable, especially in layer 1 and 3. In total, 28 of the companies have US origins, of which constitutes 50% of layer 1 and 56.5% of layer 3. With 15 companies, Europe is the second biggest group, followed by the

Japanese (9). While network operators (layer 2) are the majority in the Europe list, the Japanese are strong both in layer 1 and layer 2. On the other hand, each having 4 companies, India and China emerge as the global challengers.

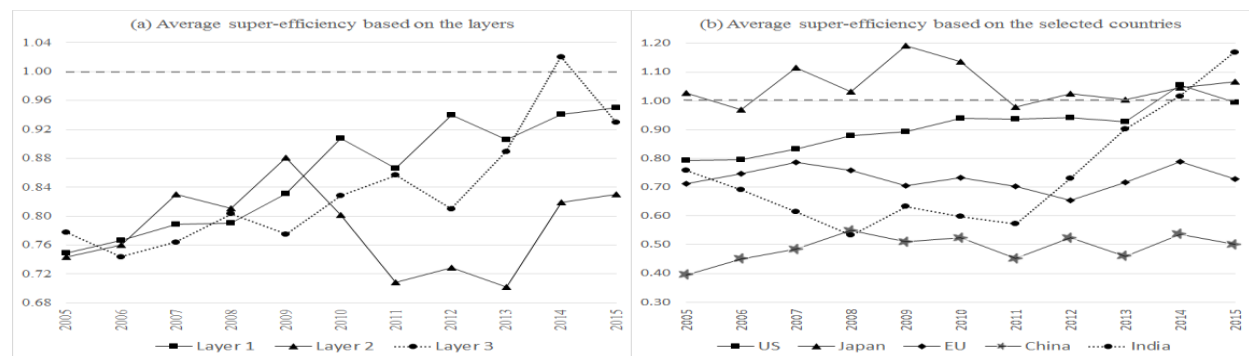
Table 4 exhibits the average super-efficiency scores of the ICT companies based on the selected countries/regions of origin (i.e. US, Japan, Europe, China and India). Average super-efficiency of Japan is 1.052 for the whole period, which declares Japan as the efficiency leader by far. With the average of 0.905, US companies are in the second place, where they consistently moved from the level of 0.80 in 2005 to almost 1.00 in 2015

**Table 4: Average super-efficiency scores of the selected countries/regions**

	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	Average
US	0.794	0.795	0.832	0.878	0.893	0.938	0.938	0.942	0.928	1.055	0.995	0.905
Japan	1.027	0.969	1.115	1.031	1.193	1.137	0.979	1.024	1.003	1.045	1.066	1.052
Europe	0.712	0.747	0.787	0.757	0.705	0.733	0.702	0.655	0.716	0.788	0.728	0.729
China	0.394	0.451	0.484	0.549	0.510	0.524	0.452	0.524	0.460	0.537	0.500	0.487
India	0.757	0.691	0.614	0.533	0.632	0.598	0.572	0.730	0.901	1.016	1.169	0.724

Source: Own calculations of authors. (Note: Average is the geometric mean of each row.)

As also seen from Figure 1 (b), with the average score of 0.729, European firms move along a narrow efficiency corridor during the whole timeline, where 0.790 is the ceiling. On the other hand, having the poorest average scores, China is stuck in an efficiency band of 0.45 – 0.55. We should admit that India deserves a special look in her efficiency progress. The average efficiency score of India was the penultimate until 2011, since when soared steadily above 1.00. With the avg. of 1.169, the country was the champion of the efficiency race in 2015.



**Figure 1: (a) Average super-efficiency scores based on layer. (b) Average super-efficiency scores based on selected countries**

## CONCLUSIONS

This study extends the research on the efficiency of ICT industry to a cross-layer platform, where the top ICT companies worldwide are analyzed at aggregated layers. As the force of convergence constantly transforms the nature of the value chain in the industry, the study benchmarks the efficiency of the different layers of ICT and provides important insights for the ICT companies on the dynamics of the competition within the ICT ecosystem.

Our super-efficiency DEA results exhibit that the overall efficiency of the ICT industry is increasing throughout the whole analysis period between 2005 and 2015. With the average super-efficiency score of 0.855, layer 1 (networked elements) had been a consistent efficiency leader within the ICT industry after 2010. Equally important, after the remarkable jump of 21% from 2012 to 2014, layer 3 companies (platforms, e-commerce, content, service and software) emerged as the strong challengers for the leadership of the efficiency realm in the ICT industry. On the contrary, layer 2 (network operators) have been experiencing a steep fall (18%) in average efficiency after 2009. Although the average efficiency of the operators has increased in 2014/2015 period, the efficiency gap with the other layers is still significant. This is a warning signal for the sustainable development of the ICT industry. As Thore et al. (1994) underline, companies having consistently less efficient scores, would suffer large financial losses and experience declining market share. Therefore, the inferior average efficiency scores of the network operators put pressure on the operators' ability to invest, innovate and play a critical role in building the networks on which ICT rises.

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# MADM-BASED METHODS TO EVALUATE THE CARGO INSURANCE PRODUCTS

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## ABSTRACT

*Exporting and importing helps grow national economics and expands the global market. Cargo insurance provides coverage against physical damages or loss of goods during shipping, which plays an important role in doing such economic activities. Since there are various companies providing such services with different offers, selecting the best company product can be considered as a MADM problem. The goal of the paper considering criterion which selected (including premium, all risk, experience and financial capability) to utilizes approaches (such as DEA, TOPSIS and ELECTRE) in order to choose the most preferable company product.*

**Keywords:** MADM, DEA, TOPSIS, ELECTRE, Cargo insurance

## INTRODUCTION

Multiple Attribute Decision Making (MADM) is the most well-known branch of decision making. It is a branch of general class of operation research models which deal with decision problems under the number of decision criteria. Decision-making processes involve a series of steps distinguish between ordinal and cardinal methods. General procedures for ordinal methods (ranking methods: e.g. borda method, condorcet) and for *cardinal methods* (interval: e.g. weights / preference criteria):

- Determine the subject: person, group, election...
- Setting up goal / purpose: insurance product (choice of best product) ...
- Determine the alternatives: insurance product...
- Determine the criteria: claim, premium, pay-off...
- *Evaluate the weights.*
- *Transform of decision making matrix: criteria minimum to maximum: normalized values of criteria.*
- Create the decision-making matrix.
- Aggregation procedure: utility function, distance, preference.
- Ranking alternatives.

Normally, MADM refers to making decisions in the presence of multiple, usually conflicting, attributes. Problems for MADM are common occurrences in every aspect of life. General characteristics of MADM problems considering alternatives and multiple attributes. Hence, the goal of this paper considering selected criterion to utilizes approaches in order to choose the most preferable company product. It illustrates various non-life insurance companies' products which providing cargo services with different offers, aim of selecting the best preferable company's product can be considered as a MADM problem.

## LITERATURE REVIEW AND ANALYSIS OF MADM METHODS

According to Hwang C.L., Yoon K. (1981), MADM method is a procedure that specifies how attribute information is to be processed in order to arrive at a choice. General to say, there are two major approaches in attribute information procession:

- Non-compensatory model: models don't permit tradeoffs between attributes (e.g. dominance, maxi-min, maxi-max, conjunctive constraint method, disjunctive constraint method, lexicographic method).
- Compensatory model: models permit tradeoffs between attributes - scoring model (e.g. simple additive weighting, hierarchical additive weighting, interactive simple additive weighting); compromising model (e.g. TOPSIS, LINMAP, non-metric MDS); Concordance model (e.g. permutation method, linear assignment method, ELECTRE)

#### Data Envelopment Analysis (DEA)

DEA is a method for measuring efficiency, bench-marking and continuous improvement. It developed by Charnes, Cooper and Rhodes (1978). It is a method used in the evaluation of performance of DMUs. All DMUs under comparison are assumed to operate homogeneously: they receive the same inputs and produce the same outputs (in differing quantities) and these inputs and outputs are representative of the whole population. Hence, basic formula written by following

$$efficiency = \frac{output}{input}, \quad (1)$$

A common measure for relative efficiency is

$$efficiency = \frac{weighted\ sum\ of\ outputs}{weighted\ sum\ of\ inputs} \quad (2)$$

which introducing the usual notation can be written as

$$efficiency\ of\ unit\ j = \frac{u_1 y_{1j} + u_2 y_{2j} + \dots}{v_1 x_{1j} + v_2 x_{2j} + \dots} \quad (3)$$

Units might value inputs and outputs differently and therefore adopt different weights, and proposed that each unit should be allowed to adopt a set of weights which shows it in the most favorable light in comparison to the other units. Efficiency of a target unit  $j_0$  can be obtained as a solution to the following

$$\text{problem: Max } h_0 = \frac{\sum_r u_r y_{rj_0}}{\sum_i v_i x_{ij_0}}; \text{ Subject to } \frac{\sum_r u_r y_{rj}}{\sum_i v_i x_{ij}} \leq 1 \text{ for each unit } j, \text{ where } u_r, v_i \geq \varepsilon. \text{ Maximize}$$

the efficiency of unit  $j_0$ , subject to the efficiency of all units begin  $\leq 1$ . The variables of the above problem are the weights and the solution produces the weights most favorable to unit  $j_0$  and produce a measure of efficiency. The DEA model is a fractional linear program. To solve the model, it is necessary to convert it into linear form so that the method of linear programming can be applied.

#### The Technique for Order Preferences by Similarity to an Ideal Solution (TOPSIS)

TOPSIS was proposed by Hwang and Yoon (1981) to determine the best alternative based on the concepts of the compromise solution. The compromise solution can be regarded as choosing the solution with the shortest Euclidean distance from the ideal solution and the farthest Euclidean distance from the negative ideal solution. The procedure of TOPSIS are:

- 1) Construct the normalized decision matrix.
- 2) Construct the weighted normalized decision matrix.
- 3) Determine ideal and negative-ideal solutions.

- 4) Calculate the separation measure.
- 5) Calculate the relative closeness to the ideal solution.
- 6) Rank the preference order.

TOPSIS can be only used when criteria of decision making are monotonously increasing or decreasing.

#### Elimination Et Choice Translating REality (ELECTRE)

ELECTRE was introduced by Roy (1968) and Benayoun et al. (1966), it originally used the concept of outranking relations. The outranking relationship of  $A_k \rightarrow A_l$  says that even though two alternatives  $k$  and  $l$  do not dominate each other mathematically, the DM accepts the risk of regarding  $A_k$  almost surely better than  $A_l$ . Through the successive assessments of the outranking relationships of the other alternatives, the dominated alternatives defined by the outranking relationship can be eliminated. But the construction of this partial order is not an unambiguous task for the DM. ELECTRE sets the criteria for the mechanical assessment of the outranking relationships.

Roy (1977,1978) developed ELECTRE III, extending the crisp outranking relations for modeling decision makers' preferences in fuzzy conditions. The exploiting ranking procedure used in ELECTRE III generally consists of the following steps (Belton and Stewart 2002):

- 1) Construct a complete preorder  $Z_1$  by descending distillation procedure.
- 2) Construct a complete preorder  $Z_2$  by an ascending distillation procedure.
- 3) Construct the partial preorder  $Z = Z_1 \cap Z_2$  as the final result.

ELECTRE III is more appropriate for the evaluation of real world problems. For this reason, in the following example illustrates the ELECTRE III as MADM method to evaluate. Summary of selected MADM method which evaluate by following illustration.

**Table 1: Summary of MADM-based methods**

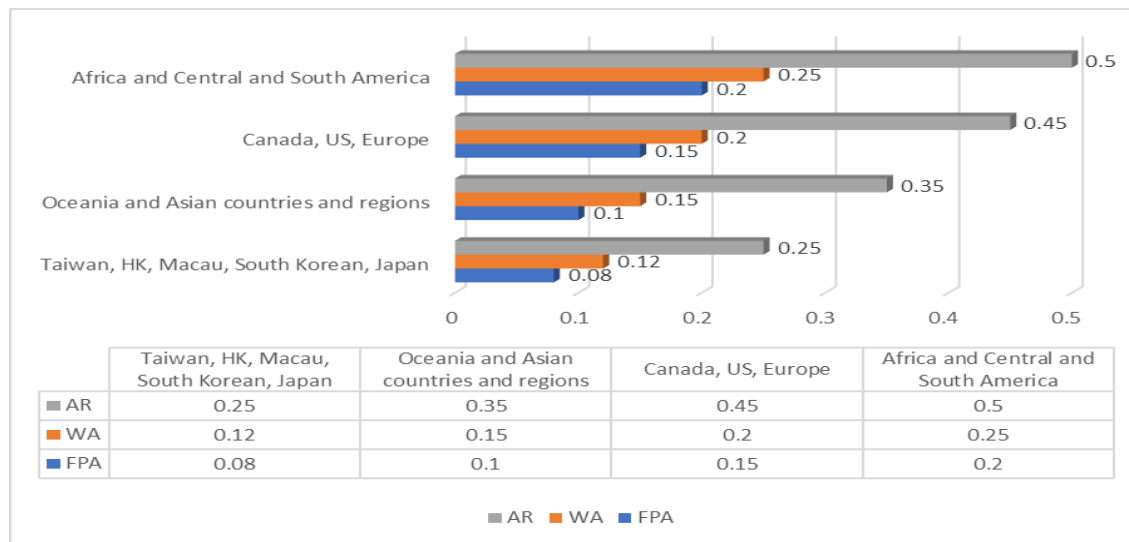
Method	Advantages	Disadvantages	Areas of Application
DEA	Capable of handling multiple inputs and outputs; efficiency can be analyzed and quantified.	Does not deal with imprecise data; assumes that all input and output are exactly known.	Economics, medicine, utilities, road safety, agriculture, retail, and business problems.
TOPSIS	Has a simple process; easy to use and program; the number of steps remains the same regardless of the number of attributes.	Its use of Euclidean Distance does not consider the correlation of attributes; difficult to weight and keep consistency of judgment.	Supply chain management and logistics, engineering, manufacturing systems, business and marketing, environmental, human resources, and water resources management.
ELECTRE	Takes uncertainty and vagueness into account.	Its process and outcome can be difficult to explain in layman's terms; outranking causes the strengths and weaknesses of the alternatives to not be directly identified.	Energy, economics, environmental, water management, and transportation problems.

Source: Velasquez and Hester (2013)

#### MADA-based methods to evaluate the cargo insurance - results and discussions

Cargo insurance covers approved merchandise that as

insureds are obliged to insure under terms of sale. Under an open cargo policy, goods can be insured All Risk (AR), Free of Particular Average (FPA), With Average (WA). Following Figure presents China marine cargo insurance rate table.



**Figure 1: China marine cargo insurance rate table**

#### *Illustration – (marine) cargo insurance for transporting wine*

Wine drinking is becoming more and more popular in China and the prices of wine can vary considerably from brand to brand. Hong Kong has long been one of the world's key shipping hubs, and for many years was the home to the world's busiest port (now located in Singapore). Due to the low import duty in Hong Kong, most wines are imported from around the world to Hong Kong and then re-exported to China. As the first leg of the shipment to Hong Kong is usually insured by the seller, Hong Kong customers only need to insure the second leg of the shipment from Hong Kong to China. Based on background, assumed for transport the wine from Hong Kong to mainland of China, it approximately value of \$5357150. Input data presents by following Table.

**Table 2: Input data**

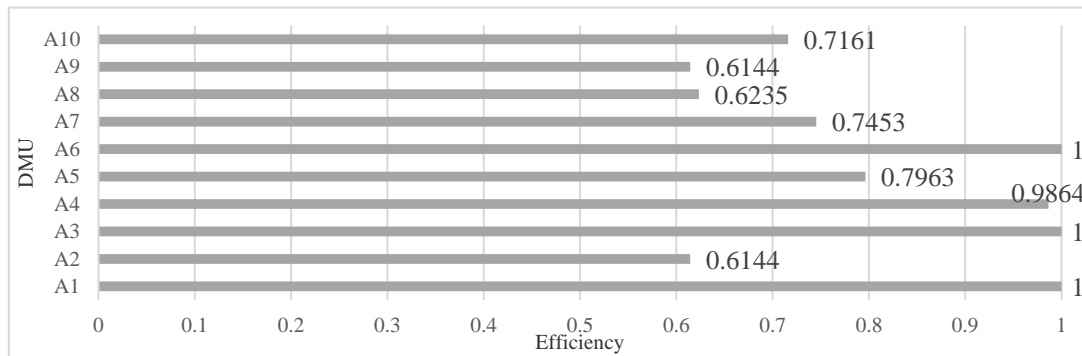
Alternatives		Premium	All risk	Experience	Financial capability
AIG	A1	4,929	18,750	81	51
Allianz insurance	A2	4,821	16,071	7	100
AXA	A3	4,554	27,857	56	107
China Pacific insurance	A4	3,987	19,560	41	111
China Taiping insurance	A5	4,553	13,392	12	108
HSBC	A6	5,250	21,428	30	217
ING	A7	4,285	24,107	11	132
MSIG	A8	5,142	18,214	9	115
The people's insurance company of China	A9	4,928	16,071	13	100
Zurich insurance	A10	4,392	19,821	5	130

### Method 1 – DEA

The calculation of DEA method based on previous discussion. Following table presents the DEA evaluation result that considering CCR model.

**Table 3: DEA CCR model**

DMU	Score	Rank	V(1)* Premium	v(2)* All risk	u(1)* Experience	u(2)* Financial capability
A1	1	1	1	0	0.83467	0.16533
A2	0.6144	9	0	1	0	0.61444
A3	1	1	1	0	0.62458	0.37542
A4	0.9864	4	0.95449	0.04551	0.50815	0.47822
A5	0.7963	5	0	1	0	0.79634
A6	1	1	1	0	0	1
A7	0.7453	6	1	0	0	0.74529
A8	0.6235	8	0	1	0	0.62347
A9	0.6144	9	0	1	0	0.61444
A10	0.7161	7	1	0	0	0.71611



**Figure 2: DMUs efficiency**

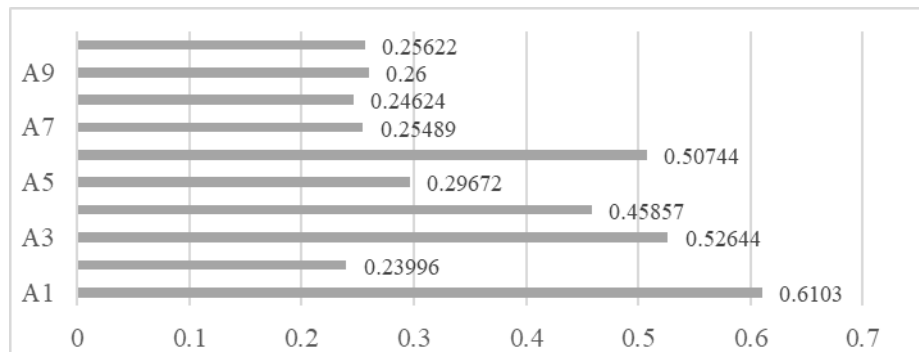
Obtaining the utilities and the preferred order of alternatives can be expressed as A1, A3, A6>A4>A5>A7>A10>A8>A2, A9.

