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Albrecht Dürer's personal Underweysung der Messung

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Abstract The article discusses the handwritten revisions and drawn additions by Albrecht Dürer in his own copy of the treatise on geometry, *Underweysung der Messung* (1525). Situating Dürer's interest in mathematics within the scholarly milieu of Renaissance Nuremberg, the article addresses the shifts in style and content that Dürer proscribes and offers new perspectives on the artist's relation to his own late work. The article concludes by displaying several drawings bound into the edition by Dürer that illustrate variations on perspectival apparatuses. These lesser known drawings illuminate the evolution of Dürer's conceptualization of the dynamics between artist and subject, as well as the tools used to facilitate this interaction.

Keywords Albrecht Dürer, editing, geometry, measurement, perspective, Platonic solids

Albrecht Dürer's (1471-1528) most important contribution to sixteenth-century geometry remains the first of his two major treatises on measurement and proportion, Underweysung der Messung (1525), a text which synthesized the internal structure, principles, and precision of Euclid's Elements with various classical and contemporary geometrical texts available to him in his home city of Nuremberg.¹ Beginning with Euclidean principles and spreading into a diverse set of "mixed-mathematical," or applied-geometrical applications, such as column design, sundials, calligraphy, the modeling of polyhedra, and perspective, Underweysung systematically organizes a wealth of knowledge accumulated over a lifetime of artistic practice, and was intended, as Dürer declared in Underweysung's introduction, to be "not only for painters, but also for goldsmiths, sculptors, stonemasons, carpenters, and all those for whom using measurement is useful."² Envisioned more as a pedagogical provocation than a personal dissection of his own creative process, Underweysung would go on to form the theoretical base upon which the next generations of German artists built their approach to geometrical research and experimentation. Artists like Augustin Hirschvogel (1503-53), Wenzel Jamnitzer (1507/8-85), Heinrich Lautensack (1522–90), Lorentz Stöer (c.1530–after 1621), and others developed proficiencies in geometricality by adopting the physicality and tactility of Dürer's geometry, in the process altering the nature of geometrical knowledge from the abstract and theoretical discussions of Euclid into an intimate and approachable field of investigation. This in turn generated new geometrical objects intended to be touched, handled, and manipulated, such as the many three-dimensional polyhedral solids that appear in sixteenthcentury German print culture and decorative arts.

Nevertheless, *Underweysung* never became as popular with historians as Dürer's second major treatise, the posthumously published *Vier Bücher von menschlicher Proportion* (1528), which had been conceptualized as a more advanced treatise on drawing to be studied after *Underweysung* and deployed geometry as a stereometric aid in the representation of the human body.³ After all, *Vier Bücher von menschlicher Proportion* reveals Dürer's developing and experimental interest in the use of proportional

ratios as a means of determining the human body—a set of theoretical concerns that potentially tracks back more easily to his artistic œuvre. Many of Dürer's extant working drawings of figures are designs for prints in *Vier Bücher von menschlicher Proportion*, which show Dürer grappling with generating geometrical ratios of body proportions using a compass, though he eventually moves towards systems of ratios that balanced the dimensions of body parts to each other, sometimes even surveying the ratios of the body's parts together in elaborate detail.⁴

Perhaps the relative lack of interest in divining the permutations of Dürer's first geometrical work explains the scarcity of scholarship on Dürer's extraordinary personal copy of Underweysung der Messung, which has been located at the Bayerische Staatsbibliothek in Munich since 1839 and can be consulted under the call number 4° L.impr.c.n.mss. 119.5 Still, this fascinating volume deserves another look. The book is a complete rendition of Dürer's own edits and additions to Underweysung, made sometime after its 1525 printing and before his death on April 6, 1528. The volume includes numerous unpublished geometrical diagrams (some of which have been reproduced in this article) as well as two very significant, yet widely unknown, drawings of figures that have nearly escaped attention.⁶ Dürer's notes, marked directly in the margins in his own hand, bound together as separate pages, and contained on small scraps of paper glued onto the pages they reference, attest both to Dürer's prolonged and passionate engagement with the treatise even after its initial publication and to the authority he commanded as an author.⁷ There is no "range of interpretive possibilities" for the workshop of Hieronymous Andreae (d. 1556)-the printer of the first and second posthumous 1538 edition of Underweysung as well as Dürer's monumental Triumphal Arch (first printed 1526) for Emperor Maximilian I (1459–1519); no leeway to "schreibt hrein, was Ir wollt," as has been discussed in relation to the blank sign in Dürer's drawing The Recording of the Thoughts of the Pious and the Wicked (c.1500-15).⁸ All Dürer's edits were carried over and faithfully reproduced.9

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Tracing the contours of Dürer's acute investment in his geometrical treatise provides a yet untapped access into the mind of the artist in the last years of his life, and, moreover, indicates a route towards extrapolating more general conclusions about how Dürer's revisions and edits are at once reflective, and in excess, of the cultural expectations and material practices of the Renaissance print workshop. By triangulating between all three versions of Underweysung-the 1525 original, Dürer's copy, and the 1538 second edition-it is possible to reconstruct the evolution of the treatise and Dürer's thoughts on it; to confirm the extent to which Dürer's edits and proscriptions were posthumously followed by the printing house; and their relation to the numerous fragments of preparatory material for Underweysung scattered throughout the world, as well as direct attention to the numerous drawings in 4° L.impr.c.n.mss. 119.10 Beginning by contextualizing Dürer's lifelong interest in geometry, and Euclid in particular, the article centers on the corrections and insertions that Dürer made to his personal copy of Underweysung der Messung, commencing with the fastidious editing of the text he undertook and concluding with his first drafts for his famous perspectival scenes printed in the 1538 edition. As Dürer himself notes in a handwritten inscription on the title page of 4° L.impr.c.n.mss. 119:

I have properly corrected this book. If one wants to print a further new copy then close attention should be paid to the corrections and care taken throughout to properly arrange the old and new diagrams relative to the appropriate text.¹¹

Let us begin by accepting Durer's invitation.

Dürer's library

Upon the death of the great mathematician and astronomer Johannes Regiomontanus (1436-76), who had lived in Nuremberg from 1471 to 1475, the working library he had amassed there was turned over to Bernhard Walther (c.1430-1504), a local merchant and proficient astronomer in his own right who had collaborated with Regiomontanus on astronomical observations and in the establishment of a printing press tasked with the purpose of printing astronomical texts revised with new observational data. Attempts by King Matthias I of Hungary and Croatia (1443-90) to convince Walther to part with Regiomontanus's books and scientific instruments so that they could be housed in the royal library in Buda were unsuccessful, and the collection remained in Regiomontanus's house, which Walther himself had purchased.¹² Shortly before his own death in 1504, Walther decreed that the entire collection of books and instruments were only to be sold all together, and with the exception of several books which were sent to Krakow and Italy in 1512 and a selection of the brass instruments which were stolen in 1514, the collection remained remarkably intact over the next fifteen years, though not likely in situ, and under the auspices of the city of Nuremberg.

During this time, the "Regiomontan-Waltersche Bücherei" became an intellectual resource and reference library for a new generation of Nuremberg-based scholars such as Johannes Werner (1468–1522), who worked on spherical trigonometry and conic sections; Dürer's close friend the humanist Willibald Pirckheimer (1470–1530); Joachim Camerarius (1500–74), the classical scholar, Dürer's biographer, and the first translator of Dürer's *Underweysung* into Latin; the influential globe-maker and cosmographer Johannes Schöner (1477–1547), and others.¹³ It was only in 1519, after a failed attempt to sell the books to Elector Friedrich the Wise of Saxony, that a portion of the books was sold for the sum of 150 *gulden*, most of which went to Pirckheimer, who recorded the purchase in his personal manuscripts.¹⁴

Dürer's status as Nuremberg's most celebrated artist would have been sufficient to provide him with access to the Regiomontanus-Walther library, irrespective of his close association with Pirckheimer and the other Nuremberg scholars. But given his direct connections to Walther, he may even have had access to the library while Walther was alive. His parents had known Walther personally and Walther's wife had been the godmother to Dürer's sister Christina, who was born in 1488. That Dürer was intimately familiar with the house itself is attested to by the fact that in 1509, five years after Walther's death, he purchased the house from Walther's heirs and moved into it with his wife Agnes (1475-1539), living there until his death in 1528 when he was said to have been found with books from the library scattered near him.¹⁵ Perhaps some of these books were those he had bought in 1523 from the library because of their "usefulness to painters," when he had paid ten florins for a selection of ten.¹⁶

Due to the library's unique status as a substantial mixedmathematical resource in Nuremberg, it is extremely likely that Dürer would have used the library for the purpose of deepening his understanding of geometry and perspective and that he could have easily gained entry, either as a family friend or as an inquisitive artist, from very early on in his career, most certainly should he have desired prior to his second extended 1505-07 trip to Italy. Dürer also would have had contact with all of the above-mentioned scholars and could have approached them for help in solving the more complex geometrical issues found in his Underweysung. Thus, given Dürer's intimate familiarity with Pirckheimer and his extended humanist circle and in combination with Dürer's probable access to the library, it is difficult not to defend the characterization of Dürer as having lived within a learned community in Nuremberg capable of answering any or all the questions he might have had on the mathematical issues that pertained to his work on geometry and measurement.