### Method 2 – TOPSIS

The calculation of TOPSIS method based on previous discussion. Following table presents the TOPSIS evaluation result.

**Table 4: TOPSIS**

	MIN	MIN	MAX	MAX			
	Premium	All risk	Experience	Financial capability	di+	di-	ci
A1	0.08292	0.07439	0.17852	0.03263	0.10947	0.17144	0.61030
A2	0.08110	0.06376	0.01543	0.06398	0.18031	0.05693	0.23996
A3	0.07661	0.11052	0.12342	0.06846	0.10665	0.11855	0.52644
A4	0.06707	0.07760	0.09036	0.07102	0.11389	0.09646	0.45857
A5	0.07659	0.05313	0.02645	0.06910	0.16757	0.07070	0.29672
A6	0.08832	0.08501	0.06612	0.13884	0.11875	0.12234	0.50744
A7	0.07209	0.09564	0.02424	0.08446	0.16909	0.05784	0.25489
A8	0.08650	0.07226	0.01984	0.07358	0.17373	0.05676	0.24624
A9	0.08290	0.06376	0.02865	0.06398	0.16861	0.05924	0.26000
A10	0.07389	0.07863	0.01102	0.08318	0.17847	0.06148	0.25622
Weights	0.25000	0.25000	0.25000	0.25000			
Ideal	0.06707	0.05313	0.17852	0.13884			
basal	0.08832	0.11052	0.01102	0.03263			

**Figure 2: TOPSIS results**

Obtaining the utilities and the preferred order of alternatives can be expressed as  $A1 > A3 > A6 > A4 > A5 > A9 > A10 > A7 > A8 > A2$ .

### Method 3 – ELECTRE III

**Table 5: ELECTRE III indifference classes**

Alternatives	Rank
A1	5
A2	5
A3	4
A4	2
A5	1
A6	5
A7	3
A8	5
A9	4
A10	4

The calculation of ELECTRE III method based on previous discussion. Following table presents the ELECTRE III evaluation result.

Obtaining the utilities and the preferred order of alternatives can be expressed as  $A5 > A4 > A7 > A3$ ,  $A9$ ,  $A10 > A1$ ,  $A2$ ,  $A6$ ,  $A8$ .

#### Result by ranking method – Borda

Using borda method to rank the results obtain the most appropriate insurance. The borda method is an election method in which voters rank options or candidates in order of preference: the highest borda count wins. Following Table presents the borda method result for optimal cargo insurance.

**Table 6: Borda method for optimal cargo insurance**

	DEA	TOPSIS	ELECTRE	Rank
A1	1	1	5	1
A2	9	10	5	9
A3	1	2	4	1
A4	4	4	2	3
A5	5	5	1	4
A6	1	3	5	2
A7	6	7	3	5
A8	8	9	5	8
A9	9	8	4	7
A10	7	6	4	6

Based on the borda method result, both A1 and A3 win with highest borda count, hence the optimal cargo insurance are A1 and A3 for this illustration case.

Considering borda method's limitation, further research will use DEA super efficiency ranking method for the double check. DEA super efficiency model is a modified version of DEA which based on comparison of efficient DMUs relative to a reference technology spanned by all other units is developed. The procedure provides a framework for ranking efficient units and facilitates comparison with rankings based on parametric methods. The ideal of DEA super efficiency ranking method based on the Andersen and Petersen (1993) and Jablonský (2016).

## CONCLUSIONS

The aim of this paper present selected MADM methods and application in case of cargo insurance decision problems. Various decision making methodology techniques have been developed and used in the course of recent years. The blend of numerous techniques addresses gaps that might be found in specific strategies. These strategies, alongside the techniques in their unique structures, can be a great degree fruitful in their applications just if their qualities and shortcomings are appropriately surveyed.

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# ON THE CONCEPT AND MEASUREMENT OF FRONTIER SMOOTHNESS IN DEA MODELS

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## ABSTRACT

*Computational experiments with DEA models show that many inefficient units are projected onto the weakly efficient parts of the frontier when efficiency scores are computed. This fact disagrees with the main concept of the DEA approach, since efficiency scores of inefficient units have to be measured relative to efficient units. As a result, inaccurate efficiency scores may be obtained. In our previous work, we developed an algorithm for smoothing the frontier based on using the notion of terminal units. Moreover, it turned out that the smoothness of the frontier can be measured. For this reason, we introduced the notion of a smoothing factor in order to measure the smoothness of the frontier. This factor has to satisfy the following principles: a) it does not depend on units of variables measurement in DEA models; b) the more smoothness corresponds to the less value of the smoothing factor. Our theoretical results are confirmed by computational experiments using real-life data sets.*

**Keywords:** Data envelopment analysis (DEA); Terminal units; Anchor units; Smoothing factor

## INTRODUCTION

In the first decade after the seminal contribution on DEA (Charnes et al. 1978) concerns about the empirical results were raised. The main problem was that too many output and input variables had the minimal non-Archimedean  $\varepsilon$ -value of shadow prices, or weights, raising doubts about the validity of efficiency scores in empirical applications. To solve the problems constraints were introduced on the weights or dual variables using much higher numbers than  $\varepsilon$  (Thompson et al. 1986; Dyson and Thanassoulis 1988; Charnes et al. 1990). However, working in dual space is a challenge for people used to working in input-output space. The restrictions in dual space of weights using domination cones can be implemented by introducing artificial units in the primal space of input and outputs as proved in Krivonozhko et al. (2009). Artificial units had been used as early as in the seminal paper Farrell (1957) in order to secure convex isoquants. Thanassoulis and Allen (1998); Allen and Thanassoulis (2004) motivated introducing artificial units by claiming that such units can capture value judgements about the importance of inputs. The concept *anchor unit* was introduced by Thanassoulis and Allen (1998) for the class of observed units that would be the point of departure for constructing artificial units. The aim was to get a proper envelopment of inefficient units defined as getting weights of meaningful values meaning numbers greater than  $\varepsilon$ . A method for finding the set of anchor units was sketched, but a model with only one input and multiple outputs exhibiting constant returns to scale was used. Bournol and Dulá (2009) extended the search for anchor units in DEA models exhibiting variable returns to scale (Banker et al.

1984) (hereafter BCC), redefined anchor units, and introduced an algorithm to identify them. Thanassoulis et al. (2012) also used a standard BCC model and introduced their new definition of anchor units and an algorithm for how to find them, building on the super-efficiency concept. In Thanassoulis and Allen (1998); Allen and Thanassoulis (2004); Thanassoulis et al. (2012), the construction of artificial units was based on involving decision makers for the units in question. How this process was supposed to work was not detailed.

Krivanozhko et al. (2015) worked out a new definition of units called *terminal* units, that should be the point of departure for constructing artificial units. The production possibility set of the BCC DEA model is a convex polyhedral set spanned by vertices. A unit is a terminal one if an infinite edge goes out from this unit. In a somewhat imprecise way, the terminal units are found at faces constituting the outer parts of the set. Theorems for finding terminal units were established in Krivanozhko et al. (2017). The differences between the various different sets of anchor units, *exterior units* (Edvardsen et al. 2008), and terminal units were revealed, and a comparisons between the different proposals for sets found strict relations between the sets based on a number of theorems. The set of terminal units was the superior one as a point of departure to construct artificial units.

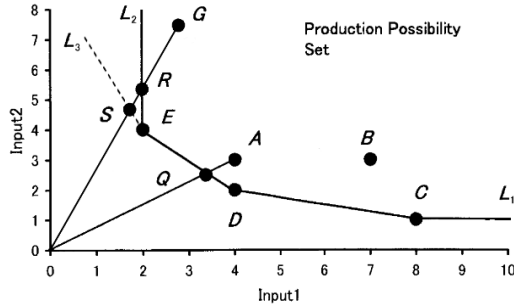
In Krivanozhko et al. (2015) the artificial units were formed with the help of an expert on the production activity at hand (banks). But to find the right experts is not an easy task, and it is also difficult to find a procedure to elicit information. Therefore this idea of using an expert, or a decision maker as in the case of Allen and Thanassoulis (2004), was abandoned in Krivanozhko et al. (2016). Instead, an algorithm was established that had as the objective to establish artificial units in such a way that all units that were strongly efficient before inserting artificial units in the dataset remained strongly efficient, i.e. particularly the terminal units, and that the projections of all inefficient unit would now be on efficient faces. This resembles the quest for “proper envelopment” of inefficient units as expressed by Thanassoulis et al. (2012), but the strategy for forming artificial units is explicit and transparent in Krivanozhko et al. (2016). Using artificial units suggested by experts may destroy the frontier.

## METHODS

The introduction of a measure of smoothness of the frontier in a BCC model is based on the approach of constructing artificial units in Krivanozhko et al. (2016). When all projection points on to the extended frontier are on efficient faces then smoothness is defined based on the angle between the second to the last and the last face towards a pair of axes, and then these angles are summed over all pairs.

## RESULTS AND DISCUSSION

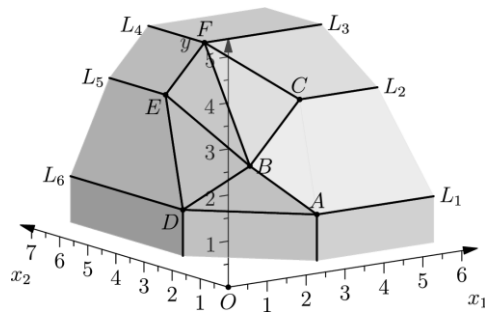
A simple illustration may help to explain the background for introducing artificial units. In the two-dimensional Fig. 1 in input space,  $L_2$ - $L_1$  is the frontier isoquant for units  $G$  and  $A$ . Units  $E$ ,  $D$  and  $C$



**Figure 1: Farrell (1957) measures of technical efficiency**

are the efficient units and the segments  $E-L_2$  and  $C-L_1$  are the weakly efficient faces. There is no support in the data for these two vertical and horizontal segments, respectively; they just represent the default routine of the estimation of the frontier of providing the most pessimistic estimate of the frontier technology given the constraint of convexity. This means that the efficiency score for unit  $A$ ,  $OQ/OA$  where the projection point of unit  $A$  onto the frontier is  $Q$ , is the best estimate of efficiency the DEA model will give. However, the efficiency measure for unit  $G$  is  $OR/OG$ , and  $R$  is not on an efficient segment, but on a weakly efficient segment. This segment being the most pessimistic estimate of the frontier technology yields the efficiency measure as the most optimistic one. But since the segment  $L_2E$  is an arbitrary choice without basis in data the efficiency measure may be way off from a true measure. If the true technology is expressed by the segment  $L_3E$  then the efficiency score for unit  $G$  is reduced to  $OS/OG$ .

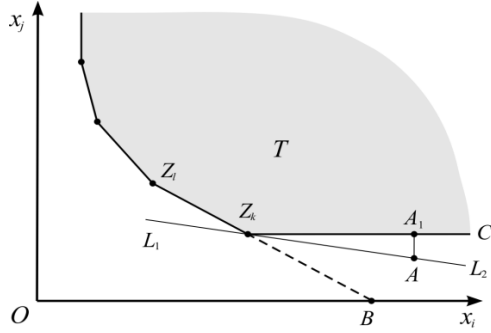
The concept of a terminal unit is illustrated in Fig.2 for the case of two inputs and one output. We see



**Figure 2: Units  $A, C, D, E$  and  $F$  are terminal ones with infinite edges going out from these points**

that infinite edges  $L_1$  to  $L_6$  go out from units  $A, C, D, E$  and  $F$ . We have eight weakly efficient faces and four efficient faces. In this simple figure the weakly efficient faces are all constituting the boundary faces of the production possibility set. The figure illustrates the typical empirical results that the majority of units spanning the set turns out to be terminal units, there is only one unit  $B$  that is a normal efficient unit. It is also an empirical result, especially with datasets having many observations and dimensions, (Bougnol and Dulá 2009) that a majority of projections to the frontier of inefficient units are onto weakly efficient faces.

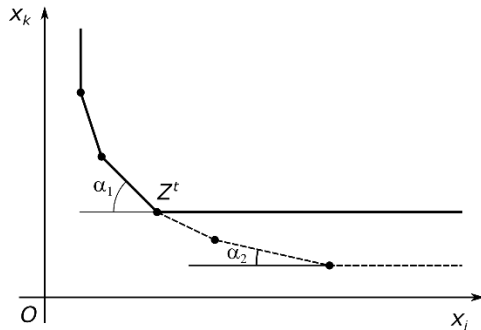
Fig. 3 can give an idea of how the algorithm for constructing artificial units works. A section of the



**Figure 3: Creating an artificial unit based on terminal unit  $Z_k$**

general production possibility set  $T$  can be created with a two-dimensional plane in the input space, going through the terminal unit  $Z_k$  and being spanned by axes  $Ox_j$  and  $Ox_i$ . The rays  $Z_kC$  and  $Z_kB$  and the axis  $Ox_i$  limit the region of where the artificial unit is to be located due to the demand that no unit being efficient before inserting artificial units shall become inefficient after the insertion. Point  $A$  on the line  $L_1L_2$  can be a starting point. Including unit  $A$  the point  $Z_k$  is no longer a terminal one, but becomes an ordinary efficient point. Then new efficiency scores are calculated for all units. If conditions of no initially efficient units becoming inefficient are violated a new artificial unit has to be substituted. The point  $A_1$  is the point the artificial unit  $A$  must be moved towards if inserting  $A$  in the dataset violates the constraints we have imposed. All the possible two-dimensional sections in input space, output space and the space of an output and an input have to be gone through, and the algorithm stops when all inefficient units initially having projections on the frontier get projection units on the new parts of efficient frontier with artificial units in the dataset.

The definition of the smoothing factor is illustrated in Fig. 4 using a two-dimensional projection in input space  $x_k$  and  $x_i$ .  $Z^t$  represents in general all the elements  $z_i^t$  in input space. Initially  $Z^t$  represents



**Figure 4: The construction of the smoothing factor**

terminal unit due to the infinite horizontal edge going out from the units. Solid lines draw the initial isoquant. Broken lines draw the new parts of the isoquant after the algorithm has found the artificial units satisfying all constraints.  $\alpha_1$  is the angle between the efficient segment to the left of  $Z^t$  and the horizontal infinite edge going out to the right.  $\alpha_2$  is the angle between the efficient segment to the left of the last unit of the new isoquant using the artificial units and the horizontal broken line going out from this unit.

Notice that the new terminal unit is also an artificial unit. The purpose is not to eliminate all terminal units, but to get all projection points of inefficient units to be on efficient faces.

The definition of the smoothing factor is the tangent to the angle between the left-hand segment of a terminal unit multiplied with the ratio of the terminal units in  $x_k$  and  $x_i$  space in order to make the factor independent of the units of measurement of the variables. The initial smoothing factor,  $SF^{ini}$ , and the smoothing factor for the new artificial terminal unit after all changes have been made,  $SF^{new}$ , are:

$$SF^{ini} = tg\alpha_1 \cdot \frac{z_i^t}{z_k^t}, \quad SF^{new} = tg\alpha_2 \cdot \frac{z_i^t}{z_k^t} \quad (1)$$

In the general multidimensional case we have to sum over all the pairs of two-dimensional sections in input space for the terminal unit  $Z^t$ :

$$SF_t = \sum_i \sum_{\substack{k \\ k > i}} \left| \frac{\partial z_i^t}{\partial z_k^t} \cdot \frac{z_k^t}{z_i^t} \right|_{z^t \in \text{Sec}(z^t, e_i, e_k)} \quad (2)$$

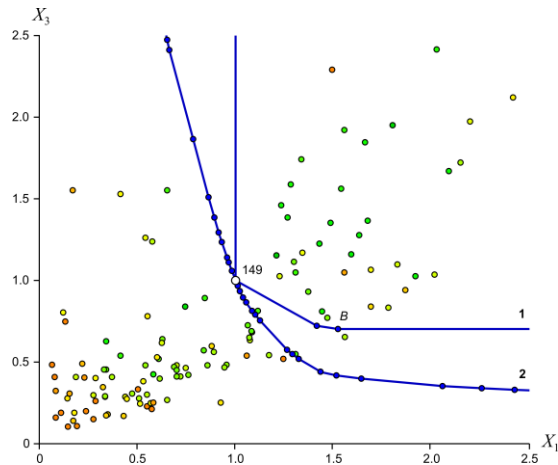
where  $\text{Sec}(z^t, e_i, e_k)$  is a section going through the point  $z^t$  and is spanned by vectors  $e_i$  and  $e_k$ . Similar calculations are to be done for two-dimensional sections of output isoquants and dependence between an output and an input. The general smoothing factor for all sections is

$$SF = \sum_{t \in T^{term}} SF_t \quad (3)$$

$T^{term}$  is the set of all terminal units (different for the initial case and the case after inserting the artificial units obtained by the algorithm).

Smoothing factor was estimated on a dataset for 174 Russian banks year 2008. There are three outputs (equity capital; liquid assets; fixed assets) and three inputs (working assets; time liabilities; demand liabilities). The number of efficient units initially were 26 and inefficient units 148. Almost all inefficient units are projected onto weakly efficient faces; 146 in all. The algorithm constructed 412 artificial units. All projections of inefficient units then ended up on efficient faces.

We will show results for one case of input isoquants. Fig. 5 shows two input isoquants, before and after insertion of artificial units. The isoquants are the intersection of the six-dimensional production possibility set with a two-dimensional plane for the efficient unit 149. Isoquant 1 is the initial isoquant, with unit  $B$  as the second terminal unit and isoquant 2 is after the inclusion of the artificial units.



**Figure 5: Input isoquants before and after for efficient unit 149**

According to the rule of preserving the full efficiency of units bank 149 is located on both isoquants. We see a marked smoothing of frontier 2.

The overall calculation of smoothing factors for all three types of two-dimensional sections is presented in Table 1. We see that inserting 412 artificial units, has a significant impact on the smoothing factors; the decrease in the total factor is about 20 times.

**Table 1: Smoothing factors before and after improving the frontier**

Sections	$SF^{ini}$	$SF^{new}$
Input isoquants	42.39	1.32
Output isoquants	132.48	8.09
Input-output sections	216.39	9.28
<b>Total</b>	<b>391.26</b>	<b>18.69</b>

## CONCLUSIONS

A problem with accepting DEA efficiency score results is that inefficient units get their relative efficiency measures by using comparison points being located at weakly efficient faces and are thus not compared with efficient points. A way to solve this problem is to extend the efficient frontier by introducing artificial units. Starting out from the assumption that all units found efficient shall remain so also after the insertion of artificial units, terminal units are found to be the crucial units to base the construction of artificial units on. The different concepts of anchor units in the literature do not match the set of terminal units. The second requirement of our algorithm for determining the most suitable artificial units is that all projections of inefficient units are onto efficient faces. The procedure for using anchor units as a part of departure and engaging experts to find artificial units may destroy the initial frontier. The shadow prices of the variables of the inefficient units in question initially will then all become positive. However, this does not mean that there are no zero shadow prices in the optimal solution with artificial units. We are addressing the question of how to measure the smoothness of the frontier. Our measure is based on the angles between the last efficient segment and the infinite edge, measured in two-dimensional cuts of the frontier showing input-input, output-output and input-output combinations. The empirical results show a quite overwhelming increase in smoothness of the frontier. This is crucial for calculating trade-offs

between inputs and outputs, respectively that are more acceptable than having to deal with infinity or zero values.

## ACKNOWLEDGEMENTS

This research was supported by Russian Science Foundation (project No. 17-11-01353).

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# PERFORMANCE BENCHMARK OF THE TOP TELECOM OPERATORS IN THE MOBILE ERA

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## ABSTRACT

*Telecom operators play a critical role in the sustainable future of the ICT industry. Utilizing sequential Malmquist Productivity Index (MPI) analysis, this study investigate the productivity of the top operators during the mobile revolution and benchmark their performance along with two mobile transition waves between 2000 and 2015. Our MPI results show that the average productivity score of the operators for the whole period is positive during the growth phases of both waves, but a sharp fall is observed in the second one. The decompositions of the MPI exhibit that technological change is the main driver of the productivity gains in the industry in both waves. The average efficiency change score, which is hardly over 1.00 line in the second wave, signals a potential risk for the operators in their catch-up ability. Besides, business cycle component hinders the productivity of the telecoms for the whole period. On the other hand, US operators had a productivity increase in the second wave on average, while Europeans experienced a severe drop.*

**Keywords:** Telecommunication industry; performance; Malmquist Index Analysis; sequential DEA

## INTRODUCTION

Network operators play a dominant role in undertaking the investment burden of the network on which information and communications technologies (ICT) rises (Fransman, 2010). Therefore, the financial productivity is an important aspect on the ability of the operators to invest, innovate and continue to play a critical role in the sustainable development of the ICT industry.

Starting by mid 1990s, the landscape of the telecommunications has changed dramatically in terms of regulations, market/competition and technology. Liberalization of the telecom market made it possible for fixed/mobile phone, ISP or cable operators to provide similar services to consumers and changed the nature of the competition and the market (Curran and Leker, 2011). On the other hand, wireless technologies has transformed the previous paradigm of communication (O'Regan, 2012). As Figure 1 (a) reveals, the mobile revolution came with two important waves, first in voice then in data. Started to be a challenger even by the second half of 1990s, mobile phones rapidly moved from niche to mainstream. Even by 2000, the worldwide subscription rate of mobile phones rose to 12.05%. In 2015, this rate was 6.81 times more than the worldwide fixed-line phone subscriptions (97.89% vs. 14.38%). However, the growth period of mobile phones ended around 2007, the year which the increase rate of worldwide subscriptions started to drop. The building blocks of the second mobile wave were being laid by the telecom operators since early 2000s by investing heavily on upgrading their networks to 3G. But the real take-off of the mobile internet (data) was around 2007, which is the year Apple Inc. has introduced its game changing smart phone to the market, iPhone. Smart phones allowed people to produce and access the digital content via mobile networks, after which mobile broadband subscriptions surged impressively.



As exhibited by Figure (a), worldwide mobile broadband subscriptions surpassed the fixed-line broadband subscriptions even by 2008 (6.24% vs. 6.13%) and quadrupled in 2015 (43.98% vs. 11.16%).

Previous research exists on the productivity change of telecom industry utilizing Malmquist Productivity Index (MPI) analysis. For example, Calabrese et al. (2002) employed MPI for evaluation of the productivity change of the telecom sector in 11 OECD European countries between 1979 and 1998. Along with DEA, Diskaya et al. (2011) used MPI for understanding the productivity change of nine telecom companies in G8 countries and Turkey for the years between 2007 and 2010. Uri (2001) tried to understand the effect of incentive policies on the productivity of the local exchange carriers (LEC) in US between 1988 and 1998.

In the telecom industry, different technologies co-exist and technological regress is practically not possible. Therefore, this study utilizes sequential MPI to investigate the productivity change of the worldwide top telecom operators in mobile take-off and benchmark their performances during the growth phases of the two mobile transition waves, in the periods of 2000-2007 and 2008-2015 respectively.

## METHODS

MPI is introduced by Malmquist (1953) and improved by Caves et al. (1982) and Färe et al. (1992). Based on data envelopment analysis (DEA), MPI measures the productivity change between two data points by the ratio of the distances of each data point with respect to a common technology. The output oriented MPI ( $M_o$ ) can be expressed as:

$$M_o(x^{t+1}, y^{t+1}, x^t, y^t) = \frac{D_o^{t+1}(x^{t+1}, y^{t+1})}{D_o^t(x^t, y^t)} \left[ \frac{D_o^t(x^{t+1}, y^{t+1})}{D_o^{t+1}(x^{t+1}, y^{t+1})} \frac{D_o^{t+1}(x^t, y^t)}{D_o^t(x^t, y^t)} \right]^{1/2} \quad (1)$$

The ratio outside of the brackets in (1) is the change in efficiency (EFFCH), which represents the change in the relative distance of the observed output level from the maximum potential output between two periods. On the other hand, the geometric mean of the two ratios inside the brackets of (1) captures the shift in technology (TECH) from  $t$  to  $t+1$ . TECH reflects the capability of the company to innovate its production processes, which result in a shift in the efficiency frontier line. And EFFCH is related with the catch-up ability of a company to mimic the efficiency frontier as identified by the DEA.

Unlike the contemporaneous DEA, the sequential DEA model assumes that the technologies of the preceding periods are nested in each period under analysis. The same assumption is applied to calculate the sequential Malmquist index and its decompositions, where sequential TECH only indicates technological progress and regress is assimilated in the inefficiency (Tulkens and Vanden Eeckaut, 1995). Shestalova (2003) suggested a composite model, which is a synthesis of the contemporaneous and sequential models, where the decomposition formula of Malmquist index can be rewritten as:

$$M_s = TECH_s \times \frac{EFFCH_s}{EFFCH_c} \times EFFCH_c \quad (2)$$

where the subscript C and S refer to contemporaneous and sequential frontiers respectively. The second item is called as the business cycle (BC) component, which measures the productivity change that is attributable to the business cycle via capacity utilization and labour hoarding (Shestalova, 2003). In this study, sequential Malmquist index analysis is used to analyse the TFP change of the telecom industry, and Shestalova's composite model in (2) is utilized for further decompositions of the Malmquist Indices.

## DATA

22 telecom operators are selected from 2015 Financial Times Global 500 (FT500) lists based on the availability of financial data. The financial data of the companies are collected from company annual reports/SEC filings, then deflated to 2010 values. The outliers are removed in iterations.

Input/output variables are selected, which have the same meaning across different tax regimes and reporting standards and are publicly available. Therefore, *revenue* is used as the output (Calabrese et al. 2002, Chen and Ali 2004, Diskaya et al. 2011, etc.), whereas *total assets* (Chen and Ali 2004), *capital expenditure* (Chou et al. 2012, Diskaya et al. 2011 etc.), *total equity* (Chen and Ali 2004) and *number of employees* (Calabrese et al. 2002, Chen and Ali 2004, Diskaya et al. 2011 etc.) are the inputs.

As can be observed from Figure 1 (b), in terms of revenue, assets and employee size, the operators in our sample are bigger in wave 2 than in wave 1. However, revenue increase rate is retarded significantly in wave 2 compared to the first one, while operators had to increase their asset base and capital expenditures faster in wave 2.

## RESULTS AND DISCUSSIONS

Table 1 lists the productivity scores of our sample telecom operators, which are calculated by employing sequential MPI with an output oriented variable returns to scale (VRS) DEA model. America Movil (1.095), Softbank (1.079), China Unicom (1.078), AT&T (1.056) and Verizon (1.055) are the top five operators of our sample. Although the average  $M_s$  of the sample is positive (1.033) for the whole period, it has a negative slope, where the productivity drop is more obvious especially after 2011.

**Table 1: Sequential Malmquist Index Scores ( $M_s$ ) for the years between 2000 and 2015**

No	Company	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	Mean
1	America Movil	1.405	1.147	1.159	1.116	1.196	1.030	1.223	1.067	1.014	1.011	1.072	0.994	1.050	1.012	0.997	1.095
2	Softbank	1.000	0.918	1.185	1.027	1.300	1.021	1.210	1.039	1.073	1.124	1.087	1.044	0.749	1.467	1.114	1.079
3	China Unicom	0.965	1.166	1.639	1.182	1.122	1.084	0.872	1.059	0.885	1.062	1.200	1.110	1.167	0.950	0.907	1.078
4	AT&T	0.962	1.276	1.027	0.990	0.933	1.114	1.435	1.047	1.030	1.015	1.011	1.021	0.975	1.018	1.085	1.056
5	Verizon	1.082	1.072	1.031	0.984	0.986	1.064	1.031	1.071	1.077	1.040	1.043	1.046	1.044	1.271	1.012	1.055
6	KDDI	1.154	1.155	1.132	1.034	1.092	1.041	1.007	1.007	1.010	1.008	1.048	1.106	1.030	0.923	1.043	1.051
7	China Mobil	1.229	0.914	1.202	1.051	1.156	1.109	1.103	1.077	1.017	0.955	1.035	1.013	0.998	0.947	0.972	1.048
8	Orange	1.986	1.123	1.144	1.019	0.980	1.012	1.037	1.059	0.908	0.979	0.997	0.929	0.994	0.985	0.855	1.046
9	Telenor	1.379	0.957	1.389	0.925	0.920	1.095	0.988	1.093	1.016	1.026	1.092	0.880	1.007	0.993	1.051	1.045
10	Vodafone	1.042	1.394	1.215	1.316	1.023	0.839	1.202	1.051	0.839	1.076	1.070	0.928	0.820	1.139	0.902	1.044
11	Deutsche Tel.	0.931	1.460	1.341	1.030	0.888	1.059	1.158	0.944	1.036	0.930	0.958	1.087	0.927	0.919	1.021	1.035
12	NT&T	1.120	1.095	1.037	1.017	1.031	1.005	0.995	0.975	1.082	1.049	1.034	1.094	1.038	0.937	1.031	1.035
13	Telefonica	0.912	1.641	1.073	1.068	1.018	1.082	1.063	1.040	1.015	0.874	1.037	1.019	0.970	0.844	0.925	1.027
14	Telstra	0.956	1.162	1.064	1.191	0.991	0.996	0.968	1.107	0.875	1.209	1.034	0.981	1.004	0.945	0.933	1.023
15	Singtel	0.717	0.679	2.147	1.415	0.981	0.909	0.969	1.097	0.972	1.119	1.034	0.966	1.018	0.923	0.957	1.021
16	Bharti Airtel	NA	NA	NA	NA	NA	NA	NA	NA	1.042	0.946	0.985	1.183	1.065	1.268	0.738	1.019
17	T-Mobile US	NA	NA	NA	NA	NA	1.009	0.832	0.619	1.453	1.235	2.014	0.939	0.637	1.060	1.017	1.019
18	BCE	0.954	1.228	1.030	1.017	0.966	1.101	0.941	1.066	0.964	1.074	1.031	0.960	0.983	0.947	1.003	1.015

19	NTT DoCoMo	1.036	1.055	1.037	1.125	0.967	0.934	1.033	1.076	0.960	1.036	1.090	1.010	0.942	0.920	0.928	1.008
20	Telia	NA	NA	1.392	0.913	0.983	1.074	0.890	0.960	1.119	0.965	0.952	1.004	0.998	0.781	0.944	0.989
21	Swisscom	1.055	1.215	1.036	0.806	1.087	0.883	0.706	1.135	0.940	1.044	1.053	0.884	1.022	0.994	0.998	0.982
22	MTN Group	NA	NA	0.901	0.927	0.809	0.987	1.000	1.101	0.905	1.224	1.146	0.722	0.983	1.142	0.794	0.961
Average		1.105	1.148	1.209	1.058	1.021	1.021	1.032	1.033	1.011	1.046	1.092	0.996	0.974	1.017	0.965	1.033

Source: Own calculations of authors (Note: Mean is the geometric mean of each row. Average is the arithmetic mean of each column.)

Based on the Shestalova's model (2003), Table 2 benchmarks the average  $M_S$  scores and decomposition of the Malmquist Indices for the growth periods of the two waves of the mobile revolution. Although it is still positive, the average productivity of the telecom operators fell sharply in P2 (1.009) compared to P1 (1.061). MTN Group, Swisscom, T-Mobile US and Verizon are the only operators which have a higher average  $M_S$  score in P2. As the "Average" row of Table 2 indicates, TECH is the main factor of the productivity increase in both periods (1.064 and 1.025). This means that, the productivity increase of our group is mainly the result of the shifts in the efficiency frontier lines due to the innovations in the production process of the operators. We have to note that TECH dropped almost 4% in P2 compared to P1. On the other hand, average EFFCH played a positive but moderate role in the productivity of the operators. We have to note that, residing almost on the border, average EFFCH (1.005) in P2 signals a potential risk for the telecom operators in their catch-up ability to mimic the efficiency frontier as identified by the DEA. In P2, Bharti Airtel (1.059), MTN Group (1.062) and Telenor (1.063) are top three EFFCH<sub>C</sub> scorers. Furthermore, the average business cycle (BC) component, had a negative impact on their productivity in both periods (0.983 and 0.980).

**Table 2: Decomposition of the Malmquist Indices**

Company	Country of Origin	$M_S$		TECH <sub>S</sub>		BC		EFFCH <sub>C</sub>	
		P1	P2	P1	P2	P1	P2	P1	P2
America Movil	Mexico	1.178	1.027	1.049	1.027	1.000	1.000	1.122	1.000
AT&T	US	1.093	1.025	1.092	1.025	1.000	1.000	1.001	1.000
BCE	Canada	1.030	1.002	1.070	1.011	0.970	0.987	0.992	1.004
Bharti Airtel	India	NA	1.019	NA	1.022	NA	0.941	NA	1.059
China Mobil	China	1.105	1.001	1.037	1.016	0.996	0.989	1.070	0.996
China Unicom	China	1.127	1.036	1.072	1.017	0.997	0.993	1.055	1.027
Deutsche Tel.	Germany	1.107	0.976	1.107	1.007	1.000	0.990	1.000	0.979
KDDI	Japan	1.086	1.021	1.086	1.026	1.000	0.995	1.000	1.000
MTN Group	S. Africa	0.922	0.987	1.039	1.003	0.977	0.927	0.908	1.062
NT&T	Japan	1.042	1.029	1.043	1.034	0.999	0.995	1.000	1.000
NTT DoCoMo	Japan	1.025	0.993	1.035	1.012	0.990	0.981	1.000	1.000
Orange	France	1.151	0.961	1.065	1.015	0.999	0.986	1.082	0.961
Singtel	Singapore	1.036	1.009	1.113	1.014	0.931	0.994	1.000	1.000
Softbank	Japan	1.087	1.072	1.087	1.072	1.000	1.000	1.000	1.000
Swisscom	Switzerland	0.955	1.006	1.041	1.016	0.954	0.958	0.962	1.035
T-Mobile US	US	0.917	1.047	1.005	1.086	0.912	0.998	1.000	0.966
Telefonica	Spain	1.105	0.963	1.084	1.018	1.000	0.973	1.020	0.972
Telenor	Norway	1.078	1.018	1.081	1.007	0.991	0.951	1.006	1.063
Telia	Sweden	1.036	0.961	1.028	1.018	0.937	0.966	1.076	0.978
Telstra	Australia	1.044	1.006	1.073	1.012	0.995	0.952	0.977	1.044
Verizon	US	1.035	1.073	1.035	1.073	1.000	1.000	1.000	1.000
Vodafone	UK	1.133	0.972	1.095	1.020	1.000	0.989	1.035	0.963
Average		1.061	1.009	1.064	1.025	0.983	0.980	1.015	1.005

Source: Own calculations of authors. (Notes: P1: 2000-2007. P2: 2008-2015. Values are the geometric mean of each period. Average is the arithmetic mean of each column.)

Table 3 compares the average productivity of the operators headquartered in US, Japan and Europe. We see that in the first mobile wave in P1, average productivity score ( $M_S$ ) of the European operators is the

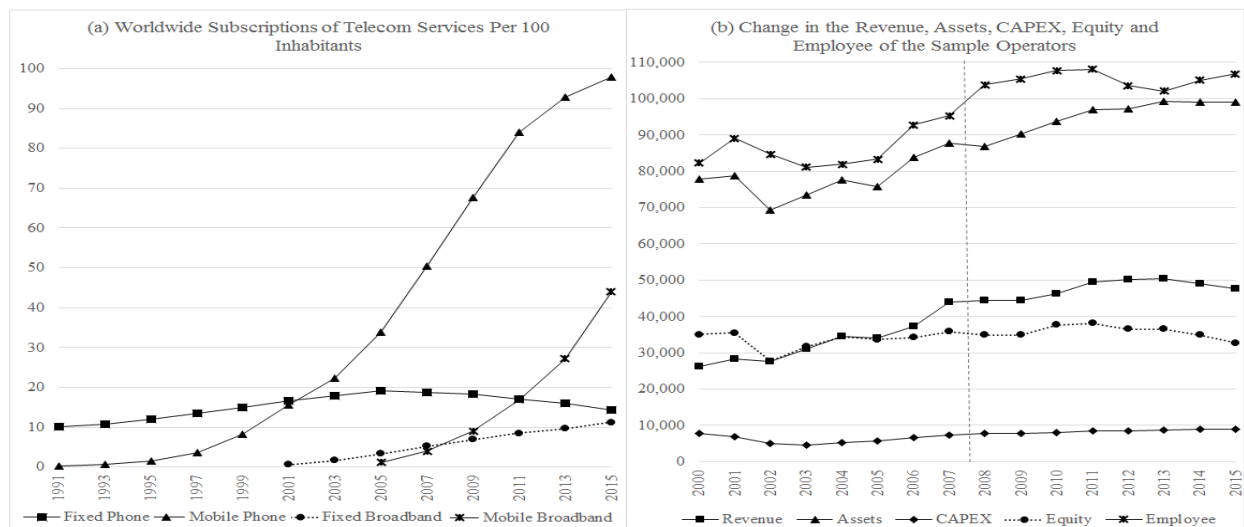
higher than operators of US and Japan. With the score of 1.077, TECH has a major impact on productivity of Europeans, where EFFCH is also a significant contributor. We must note that, TECH is distinguished as almost the sole source of productivity gains of US and Japanese operators in P1. Interestingly, the ranking is upside down during the second mobile wave in P2. US operators are the productivity leaders in P2, which is the only group able to increase the average  $M_s$  score compared to P1. With 1.067 average TECH score, US operators are the innovation leaders in the second wave, where TECH is again the main component of their productivity gain. On the contrary, the rise in average BC of the US operators to 1.006 is balanced with the fall of EFFCH to negative levels (0.989). This points out an efficiency issue of the US operators in utilizing the inputs to generate the revenue (output) in P2.