Despite having access to this mathematical literature, Dürer might very well have also needed help reading the ancient texts, such as the newly published translation into Latin, from the original Greek, of Euclid's Opera (1505) by Bartolomeo Zamberti, which Dürer acquired in Venice for one ducat.¹⁷ His contemporary biographer Camerarius claims that although Dürer had acquired skills in "naturalium et mathematicarum rerum scientiae," he had not undertaken literary studies, which meant in this context "the analysis and also the productive imitation of classical Latin writers."18 Camerarius does admit, however, that the natural and mathematical sciences were predicated upon ancient texts which Dürer had "almost learned" (fere didicerat), and that Dürer's geometrical treatises evidenced his understanding of scientific principles and his ability to set them down into words.¹⁹ Pirckheimer would have been an inimitable asset here. There are numerous examples of Pirckheimer's writing in Dürer's voluminous literary remains that confirm their close collaboration on mathematical issues and translations, including corrections made by Pirckheimer to drafts of Dürer's Underweysung.²⁰ Pirckheimer had studied law in Italy from 1489 to 1495, was fluent in Greek and Latin, and routinely acted as an intermediary between Dürer and Emperor Maximilian I, even going so far as to act as an advisor of antique iconography for some of Dürer's highest profile commissions including the Triumbhal Arch or Ehrenpforte Maximilians I (first edition printed 1517–18) -a monumental composite image printed on thirty-six large sheets of paper from 195 individual wood blocks and saturated with trans-historical references.²¹ Pirckheimer also possessed his own well-appointed library which, he claimed in a 1503 letter, held a copy of almost every text printed in Italy.²² Dürer surely had access to this library as well, including the volumes on geometry and perspective, such as Luca Pacioli's (c. 1445-1514) Somma di aritmetica, geometrica, proportioni e proportionalita (1494). The solution to the Delic Problem contained in Underweysung also appears to be based upon the Eutokius manuscript in Pirckheimer's library, and Dürer is known to have personally illuminated a selection of his friend's books.²³

Two inventories of the Regiomontanus-Walther collection were compiled by Pirckheimer in 1512 and 1522, the second inventory listing 145 remaining books mainly comprised of classics from antiquity and the Middle Ages on geometry, perspective, astronomy, and astrology (e.g. Ptolemy, Sacrobosco, Ibn Al-haitan, and Archimedes) as well as contemporary works of relevance including calendars and astronomical tables.²⁴ Of note in the context of Dürer's Underweysung are the many copies of Euclid that were still in the collection, and to which Dürer would presumably have been able to consult, as well as the imprint Underweyung bears from Dürer's reference to several prominent mixed-mathematical texts, including Fialenbüchlein (1486) by Hans Schmuttermayer-who was an acquaintance of Dürer's father, Püchlein von der filialen Gerechtigkeit (1486) by Matthis Roriczer, and the anonymous Geometria deutsch, aus der geometry etliche nutzparliche stuck (c.1472-84), possibly attributable to Roriczer.²⁵ The library also contained the first printed edition of what had been the most popular version of Euclid's *Elements* up until the sixteenth century, the *Preclarissimu*[*m*] *opus Elementor*[*um*] *Euclidis Megaare*[*n*]*sis* (1482) referred to as "Euclides. Impressus. (Geometria)" in the inventory —which was based upon the medieval text by the Italian mathematician Campanus of Novara (*c*.1220–96) and had been printed for the first time with accompanying geometrical diagrams in Venice by the Augsburg printer Erhard Ratdolt (1442–1528); a rare manuscript of the first translation of Euclid from Arabic into Latin by the English monk and natural philosopher Adelard of Bath (*c*.1080–*c*.1152); and a manuscript of Leon Battista Alberti's *De pictura*, "Liber de picture L. Baptiste de Albertis. (Geometria)," referred to as "De picture babtis" in the 1512 inventory.²⁶

Although Dürer might very well have bought Alberti's De pictura as one of his ten "artistically useful" purchases in 1523, the only book from the collection identified as having been in Dürer's possession was Regiomontanus's own manuscript copy of Euclid's *Elements*.²⁷ In fact, it is not surprising that Dürer would have consulted multiple copies of *Elements* in addition to the copy he bought in Venice, even if the manuscript contained an older translation. Dürer's letters contain what I suspect to be among the first translations into German of eleven suppositions and forty theorems from Euclid's Perspectiva Naturalis, some of which are in Pirckheimer's hand, alongside numerous hardline geometrical sketches and illustrations copied from the printed diagrams in Zamberti's Euclid.²⁸ Although he owned at least two Euclids by 1523 and had continual access to the precious Adelard of Bath edition as well as Ratdolt's 1482 first edition based on Campanus in the Regiomontanus-Walther library, Dürer evidently maintained a lifelong fascination with Euclid and remained interested in the most up-to-date translations. In a letter dated December 5, 1524, to the German mathematician Nikolaus Kratzer, who was living in London at the time, Dürer inquires how long it would take until Kratzer's planned new German translation of Euclid would be ready.²⁹

As we have seen, the wealth of mathematical knowledge locally available to Dürer in Germany, not least of which were several versions of Euclid and other ancient and contemporary German texts from the Regiomontanus-Walther library, would form the basis of his Underweysung.³⁰ Aside from the copies of Euclid in Nuremberg by Adelard of Bath and Regiomontanus' own manuscript, the popular Euclid printed by Ratdolt had been in circulation since 1482 and had most likely made its way to Nuremberg long before Dürer's first trip to Italy (1494-95). Yet Dürer's engagement with Euclidean geometry has often been co-opted into the historical story of his "discovery" of perspective in Italy.³¹ While it may be true that Dürer came back from his second trip to Venice acquainted with Italian geometrical-perspectival techniques and an additional copy of Zamberti's 1505 edition of Elements, and that in Bologna Dürer may have met Pacioli, who would have been working on his own annotated edition of Euclid at the time, Dürer had an abiding interest and long-standing access to Euclidean geometry that predated his engagement with Italian perspective.³²

Second thoughts

Underweysung der Messung is a treatise on Euclidean geometry that aims to be a textbook for artists and craftsmen, indeed "everyone desirous of learning about art," and a definitive "source for learning about measurement [Messung] with ruler and compass."33 The treatise is divided into four books that straddle a range of geometrical topics in applied geometry and the visualization and invention of geometrical figures, emphasizing the construction of complex shapes/forms from basic elements.34 Book I covers the definition and construction of lines-ranging from parallel lines to multiple ways of constructing spirals, spiral projections, hyperbolic lines, conic sections, and parabola. Book II discusses plane surfaces, the construction of polygons, tile patterns, and the Pythagorean theorem. Book III covers a variety of applied-mathematical topics, including column and monument design and construction, sundials, and the geometrical construction of the letters of the alphabet. Book IV returns to the Platonic and Archimedean solids and concludes with the Delic Problem, studies of proportional lines, and various perspectival theories and apparatuses.

Dürer completes the first edition of *Underweysung* with an errata list of printing errors, a practice not uncommon in sixteenth-century books.³⁵

Even though I diligently and skillfully tried to ensure that this book would be well and properly corrected, because of the removal and resetting of the type, honest mistakes in word choice were committed. For this reason, I want to show here in one section the most noteworthy errors. An honestly intelligent reader will easily be able to correct the other words.³⁶

Dürer's statement suggests that the "honest mistakes" (etlich *yrrtum*) were due to the prolonged process of arranging and rearranging the layout of the book, alluding to the experience of repetitively combing through its content, searching for errors, and correcting geometrical diagrams, assessing how the sentences flowed when formalized and concretized in print, and then working closely with the Andreae workshop to ensure that his changes were reset into the type. For instance, Dürer states that a certain "durchschneyden" should be replaced with a "durchschnitten," a "rechten" for a "gleychen," a "schneydan" for a "schneydet."37 No subsequent reader or any of the professional correctors employed by the printing house would have to be able to make these corrections because they would never have appeared to be errors in need of correction.38 Rather, these new words seem to have more precisely articulated the intricate and personal textures of Dürer's thoughts than their predecessors, slotting into the overall flow of Underweysung to fine-tune the exact balance of the text's tone and timbre.

The specificity of the mistakes listed in the addendum, bound together into a category of error noteworthy enough to be singled out and mentioned, is undercut by its provisional last sentence, which opens up an infinite possibility of words in need of similarly delicate alteration. It is as if Dürer's existential commitment to the minute valences of his personal locution, revision after revision, had finally drained him of the ability to hear himself. And that even though the text was ostensibly finished and wholly bereft of perceivable errors, he had been left with a nagging if placeless sense that it was unfinished, wishing for a "yetlicher verstendiger," someone who could access his thoughts and intuit all the refining work that he knew remained but was no longer able to access as the book went to print.

The perceived if elusive imperfections in the treatise must have continued to rankle, in that their very presence, invisible as they may have been to others, constituted a direct and personal threat to the reliability of Dürer's geometrical knowledge. Indeed, the errata list would have served as a natural blueprint for the first set of revisions to any later edition, revisions that were subsequently completed by the time the second edition went to print in 1538 as signified by the large 'X' across Dürer's personal copy (figure 1). Having spent decades compiling, synthesizing, and formulating his theories on geometry and measurement, the panacea of embedding a public acknowledgment of Underweysung's flaws within Underweysung itself, no matter if it was common practice for many 16th century books, must have continued to rankle. Dürer's dedication to Pirckheimer in Underweysung's introduction decries the state of German art as a milieu in which "the sole reason that painters haven't learned the art of measurement is because they have taken pleasure in their errors and thus they cannot and will not ever be true artisans." And yet the contradiction of professing to inculcate a much needed precision in German art through a text that could not even manage to contain the chaos of its own creation may well have seemed to Dürer to have critically undermined the intellectual integrity and indeed the pedagogical remit of the text.39

Dürer's personal copy in Munich evidences that for a sustained period following Underweysung's 1525 publication and prior to his death in 1528, Dürer reclaimed the patience and fortitude once again, and conclusively, to spread his consciousness throughout the text. His Underweysung is replete with an intimate carpeting of edits and editorial commentary, a small selection of which have been subsequently reproduced in this article, in particular where the edits involve new drawings, and which, in total, serve as a model of uncompromising, epistemological integrity. To be a "rechter werckman" (a true artisan) was to be impervious to seduction by the *appearance* of correctness and accuracy, and steadfastly to pursue the full embodiment of an artistic vision down to the microscopic level. In contradistinction, "the taking of pleasure in error" (wolgefallen in iren yrthumben) generated a panoply of "dishonest mistakes"-dimensions or components of a work that although they might be imperceptible to anyone other than the artist, or

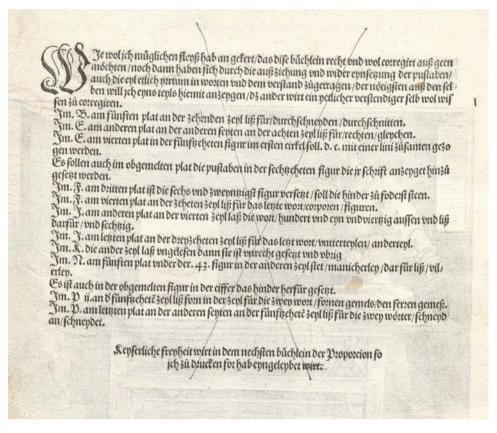


Figure 1. Albrecht Dürer, crossed out errata list from the last page of *Underweysung der Messung* (1525), 4° Limpr.c.n.mss. 119. Bayerische Staatsbibliothek München (BSB), urn:nbn:de:bvb:12-bsb00084858-4.

Sife offenber Dis fie bie greaten timber Beeidelei out woon en onder ab Muriben taften die erfen finender man ab duas die mand mie bas faar omgenen in Ander die andern meuslei withers werden great hie roat ab mit finden werd minaste bed mi Die andern finden in om duast in great bie roat ab mit finder bei bei wird Sie gerabenlinien bie gegen einander abgefonnten werden, die find die bei wird die wer Sorten gefehlecht macht man burch sal bie erften burch ein gerade sal ber romset ift /2 / bie anbten ber burch ein ongerade sal ber rourset ift/3/ Samaer halten fich biefnien einer fölchen mali gegen einander die man durch die salnie mag auf fprechen die dren ding verftee alfo, Sum erfter feit fabe pier linienineben einander/ond brauch dich in der abreihung der geraden sal die erft lini halt. 2. bie ans ber. 4. bie bitt/8/bie viert/16/21ber die ungeraden andern vier linien/bie erft belt. 3. bie ander/9/bie bit 27/bie viert/8/21/fomag man die gerad und ungerad sal in bederleg gefchlecht ber linien mern oder uninger nyndern vind nit alweg doppel gegen einander brauchen/wie das ein ytlicher der innen internet Ean leichdich verfteen wirdt/DRan mag auch mit einer einfachen sal durch die linien fteigenn/Über wie man die linien die man durch die sal nit teylen fan/teylen foll/ift in der negften feborn figurati-arteigind bie werken verblie die sal nit teylen fan/teylen foll/ift in der negften feborn figuratigeseigt ond bie sween gefchlecht der geraden und ungeraden sall in linien zu brauchen /babich unden anfgeroffen der mägen fich auch die Steinnersen gebrauchen in den außaugen/Auch mag man dis Relitien lenter weder ist der die Greinnersen gebrauchen in den außaugen/Auch mag man dis felinien lenger underfich bynab syeben auf ein swerch lini fo halten fie fich aber anderft dann vor/ger gen einander.

Figure 2. Albrecht Dürer, paragraph with Dürer's handwritten edits from above Diagram 49, Book I, 23^v, *Underweysung der Messung* (1525), 4^o L.impr.c.n.mss. 119, BSB.

might possess unintentionally pleasurable qualities, were born from accidents in the creative process.⁴⁰ These uninvited guests, who had taken up residence in a work and sought to convince the artist that their place was rightful were pernicious entities for Dürer signifying a general infatuation with impressions, a malaise of imprecision in German art that could only be expunged through a steadfast commitment to structuring direct reciprocities between thought/vision and content/image. The depth of Dürer's fastidious attention to language mirrors his faith in the foundational veracity of "the art of measurement" (*die kunst der messung*), an art whose business it was to align calculable quantities with graphic qualities and in which his *Underweysung* sought to provide much needed instruction.

It is worth noting that Dürer was striking out into new linguistic, and mathematical, territory-one that tested the limits of the German language while exploiting its capacity to convey meaning through description. As Erwin Panofsky notes, German did not yet even have words for many of Dürer's geometrical constructions, thus requiring Dürer to invent a new scientific prose of descriptive-graphical terms. The use of words like "Fischblase' (fish's bladder) and 'der neue Mondschein' (crescent) for the figures resulting from the intersection of two circles," as well as new terms like "Gabellinie' (fork line) for hyperbola ... 'Schnecken linie' (snail line) for spiral'' meant that the typesetters, who would have given a first treatise by Germany's most famous living artist their undivided attention, may not have been able to always follow the challenging logic of the text/image relations.⁴¹ The innovative interplay and relentless alternating of text and image made it a much more challenging book to print than other contemporary mixed-mathematical textbooks-such as Zamberti's edition of Euclid's Opera (1505) or De divina proportione (1509) by Pacioli, in which images are relegated to the margins or an appendix. The text alone would not have been simple for the printers and typesetters to understand. Any mathematical errors surfacing in Dürer's many geometrical constructions would have been difficult to catch without an intimate knowledge of the mathematical content Dürer aimed to describe.

Errors, explicit or implicit, are only one of the categories of corrections that Dürer addresses in his fastidious edits. Scattered throughout the text are interventions that convey the ongoing evolution of Dürer's own intimate relationship with the treatise a relationship that evidently did not end with the publishing of the 1525 edition (figure 2).42 Written in margins or glued into the book on small slips of paper are additional sentences and geometrical diagrams. On the final page of Book I, Dürer pasted a small piece of paper including an additional method of determining proportional lines, directly translated into the 1538 edition as Diagram 51 (figure 3). A diagram describing a method for multiplying the size of a cube has also been sketched on the back of an insert added after Dürer's description of proportionally increasing cannonball weight, added along with a textual description as Diagram 53 in the 1538 edition (figure 4).43 There are two diagrams inserted by Dürer into a typology of surfaces contained in Book II, Diagrams 7 and 8 (figure 5).44 Following three tiling patterns in Book II, Diagram 23, Dürer draws a small sketch of an additional tiling pattern overlaid with the words "Dy form gehört noch hir zu setzen" (the form still belongs here) (figure 6).⁴⁵ Below this he writes, as if for further emphasis. "Die form soll auf die seiten 'a' gistiht werden" (the form should be placed on the side with "a"). On the right side of the third tile pattern, he has drawn a small 'a' to signify that his new tile drawing should be added into future

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publications. The following page includes three drawings of tile patterns that have been bound into the Munich copy on two small scraps of paper attached to each other. They are not drawn in hardline and, rather, are gestural or notional. The 1538 edition includes all three of these designs in an expanded version of Diagram 23, while the descriptive text remains unchanged, in which Dürer states that his aim is "to combine separately several five, six, seven, and eight cornered [figures]" (in other words pentagrams, hexagrams, heptagons, and octagons).⁴⁶

Dürer also combs through the text, micromanaging the layout of the document, as in the sentences beneath Book IV, Diagram 58, depicting a cube in perspective. Dürer has written the word "linin" and two pluses in the right margin, specifying that these lines of text should be spaced out in the next edition-an instruction which, like all Dürer's instructions, was followed to the letter (figure 7). Dürer was prone, too, to changing his mind about the use of certain words, substituting and crossing out as he saw fit. After contemplating the first edition, Dürer altered the treatise by removing the first person singular pronoun "ich" from the text, as in his representative edits to a paragraph describing the construction of a spider line (spinen lini) show (figure 8). Altering the delivery of geometrical information by reconstituting the narrative voice into a disembodied imperative was intended to alter the valences of the knowledge content in the treatise, making it more prescriptive and less reliant upon Dürer's own experiences.⁴⁷ It is possible to imagine that having pored over the newly printed treatise, Dürer must have felt that the version read like too much of a reflection of his own personal process of geometrical investigation and discovery. And furthermore, having been translated into print from out of a lifetime of engagement with geometry in his artistic practice, the knowledge he intended to convey deserved the kind of definitive certitude that accompanied recipes or other sets of instructions in which the outcome has already been rigorously tested and predetermined. Perhaps Dürer may have been seeking to model his authorial voice or even professorial tone on Alberti, who deploys a similar narrative voice in his writing.48

While errors in printing were not uncommon in Renaissance books, the vast majority of modifications made by Dürer are subjective, reflecting his refinement of language and his desire for irrevocable typographic and graphic clarity, little of which could have been anticipated by the Andreae workshop. The sum total of Dürer's edits gives the impression of an author deeply occupied and personally involved with perfecting and augmenting the content of Underweysung. Although Dürer died ten years before the publication of the second edition in 1538, the Munich copy strongly suggests that Dürer was anticipating an imminent second edition and approached his revisions with befitting intensity. His printed admonition of potential plagiarists in the treatise's concluding paragraph, which might otherwise have been interpreted as a warning to those who might want to produce a new version without authorization, may in fact suggest that he always intended Underweysung to be further embellished and developed:

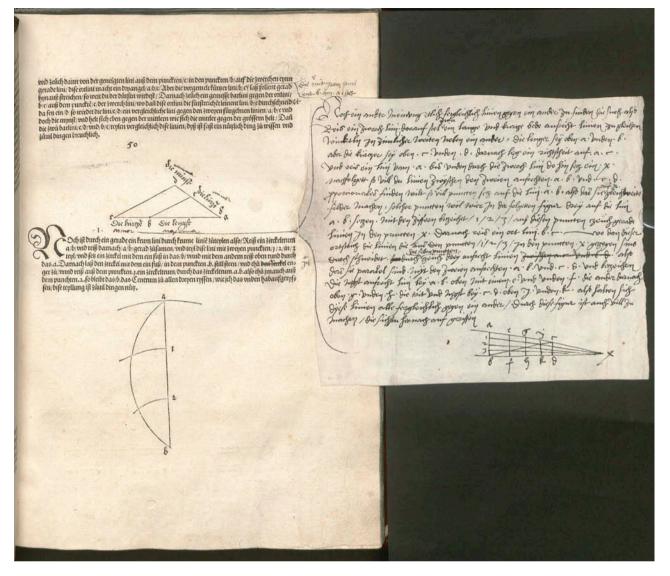


Figure 3. Albrecht Dürer, additional content inserted by Dürer into Diagram 50, Book I, 24^r, and reprinted as Diagram 51 in the 1538 edition, *Underweysung der Messung* (1525), 4° L.impr.c.n.mss. 119, BSB.