**Table 3: Productivity comparison of operators from US, Japan and Europe**

	$M_s$		TECHs		BC		EFFCHc	
	P1	P2	P1	P2	P1	P2	P1	P2
US Average	1.046	1.065	1.060	1.067	0.986	1.006	1.002	0.989
Japan Average	1.062	1.033	1.065	1.036	0.998	0.994	1.000	1.000
Europe Average	1.101	0.982	1.077	1.014	0.988	0.977	1.037	0.999

Source: Own calculations of authors. (Notes: Values are the geometric means of the avg. of the operators in US, Japan and Europe at each year.)

Still continuing well above the border line of 1.00, Japanese telecoms experienced a 2.9% productivity loss in P2 on average, almost which is attributed to drop in average TECH. But the drastic fall in average productivity of European operators is remarkable in P2, where it fell 11.9% in P2 of which 6.3%, 1.1% and 3.8% can be attributed to TECH, BC and EFFCH respectively.



**Figure 1: (a) Worldwide subscriptions of telecommunication services per 100 inhabitants, (b) Change in the revenue, assets, CAPEX, equity and employee of the sample operators**

(Notes: Subscription data is collected from Worldbank, ITU and Kelly and Rosotto (2012). Revenue, Assets, CAPEX and Equity values are in million US dollars)

## CONCLUSIONS

This study investigates the productivity of the top 22 telecom operators worldwide during 2000 and 2015 and benchmarks their performance during the growth periods of the two waves of the mobile revolution.

The productivity of the sample telecom operators has grown annually by 3.3% during the whole analysis period but with a negative slope which is steeper especially after 2011. On average telecom operators succeeded higher productivity scores during P1, whereas the average  $M_s$  score fell sharply from 1.061 in P1 to 1.009 in P2. The decompositions of the Malmquist Indices exhibits that, innovation (TECH) is the main driver of the productivity gains of the telecom operators. However, the EFFCH component which fell almost to the border line of 1.00 in P2 is a sign of issue for the catching-up capacity. With average value of 0.98 in both periods, Business Cycle (BC) component, which is an indicator of capacity utilization and labour hoarding, had a negative impact on the productivity of the operators. On the other hand, the average productivity of the Europeans was higher than the operators of US and Japan in P1. But with a productivity drop of 11.9%, Europeans failed to sustain their position in P2, when the US telecoms took over the leadership.

The negative direction in the average productivity and the efficiency of the telecom operators in P2 is an expected result of a dilemma which the operators have to face. On one side, intense competition in the worldwide telecom markets continuously pushes the communication prices down and put pressure on the revenues. On the other side, sustaining the level of service quality and responding the increasing expectations of the customer force operators to continue to invest heavily in their network infrastructure.

Operators need to continue to invest on the sustainable future of the whole ICT industry. We must note that, the productivity of the telecom operators is an important aspect of this equation. Within this market context and dilemma, the operators, especially the ones which have productivity scores below the industry average, may think of more R&D investment, innovation (technological change component) and introduction of new value added services to increase the revenue and to leverage their productivity.

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# PERFORMANCE EVALUATION USING DATA ENVELOPMENT ANALYSIS: A CASE OF A PHILIPPINE UNIVERSITY

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## ABSTRACT

*Performance appraisals are a part of career development and consist of regular reviews of employee performance within organizations (Muchinsky, 2012). This study assessed the performance of the eight colleges of a University vis-à-vis their efficiency along instruction (faculty and students), research, extension services, and program requirements. Data Envelopment Analysis (DEA), a non-parametric technique, was utilized to measure the efficiency of the different colleges. It compared the efficiency and performance of the different colleges in terms of the decision-making units (DMUs); the best practice performers that served as the benchmark on which the performances of others were to be evaluated. The data (inputs and outputs) were treated with xDEA software in Microsoft Excel. It determined the efficiency of the different colleges; the peer groups and weights of the colleges; the virtual inputs/outputs or improvements of the colleges to be in the efficient frontier; and the reference set proposed for inefficient colleges for benchmarking /efficient colleges. The colleges with the best practices in the different performance indicators were identified. It was found out that there is no particular single college which is “fully efficient” in all the indicators.*

**Keywords:** performance evaluation, Data Envelopment Analysis, efficient frontier, peers and weights, virtual inputs and outputs

## INTRODUCTION

All over the globe, the Higher Education Institutions (HEIs) play very important role in the economic and social development of the country. In the Philippines, the higher education system has a tri-fold function of instruction, research, and extension services. Every institution's goal is to create a “culture of excellence” in order to obtain autonomy. Performance appraisals are a part of career development and consist of regular reviews of employee performance within organizations. According to Manasa and Reddy (2009) and Abu-Doleh & Weir (2007), a performance appraisal is a systematic and periodic process that assesses an individual employee's job performance and productivity in relation to certain pre-established criteria and organizational objectives.

Many schools, colleges and universities have employed some performance efficiency evaluation models, including Data Envelopment Analysis (DEA). Wolszczak-Derlacz and Parteka (2011) evaluated the relative technical efficiency of European higher education institutions in a comparative setting and determined the external determinants of their performance. Alexander et al. (2010) also used DEA to analyze the secondary school sector in New Zealand. Sagarra, et al (2016) combined traditional ratios together with DEA models to assess how Mexico's “Educational Modernisation Programme” has affected efficiency in teaching and research at Mexico's universities. DEA was used to measure the relative efficiency of academic departments and faculties by Sirbu, et al. (2016); Awadz, Ramasamy, Akhir & Loy (2012); Badri and El Mourad (2012).

The study conducted by Castaño, et.al (2007) used DEA Malquist and Input Oriented CRS approaches, determined the sources of productivity and efficiency growth of different state universities and colleges in the Philippines; Baldemor (2009) utilized the CRS Input Oriented Multi-stage DEA Model and determined the performance of 16 colleges and institutes of Don Mariano Marcos Memorial State University.

The present study is seated on human capital theory where it presupposes that investment in an individual's education and training is similar to business investments in equipment (Becker, 1994). In their study "*Human capital and rates of return*", Psacharopoulos and Patrinos (2002) conclude that educational quality, measured by cognitive skills, has a strong impact on individual earnings; moreover, educational quality has a strong and robust influence on economic growth. This study also picks up on the concept of Gleich, et al, (2008) that in today's competitive environment, better performance and commitment to achieve a competitive advantage is essential for universities as the places for knowledge worker, knowledge direction and education. It also relates on the concept of Senses (2007) who maintains that quality issue in higher education can be evaluated based on several categories such as education and research. Furthermore, Yilmaz and Kesik (2010) explain that quality administration requires assessment and evaluation, among others. According to Ozer, Gur, and Kucukcan (2010), quality assurance is concerned with instruction, research, publication, academic achievement, project development and all processes used in other processes by higher education institutions.

The foregoing literature and theories led the researcher to conduct this study in a University in the Philippines. DEA will help XYZ University in its performance evaluation system that will regularly identify and analyze the school's strengths, weaknesses, opportunities, and threats in order to develop a full awareness of all the factors involved. There are several accrediting agencies (external evaluators) but no internal performance evaluation on the efficiency of the colleges has been adopted by the University. Thus, this study was conceptualized to evaluate the performance of the undergraduate programs in XYZ University along selected areas, through the Data Envelopment Analysis. This is the only study in the Region that uses DEA.

It was the objective of this study to assess the performance of the eight colleges of XYZ University vis-à-vis their efficiency along instruction (faculty and students), research, extension services, and program requirements for the year 2014; also to determine the peer groups and weights of the colleges; the virtual inputs/outputs or improvements of the colleges to be in the efficient frontier; and what reference set is proposed for other colleges for benchmarking efficient colleges.

## **METHODS**

A non-parametric technique called Data Envelopment Analysis (DEA) introduced by Charnes et al (1978) was utilized to measure the efficiency of the different colleges of XYZ University. The main developments of DEA in the 1970s and 1980s are documented by Seiford & Thrall (1990). DEA compared the efficiency and performance in terms of the decision-making units (DMUs); the best practice performers serve as the benchmark on which the performance of others is to be evaluated. Virtual inputs and outputs were used in estimating performance and efficiency. It also identified the best "virtual



performing unit”. DEA identifies a “frontier” on which the relative performance of all utilities in the sample can be compared; DEA benchmarks colleges only against the best producers. It can be characterized as an extreme point method that assumes that if a college can produce a certain level of output utilizing specific input levels, another college of equal scale should be capable of doing the same. The most efficient producers can form a 'composite producer', allowing the computation of an efficient solution for every level of input or output. Where there is no actual corresponding firm, 'virtual producers' are identified to make comparisons (Berg 2010). The main advantage to this method is its ability to accommodate a multiplicity of inputs and outputs. It is also useful because it takes into consideration returns to scale in calculating efficiency, allowing for the concept of increasing or decreasing efficiency based on size and output levels. A drawback of this technique is that model specification and inclusion/exclusion of variables can affect the results (Berg, 2010). Table 1 shows the eight colleges, the indicators, the inputs, and the outputs.

**Table 1. Instrument for data collection**

College	Indicators	Sub-indicators	
		Input	Output
CA	1. Instruction (Faculty)	a) Number of faculty	a) Academic Rank of Faculty b) Highest educational attainment of faculty
CAS		b) Seminars/trainings attended	
CEAT	2. Instruction (Students)	a) Student enrolment	a) Number of graduates
CF		b) Number of scholars	b) Number of student awardees
CHET	3. Program Requirements	Number of Programs	a) Number of accredited programs b) Accreditation Status
CN			
CTE	4. Research	Number of completed researches	a) Number of researches presented b) Awards received
CVM			
	5. Extension	Number of trainings conducted	Number of clients served

The data gathered were tallied using counts or frequencies to be the virtual inputs and outputs. Then they were treated with the xDEA software, an add-in in the Microsoft Excel, wherein the DMUs were assessed whether “fully efficient”, “weak efficient”, or “Inefficient”.

## RESULTS AND DISCUSSIONS

Table 2 shows the scores which resulted from treating the input and output scores with xDEA under the sub-indicator Faculty. Two DMUs/colleges or 28.57% emerged to be “fully efficient”. It is noted that the efficient units, CA and CTE, have the highest percentage of faculty members with doctorate degree. The finding confirms the findings of some researches which suggest that in general, higher levels of teacher education are associated with higher overall classroom quality, more positive teacher behavior in the classroom, and greater gains in cognitive and social development in children (Bowman, et al., 2001; and Whitebook, 2003).

Johnson (2000) also found out that percentage increases if the teacher holds an advanced degree in math or science which led him to conclude that teachers who are more qualified in a subject transmit the more

advanced concepts. The other six DMUs are rated as “inefficient”. This means that there is a need for these DMUs to improve especially on the highest degree earned; this will translate to a corresponding increase in their academic rank.

**Table 2. Efficiency scores of the college as to instruction along the area “Faculty”**

COLLEGE	EFFICIENCY SCORES	DESCRIPTION
College of Agriculture (CA)	1.0000	Fully Efficient
College of Arts and Sciences (CAS)	0.9426	Inefficient
College of Engineering and Agricultural Technology (CEAT)	0.9394	Inefficient
College of Forestry (CF)	0.9123	Inefficient
College of Home Economics and Technology (CHET)	0.8263	Inefficient
College of Nursing (CN)	0.8581	Inefficient
College of Teacher Education (CTE)	1.0000	Fully Efficient
College of Veterinary Medicine (CVM)	0.8310	Inefficient

Tables similar to Table 2 were constructed for the other indicators. Under the sub-indicator Students, Six DMUs or colleges (75%) are considered “fully efficient” while two DMUs are rated “inefficient”. The other two lack some points to reach the efficiency score of 1.000 to attain efficient frontiers, particularly in the number of students who finished within the prescribed period and the number who received awards. This implies that there are factors that deter the students from finishing their program of coursework; this may be addressed by intensifying the career guidance counselling and developing a better communication system between the administration and the student organization.

Along “Research”, only one DMU (12.5%) is “fully efficient” and the others are “inefficient”. The finding implies that the colleges except CTE need to exert more effort to conduct more researches and present them also in national and international fora and strive to garner awards for the researches presented. This further implies that more workshops and trainings on how to write a publishable paper are needed. The importance of Research in the efficiency of a firm is affirmed by a report chaired by Wakeham (2010) *Financial Sustainability and Efficiency in Full Economic Costing of Research in UK Higher Education Institutions* which made a number of recommendations relating to the need for the research base to continue making progress in the efficiency of its operations.

In addition to research and instruction, extension completes the trifocal functions of higher education institutions. Only one DMU (12.5%) is “fully efficient” and the others are “inefficient”. The inefficient DMUs lack points to be in the efficient frontier. They need to plan ways and means to strengthen their extension performance. The finding implies that Extension is another weakness of the University, besides Research. There may be a need for the colleges to be resourceful and productive in their extension programs. This finding affirms the observation of Cañares (2008) that in the accreditation processes of higher education institutions (HEIs) in the Philippines, the importance of community extension is, to an extent underrated (relative to the other two) and vaguely defined.

Meanwhile, under the sub-indicator ‘program requirements’, five (62.5%) are “fully efficient” while the other three are “inefficient”. The finding implies that there is a need for the colleges to submit their programs for accreditation (if not yet accredited) or to acquire higher accreditation levels.

The finding corroborates with Madriaga (2015) who conducted a study on private HEIs in which out of twenty participating HEIs, only 40% HEIs were engaged in extension service. The finding also

corroborates the findings of a survey by NEASC (2006) wherein over 90% of school leaders surveyed believe that participation in the accreditation process has enhanced the overall quality of education, has improved the quality of classroom instruction, and will improve teaching and learning at their school in the future.

#### Peer groups and weights of the colleges

Peers and Weights indicate the percentage values needed by “inefficient” DMUs compared to the “fully efficient” ones. This refers to the best practices of the DMUs in the efficient frontiers wherein the “inefficient colleges be more vigorous in their pursuit to become “fully efficient”.

The “fully efficient” colleges had their own peers and weights since they do not need target values to be in the efficient frontier. The “inefficient” colleges posted different peers and weights. Table 7 presents the peers and weights of the inefficient DMUs. It also indicates the percentage values of the best practices of the “fully efficient” colleges. The “fully efficient” DMUs have their own colleges as their peers – these are the references of the “inefficient” colleges. The percentages were computed -these indicate that which could be adapted by the “inefficient” DMUs to become “fully efficient” or to catch up the efficient score frontiers.

#### Virtual Inputs/Outputs (Virtual IOs)

Different “inefficient” DMUs had different virtual IOs in their respective inputs/outputs in all the performance indicators. The “fully efficient” colleges did not need virtual IOs. The colleges, mentioned in the foregoing section, are with the best practices in the different performance indicators.

### **CONCLUSIONS**

There was no particular single college which is “fully efficient” in all the indicators. The “fully efficient” colleges, having their own peers and weights in the different performance indicators, become the peers and references of the “inefficient” ones. The “fully efficient” DMUs or colleges do not need virtual IOs, while the “inefficient” units have different virtual IOs in their respective inputs/outputs in all the performance indicators.

The faculty and students have to exert more effort to obtain awards in their fields of specialization. The faculty may pursue higher level of education and conduct more researches to be presented in research fora and publish in refereed journals for the consumption of the intended population. To become more efficient, each college need to establish more linkages and have a broader perspective of Extension. The “fully efficient” colleges may share their best practices to the other colleges for the latter to implement what is applicable to their college. The “fully efficient” colleges should continue performing their best. The “inefficient” colleges should adapt the best practices of their peers or references in the different performance indicators in order to catch up points to be in the efficient frontier – all colleges are encouraged to aim to be in the efficient frontier.

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# PRODUCTIVITY GROWTH, TECHNICAL PROGRESS AND EFFICIENCIES IN THE INDIAN URBAN WATER SUPPLY SECTOR

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## ABSTRACT

*DEA-based Malmquist approach for evaluating total factor productivity (TFP) and technology change in 21 Indian water supply services over 1999-2009 is employed in this paper to answer the following: a. Are Indian water supply services efficient? b. Are water supply operations improving over time? c. How should the present tariffs be restructured in order to overcome losses in the sector? The paper employs DEA to estimate relative efficiencies and Malmquist productivity index to evaluate productivity and X-Factors over time. The Sample CRS mean efficiencies were found to be 53%, with individual municipalities performing as low as 14%. The TFP growth model indicated that over the time-period 1999 to 2009 increasing inefficiencies were witnessed for a majority of municipalities, implying progressive deterioration in services amounting to -3.2% annual productivity decrease per year over the 10-year period. The study further focuses on determining X-factors which help in price-cap regulation. X-factors calculated from TFP model were found to average 6.12% indicating that an average municipality should increase tariffs by 6.12% annually over the next 5 year period to catch-up 50% of the gap with best-practices at the end of 5 years for improving annual productivities, ultimately leading to financial sustainability of the services.*

**Keywords:** DEA-based Malmquist approach; total factor productivity; Indian water supply services; relative efficiencies; productivity.

## INTRODUCTION

Water-supply services have emerged as profitable industries across developed nations wherein water-tariffs are regulated and set scientifically by a Regulator. However, the Indian urban water supplies are still a monopoly of the government, wherein the policies are focused on drinking water provisioning for growing populations to the complete neglect towards making operations efficient and profitable. This results in financial losses to the municipalities that supply water increasing their dependence on the government subsidies on one hand, and in wide dissatisfaction with the services at the consumer end. A general perception exists that Indian water supply services are inefficient, though these perceived inefficiencies remain largely unmeasured. Therefore the present work has been undertaken to measure productivities and efficiencies of Indian municipal water supply services to answer the following specific questions:

- a. Are Indian water supply services efficient?
- b. Are water supply operations improving over time?
- c. How should the present tariffs be restructured in order to overcome losses in the sector?