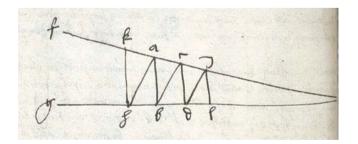


Figure 4. Albrecht Dürer, diagram illustrating a method of proportionately multiplying a cube (the cube-square law), reprinted as Diagram 53, Book IV in the 1538 edition of *Underweysung der Messung*, 82^v, *Underweysung der Messung* (1525), 4^o L.impr.c.n.mss. 119, BSB.

By the grace of God the books I have written on human proportion and other related matters will perhaps be eventually printed.⁴⁹ And at the same time I wanted to warn anyone who would have the audacity to copy this published book, which I myself want to print again with more and greater additions than have been presently articulated.⁵⁰

The second edition may not even have necessarily been prompted by the book's commercial success, as much as by the desire of his widow and Pirkheimer, in cooperation with Andreae the publisher, to posthumously fulfill Dürer's vision for the treatise, which might explain why the new edition very conspicuously does not contain an errata list.⁵¹

Whether or not Dürer knew another edition was forthcoming, his edits would eventually become advertised as the second edition's main selling point to potential buyers, though, it

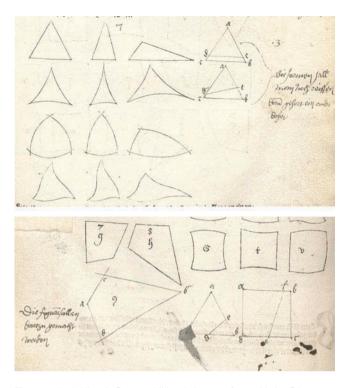
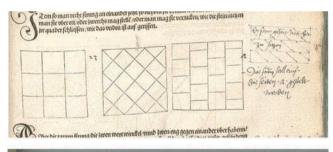


Figure 5. (top) Albrecht Dürer, additional planar surfaces added to Diagram 7, Book II, 27^r; and (bottom) additional planar surfaces added to Diagram 8, Book II, 27^v, *Underweysung der Messung* (1525), 4° Limpr.c.n.mss. 119, BSB.



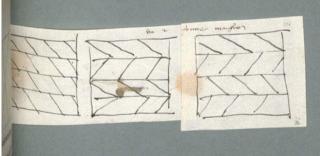


Figure 6. (top) Albrecht Dürer, tiling pattern drawn in the margins of Diagram 23, Book II, 31^r; and (bottom) three tiling patterns inserted on a slip of paper, 31^v, *Underweysung der Messung* (1525), 4° L.impr.c.n.mss. 119, BSB.

appears, these edits were either not enough or never intended to be used in order to secure a renewal of the imperial privilege:^{5^2}

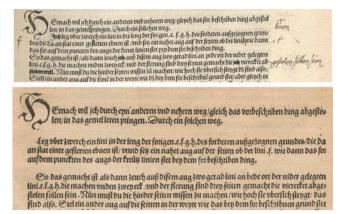


Figure 7. (top) Albrecht Dürer, paragraph with personal edits, text from beneath Diagram 58 Book IV, 86^r; *Underweysung der Messung* (1525), L.impr.c. n.mss. 199, BSB; and (bottom) the same paragraph as printed in the 1538 edition, which evidences that Dürer's formatting changes were posthumously adopted, Diagram 58 Book IV, 88^r, *Underweysung der Messung* (1538), VD16 D 2858, ETH-Bibliothek Zürich, Alte und Seltene Drucke (ETH).

Instruction of measurement with a compass and ruler in Linear planes and whole bodies, drawn together with Albrecht Dürer and improved in many places by Dürer himself while he was still on this earth. In particular, 22 figures have been added, which have also been drawn by his own hand, as can be recognized by every craftsman. Now available in print for the use of all lovers of art.⁵³

The major interventions

The majority of Dürer's additions to *Underweysung* are located in its fourth and final book, a section that covers the innovative unfolding of Platonic and Archimedean solids into polyhedral "nets" intended to be cut out and folded together, alongside Dürer's theories on perspectival drawing.

Polyhedra included two major groups in the Renaissance the so-called Platonic solids, which Plato first describes in *Timaeus* (c.360 BCE), and the Archimedean solids: thirteen convex and uniform geometries collected together, for instance, by Johannes Kepler (1571–1630) in *Harmonices mundi libri V* (1619). The Platonic solids are the tetrahedron (pyramid), hexahedron (cube), octahedron, dodecahedron, and icosahedron. Dürer enumerates his rationale for providing unfolded surfaces (ganz offen) for the Platonic solids rather than adhering to the more traditional Euclidean diagrams or the perspectival stylings of Pacioli's *De divina proportione*:

Should you want to make other handsome bodies, which touch a hollow sphere with all of their corners but have dissimilar surfaces, I will draw them completely open in the following section so that each of their surfaces touches the other. Whoever wants to replicate these surfaces should trace the larger (i.e. unfolded) figure on a duplicate piece of paper attached to the original and cut this paper along the figure with a sharp knife so that all the lines on the original paper

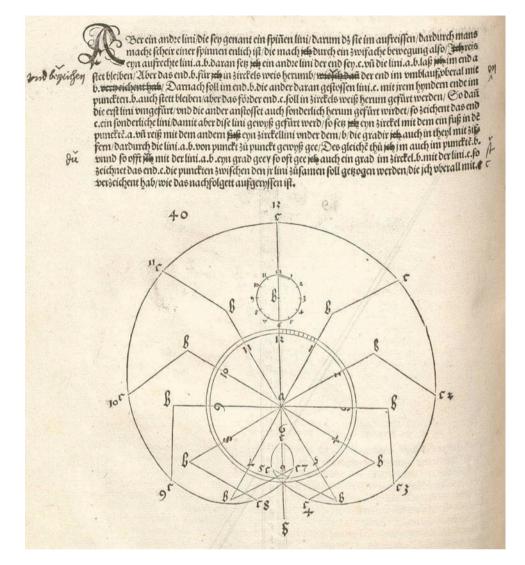


Figure 8. Albrecht Dürer, editing of the text describing the construction of a "spider line," Diagram 40, Book I, 19^v, Underweysung der Messung (1525), L.impr.c.n. mss. 199, BSB.

remain untouched. Then fold the body together along the lines of the drawing. One should pay attention to this technique because it will be useful for the following figures.⁵⁴

Book IV of the 1525 edition begins with the first unfolded solid— Diagram 29, the pyramid (*dryanglich corpus*). It is flanked by two definitions: its own definition, which begins on the second paragraph, is appended to the introductory text above it and is missing the ornate first letter that otherwise starts the first word of each geometrical definition (figure 9).⁵⁵ The doubling of definitions on this first page creates an intermittent staggering of visual and textual definitions that confounds the desire to read the text as captions to the images. The pages containing Diagrams 31–33 in both editions of the treatise each include one image of an unfolded solid—the icosahedron, the hexahedron (cube), and the dodecahedron—above the textual description of the solid that follows on the next page. As examples, the description of a cube is included below the image pertaining to an icosahedron (figure 10), the description of a dodecahedron is beneath the image of the cube, the description of the sphere is beneath the image of the dodecahedron, etc. Dürer continues with seven truncated solids (a selection of seven of the Archimedean solids), a polyhedron of his own invention, and a six-sided prism. Many of the following pages juxtapose the unfolded polyhedral net of an Archimedean solid with the definition of the next solid in Dürer's sequence.⁵⁶

After the last unfolded polyhedral net (Diagram 42) in the Munich copy, Dürer has inserted three new pages, all containing geometrical drawings that have not been reproduced before in a modern publication. The first drawing in the 1538



Figure 9. Albrecht Dürer, unfolded net of a pyramid, Diagram 29, Book IV, 69[°], *Underweysung der Messung* (1525), Limpr.c.n.mss. 199, BSB.

edition is labeled (inaccurately) as Diagram 43 and depicts an unfolded net of a truncated icosahedron (figure 11). Diagram 43a is an unfolded icosidodecahedron, Diagram 43c (there is no 43b) is a faceted sphere composed of forty-eight quadrangular and twenty-four triangular facets, and the last unnumbered figure describes a method for finding the circumference of a circle.

As with the earlier descriptions of unfolded solids, the text accompanying the new truncated icosahedron describes the component parts of its net. The drawing is executed precisely and is sandwiched between its geometrical description, written in Dürer's distinctive hand, and the geometrical description of the following icosidodecahedron. The description of the truncated icosahedron, which was eventually printed, has been exactly copied from Dürer's written text, though the corresponding drawing has been rotated ninety-degree counterclockwise in the 1538 edition.⁵⁷

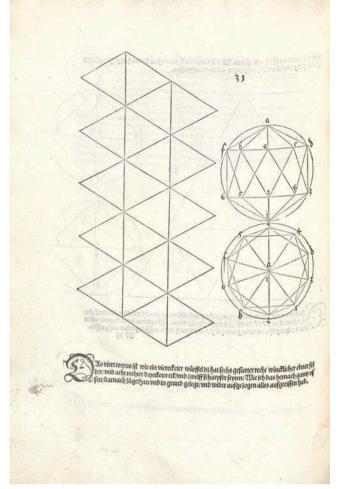


Figure 10. Albrecht Dürer, unfolded net of an icosahedron, Diagram 31, Book IV, 70[°], *Underweysung der Messung* (1525), L.impr.c.n.mss. 199, BSB.

Unlike the truncated icosahedron, the drawing of the icosidodecahedron is sketchier and the print rotated 180 degrees in the 1538 edition (figures 12 and 13). The woodblock seems to have preserved some of the sketchiness of the drawing and lacks the appearance of visual precision that the other solids display. This is the only unfolded net to exhibit this kind of uneven line, which though it accurately reproduces the quality of Dürer's sketch nevertheless in its wobbliness somewhat undermines his professed agenda to provide a precise and clean geometrical figure able to be easily cut out and folded up.

The diagram describing the circumference of a circle in the Munich copy (figure 14) is bereft of the alphanumeric notations that would subsequently be added in the later printed edition.⁵⁸ Crucially, Walter L. Strauss includes two additional versions of this drawing in the appendix to his modern edition of *Underweysung*, which seems to strongly suggest that while Dürer made smaller editing corrections directly in the text of his own copy of *Underweysung*, he worked through various iterations of his

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Figure 11. Albrecht Dürer, drawing of an unfolded net of a truncated icosahedron, 76a^r, Underweysung der Messung (1525), L.impr.c.n.mss. 199, BSB.

major additions on loose sheets of paper before cleaning up the information and recopying it on the pages that were inserted into the Munich copy.⁵⁹ Although most of the portion of Dürer's literary remains that cover *Underweysung* belong to the phase leading up to the formulation and development of its first publication, there are indeed a few existent examples of textual fragments pertaining to the 1538 edition outside of Munich.⁶⁰ For instance, in London there is an earlier version of the text describing the new unfolded net of the truncated icosahedron—further evidence that the Munich copy, which bears the final version of the description, was a self-conscious final compilation of all these notes and additions, re-transcribed and corrected to be as close a reflection as possible to what Dürer imagined the final state of his treatise to be.⁶¹

Given the close attention paid by Dürer to editing his copy of *Underweysung*, and the reverence accorded these edits by Andreae's workshop when the text posthumously went to

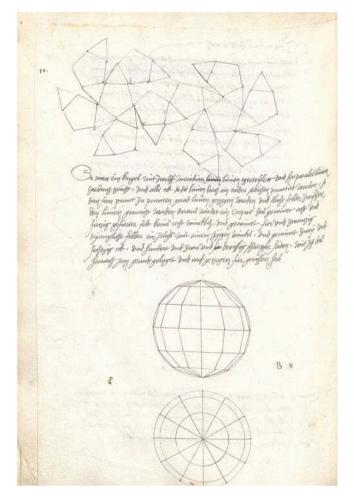


Figure 12. Albrecht Dürer, drawing of an unfolded icosidodecahedron and a faceted sphere, $76a^{v}$, *Underweysung der Messung* (1525), L.impr.c.n.mss. 199, BSB.

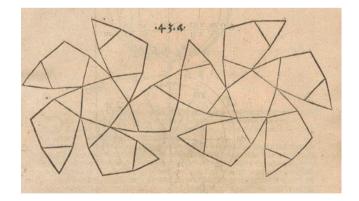


Figure 13. Print of an unfolded icosidodecahedron based upon Dürer's drawings, Book IV, Diagram 45a, 78, *Underweysung der Messung* (1538), VD16 D 2858, ETH.

print again thirteen years later, the fact that Dürer did not address or correct the misleading proximities of polyhedral diagrams and definitions might very well imply that he did

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Figure 14. Albrecht Dürer, diagram describing the circumference of a circle, 76b^r, *Underweysung der Messung* (1525), L.impr.c.n.mss. 199, BSB.

not find any problems with the information as it was displayed. Dürer conceived of his unfolded polyhedral solids as a continuous scroll of geometrical information unbound by the printed page. The reader was simply to read the text and look at the image following it, regardless of whether that image rested on the same page as the text or on the adjacent page.

On the other hand, this approach to the displaying of geometrical information may have been the cause of the variations in the layout of the second edition, which in addition to reprinting (and editing) the text did not consistently reproduce the visual arrangements of the original edition. While the scroll-like order of the polyhedral nets was preserved, the page breaks in the 1538 edition parcel up the information differently.⁶² The logic for the differentiation does not seem to have been a response to any edits by Dürer nor based upon a desire to unify self-similar geometrical information on one page. Ostensibly, both editions evidence a faith in a reader being able to follow the often-confusing proximities of text and image. An imaginary reader was meant to be able to bridge the gap between text and image in order to unite the polyhedral net with the description of its component geometries, often even while looking at a page with seemingly mismatched information.

And yet, Dürer may have been overestimating the faculties of his readers, contemporary to him and modern.⁶³ In expanding and developing the technique of unfolded polyhedral geometry, later German Lehrbücher, such as Augustin Hirschvogel's (1503-53) two-volume Geometria (1543), were exceptionally careful to include self-similar information on the same page and to accurately cross-reference image and text, perhaps in response to the natural confusion that might have arisen from Underweysung. Hirschvogel, in fact, published all the text to his treatise in a first volume-entitled "Ein eigentliche und grundtliche anweysung in die Geometria"and all the images in a second.⁶⁴ He uses the words "Rete" und "Netz," both meaning "net," neither of which is present in Underweysung, to refer to the unfolded polyhedra and added diagrammatic three-dimensional solids on each page to further substantiate the final shape of the net once folded up. The typical page from Geometria (figure 15, center) depicts an unfolded solid with construction lines overlaid on top of it and one or two drawings of the same solid rendered from two different perspectival angles, each also shown in a wireframe view.⁶⁵ In the text accompanying the "twenty-six sided body" (the same geometry as Diagram 39 in Underweysung, the rhombicuboctahedron or the "fifth irregular solid" as Dürer called it; figure 15, left), Hirschvogel labels the net and each perspective view with a different number corresponding to the text in the first volume. In contrast to Dürer's continuous flow of information, Hirschvogel was hyperconscious of the potential for error arising from any relational ambiguity between word and image, and uses the two-volume format and the space of the individual page, to design out as much confusion as possible. Nevertheless, his enthusiastic inclusion of all the supporting construction lines overlaid on the net, all unfortunately printed with the same line thickness as the net itself, may also have resulted in further unforeseen illegibilities. In the first book of his De Varia commensuración para la escultura y arquitectura (1585), Juan de Arfe (1535–1603) copies Hirschvogel's polyhedral drawings but chooses to temper the overabundance of construction lines by stripping away half of them to reveal the net on its own, much as Dürer had originally chosen to display it (figure 15, right).

The differences between the first and second editions of Underweysung are most apparent in the substantial additions made by Dürer to the rather succinct section on perspectival apparatuses in the 1525 edition, a section that appeared somewhat underdeveloped given the extensive detail Dürer devoted to more elementary geometrical and graphic operations. The Munich copy contains four-and-a-half continuous pages of new text, all of which were printed in the 1538 edition, that flesh out

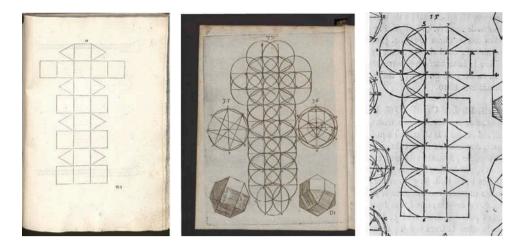


Figure 15. Three sixteenth-century rhombicuboctahedra: (left) Albrecht Dürer, "Fifth irregular solid," Diagram 39, Book IV, 75, Underweysung der Messung (1525), 4° L.impr.c.n.mss. 119, BSB; (center) Augustin Hirschvogel (1503–53), Geometria (1543), BSB, 4 Chalc. 131, f. DI, urn:nbn:de:bvb:12-bsbooo01758-4; and (right) Juan de Arfe y Villafane (1535–1603), Plate of a Solid [rhombicuboctahedron], 42, Book I, Varia Commensuracion para la Escultura, y Arquitectura (1585), 6th edition (1773), Universitätsbibliothek Heidelberg, urn:nbn:de:bsz:16-diglit-256855.

techniques for drawing a three-dimensional object from life onto a pane of glass as well as instructions for building the perspective devices themselves.⁶⁶ Initial sketches of what would become three new prints were also subsequently added.

The first sketch is embedded directly into the flow of the text (figure 16). Here the triangle is a conceptual diagram where point "o" represents the eye of the artist and line ao is the line of sight (*Auglini*). In its materialization as a physical device, point "o" is displaced from the eye and becomes a fastener on a wall behind the artist to which a silk cord is pulled taut and secured. The cord runs from the wall and into the device's base—as can be seen more clearly represented by points "d" and "f" in the printed image, where the length of cord is shown running through the base (at "d") and emerging from its far end (at "f"), and where line *fo* is the line of the cord, or the "Schnurlini."

In the 1538 Underweysung der Messung, the woodblock prints of the two final perspectival scenes are included on the same page, one above the other. The first is an image of a man drawing a vase (figure 17, bottom), which is shown in development in the Dresden *Skizzenbuch* at the Sächsische Landesbibliothek, while the second is an image of a nude perspectival subject on a table that does not appear in the sketchbook or in any other collection of Dürer's preparatory drawings.

In the first print, the artist holds the instrument in his left hand as if it were an extension of the silk cord and peers through a hole drilled into an eyepiece mounted onto the base (figure 17, top). The silk cord is attached to the wall and the excess string dangles from the end of the device in the figure's left hand. Dürer has drawn him from his left side, his chin tucked back against his neck and squinting through the eyepiece. His head is slightly rotated towards the viewer to reveal his right eye and he holds a stylus up against the glass pane, ostensibly tracing the vase sitting on the table before him. The device terminates in a pointer (*ein spitzig*), whose tip is an extension of the line of sight. In the drawing and print this is represented by point "a." Dürer is extremely careful to note that point "a," the tip of the device, lines up with the eyepiece, and that there is a discrepancy that needs to be accounted for between the line traced by the cord through the base of the device and the imaginary line of sight. This difference is demarcated as line bd—the difference between the cord and the position of the eye—as represented in the triangular diagram.

The drawings of perspective apparatuses in Dürer's Dresden Skizzenbuch (figure 18) depict further developed scenes of the same figure drawing the vase and a larger rendering of the device itself.⁶⁷ The drawings in Munich were in all likelihood the first attempts at visualizing these concepts, and before the production of the final print, further drawings like those in Dresden would have been required to bridge the gap from initial sketch to print.⁶⁸ In the Munich drawing, the artist's left hand loosely holds the perspective device as an extension of the silk cord. His eve is aligned with the viewing hole and he is gazing at the pointer, which resembles a cursorily drawn spike close to his right hand. In the print equivalent, the cord is attached to the wall at a height above the head of the figure, out of alignment with his eyes. The artist grasps the device upside down, his left palm facing up and his wrist hyperextended. The physical strain of this position is substantially different from the comfort with which the earlier drawn figure surveys his subject and there is no hint in Dürer's text, which is otherwise extremely detailed, of the device needing to be utilized in this manner.