## METHODS

### Productivity Measurement

When appropriate panel data with time series having at least 2 datasets is available, the DEA frontier construction methods can be used to obtain estimates of Malmquist TFP (Total factor productivity) index numbers (Fare et al., 1994; 1998). The TFP indices may be decomposed into 2 specific components – The Technical Change or TC (also called the frontier-shift), and the Technical Efficiency Change or TEC (also called the Catch-up). The TFP change between two data points is measured using the Malmquist TFP index, which calculates the ratio of the distances of each data point relative to a common technology. The methodology used in this paper follows that given by Coelli and Walding (2005).

In this paper, the solution employed to solve the linear programming problem equations for finding technical efficiencies is adopted under the conditions of constant return to scale technology, because this is believed to be more reliable than a variable return to scale when calculating Malmquist TFP indices (Fox (2002)).

### Efficiency Measurement

The Linear Program problem to be solved to obtain CRS efficiency scores as mentioned by Charnes et al (1978) and VRS efficiency scores according to Banker, Charnes and Cooper (1984) were both adopted to evolve efficiencies under the constant and variable returns to scale conditions respectively.

### Data Employed and Model Specifications

A simple model for the analysis is framed based on the cost drivers available in the sector. As Input variable, the Operational expenditure (OPEX) (Indian Rs. Millions (1US\$=64.28 Indian Rupees) was employed, whereas the Output variables were the Total quantity of water supplied in Million liters per day (MLD) and the Numbers of water supply connections.

## RESULTS AND DISCUSSIONS

Table 1 summarises the data characteristics for the sample of 21 municipalities employed in the analysis.

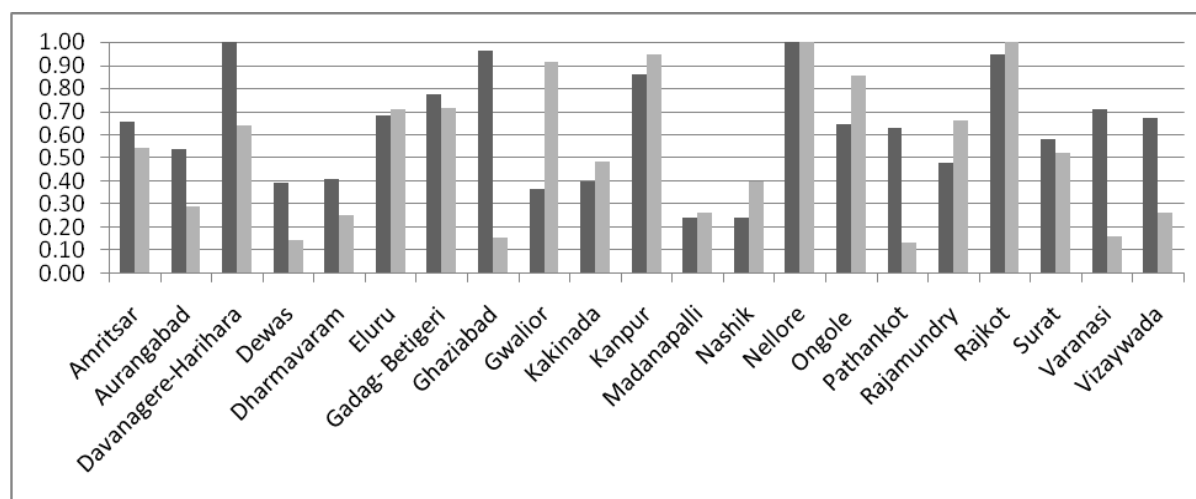
**Table 1: Data Characteristics for sample municipalities**

Variable description	Mean	Standard deviation	Maximum	Minimum
Year 1999				
Total water supplied (MLD)	97.00	97.93	320.00	8.00
Operating expenditure (Rs. Millions)	510.42	547.05	1941	58.80
No. of connections	47212	48396	176985	4327.00
Year 2009				
Total Water supplied (MLD)	152.09	157.47	540	6.41
Operating expenditure (Rs. Millions)	2087.35	2626.34	10770.7	121
No. of connections	74235.42	78598.55	303930	8605

Data Source: CPHEEO (2005),CPCB (2009), Town and Country Planning Organisation (2017)

Figure 1, shows the CRS TE scores for two periods. It is evident that a majority of municipalities witnessed lesser efficiency scores in year 2009 as compared to 1999, indicating that over the time period 1999 to 2009 increasing inefficiencies were witnessed in the sector implying further deterioration in

services over time. For policy makers this would be a very alarming finding as performances seem to be deteriorating over time for more than 50% of the sample municipalities, necessitating urgent policy intervention.



**Figure 1. CRS Technical Efficiencies for the two time periods (TE-1999 in Black & TE-2009 in Grey).**

Table 2 presents the Productivities, Efficiency measures, the Returns to scale and X factors determination for the 21 sample municipalities. It is evident that the average water utility level productivity change was -32%, implying significant overall negative productivity for the sample on an average (amounting to -3.2% annual productivity decrease per year over these 21 municipalities over 10 years) indicating large deterioration in services with time. This is a very serious outcome and reflects the ailing nature of illness plaguing the municipal services in India.

Table 2 also shows the measures of efficiency for the financial year 2009. The CRS mean TE score for the sample is 0.53, while the VRS TE score had an average of 0.67, which indicates that the average firm could reduce the usage of input by 33% and still produce the same level of output. The mean scale efficiency score for the sample is 0.80 (Table 2), indicating that the average utility should be able to reduce its input usage per unit of output by 20%, if it was able to change its scale of operations.

The returns to scale category in Table 2 indicate that majority of municipalities showed decreasing returns to scale indicating that these services would give proportionately decreasing returns of outputs for a unit increase in the input level. A restructuring of these municipalities may therefore be required for scaling down their operations for ensuring better efficiencies.

In the present sample, the average Malmquist TFP value stood at 0.68, implying 32% decrease in productivity over a decade, or 3.2% annual productivity loss. To offset this productivity lag, the regulator may require all the municipalities to achieve a mean annual productivity growth of 3.2% (generally referred to as Total Factor Productivity Change (TFPC)), assuming that this could possibly prevent the municipalities from remaining in the negative zone by enabling them to focus on demand management by improving efficiencies. Furthermore, it could require water municipalities with DEA technical efficiency scores below one to catch up 50% of the way to the frontier over the next five years (Coelli and Walding,



2005). These values are depicted in the Catch-Up column of Table 2. VRS TE scores are used in determining catch up given that most of the municipalities are scale inefficient.

**Table 2: Productivities, Efficiency measures, the Returns to scale and X-factors for sample utilities**

City	Productivity Measures (1999-2009)			Efficiency Measures (2009)					X factor	
	MI	TEC	TC	TE CRS	TE VRS	SE (CRS TE/VRST E)	RTS	TFPC Annual(%)	Compounded Catch-up Annual(%)	X factor Annual (%)
Amritsar	0.68	0.83	0.81	0.55	0.56	0.98	DRS	3.2	4.05	7.25
Aurangabad	0.52	0.54	0.96	0.29	0.30	0.96	DRS	3.2	6.19	9.39
Davanagere-										
Harihara	0.10	0.13	0.74	0.64	0.74	0.86	IRS	3.2	2.44	5.64
Dewas	0.28	0.37	0.76	0.15	0.17	0.85	IRS	3.2	7.18	10.38
Dharmavaram	0.49	0.61	0.80	0.25	0.49	0.52	IRS	3.2	4.65	7.85
Eluru	0.99	1.04	0.96	0.71	1.00	0.71	IRS	3.2	0	3.20
Gadag- Betigeri	0.71	0.93	0.76	0.72	1.00	0.72	IRS	3.2	0	3.20
Ghaziabad	0.16	0.16	1.02	0.16	0.16	0.98	DRS	3.2	7.27	10.47
Gwalior	2.20	2.51	0.87	0.92	0.93	0.99	DRS	3.2	0.69	3.89
Kakinada	1.13	1.22	0.93	0.48	0.55	0.87	IRS	3.2	4.11	7.31
Kanpur	1.14	1.10	1.03	0.95	1.00	0.95	DRS	3.2	0	3.20
Madanapalli	0.83	1.11	0.75	0.27	0.60	0.44	IRS	3.2	3.72	6.92
Nashik	0.32	0.37	0.88	0.40	0.91	0.44	DRS	3.2	0.84	4.04
Nellore	1.13	1.00	1.13	1.00	1.00	1.00	CRS	3.2	0	3.20
Ongole	1.10	1.33	0.83	0.85	1.00	0.85	IRS	3.2	0	3.20
Pathankot	0.21	0.22	0.97	0.14	0.14	0.95	DRS	3.2	7.39	10.59
Rajamundry	0.12	0.15	0.89	0.66	0.71	0.93	IRS	3.2	2.76	5.96
Rajkot	0.80	1.06	0.75	1.00	1.00	1.00	CRS	3.2	0	3.20
Surat	0.75	0.90	0.83	0.52	1.00	0.52	DRS	3.2	0	3.20
Varanasi	0.23	0.23	1.05	0.16	0.17	0.96	DRS	3.2	7.21	10.41
Vizaywada	0.11	0.09	1.25	0.26	0.71	0.36	DRS	3.2	2.72	5.92
<b>Mean</b>	<b>0.68</b>	<b>0.76</b>	<b>0.90</b>	<b>0.53</b>	<b>0.67</b>	<b>0.80</b>			<b>2.92</b>	<b>6.12</b>

*Note: MI=Malmquist Total Factor Productivity Index, TEC=Technical efficiency change, TC=Technical change, TFPC=Total factor productivity change, TE=Technical efficiency, SE=Scale efficiency, RTS=Returns to scale: IRS = Increasing returns to Scale, DRS= Decreasing returns to scale, CRS= Constant returns to scale*

Considering the above rules, calculations for X factors (TFPC+ Catch up) are made for 21 municipalities. The X factors in Table 2 range from 3.2% per year for the frontier municipalities (as they don't have to do any Catch-Up), to 10.59% per year for most inefficient utility. The average X factor for the sample municipalities was 6.12% per year with an average compounded catch-up of 2.92%  $((1-0.67)/2 = 0.165$  or 16.5% over 5 year period which is  $(1.165)^{1/5} = 1.0292$  or 2.92% compounded catch up per year), implying that the average sample utility must reduce unit costs in real terms by 6.12% per year or must increase its tariffs by this amount to offset inefficiencies. If this happens, in the long-run most of the municipalities having efficiency scores less than one should stand a reasonable chance to catch up significantly on the way towards the efficiency frontier by following the best practices, leading to effective services and better sector management, which would surely reflect in satisfied consumers and sustainable water supply services.

## CONCLUSIONS

The study endeavored to measure of productivity and efficiency for 21 urban municipalities in India over a 10 year period. The results indicated large prevalence of inefficiencies, with decreasing productivities

over time, thereby confirming the consumer perception that water supply services in urban India remain dissatisfactory. At present, the Indian water supply sector is not regulated by an independent regulator, and hence the illustrated framework may comprise a first step in the direction of bringing regulatory reforms in a sector which otherwise remains an essential monopoly of government municipalities having large operational inefficiencies, as revealed by the results. The results demonstrated the fact that majority of municipalities showed decreasing productivities over time which has implications over possible future privatization and for induction of competition in the monopolistic sector, as water services need lumpy investments and no private company would be forthcoming to sink its investments in municipalities that are not just inefficient, but also have been having decreasing productivities with time. The X factors calculated in the study, if implemented, may help the inefficient municipalities to catch up nearly 50% of the way towards the frontier over the next five year period. This may help municipalities to improve their annual productivity growths, and make a turnaround in accordance with the reductions made in their input levels so as to generate internal revenues that cover operational costs incurred by the government, ultimately leading to financial sustainability of the municipalities.

## ACKNOWLEDGEMENT

The authors are grateful to the Ministry of Human Resources Development (MHRD) and the National Institute of Technology, MANIT-Bhopal, India for providing the financial support as to enable this work to be presented at DEA-2017.

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# RURAL HOSPITALS IN GEZIRA STATE: PRODUCTIVE EFFICIENCY AND ITS DETERMINANT FACTORS IN 2016

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## ABSTRACT

*Efficient utilization of scarce resources is crucial for better performance of health care institutions specially for developing countries. This is the first study that uses data envelopment analysis(DEA) to assess the productive efficiency of rural hospitals in Sudan and to determine its contributing factors . Data from a total of 51 rural hospitals in Gezira State in 2016 was analyzed using two-stage DEA model. The study revealed that 28 rural hospitals (55%) in Gezira State were efficient and 23 (45 %) were inefficient under variable returns to scale (VRS). While 23 rural hospitals (45%) were efficient and 28 (55%) were inefficient under constant returns to scale (CRS). The mean productive efficiency scores were found to be 0.86 and 0.81 for (VRS) and (CRS) respectively. Population in the catchment area of the rural hospital, distance from the nearest teaching hospitals and the squared size of the rural hospital were positively affecting inefficiency of rural hospitals, while negative factors were found to be the multiple medical specialties, presence of radiological equipments and the number of non medical staff.*

**Keywords:** *productive efficiency, data envelopment analysis, rural hospitals, variable returns to scale, constant returns to scale.*

## INTRODUCTION

Efficiency is an important aspect in health care (Murray & Frenk 2000). In addition to that it represents one of the desirable goals for the health system. A rural hospital is defined as small hospital with 50 beds or less and provides basic diagnostic services, minor surgery and care for patients who need nursing care ( McKee & Healy 2002) . Rural hospitals are regarded as an essential level of health care delivery for their crucial role in serving a great number of people specially in our country Sudan and in Gezira State in particular. Assessment of efficiency status of rural hospitals contributes with great deal to the efficiency of overall health system particularly in our situation where the health system suffers a lot from scarcity of resources. Increased pressures on health care resources have led policy makers, administrators, and clinicians to search for more efficient ways to deliver health services. Efficiency improvements in the health sector, even in small amounts, can yield considerable savings of resources or expansion of services for the community ( Peacock, Chan, Mangolini, & Johansen 2001). This study helped in filling an obvious gap in the field of efficiency assessment of rural hospitals in Gezira State.

Gezira State is located central in Sudan spanning about 25,549.2 km<sup>2</sup>. Its population is about 4,264,358 according to population 2015 projection on 2008 census (Sudan's Central Bureau of Statistics 2015). Rural population constitutes about 80% of the total population, scattered over 3000 villages The main causes of morbidity and mortality are infectious and parasitic diseases such as malaria, , Schistosomiasis (Malik, Abdalla, & Babiker, 2016), TB, diarrheal diseases, and malnutrition. In Gezira State the health system is governed by the State Ministry of Health (SMoH) which represents the main provider of

curative, preventive and promoted health services in the State. The total number of public hospitals is 84 of which 53 (63.1%) constitute the rural hospitals.

The study aims generally to assess the productive (technical) efficiencies of the rural hospitals in Gezira State and to determine the factors that affect their efficiencies. The specific objectives are to calculate the productive (technical) efficiency scores of the rural hospitals in Gezira State in 2016, to identify the factors affecting productive (technical) efficiency of the rural hospitals in Gezira State and to quantify the magnitudes of sufficient inputs for the rural hospitals to work efficiently.

## METHODS

This is a cross-sectional analytical study using two-stage DEA . The totally covered population of this study comprises 51 rural hospitals in Gezira State (after exclusion of two hospitals not fulfilling the inclusion criteria). These represented the decision making units (DMUs) of the study. Data for specific inputs and outputs of the rural hospitals was collected retrospectively from January to October year 2016.

**Inputs** included *number of medical doctors*: including general and specialist physicians, *number of nurses*: including qualified nurses, *number of beds*: number of inpatient beds in a rural hospital in 2016, *number of laboratory personnel*: including all technical personnel working in the laboratory department such as laboratory technicians, laboratory assistants, malaria technicians and laboratory attendants, *Operational expenditure*: which included all current expenses such as electricity, water and telephone bills, maintenances for buildings and equipments and stationeries in Sudanese pounds, and *Assets*: which included the number of durable assets of medical equipments. Outputs of this study were: *number of outpatient visits*: included mainly the common acute diseases' visits, chronic diseases follow up visits, *number of inpatients*: the total patients admitted in long or short stay wards to hospitals in 2016, *number of surgical operations*: the total of major, medium and minor surgical operations done at the hospital in 2016, *number of children vaccinated*: vaccination of children against vaccine- preventable diseases is one of the service package that is delivered through the rural hospitals. Two techniques were used as analytical tools into two stages; data envelopment analysis (DEA) and regression analysis.

### The first stage:

The stage of measuring the technical efficiencies of the rural hospitals, Data Envelopment Analysis (DEA) was used as a tool to calculate the technical efficiency scores of the rural hospitals . DEA is very ideal for measuring efficiency of hospitals with multiple inputs and outputs ( Harrison, Nicholas & Wakefield 2004). Input-oriented DEA model was used in this study two assumptions constant returns to scale (CRS) and variable returns to scale (VRS). as the majority of studies used input orientated DEA due to the fact that most of hospital managers and policy makers have more control on inputs than outputs (O'Neill, Rauner, Heidenberger, & Kraus 2008) .Since all hospitals included in this study are public so the concern was being mainly on resources (inputs).

### The second stage:

In the 2nd stage—the stage regression analysis - Tobit model was used to identify the factors that affect technical efficiency in terms of direction (nature: positive or negative effect) and magnitude. The

regression was done using stata11 soft program. The score of Productive (technical) efficiency was used as dependent variable that is regressed against independent variables which included: *Population size in the catchment area, the size of the hospital (number of beds), the distance from the nearest teaching hospital, number of non-medical staff, the number of medical specialties and presence of radiological diagnostic equipments.*

## RESULTS AND DISCUSSIONS

Under (VRS), the results of this study revealed that only 28 (55%) out of 51 rural hospitals in Gezira State in 2016 were productively efficient while the rest 23(45%) hospitals were run inefficiently (Figure 1) with the mean productive efficiency score of 0.86, standard deviation of 0.20, a maximum value of 1 and a minimum of 0.32 (Table 1). The results of (CRS) showed that 23 (45%) out of the rural hospitals were productively efficient while the rest 28 (55%) hospitals were run inefficiently, with the mean productive efficiency score of 0.81, standard deviation of 0.23, a maximum value of 1 and a minimum of 0.24 (Table 1). According to the results of this study 74% of the rural hospitals had efficiency scores ranging between 0.75 and 1.00, while 18% had efficiency scores between 0.50 to 0.74 and the rest of the hospitals ranged between 0.25 and 0.49 (Figure 2). This meant that the inefficient hospitals overused available resources to yield a level of outputs that could be reached with lesser amount of inputs. These results were similar to those of some studies conducted in the region; in Ghana 2005 Daniel Osei's study revealed that 47% of district hospitals were run inefficiently (Osei et al 2005). Also Kirigia et al in 2004 who measured the productive efficiency of public health centers in Kenya using the same DEA approach, and found that 44% of the health centers were productively inefficient (Kirigia, Emrouznejad, Sambo, Munguti, & Liambila 2004). In 2006 Eyob Zere et al did a study in Namibia for a sample of 26 district hospitals and found that more than 50% of them were inefficient (Zere et al 2006).