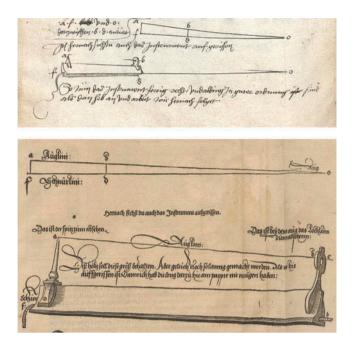


Figure 16. (top) Albrecht Dürer, drawing and description of a perspective device based upon an invention by Jacob Keser, 89a^v, L.impr.c.n.mss. 199,

However, one working drawing from the Dürer Skizzenbuch depicts a similar figure with two overlapping versions of the same perspectival device-the first attached at eye level to the wall and the second attached above his head (figure 19). Might this be an instance where Dürer sought to test out alternative configurations of the initial visualization in Munich? After all, if the silk cord were to be attached slightly above the level of the artist's head (exaggerated in the Dresden sketch), in such a way that the eyepiece would fall directly in line with the artist's eye and the device itself were to point at a large object (like a vase) centered behind the glass picture frame, the device would indeed need to be turned upside down, hence the awkward hand position in the print. Perhaps these drawings are also records of Dürer seeking to express tension in the body of the artist, changing his countenance from one of calm if concentrated contemplation in the initial sketch to a contorted posture intended to convey the difficulty, expertise, and self-control required to practice perspectival drawing.

While the basic elements have been preserved—an artist drawing a naked figure lying on a table, using a pointer to aid the translation of points on a standing frame onto an equivalent grid—the gap between the final new drawing in Book IV of the Munich copy and the equivalent print in the 1538 *Underweysung* is even more vast, and the intimacy of the other perspectival scenes, which all take place in bedrooms or private settings, is missing from the final print (figure 20).

Instead, the print relocates the initial drawing to an airy, Italianate room that is open to the elements. The gender of the male nude from the drawing has been switched in the print, transformed to a larger-than-life female with a classical profile, who has been swathed in cloth and propped up on two large pillows. The artist stares intently at the idealized beauty of his model in what has often been cited as an illustration of the "male gaze"; a gaze that "relegates women to the status of objects" and "gives the artist power over the subject."69 In contrast, the artist in Dürer's original drawing is more circumspect and reserved, even impassive. While the female nude rests somewhat unnaturally, her breasts turned towards the viewer, her right hand awkwardly tangled, or trapped, in the blanket, appearing also to be raised off the table surface in an effort to titillate the viewer by presenting her concealed genitals, to which she points with her left hand, the drawing in Munich represents a consciously realistic, if also sexualized, figure. The male model's left arm is propped against the table, allowing him to arch his chest and neck slightly as if in ecstasy. His legs are spread open, his left leg propped onto the table surface at an angle blocking his genitals from the viewer while revealing them to the artist, his right foot gently pushing against the picturing frame. The female nude is envisaged to be so large that she fills up the gridded frame, in turn requiring the artist to use a taller pointer situated closer to his eye. His line of sight is materialized as the horizon line, entombing the figure between it and the parallel line of the table surface. The pointer in the Munich drawing is located on the far side of the table grid, enabling the depicted artist to hold his head and body in a relaxed posture, his gaze gently descending onto the naked model, as signified by a drawn line running from his eye through the frame and past the body. The choreography of viewer/ artist, subject, and the various accoutrements of the perspectival scene construct a highly charged space, in which the proximities of the distributed elements symbolize a variable network of relations between the participants, vacillating between objective study of the naked male body in the drawing to the luridness of the print, where the proximity of the pointer conveys the sense that the artist is peeping illicitly through a keyhole.

Taken on its own, the print, which in its corporeal sensationalism bears no direct relation to the dry and technical description of the perspective apparatus that accompanies the image in *Underweysung*, evidently sought to amplify and heteronormativize the latent, if also objectified, eroticism of the Munich drawing.⁷⁰ But in concert with the drawing, the dual gender and spatial ambiguities of the scene become more pronounced. A mirror image of the drawing, the print retroactively locates the viewer outside of the room depicted in the drawing, recasting him/her as an observer looking through the window. Thus, the artist

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Figure 17. (top) Albrecht Dürer, initial drawing of a man using a perspective device to depict a vase, 89b, L.impr.c.n.mss. 199, BSB; and (bottom) final print for which the above initial sketch was a first draft, Book IV, 93, *Underweysung der Messung* (1538), VD16 D 2858, ETH.

and subject have exchanged both positions and genders, confounding the seemingly distinct roles taken by the print's occupants.

The gap between the Dürer drawing and the second print requires further research, both to verify the authenticity of the print as a bona fide end product developed directly from Dürer's own hand, or the extent to which the drawing may have been transformed in the hands of another artist or the print workshop of Hieronymous Andreae, and to consider how the gender switch troubles the standard reading of this iconic print. While the image depicting the perspectival device used to draw a vase does seem to relate to the developmental drawings in Dresden, the instability of the second scene may point towards someone besides Dürer, who may have sought both to accent and to signal his changes to the Dürer original by reversing the spatial orientation.⁷¹ What is very likely, however, is that while the text does not directly stipulate the type of

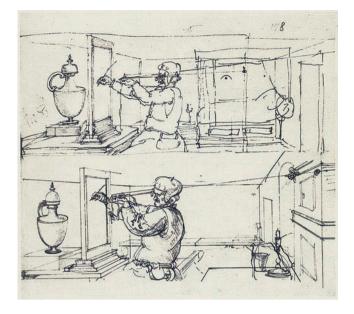


Figure 18. Albrecht Dürer, the two sketches of a man drawing a vase that most closely resemble the corresponding image in *Underweysung der Messung* (1538), Taf. 136 (178), Dresden Skizzenbuch, Signatur/Inventar-Nr.: Mscr.Dresd.R.147.f, Sächsische Landesbibliothek- Staats- und Universitätsbibliothek Dresden (SLUB)/S.B.6023, Handschriftensammlung.

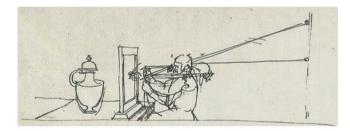


Figure 19. Albrecht Dürer, developmental sketch of a man drawing a vase showing two different attempts to visualize the perspectival setup, Taf. 137 (179), Mscr.Dresd.R.147.f, SLUB/S.B.6023.

images to include, the drawings in Munich remain the earliest existent manifestations of Dürer's attempts to visualize the expansion of his perspectival theory in *Underweysung*.

It is impossible to know conclusively why, if Dürer had seemingly intended to publish a second edition of *Underweysung*, he had not just simply delayed the printing of the first edition to enable all the material to be included. Perhaps the stress of finishing his two other treatises necessitated the rushing to print of a project that had stretched across decades, as substantiated by Dürer's personal notes and sketches on geometry. His



Figure 20. (top) Albrecht Dürer, preliminary drawing of man using perspective devise, 89b^v, L.impr.c.n.mss. 199, BSB; and (bottom) corresponding print with substantial embellishment, including a gender shift of the figure model, f. QIII, 92^v, *Underweysung der Messung* (1538), VD16 D 2858, ETH.

treatise on the subject of artillery fortifications, *Etliche underricht zu Befestigung der Stett, Schloss und Flecken* (Several instructions for fortifying towns, castles and small cities) was published in 1527 and his *Vier Bücher von menschlicher Proportion* shortly after his death in 1528, and both were well underway in 1525. In addition, though Dürer did not execute many large-scale commissions in the last few years of life, a steady stream of artistic work may have kept him from fully concentrating on *Underweysung*

just prior to its first printing. After all, in 1525 he was commissioned to make five portraits, now lost, of Albrecht of Brandenburg-Ansbach in honor of his new title of Duke of Prussia (1525–68).⁷² He was in the midst of a series of highprofile and elaborate engravings of Friedrich the Wise, Elector of Saxony (1524), Wilhelm Pirckheimer (1524), Philipp Melanchthon (1526), and Erasmus of Rotterdam (1526), and rounding out his collection of engravings of apostles with Philip (1526). Finally, in 1526 he finished a painting of the Madonna and Child (1526), a portrait of Hieronymous Holtzschuher, as well as his last masterpiece, *The Four Apostles*, a larger-than-lifesize two-panel painting held in the Alte Pinakothek, Munich.

Nevertheless, the Munich copy shows Dürer to be in top form. Supremely confident in both his mastery of mathematics and the feel and textures of the German language, he agitates for a manifestation of the treatise that would utterly and completely embody the intricacies of his thinking, even if his thinking had continued to evolve in response to seeing the text finally compiled all together in print. Underweysung der Messung never stopped being personal for Dürer, never became separate from him, though perhaps unlike Montaigne and his perpetually unfinished and reinvented Essais, Dürer seems to have been homing in on an exact vision of his ideas in print that was on the cusp of being tantalizingly realizable. In its near realizability, compromised not by common printing error but by an exactitude that only he, as the author and artist, could viscerally understand, Dürer remained unperturbed by the complications and potential expense of reprinting Underweysung again. The additions to the section on perspective brim over not as much with new ideas, as with new image-based ways to refine and further present the ideas already in the text. Through the traces of his edits and interventions, multiple overlapping portraits emerge of Dürer in the last years of his life, striking in their directness and clarity. The highly conscientious artist, adamant that the book's graphic images reflect the entire spectrum of his geometrical research; the uncompromising visionary, orchestrating yet another ambitious project of extreme technical complexity in collaboration with one of the sixteenth century's great printing workshops; the writer for whom the printed word was never final, until it was perfect.