**Table 1: Summary statistics for efficiency scores of rural hospitals in Gezira State 2016 (VRS) and (CRS)**

Statistic	EFFICIENCY SCORES(VRS)	EFFICIENCY SCORES(CRS)
Mean	0.86	0.81
Median	1	0.93
Maximum	1	1
Minimum	0.32	0.24
Std. Dev.	0.20	0.23
Observations	51	51

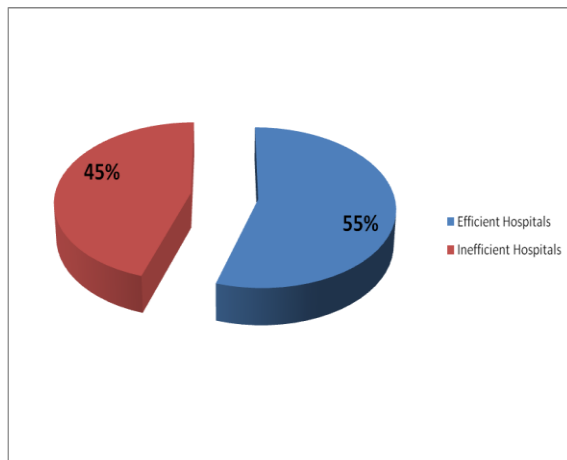
The result of regression analysis (Table 2) revealed a positive relationship between number population in the catchment area and productive efficiency of a rural hospital, and this was found to be statistically significant indicated by a p-value of 0.0000. This could be explained by the fact that as the population increased the proportion of people seeking for healthcare would increase hence contributing to more patients receiving services from the hospital which ultimately would increase the hospital output and consequently increasing its productive efficiency. The effect of the *distance* of the rural hospital from the nearest teaching hospital also found to be positively affecting hospital efficiency. This could be attributed to fact that people believed in teaching hospitals where they thought they would receive better care provided that the city where the teaching hospital was not so far from the rural area. This could be augmented by the bypass phenomenon. Regarding the size of the rural hospital, it was proved that the

productive efficiency of a rural hospital was affected positively by the squared *size* of the hospital as shown by the sign of the coefficient.

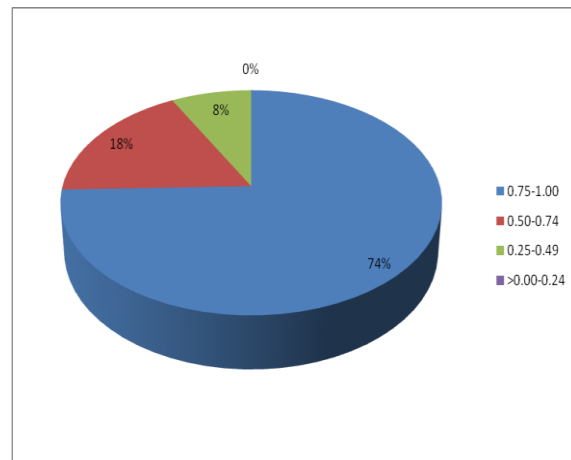
**Table 2: Result of regression analysis using Tobit model regression variables for rural hospitals in Gezira State 2016 (N = 51)**

Productive Efficiency	Coef.	Std.Err.	t	P> t
Population in the catchment area	6.41E-06	1.39E-06	4.61	0
Distance from the nearest teaching hospital	0.003437	0.001795	1.91	0.062
Squared Size	2.99E-05	1.09E-05	2.74	0.009
Non Medical Staff	-0.00864	0.003293	-2.62	0.012
Specialities	-0.04624	0.021961	-2.11	0.041
Radiological Equipment	-0.22813	0.074633	-3.06	0.004
_cons	0.857919	0.095129	9.02	0

Having a p-value of 0.0061 this effect was statistically significant. Meaning that an increase in hospital size will increase the productive efficiency of a rural hospital up to a certain level after which the efficiency decreases with more increase in size. With a statistical significance denoted by a P-value of 0.012 the effect of the number of *non-medical* personnel was shown to have a negative effect on efficiency of a rural hospital indicated by the negative sign of the coefficient. The prediction for that was as follows: if the non-medical personnel increased by one unit the efficiency would decrease by 0.008 units holding other variable constant. This could be explained by the direct effect of redundancy of staff on the operational cost. The number of medical *specialties* and the presence of radiological *equipment* in a rural hospital were used as proxies for complexity of work.



**Figure 1: Percentages of efficient and inefficient hospitals rural hospitals under (VRS) in Gezira State 2016 (N = 51)**



**Figure 2: Percentage of rural hospitals in Gezira State 2016 (N = 51) in efficiency ranges (VRS)**

The results of this study revealed that these two variables contributed negatively to the efficiency status of a rural hospital. These effects were statistically significant as the p-values of 0.041 and 0.004 indicated for *specialties* and *equipment* respectively. This would be expected since complexity of work necessitated the use of more resources both capital and human, hence contributing to lower level of efficiency unless

compensated by increase of outputs. These results were similar to what was revealed by Hsi Hui' Chang study (Chang 1998) in which the presence of service complexity in public hospitals in terms of more specialties and costly equipments supported the hypothesis that operating complexity reduces hospital efficiency.

## CONCLUSIONS

According to the results of this study 28 rural hospitals (55%) in Gezira State were found to be efficient and 23 (45 %) were inefficient under variable returns to scale (VRS). While 23 rural hospitals (45%) were efficient and 28 (55%) were inefficient under constant returns to scale (CRS). The mean productive efficiency scores were found to be 0.86 and 0.81 for (VRS) and (CRS) respectively. Excess inputs that could be saved amounted to 26 doctors, 16 laboratory personnel, 46 nurses, 258 beds, 19 items of equipments and a total of 98499 SDG from operational expenses per month. Productive efficiency of rural hospitals in Gezira State in 2016 was influenced positively by the population in the catchment area of the rural hospital, distance from the nearest teaching hospitals and the squared size of the rural hospital measured by the number of beds. The negative influential factors for productive efficiency of rural hospitals included the multiple medical specialties, presence of radiological equipments as proxies for service complexity and the number of non medical staff.

## ACKNOWLEDGEMENT

The authors acknowledge all members of Community Medicine Council at Sudan Medical Specialization Board Prof Haider Abu Ahmed, Dr Alfatih Malik, Dr Abdelgadir Bashir, Dr Samia Habbani, Dr Somia Alfadil, Dr Siham Balla and Dr Hassan Al Sheikh for worthful comments and enriching advices that added a lot to this work. The authors also would like to express deepest gratitude to Gezira State Ministry of Health with special thanks to Director General Dr. Widad Yosof and Dr. Nazar Algaddal Director of Curative Medicine General Directorate, Dr Aml Babiker the Director of Drug Revolving Fund for their financial and technical support, and all directors of rural hospitals for their endless support in facilitating all activities related to data collection from rural hospitals..

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# SUPPLY CHAINS DEA EFFICIENCY: INCORPORATION OF DECISION MAKERS' PREFERENCES

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## ABSTRACT

*Among many applications, several studies using Data Envelopment Analysis DEA have examined and studied the efficiency measurement of supply chains. However, the majority of the existing approaches dealing with this research area have ignored to take into account an important attribute, which is the preferences of decision makers. The main objective of this research is to provide consistent DEA models allowing conducting an efficiency analysis to determine the optimal allocation of resources according to the decision makers' preferences. The idea here is to provide a geometric decomposition of preferences attribution, this idea is especially inspired because a decision maker may treat each supply chain as a single non-detachable entity (horizontal attribution of preferences). Then, the case when the decision maker considers supply chains detachable will correspond to (vertical attribution of preferences). While another case could be considered, that combines the two previous situations where at the same time decision makers assign weights to the supply chain as to its members. This case will correspond to a combined attribution of preferences. Obtained results are very relevant and show that decision makers' preferences can be incorporated into DEA models without affecting the standard distribution of efficiency scores..*

**Keywords:** Data Envelopment Analysis; Dyadic Supply Chains; Efficiency; Decision-makers' preferences

## INTRODUCTION

In Supply Chain Management SCM, it is considered as an important issue to control the consumption of every input and the production of every output of each member included into the supply chain. Nevertheless, comparing with other Multi-Criteria Decision Making MCDM methods, specifically when Decision-Makers' Preferences DMsP are taken into account, we find that the literature has not frequently attacked this subject when it is associated with the DEA efficiency evaluation. This gap can be explained by several reasons; the most important reason is that into DEA models, inputs and outputs weights are considered as integrated decision variables and no additional information is needed to reach the reference point. Consequently, these models in their standard form can generate incoherent solutions with the operational DMsP. Therefore, a great deal of scientific research has been conducted to set the constraints to the weights, so DMsP can be incorporated into the assessment of efficiency. Allen et al. (1997) have presented a review of the evolution of the use of weights restrictions and value judgements in DEA. In this same context, Cook and Seiford (2009) have classified DEA models, involving weight restrictions, into five categories. Contreras (2011) proposed a procedure consisting of two stage. First, a DEA-inspired model for the aggregation of preferences is applied. Second, in order to obtain a group solution, the procedure derives a compromise solution by determining a social vector of weights for evaluating the complete set of alternatives. D.Alcaide-Lopez-de-Pablo et al. (2014) have provided a comprehensive review of the incorporation of the information on preferences in the DEA efficiency analysis. Moreover, these authors have evoked works dealing with the combination of MCDM methods and DEA where generally efficiency scores obtained by the DEA are used in the objective function of the associated mathematical programming problems.

Balfaqih et al. (2016) found on their review of supply chain performance measurement systems between 1998 and 2015 that DEA represents fourteen percent among six techniques adopted for evaluation purposes. Among studies that have dealt directly with the efficiency of supply chains using DEA, one can retain the following works: Zhu (2003) has proposed a DEA model allowing to define and measure the whole efficiency of a dyadic supply chain composed of a seller and a buyer as well as those of its members. This author has presented a set of optimal values for the intermediate variables that establishes an efficient supply chain. Using the same chain considered by Zhu (2003), Liang et al. (2006) have developed two DEA models to measure supply chain efficiency when intermediate variables are incorporated into the evaluation procedure. A first model assuming that the buyer-seller relationship is modeled as a two-stage non-cooperative game and a second model assuming that the buyer and the seller operate in a cooperative context. Chen et al. (2006) have analyzed the relations between the efficiencies of supply chain members by a model called (A Supply chain-DEA Game model). Considering a supplier-producer supply chain, the authors have proposed several supply chain efficiency functions. Recently, Tavana et al. (2015) have proposed a two-stage DEA model. The purpose of their model is to evaluate an entire three-level supply chain comprising a supplier, manufacturer and distributor.

With the aim of developing more other propositions to model supply chain efficiency, many works have combined and compared DEA with other techniques: Xu et al. (2009) have used rough-set theory and DEA techniques in a furniture manufacturing SC in China to evaluate SC network operation efficiency. Wong (2009) has combined DEA with Monte Carlo simulation to evaluate supply chain performance in a stochastic environment. Shafiee et al. (2014) have suggested a network DEA model to measure the supply chain efficiency score in the Iranian food industry. They have determined the relationships between Balanced Scorecard Approach BSC four perspectives and then used the DEMATEL approach to obtain a network structure. Recently, considering a supplier-producer supply chain containing intermediate measures, Walid and Abdelwaheb (2016) have developed different DEA models based on the differentiation between eight different situations by distinguishing between the cooperation concept and the dominance concept.

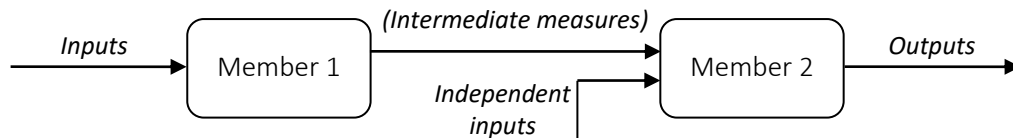
In the same context, when it is about supply chain efficiency measurement, the traditional literature mostly ignored the incorporation of DMsP. Nevertheless, even the few works, which have considered these DMsP, they had recourse to use another MCDM methods beside DEA and/or to aggregate weights representing these preferences into weights related to decision variables (inputs and outputs weights). Among others, Zhao and Sun (2008) proposed a preference restraint DEA approach for supplier selection in which, the DMsP on inputs and outputs, are given by the Analytic Hierarchy Process AHP method. Saen (2010) proposed a weight restriction DEA model for the supplier selection problem with the consideration of dual-role factors.

When trying to evaluate the efficiency of a given supply chain where several inputs are consumed and several outputs are produced, the challenge of this work is first, to take into consideration DMsP while relying only on the DEA methods, therefore, no additional alternative or method are used. Second, weights indicating these preferences should appear beside inputs and outputs weights, thus, these weights supposed to be incorporated and not aggregated into models while maintain the philosophy inherent in DEA.

## METHODS

### Methodology Proposal

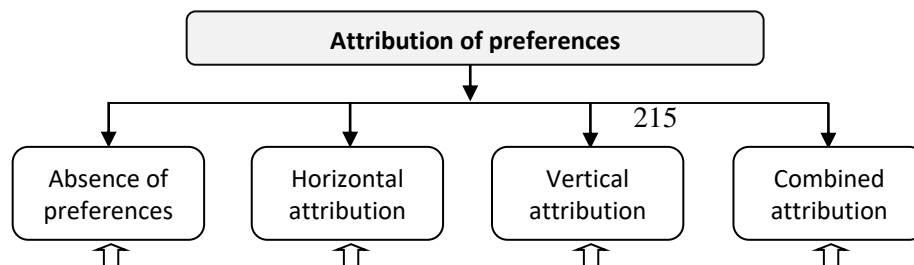
In order to achieve these goals, supply chains considered here are dyadic supply chains with independent inputs (Figure 1) which are defined as linear minimal chains composed only by two units (firms, members,...). In fact, given its simplicity, authors often advise to use it first for the performance evaluation then to iterate the reasoning to more complex chains. This work assumes the existence of several DMs controlling and managing the considered supply chains. Also, these DMs can express their judgments and interactivity by attributing weights to each member or to the totality of each supply chain.



**Figure 1. Dyadic supply chain with independent inputs**

The closest work to consider DMs when measuring supply chains efficiencies is that of Chen et al. (2006). These authors separated between two cases of supply chains control: (1) Decentralized Control System DCS where the first and the second member control their own efficiencies. (2) Central Control System CCS where a single decision maker, who is different from members making up the chain, monitors the supply chain.

In this work, the case of decentralized control corresponds to the case where, even if DMs are present, they do not affect the procedure of measuring the supply chain efficiency. Thus, DMs are supposed indifferent and neutral about all chains and members constituting these chains. Therefore, this situation assumes the absence of preferences. On the other hand, for centralized control, the author proposes to treat it differently by proceeding to a "geometric" decomposition of the DMsP attribution; DMs could allocate their preferences weights to chains and their constituent members in a horizontal and/or in a vertical manner. This idea of a geometric decomposition of preferences attribution is especially inspired because a DM may treat each supply chain as a single non-detachable entity, as a result, he could assign a relative weight to each supply chain independently of its members. This case corresponds to a horizontal attribution of preferences. Then, when that DM considers supply chains detachable, he will assign to each one a weight relative to its first member and another weight relative to its second member. This case will correspond to a vertical attribution of preferences. While another case could be generated that combines the two previous situations where at the same time DMs assign weights to the supply chain as to its members. This case will correspond to a combined attribution of preferences. Figure 2 summarizes the different ways by which preferences of DMs will be considered. Noting that each of these proposed manners will correspond to a separated DEA model. Beside the main goals, two empirical advantages can be given to these different ways of considering DMsP; first, the ability of controlling the sample size and second, the assistance in sensitivity analysis.



**Figure 2. Proposed manners to consider attribution of DMsP**

DMsP Modelling

The preferences modelling into DEA models will differ from one case to another. Infact; the case of absence of preferences assumes DMs are neutral about all chains and members constituting these chains. Then the corresponding model retains only weights representing DEA inputs and outputs, and discards any kind of weights representing preferences. The objective function considers the mean between the first member output (intermediate measures) and the second member outputs. By adopting a horizontal attribution of preferences, DMs treat each chain as a non-detachable entity, the weighted sum of DMsP is incorporated into the objective function by multiplying its expression only with the weighted sums of outputs. While the vertical attribution of preferences consists in supposing detachable chains constituting the sample under study, In the objective function of the related model, the weighted sum of DMsP regarding the first member is multiplied by the weighted sum of this later outputs (also intermediate measures), thus, the preferences weights regarding the second member is multiplied by the weighted sum of this member outputs. Finally, the case of combined attribution of preferences regroupes all the previous models into only one model in a way DMs have totally the freedom, whether they treat each supply chain as a single non-detachable entity and/or to consider it detachable. The first model, corresponding to situation of neutrality of DMs, is systematically considered.

In order to achieve the aggregation of all previous situations into one model, this work proposes

$f(\alpha_{dj}, \beta_{dj})$  to be a joint function defined as following:

$$f(\alpha_{dj}, \beta_{dj}) = \begin{cases} 0 & \text{if } \alpha_{dj} = 0 \text{ and } \beta_{dj} = 0 \\ \beta_{dj} & \text{if } \alpha_{dj} = 0 \\ \alpha_{dj} & \text{if } \beta_{dj} = 0 \\ \frac{\alpha_{dj} + \beta_{dj}}{2} & \text{if } \alpha_{dj} \neq 0 \text{ and } \beta_{dj} \neq 0 \end{cases}$$

Where,  $\alpha_{dj}$  ( $0 \leq \alpha_{dj} \leq 1$ ) is the preference weight assigned by a DM  $d$  to a given chain  $j$ , and

$\beta_{dj}$  ( $0 \leq \beta_{dj} \leq 1$ ) is the preference weight assigned by a DM  $d$  to one of the two members establishing a given chain  $j$ .

Based on this function definition, the associated model yields naturally a zero efficiency for a given supply chain when all DMs decide to set  $\alpha_{dj}$ ,  $\beta_{dsj}$  and  $\beta_{dMj}$  all equal to zero, so that when a given chain shows a zero efficiency, means that it is out of consideration. Moreover, when a DM decides to assign a preference weight to a whole given chain  $\alpha_{dj} \neq 0$ , with another preference weight regarding one of the members  $\beta_{dj} \neq 0$ , this model will consider the average between the two weights.

## RESULTS AND DISCUSSIONS

The illustration considered assumes the existence of ten dyadic supply chains, three inputs consumed by the first member, three intermediate measures, one independent input consumed by the second member and two outputs produced by the same member. On the other hand, the illustration of preferences assumes the existence of four DMs whose weights are assigned subjectively.

The solution differs from one member to another and from one chain to another; there are chains and members that have experienced either an improvement or a degradation in their efficiency according to the different considered cases. Figure 3 shows moreover this variation in chains efficiencies for each of these cases.

It is remarkable that only chain 6 has retained its position as a reference chain in all the situations of the preferences allocation, this is because each member constituting this chain has retained an efficiency score equals to one. On the other side, the lowest level of efficiency was recorded by chain 5 in the case of vertical allocation of preferences. It is normal that efficiency scores vary from one chain to another, but it is deducable that this variation is not caused by the variation of preferences weights. Infact, obtained correlation coefficients between these weights and the efficiency scores in all the proposed cases are very low. This result is in line with the objective we set which is incorporating DMs preferences without affecting the DEA mechanism.

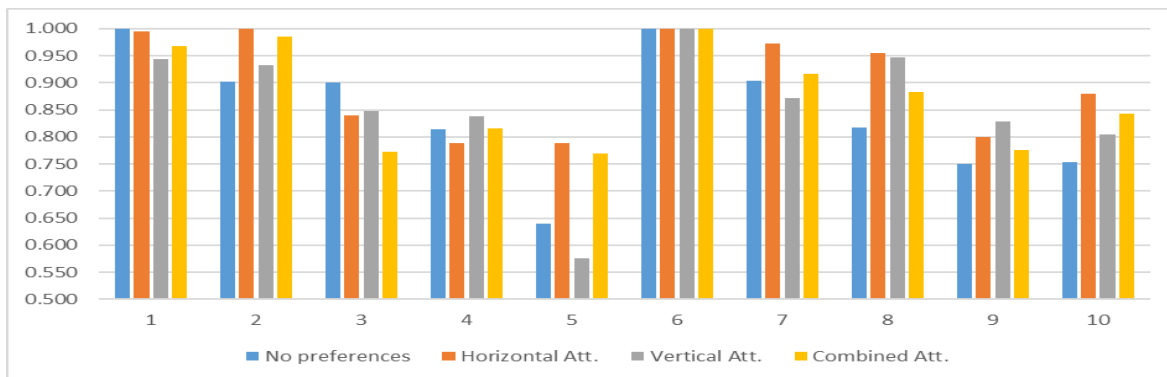


Figure 3. Chains efficiency variation according to each considered way of DMsP attribution

## CONCLUSIONS

The main objective of this research is to contribute to the improvement of procedures related to measuring with DEA the technical efficiency of supply chains by taking into account a very important attribute which is the preferences of DMs. A new theoretical framework to consider the attribution of these preferences was suggested as well as new DEA models were developed. allowing the research of best practices and strategies in order to have better efficiency. However, Although the configuration of the considered supply chain in this research is limited to simple linear two-members chains, this work as any other work is likely to be generalized in the future by trying to adopt more complex supply chains, also by proposing weights that are not subjective using methods as the AHP or the Analytic Network Process ANP, ...

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# USING DEA METHODOLOGY FOR ASSESSING THE PRICE EFFICIENCY OF THE PETROLEUM PRODUCT MARKET

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## ABSTRACT

*In this study, we use the DEA method to assess the price effectiveness of the petroleum product market. The petroleum product market has been and remains a major component of energy supplies around the economy. Price market efficiency - a complex concept that has no conventional treatment for commodity markets. Welfare economics says that pricing efficiency in terms of the welfare of society manifested in the absence of deadweight loss. Characteristically, the vast majority of the world fuel market is oligopolistic and sensitive to fluctuations in external factors. The article presents factors that influence the price performance of the petroleum products market, which reflect not only the specific characteristics of the market, such as the Herfindahl-Hirschman Index, but also a set of integral indicators such as GDP to P. Further on the basis of the data on the operation of petroleum products markets for the 13 countries of the European Union and its related markets, an assessment of price performance was carried out. The results of the evaluation by the DEA method showed that for most EU countries, the price efficiency of oil products markets is generally satisfactory.*

**Keywords:** petroleum product market; price market efficiency; assessment by DEA method

## INTRODUCTION

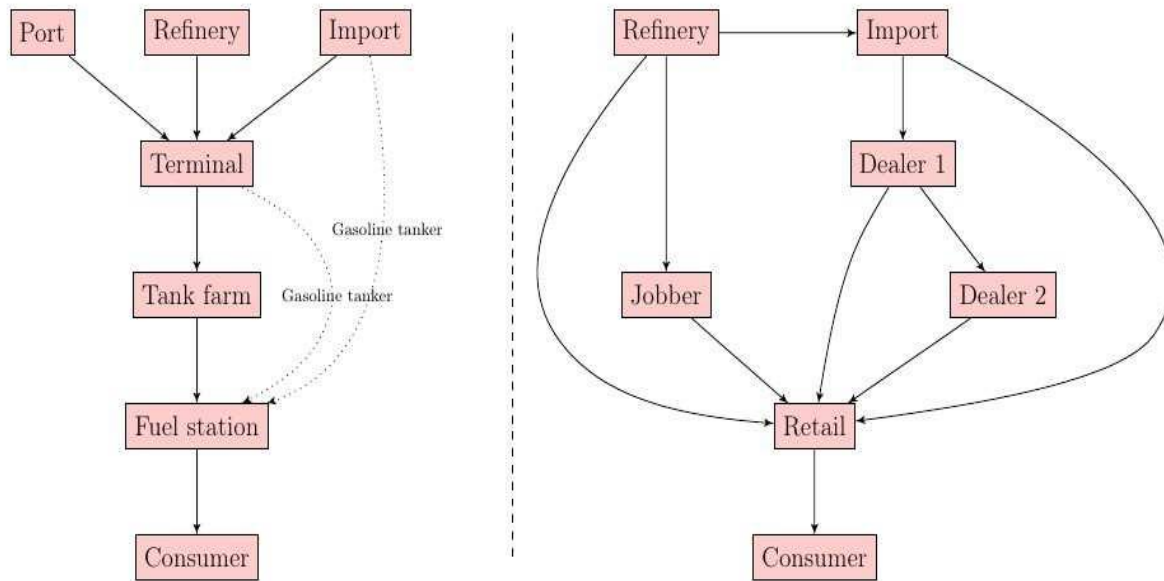
### Scheme of pricing in the market of petroleum products

The retail price of petroleum products in European countries is forming in several stages[1]. Ukraine, like most other European countries, is an importing country, importing both crude oil and finished petroleum products. The share of import of finished petroleum products is about 80% of all consumed petroleum products, which is why it is possible to consider that the basic pricing scheme is pricing, based on the import of products of production, and imported origin.

The basic pricing scheme of the market of petroleum products is presented in Figure 1.

As can be seen from the scheme, depending on the specifics of the operation of the retail company, the price may be formed in different ways, but the main ones are:

- price at the country border;
- the price of a large wholesale;
- the price of a small wholesale;
- retail price.



**Figure 1. The basic pricing scheme of the market of petroleum products**

For each country, the proportion of logistics schemes may vary[2]. For example, in Ukraine, 80% of supplies are imported petroleum products, while in Spain, almost all petroleum products pass through the refining of national refineries. But the formation of both the previous and the final price of the contract is determined as follows:

$$P_{CPT} = P_{latts} + D + L_{nat} \quad (1)$$

Where  $P_{CPT}$  - the price for imported fuel under the conditions of CPT loading (freight / carriage paid to, Incoterms-2010) at the border, \$.,  $P_{CPT}$  - aggregated values of quotations by the Platts agency, \$ / t,  $D$  - differential to Platts quotations, \$ / ton,  $L_{nat}$  - the cost of logistics from the plant to a basis on the border of the country, taking into account export costs, fees for simple tankers and other., Dollars per ton.

The general formula for the formation of the price of a large wholesale of countries outside the euro zone, in particular for Ukraine, is as follows:

$$P_{FCA} = (P_{CPT} * MB^{doll} + Az \cdot NBU_{eur} + ECO + L + M)(1 + VAT) \quad (2)$$

Where  $P_{FCA}$  is a price of fuel on the FCA (Franco-Carrier, Incoterms 2010), UAH / ton,  $P_{CPT}$  – a price on imported fuel on delivery terms CPT (Freight / Carriage paid to, Incoterms-2010) at the border of Ukraine, UAH / \$,  $MB^{doll}$  - US dollar purchase rate on the interbank currency exchange,  $Az$  - excise duty rate on petroleum products, euro / ton,  $NBU_{eur}$  - official exchange rate of hryvnia to euro,  $NBU$ ,

$ECO$ -rate of ecological tax, UAH / ton,  $L$ -costs for delivery of fuel to the loading station, UAH / ton,

$M$  - surplus of the trader and other expenses, UAH / ton,  $L$  - Expenses for delivery of fuel to the loading station, UAH / ton,  $VAT$  - is a value added tax.



The retail price is formed on the basis of the price of the wholesale, taking into account costs and market factors.

$$P_r = \left( \frac{P_{MO}}{(1 + VAT)} + C_r + Profit \right) (1 + VAT) + Acz_r^{hrn} \quad (3)$$

Where ,  $Acz_r^{hrn}$  - rate of excise tax on the retail sale of excise duties Goods, UAH / liter;  $P_{MO}$  - the price of a group of people, UAH / liter;  $C_r$  - distributed charges for the maintenance of gas stations, UAH / liter; Profit - profit, UAH / liter.

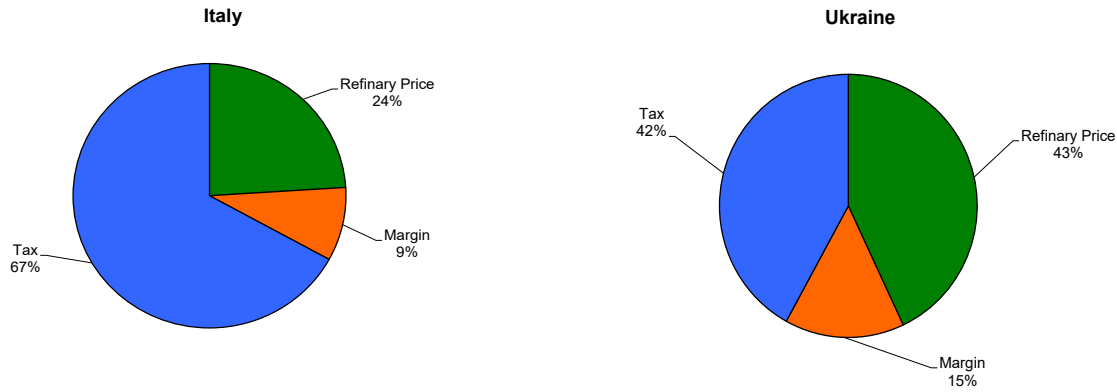
All Eurozone countries apply different tax schemes: Fixed excise tax on the production and import of petroleum products; Floating excise tax on the production and import of petroleum products; Ecological tax; VAT.

Tax rates in the countries of the euro area and the countries close to them can vary considerably.

## CHARACTERISTICS OF MARKETS OF PETROLEUM PRODUCTS

Almost all markets in the Eurozone are characterized by the presence of competition between sellers in the form of oligopoly with the number of players in the market from 3 to 7. The competitive environment of the retail market is characterized by varying degrees of concentration, which can be estimated using the Herfindahl-Hirschman index. The market for petroleum products is characterized by the significant dependence of retail prices on fluctuations in prices on world exchanges of crude oil, and for many countries, also depending on the exchange rate fluctuations between the US dollar/euro and national currencies[3]. For all markets there is a threat from collusive bargaining between sellers, parallel price behavior, high volatility of petroleum products prices and other adverse events for consumers. This circumstance raises the need for the existence of state regulatory institutions. Preliminary analysis shows that the petroleum products markets of the Eurozone countries and their adjacent ones have many common features[4]. Looking at the simplified price structure, one can see that for all national markets, the share of procurement value and taxes occupies a dominant share in the retail price. For example, Figures 2 show comparative pricing costs for Italy and Ukraine.

The proportion of the contribution of different costs may vary significantly. In this regard, there is a natural question: how well thought out the pricing system meets the needs of market participants in petroleum products, especially consumer's expectations, which in fact constitutes a perception of the level of prices that consumers will consider fair.



**Figure 2. The structure of the retail price**

The price performance of the market is a characteristic that describes the market's ability to install in the short run, the optimal, in terms of the distribution of welfare pricing for a particular resource. This is an assessment of the performance of market mechanisms. From the perspective of the welfare economy, according to which market is effective in conditions of free competition, and the price for the goods is equal to the price of marginal costs[5]. If the price of the market is different from the given point due to certain reasons, there are deadweight losses, which are characteristic of the quality of market pricing, and hence the efficiency of the market. It is also absolutely necessary to take into account other factors that lie beyond the boundaries of a particular market, for example - purchasing power of consumers, GDP per capita, the burden of customs procedures, degree of consumer orientation, etc.

## METHODS

It is obvious that in the absence of the theoretical criterion for assessing the price efficiency of a market, one can try to conduct a comparative analysis of the data of activities of national markets in terms of price effectiveness. Such an analysis can show how relevant this question is for a particular market. That is, we need a method that does not require a parametric dependence of the estimation of the initial parameters from the inputs. This method is Data Envelopment Analysis[6].

$$\begin{aligned}
 &\text{Min } \theta \\
 &\text{s.t. } \quad \sum \lambda_i X_i < \theta X_0 \\
 &\quad \quad \sum \lambda_i Y_i \geq Y_0 \\
 &\quad \quad \lambda_i \geq 0
 \end{aligned}$$

From the preliminary analysis, it follows that in order to calculate the price effectiveness of oil products markets, it is necessary to take into account a set of factors that reflect not only the internal characteristics of the market and the tax policy of the state, but also the economic situation in the country and the purchasing power of consumers. In our opinion, the minimal list of such factors has is next look:

Avg fuel - Average sales volume of fuel per one gas station per year (million liters); HHI - Herfindahl-Hirschman index ; PriceEUR – Retail price (euro / liter); GDPpc - GDP her capita; Tax - The weight of the tax component for the retail price; elast – Elasticity of demand for the price.

It is also necessary to estimate losses of consumers beforehand. The losses function of users are determined expression:

$$\text{Conlosses} = -0.5 * \text{Pr} * \text{Q} * \varepsilon * (\text{Pr} - \text{Po})^2 \quad (4)$$

Conlosses - Consumer losses, converted to 1000 liters of fuel.

The assessment of market parameters was carried out for 13 countries of the Euro-zone, as well as Ukraine and Turkey[7]-[10].

Where: Pr is the current retail price in the market, Q = 1000 - the volume of consumption brought to the same value, E - own elasticity of demand for the price, P<sub>o</sub> - wholesale prices for petroleum products in a given country (Plats quotations were used for EU countries).

Thus, user losses are an aggregate indicator that takes into account both the market condition due to the total retail price and the indirect behavior of consumers to change it. The next aggregate is the retail price excluding taxes, which reflects the effect of market mechanisms between traders and consumers. Another aggregate indicator is the ratio of GDP per capita(GDPtoP) to the retail price divided by the amount of fuel consumption at the gas station. The need to introduce such an integrated indicator is due to the need to some extent scale consumer purchasing power for different markets for petroleum products, where GDP levels may vary by one order.

$$GDPtoP = \frac{GDP_{pc}}{P_{doll} \cdot V_{perstation}} \quad (5)$$

Where GDPpc - GDP per capita, in dollars, P<sub>doll</sub> - retail price in dollars / liter, V<sub>perstation</sub> - the volume of sales of petroleum products at one gas station in the year. Thus, the DMU has the following structure:

Input: X<sub>0</sub>- Herfindahl-Hirschman Index; X<sub>1</sub>- Retail prices excluding taxes (in dollars per liter); X<sub>2</sub> - GDPtoP combined indicator.

Output: Y<sub>0</sub>- Losses of consumers by 1000 liters of fuel (in dollars) per 1000 liters.

## RESULTS AND DISCUSSIONS

Table 1 presents the calculation data for the assessment of the price effectiveness of oil products markets in a number of European countries on the proposed model.

The assesment of the price effectiveness for this model show that consumer expectations for the various national markets are generally satisfied. Although it can be noted that for two countries: Denmark and Turkey, the coefficients of price performance are somewhat lower than for others. We do not have absolutely reliable data on the reasons for this situation and the effectiveness assessment itself needs to be clarified.

**Table 1: Price effectiveness**

Country	Effectiveness for DEA
Austria	0.9018
Belgia	0.9548
Bulgaria	1.0000
Czech Republic	1.0000
Denmark	0.7439
France	0.8604
Germany	0.9059
Hungary	0.9411
Italy	0.8353
Netherland	0.8793
Poland	1.0000
Slovakia	0.9499
UK	1.0000
Ukraine	0.9016
Turkey	0.6747

However, in materials of the Round Table of EUROPEAN COMMISSION on the state of competition in the markets of motor fuels noted [11]: «Despite numerous laws and regulations, the fuel market is argued to be far from effective competition. In fact some of the clauses mentioned above have been the subject of Turkish Competition Authority's (TCA) "Liquid Fuel Sector Report", the conclusions of which were sent by the TCA as its opinion to the EMRA and the Ministry of Energy and Natural Resources with a view to promote effective competition in the market.»

With regard to the the competitiveness Danish petroleum market, then this question was devoted to a special study by the European Commission[12], which revealed an abnormal high concentration on refined petroleum products, including diesel and gasoline, which is apparently the reason for the relatively low assesment of the retail price effectiveness.

We have no direct evidence that the markets for petroleum products in Bulgaria, Czech Republic, Poland and the UK perfectly satisfy consumer expectations, but according to the European Commission, there were not any significant remarks on these markets.

Thus, such results suggest that this model, based on the DEA methodology in general, can reasonably reflect the comparative price effectiveness of the oil product market.