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NOTES

1 - The extended title reads: Underweysung der messung/ mit dem zirckel un richtscheyt/ in linien ebnen unnd gantzen corporen (Manual of the measurement of lines, areas, and solids [whole bodies] with a compass and ruler). Measurement here may have been understood to be more akin to geometry, or geometria, given that the translation of Underweysung into Latin by Camerarius in 1538 changed the word messung in the title to geometria. It appears to be Dürer's invention to use messung for what would have been understood as geometria; Albrecht Dürer, The Painter's Manual, trans. Walter L. Strauss (New York: Abaris, 1977), 10; Hans Rupprich, Dürer: Schriftlicher Nachlass, 3 vols, vol. I (Berlin: Deutscher Verein für Kunstwissenschaft, 1956-69), 310. The sixteenth century witnessed the greatest explosion of interest in what would come to be known as "mixed-mathematics"-the cluster of disciplines explicitly concerned with the application of geometry to the definition, prediction, and representation of the physical world. Linked together by the commonality of geometrical knowledge and encompassing architecture, ballistics, goldsmithery, engineering, fortification design, instrument-making, optics, and surveying, mixed-mathematics emerged as the premier driver of upward professional and social mobility in the sciences, vastly increasing the number of working practitioners and multiplying the sites in which geometry was considered indispensable.

2 – "unnd auch nicht alleyn den malern/sonder Goldschmiden Bildhaweren Steynmeßen Schreyneren und allen den so sich das maß gebrauchen dienstlich seyn mag." All translations are by the author, unless otherwise stated.

3 – In his dedication to Pirckheimer in *Vier Bücher von menschlicher Proportion*, Dürer declares: "In order that these instructions be better understood, I have issued a book about measurements which describes lines, surfaces, solid bodies, etc. [*Underweysung*] Without this book my instructions may not be fully understood. It is therefore necessary for anyone who wishes to engage in this art to first be well acquainted with measurement. He should know how to draw the ground plan and elevation of an object in the matter employed by skillful stonemasons for daily use. Lacking this, he might not comprehend my teachings in every respect"; trans. by Walter L. Strauss, *The Complete Drawings of Albrecht Dürer*, 6 vols (New York: Abaris, 1974), VI: 2384.

4 - For instance, "the foot is one seventh of the entire height. Its height equals one third of its length. The ankle is at the midpoint of the height of the foot. The toes measure a third part of the length of the foot"; trans. Walter L. Strauss, The Human Figure: The Complete 'Dresden Sketchbook' (New York: Dover, 1972), 60. In these later proportional systems, Dürer used elements from Vitruvius which he nevertheless selectively modified with drawing instruments to suit his own aesthetic sense though as has been claimed, "the principle of achieving harmony through relationships and analogies and defining distances through fractions of body length ... can also be traced to Vitruvius"; Almut Pollmer-Schmidt, "From All Sides: Man and Measurement in Dürer's Work," in Albrecht Dürer-His Art in Context, ed. Jochen Sander (Munich: Prestel, 2014), 123. Dürer mentions Vitruvius in a 1523 letter to Pirckheimer, in the context of complaining about how Jacobus, ostensibly the painter Jacobo de' Barbari (c.1460/70before 1516), "showed me how to construct a man and a woman based on measurements. ... But Jacobus, I noticed, did not wish to give me a clear explanation." Thus, Dürer took matters into his own hands and "read Vitruvius, who has written a bit about human limb proportions." "Doch nam ich mein eygen ding für mych vnd las den Fitrufium, der beschreibt ein wenig van der glidmas eines mans"; Rupprich, Dürer: Schriftlicher Nachlass, I: 102.

5 – The copy has been newly scanned and can be found online in its entirety in the digital collections of the Bayerische Staatsbibliothek, http://daten.digitale-sammlungen.de/bsb00095496/image_1.

6 – Only a handful of publications mention the Munich copy and it is not referenced in any nineteenth- or twentieth-century histories until Matthias Mende's exhibition catalogue *Mit Zirkel und Richtscheit* (Nördlingen: Dürer-Haus Nürnberg, 1986). There is a brief mention in Kaltwasser's survey of the Bayerische Staatsbibliothek's holdings, but no substantial scholarship until Thomas Schauerte, "Dürer und Spranger: Ein Autographenfund im Spiegel der europäischen Sammlungsgeschichte," *Mitteilungen des Vereins für Geschichte der Stadt Nürnberg*, Bd. 9 (2006): 25–69. See also Franz Georg Kaltwasser, *Die Bibliothek als Museum. Von der Renaissance bis heute, dargestellt am Beispiel der Bayerischen Staatsbibliothek* (Beiträge zum Buch- und Bibliothekswesen, Bd. 38) (Wiesbaden: Harrassowitz Verlag, 1999). Thomas Eser and Anja Grebe's sampling of artwork at the turn of the sixteenth century, *Heilige und Hasen: Bücherschätze der Dürerzeit* (Nuremberg: Verlag des Germanischen Nationalmuseums, 2008), devotes a single page (p. 56) to the Munich edition.

7 – The handwriting is verifiable as belonging to Dürer. Additionally, the watermarks of the inserted pages match those of the paper that Dürer was known to have used in the 1520s; Eser and Grebe, *Heilige und Hasen*, 56.

8 - Shira Brisman, "The Image that Wants to be Read: An Invitation for Interpretation in a Drawing by Albrecht Dürer," *Word & Image* 29, no. 3 (2013): 273-303.

9- The Munich copy had a colorful and potentially prominent afterlife, as a note in the beginning pages of the book indicates, having belonged to the Flemish artist Bartholomäus Spranger (1546-1611), who had apparently purchased it in Prague c.1580 along with some of Dürer's woodblocks. A note written in Dutch by a contemporary hand in the opening pages of the book says "dit boeck is by ende Neffen de Houtplaten AD [Dürer's monogram] gecocht door Bartolmeus sprangers in praag ontrent den Jaer 1580." [This book was bought together with the woodblocks AD by Bartolmeus Spranger in Prague around the year 1580.] After Spranger's death, the book passed into the possession of his nephew Gommer Spranger, and then through the hands of various prominent families in Amsterdam. It remained in the Netherlands until 1812; Schauerte, "Dürer und Spranger," 36. It is still unclear how the book came to be in Italy twenty-two years later, where it was apparently purchased by King Ludwig of Bavaria "from a Russian lady for 4000 scudi romani" in 1839 and given to the Bayerische Staatsbibliothek. Bibliographic note from the opening pages of 4° L.impr.c.n.mss. 119. Thomas Schauerte has compiled a masterful provenance of the book; Schauerte, "Dürer und Spranger," 34-37.

10 - A useful resource is Strauss's republication of *Underweysung der Messung* as *The Painter's Manual*, which includes a compilation the new prints added to the 1538 edition.

11 – "Dis puch hab Ich recht Corigirt Vnd ob man das Von Newen wider trucken wolt so hab der Corector fleisig achtung das man die form alt vnd new recht setz des gleichen die schrift überall recht einteill." Note on title page, *Underweysung der Messung*, 4° L.impr.c.n.mss. 119.

12 – Hans Petz, "Urkundliche Nachrichten über den literarischen Nachlass Regiomontans und B. Walthers 1478–1522," in *Mitteilungen des Vereins für Geschichte der Stadt Nürnberg* (1888), VII: 237–262. The city of Buda, called Ofen in German, was the former capital of the Kingdom of Hungary and is situated on the west bank of current-day Budapest.

13 - Rupprich, Dürer: Schriftlicher Nachlass, II: 9.

14 – Petz, "Urkundliche Nachrichten", 239. Johann Schöner's introduction to *Joannis de Regiomonta de triangulis, etc* (Nuremberg, 1533) also confirms that Pirckheimer had purchased many manuscripts pertaining to ancient mathematics from the libraries of Regiomontaus and Walther; Dürer/Strauss, *Painter's Manual*, 14–15.

15 – Rupprich, Dürer: Schriftlicher Nachlass, II: 10.

 $16 - (4^r: 1523$ Januar) "ad 13 ditto verkauft wir dem Albrecht Dürer 10 pücher von des Berenharts Walthers püchern, so den malleren dienstlich sein vnd durch Wilbolt Pirchamer geschetz worden vnd zalt an münz fl. 10"; Berechnung des "Gemeinen Almosen." (Nr. 42) für die Zeit vom 2. IX.

1522 bis 14. VI. 1523 im Nürnberger Stadtarchiv; repr. in Rupprich, Dürer: Schriftlicher Nachlass, I: 221.

17 – Dürer's copy of *Elements*, which he bought in 1507, is in the Herzog August Bibliothek, Wolfenbüttel. The inscription in Dürer's hand reads: "Daz puch hab ich zw Venedich vm ein Dugatn kawft im 1507 jor. Albrecht Durer." [I have purchased the book in Venice for one dugat in the year 1507.].

18 – David Price, Albrecht Dürer's Renaissance: Humanism, Reformation, and the Art of Faith (Ann Arbor: University of Michigan Press, 2003), 10.

19 – "Litterarum quidem studia non attigerat, sed quae illis tamen traduntur, maxime naturalium et mathematicarum rerum scientiae, fere didicerat. Aeque is praecipua ut intelligebat et re explicare noverat, ita et oration sciebat declarare. Testantur hoc scripta eius geometrica, in quibus quid de illa scientia possit require, quantenus quidem tractandam sibi iudicavit, non video"; repr. in Rupprich. Dürer: Schriftlicher Nachlass, I: 307-08. For the whole of Camerarius's introduction to the first Latin translation of Underweysung, see Rupprich, Dürer: Schriftlicher Nachlass, I: 307-11. Dürer in his Familienchronik (compiled 1524) states that he excelled at his lessons and was consequently brought to school by his father in order to learn how to read and write, though his father also removed him after a few years in order to apprentice in his native goldsmith trade. "Und sonderlich hate mein vater an mir ein gefallen, da er sahe, daß ich fleisig in der übung zu lernen was. Darumb ließ mein vater in die schull gehen, und da ich schreiben und lessen gelernet, namb er mich wider auß der schull und lernet mich das goltschmid handtwerckh"; Dürers Familienchronik, reprinted in Rupprich, Dürer: Schriftlicher Nachlass, I: 30. This could mean that Dürer never learned or did not learn enough Latin to be able to comprehend Ptolemy's Almagest or Sacrobosco's De sphaera materiali on his own, both books that, should he have wanted to read them, were in the Regiomontanus-Walther collection. On the other hand, Dürer himself specified that young artists should learn to "read and write well and be taught Latin in order to really understand writings", which does imply that Dürer at the very least possessed these skills himself, or believed that young artists would benefit from acquiring skills he himself lacked. On the typical curricula in Nuremberg in the period of Dürer's childhood, see Klaus Leder, "Nürnbergs Schulwesen an der Wende vom Mittelalter zur Neuzeit," in Albrecht Dürers Umwelt (Nuremberg: Selbstverlag des Vereins für Geschichte der Stadt Nürnberg, 1971), 29-34. Other scholars disagree with Leder and claim that Dürer may well have been able to read Latin; Price, Albrecht Dürer's Renaissance, 10.