## CONCLUSIONS

Presented in this paper results in the development of methodology for assessing the price efficiency of petroleum product market on the basis of the methodology DEA show that this idea may be creative. We believe this is the first step towards using the DEA methodology to benchmark the market share of petroleum products. Further research should improve the adequacy of the model through a more detailed selection of DMUs and the expansion of the statistical base.

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- [13] [http://ec.europa.eu/competition/mergers/cases/decisions/m7603\\_1139\\_3.pdf](http://ec.europa.eu/competition/mergers/cases/decisions/m7603_1139_3.pdf)

# VISUALIZATION OF CROSS-EFFICIENCY MATRICES USING MULTIDIMENSIONAL UNFOLDING

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## ABSTRACT

*The current study suggests visualization of the DEA cross-efficiency matrices using multidimensional unfolding(MDU). Cross-efficiency matrix is a DEA comprehensive and information-rich data object which includes not only the simple efficiency scores, but also the peer-evaluations for each decision making unit(DMU). Hence, the final visual configuration is a rich, holistic map that is able to reveal the similarity and dissimilarity between units, according to their optimum weights as well as levels of inputs and outputs. In general, the visual configuration can be used as an exploratory tool in order to gain insight into the CEM, and in order to discover patterns, and regularities such as homogenous clusters, as well as irregularities, such as maverick and outlier units. The suggested method is illustrated by means of one artificial and one real dataset.*

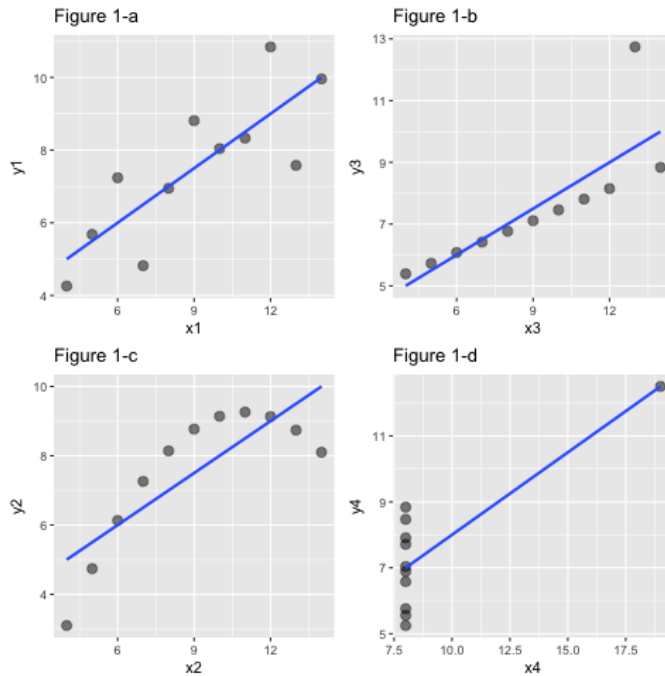
**Keywords:** Data Visualisation; Cross Evaluation; Cross-efficiency Matrix; Multidimensional Unfolding; Anomaly detection

## INTRODUCTION

Data visualization is representation of data, either numeric or categorical, in a graphical form in order to gain insight into that data. In other words, data visualization is about "looking at data to see what it seems to say", and making the data "more easily and effectively handleable by minds." Tukey(1977)

Using system science terminology, one of the goals of data visualization is "holistic" evaluation of the dataset. To have a holistic approach makes researchers able to simultaneously investigate the components, e.g. decision making units, and their relations from a bird's eye viewpoint. Having such "big picture" of the dataset can reveal some evidences in the data that otherwise would remain concealed. In contrast to this holistic view, analytical measures are generally reductionist approaches to the data. Cleveland(1985)

This contrast between a holistic data visualization and a reductionist measure, is well illustrated by Anscombe(1973) in the following example known as "Anscombe quartet", shown in figure1:



**Figure 4 - Anscombe Quartet**

All four datasets have the identical mean of x, mean of y, correlation between x and y, as well as sample variance of x and sample variance of y up to two decimal precisions. Hence, considering only these measures, all datasets seem exactly the same, however, as we can see in the figure1, there are considerable differences between each pair. Such decisive differences are revealed and detected "easily and effectively" through visualization.

Beside having the big picture and seeing "emergent properties" of the observations and their relations, discovering patterns, regularities, and thus irregularities are other motivations of data visualization, according to Keim(2002). In the domain of DEA, the example of regularities are homogenous clusters of DMUs, and irregularities can refer to outlier and maverick units.

Since in this study, data visualization is used as an exploratory tool, generation of further questions from the dataset, and thus shaping new hypotheses can be another motivation. While exploration of a dataset can be done in order to answer some specific questions, it may be used to address more "exploratory, open-ended scenarios" of "discovering the unknown", as stated by Telea(2014)

Nevertheless, the exploratory data visualization is a complementary tool to other approaches of data analysis. In the words of Tukey(1977) : "Exploratory data analysis can never be the whole story, but nothing can serve as the foundation stone- as the first step".

## DEA VISUALIZATION: A BRIEF LITERATURE REVIEW

Data visualization is a relatively neglected topic in DEA literature, considering the importance of the topic, and the total number of published DEA studies. The absence of data visualization, as an exploratory step, is more notable among the DEA case studies, and the reason of such absence may be due to high-

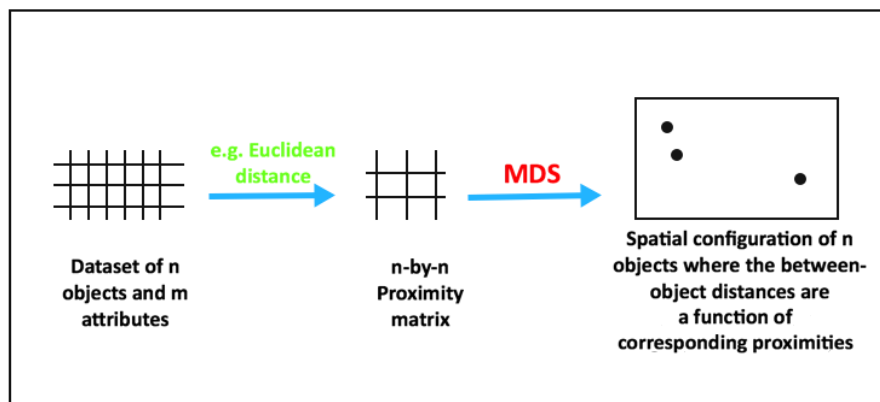
dimensionality of DEA problems which makes graphical representation more difficult, or may be due to disregarding the advantages of data visualization. Nonetheless, the DEA visualization toolbox is not empty. To visualize different DEA high-dimensional datasets, Weber and Desai(1994) suggest using parallel coordinates of input and output levels.

Serrano-Cinca, et al.(2005) propose to use principle component analysis(PCA) to reduce the dimensionality of a matrix whose rows are DMUs and columns are various DEA models, and super-impose the final map with model vectors. Porembski et al.(2005) use non-linear projection of Sammon(1969), a variation of multidimensional scaling, to map the DMUs based on their input and output profiles, and enrich the final configuration with dual multipliers. Adler and Raveh(2008) propose a biplot based on output over input ratio profiles of the DMUs, and super-imposed ratio vectors. Also Carboni and Russu(2015) offer using self-organizing maps in DEA visualization.

In order to devise a DEA visualization method, one has to address the DEA visualization dataset as well as the mapping technique, since almost all DEA datasets are high-dimensional. The current study suggests a visualization method for cross-efficiency matrix(CEM) using multidimensional unfolding(MDU). Talluri et al(1999) suggest visualization of CEM using boxplots of columns of the matrix, as well as a scatter plot of average cross-efficiency and simple efficiency. The current study however benefits from a non-linear projection technique to graphically represent all DMUs and their relations according to their corresponding CEM in a bi-dimensional map. As a pre-requisite of illustration of this DEA visualization method, MDU is introduced in the following section. To spare space, introduction to cross-evaluation is omitted from this report, and interested readers are referred to Ruiz and Sirvent(2016).

## MULTIDIMENSIONAL UNFOLDING

In order to graphically represent any CEM in a bi-dimensional plot, the dimensionality of the matrix must be reduced. Doing so can be done using multidimensional unfolding(MDU), a variety of multidimensional scaling(MDS), that can cope with asymmetric matrices such as CEM.<sup>7</sup>



**Figure 5 - Multidimensional Scaling Procedure**

<sup>7</sup> Any CEM is inherently asymmetric since in general any  $e_{ij} \neq e_{ji}$



MDS is called to a family of techniques which can project high-dimensional data points into low dimensional space by preserving the inter-point distances as much as possible. In fact, MDS reproduces a low-dimensional spatial configurations of the objects, based on their (dis)similarity matrix such that the distance between every pair of objects in the final configuration is a function of the (dis)similarity measure of that pair. This (dis)similarity matrix is called proximity matrix, and the (dis)similarity between objects can be measured using Euclidean distance, as an instance. In the final configuration, similar objects are ideally located closer to each other than dissimilar objects. Figure2 illustrates the process.

While a proximity matrix is usually symmetric, i.e. the objects on the rows are identical to the objects on the columns, a proximity matrix can be asymmetric, i.e. the row objects and column objects are not identical. In other words, proximity matrix usually has one mode, but there are proximity matrices with two-modes. A classical example is the matrix of a set of judges, in the rows, and their ratings given to a set of wines, in the columns. Such preference matrix is a two-mode proximity matrix. Similarly, CEM is a matrix of set of DMUs as judges who evaluate the same set of DMUs. While the two sets are apparently the same, the difference of their roles makes the two sets different and thus the CEM is a matrix with two modes. MDU is a proper technique to visualize such matrix in a spatial map with two different set of points, one for the row objects and one for the column objects. In the next section, the process is illustrated through an artificial example as well as a real dataset.<sup>8</sup>

## VISUALISATION OF CEM USING MDU

In order to illustrate the visualization of CEM using MDU, two numerical examples are presented.<sup>9</sup> The first dataset is a fabricated CEM with three DMUs with the goal of explanation of the visual configuration. The second dataset is from Bahari and Emrouznejad(2014) and it is composed of 38 hospitals, as DMUs. In order to save space, the CEM of the latter dataset is not presented in this report. The input and output parameters and their values can be found in the cited source.

Consider the cross-efficiency matrix of table1. The cross-efficiency scores have been assigned manually in order to highlight some important features of the unfolding map, and they have no ground in reality.

**Table 1 - An artificial CEM**

	DMU1	DMU2	DMU3
DMU1	1.0	0.4	0.1
DMU2	0.9	0.5	0.4
DMU3	0.2	0.4	0.9

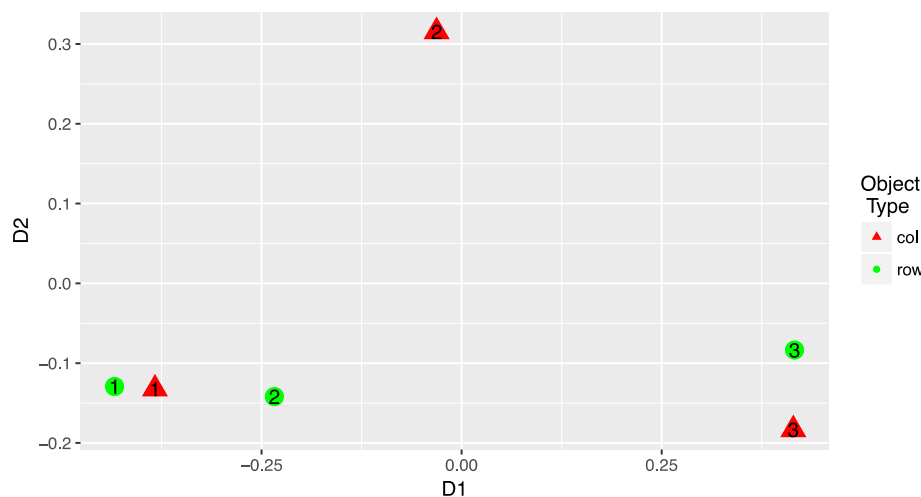
Before investigation of the visual configuration, we can investigate the CEM of table1, since the number of DMUs is small. While the set of DMUs on the rows seemingly identical to set of DMUs on the columns, they play two different roles. On the rows, they evaluate and appraise their peers, and on the columns they are passively evaluated by their peers. The diagonal scores

<sup>8</sup> Interested readers in MDS and MDU are referred to Borg and Groenen (2005)

<sup>9</sup> All the computations and visualizations have been done using Smacof package De Leeuw and Mair (2009), ggplot2 Wickham(2009) , ggrepel Slowikowski(2016) in R statistical software R Core team(2016)

are simple efficiencies, self-appraisals, and thus the highest score of each column. MDU tries to find a spatial configuration for the three row objects, and the three column objects of this matrix such that the distance between each pair of these objects reflects the dissimilarity between the pair. Therefore, if two objects, i.e. two DMUs, are dissimilar, then they are located far from each other on the map, and vice versa. While the distance between each row and each column object is a function of the corresponding cross-efficiency score, the distance between a pair of row objects is based on the dissimilarity of their row profiles. Similarly, the distance between each pair of column objects is a function of the dissimilarity of their column profiles.

Considering the stated explanations, DMU1 and DMU2 from the row profiles are relatively similar, and DMU3 is a incongruous unit here. Both DMU1 and DMU2 highly endorse DMU1, and they find DMU3 the least attractive unit. In contrast, DMU3 endorses itself, while deeming DMU2 and DMU1 very inefficient units. So we expect to have row objects of DMU1 and DMU2 close to each other, and DMU3 far from them. On the other hand, the DMU1 and DMU2 are not similar from the column aspect. While DMU1 in its column role is highly praised by both DMU1 and DMU2, and is highly disapproved by DMU3, all units almost similarly rate DMU2 with an average score. DMU3 is approved by itself as a unit with 90% efficiency, while it is the least preference for the two other units. Figure X is visualization of these entities and their relations.

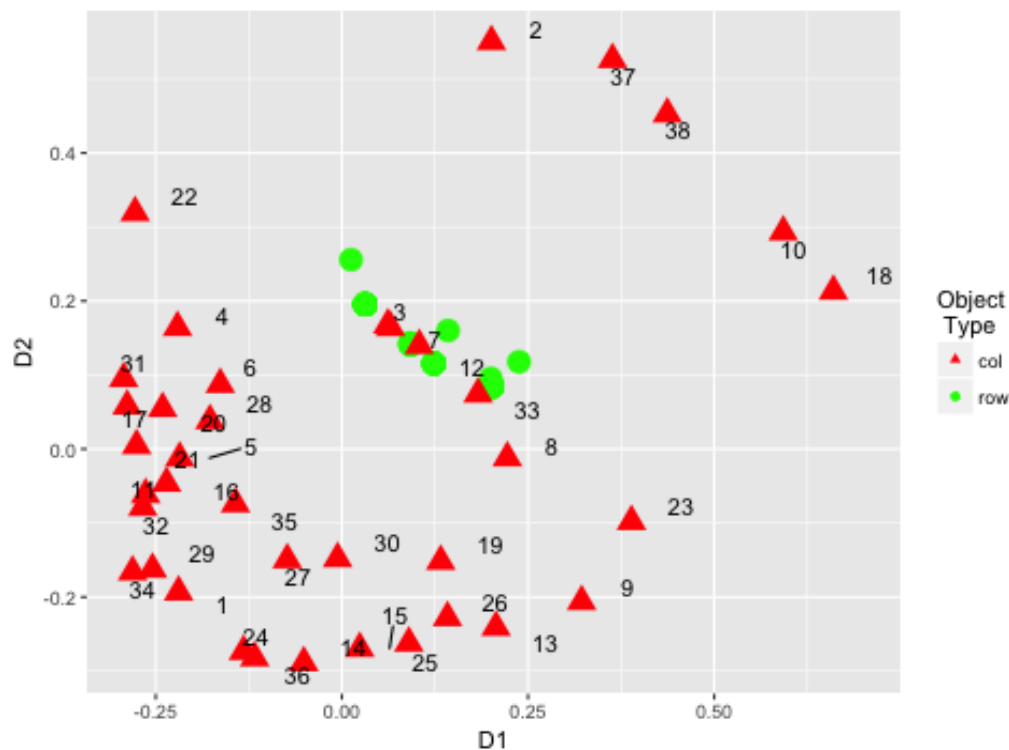


**Figure 6- Visualization of Table1 CEM**

The row objects are shown with green dots, and the column objects as red triangles. It is conspicuous from the first glance that DMU3 is totally discordant from both row and column aspects. DMU1 and DMU2 are close to each other from the row aspects, thus they have similar optimum weights, and they are close to column object 1, so they both highly endorse DMU1 as an efficient unit. The column object of DMU2 is far from the crowd and almost equi-distantly far from all row objects, which means this DMU is not approved by the any of the units. In the same manner, the relative locations of the objects can be assessed.

Figure 4 is the visualization of benevolent cross-efficiency matrix of 38 hospitals of Bahari and Emrouznejad(2014). In order to avoid overcrowding the map, the labels of the overlapping row objects are omitted. Doing so does not harm the purpose of this visualization since the row units are very homogenous, and they are the column objects which have the interesting points here.

Figure 4 can be assessed from different perspectives, however here we focus on the "highly praised" column objects of 3,7,12,33,8. These units are very close to the crowd of row objects, which means that the row objects strongly prefer these units, and endorse them as efficient units. This is not the case for the rest of column objects, as they are located far from the green crowd. We have found some "evidence" that the DMU set of 3,7,12,33, and 8 are "efficient outliers". Interestingly, this finding is compatible with the analytical method of Bahari and Emrouznejad(2014) as they have introduced units 3,8,12,19 and 33 as potential outliers.



**Figure 7 - Visualization of Benevolent CEM of 38 hospitals, presented in Bahari and Emrouznejad(2014)**

## CONCLUSION

Data visualization can play an important role in gaining insight into any data oriented problem, and DEA problems are not exceptions. Nevertheless, data visualization is a relatively neglected step in DEA studies, even though the DEA visualization toolbox has several effective visualization techniques. Such techniques can be used as a data exploratory tool for general exploration in order to "literally look at the data" and "discover the unknown", as stated by Telea(2014).

In this study, we suggested a novel method for visualization of cross-efficiency matrices(CEM), using multidimensional unfolding(MDU). Cross-efficiency evaluation is one of the current research fronts in DEA Liu et al.(2016), and this visualization tool can improve and ease exploration and comprehension of such matrices. In addition to general exploration, the suggested graphical representation of CEM can help in detection of anomalies, such as outliers and maverick units, as well as homogenous clusters.

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