20 – Rupprich, *Dürer: Schriftlicher Nachlass*, III: 328. The contemporary historian Johann Neudörffer (1497–1563) records that Pirckheimer was the negotiator when the artist Hans Sebald Beham gave his son the fifteen books of Euclid translated into German by the mathematician Johannes Werner; Johann Neudörffer, *Nachrichten von Künstlern und Werkleuten* [1547] (Vienna: W. Braumüller, 1875), 48.

 21 – Price, Albrecht Dürer's Renaissance, 14. For a survey of Dürer's projects for Maximilian I, see Ernst Rebel, Albrecht Dürer, Maler und Humanist (Munich: C. Bertelsmann, 1996), 305–23.

22 – Leona Rostenberg, "The Libraries of Three Nuremberg Patricians, 1491–1568," *Library Quarterly* 13, no. 1 (1943), 21–23. Most of Pirckheimer's Italian books were presumably in Latin and Greek. The original letter was sent to Konrad Celtis in 1503; and is reprinted in Bernhard Hartmann, "Konrad Celtis in Nürnberg," in *Mitteilungen des Vereins für Geschichte der Stadt Nürnberg* (Nuremberg, 1889), VIII: 66–67.

23 – The Delic or Delian problem concerns how to double the volume of a cube given the dimension of one of its edges using only graphic tools such as compass and ruler. On the Delic Problem and Dürer's use of the Eutokius, see Dürer/Strauss, *Painter's Manual*, 24, nn. 50–54, 34. On Dürer's illuminations for Pirckheimer, see Erwin Rosenthal, "Dürer's Buchmalereien für Pirckheimers Bibliothek," *Jahrbuch der Preussischen*

Kunstsammlungen, vol. 49 (Berlin, 1928): 1–54; and Willehad Paul Eckert, Willibald Pirckheimer: Dürers Freund im Spiegel seines Lebens, seiner Werke, und seiner Umwelt (Cologne: Wienand, 1971), 84–88.

24 – Hans Rupprich, "Die kunsttheoretischen Schriften L. B. Albertis und ihre Nachwirkung bei Dürer," *Schweizer Beiträge zur Allgemeinen Geschichte*, vol. 18 (1960), 219–239. The 1522 inventory is reprinted in Petz, "Urkundliche Nachrichten," 247–62. A later 1563 inventory is published in Ernst Zinner, *Regiomontanus: His Life and Work*, trans. Ezra Brown (Amsterdam: North-Holland, 1990), 161–68.

25 - Ethan Matt Kavaler, Renaissance Gothic: Architecture and the Arts in Northern Europe, 1470-1540 (New Haven and London: Yale University Press, 2012), 43; Dürer/Strauss, Painter's Manual, 16. Dürer directly copied certain diagrams from these fifteenth-century books. For instance, Diagram 16, Book II in Underweysung der Messung, is a copy of Diagram 27 in Roriczer's Geometria deutsch; Lonnie Royce Shelby, Gothic Design Techniques-The Fifteenth-Century Design Booklets of Mathes Roriczer and Hanns Schmuttermayer (Carbondale: Southern Illinois University Press, 1977), 117. Strauss notes that Dürer adopted the construction of a right angle and the method of locating the center of a circle from Roriczer and the construction of a pentagon from Ptolemy; Dürer/Strauss, Painter's Manual, 16-17. For further information on Roriczer and Schmuttermayer, see Shelby, Gothic Design Techniques, 7-28; and Rupprich, Dürer: Schriftlicher Nachlass, III: 309-10. For a good summary and explanation of the additional mathematical texts imbedded in Underweysung, see Albrecht Dürer 1471-1971 (Ausstellung des Germanischen Nationalmuseums) (Munich: Prestel, 1971), 341-54.

26 – The Latin translations from Arabic editions of Euclid's *Elements* by Adelard of Bath and Campanus of Novarra were the main references for Euclid in the Middle Ages. The first edition of Euclid to be printed in Greek was published in 1533 on the basis of recovered manuscripts that dated back to Theon, a fourth-century Greek mathematician and astronomer; Kirsti Andersen and Henk J. M. Bos, "Pure Mathematics," in *The Cambridge History of Science*, ed. Lorraine Daston and Katharine Park (Cambridge: Cambridge University Press, 2006), III: 696–724.

27 – Rupprich, 1960, 219–39; Rupprich, *Dürer: Schriftlicher Nachlass*, I: 222. 28 – Rupprich, *Dürer: Schriftlicher Nachlass*, II: 374–77; Strauss, *Complete Drawings of Albrecht Dürer*, VI, 2817–23. Figure 3 is a reproduction of a drawing found in the Sloane Collection at the British Museum: Sloane 5228/213^r; and repr. in ibid., 2819. Dürer also translated portions of a text on perspectival theory copied from Piero della Francesca's *De Prospectiva Pingendi*. "Item prospectiua ist ein lateinisch wort, pedewt ein durchsehung"; Rupprich, *Dürer: Schriftlicher Nachlass*, II: 373. See Erwin Panofsky, *Dürers Kunsttheorie* (Berlin: Georg Reimer, 1915), 42–43; and papers at the British Museum, Sloane 5228/202.

29 – "Item als jr mir zw sagett, so jr weill möcht haben, wolt jr den Ewklide in tewczsch bringen, wolt jch geren wissen, ob jr etwas doran gemacht het." Dürer also briefly covers the religious climate in Nuremberg in the letter and confirms Pirckheimer's planned procurement of a measurement instrument for Kratzer; Rupprich, *Düre: Schriftlicher Nachlass*, I: 113.

30 – In addition to the Regiomontanus-Walther and Pirckheimer libraries, Dürer was surrounded by the thriving book trade in Nuremberg and might well have seen books on geometry and mixed-mathematics from Anton Koberger, the publisher responsible for the Nuremberg Chronicle and Dürer's godfather; Jeffrey Chipps Smith, "Albrecht Dürer as Collector," *Renaissance Quarterly* 64, no. 1 (2011): 17.

31 – For instance, William Martin Conway, *Literary Remains of Albrecht Dürer* (Cambridge: Cambridge University Press, 1889), 208–9.

32 – Dürer/Strauss, *Painter's Manual*, 13. Panofsky claims that on his trip to Italy, Dürer learned Piero della Francesca's method of using foreshortening to create perspectival figures; Erwin Panofsky, *The Life and Art of Albrecht Dürer*, 4th ed. (Princeton: Princeton University Press, 1955), 251.

33 – Albrecht Dürer, "Introduction" to Underweysung der Messung (Nuremberg, 1525).

34 – Euclid provides three primitive constructions: a [unique] straight line drawn from any point to any other point; a straight line segment that can be

continuously extended by a finite amount to produce another straight line segment; a [unique] circle centered on any point with any radius. All the geometrical propositions contained in the thirteen books of the *Elements* are derived using only the ability to construct line and circles; Peter R. Cromwell, *Polyhedra* (Cambridge: Cambridge University Press, 1997), 61.

35 – Dürer's list is situated quite early in the history of publishing errata in printed books. See Ann Blair, "Errata Lists and the Reader as Corrector," in *Agents of Change: Print Culture Studies after Elizabeth L. Eisenstein*, (Amherst and Boston: University of Massachusetts Press, 2007), 21–41.

36 – "Wie wol ich muglichen fleyß hab angekert/ das dise buchlein recht und wol corregirt auß geen mochten/ noch dann haben sich durch die auß ziehung und wider eynsetzung der pustaben/ auch die eyl etlich yrrtum in worten und dem verstand zugetragen/ der notigsten auß den selben will ich eyns teyls hiemit anzeygen/ dz ander wirt ein yetlicher verstendiger selb wol wissen zu corregiren"; Dürer, *Underweysung der Messung*, appx.

37 - Ibid.

38 – On the history of early modern editing and the role of the corrector in the printing house, see Anthony Grafton, "Correctores corruptores? Notes on the Social History of Editing," in *Editing Texts = Texte Edieren*, ed. Glen Most: *Aporemata*, 2 (Göttingen: Vandenhoeck & Ruprecht, 1998) 54–76; Johan Gerritsen, "Printing at Froben's: An Eye-Witness Account," *Studies in Bibliography* 44 (1991): 144–63.

39 – "Das aber solche maler wolgefallen in iren yrthumben gehabt/ ist alleyn ursach gewest/ das sie die kunst der messung nit gelernt haben/ an die keyn rechter werckman werde oder seyn kan." The anxiety about the devolution or alteration of meaning occurring through the copying and translation of texts, both classical and contemporary, is a theme that runs throughout the history of Western scholarship, as Anthony Grafton, Ann Blair, and others have shown. For Dürer, printing error or editing oversight does not so much alter the meaning of an originary text as much as it acts as a proxy that renders Dürer himself, author and artist, incapable of delivering on his fundamental promise to integrate precision into the conceptualizing and production of German art.

40 – Dürer is already broaching Kant's basic concept of the third critique of judgment, namely disinterested pleasure (*interessenloses Wohlgefallen*); Immanuel Kant, *Critique of the Power of Judgment*, ed. Paul Guyer, trans. Paul Guyer and Eric Matthews (Cambridge: Cambridge University Press, 2000), 90.

41 – Panofsky, Life and Art, 245. For Panofsky on the Underweysung der Messung, see ibid., 254–60.

42 – The entire type needed to be reset for the second edition in order to accommodate the sum total of Dürer's corrections. While some of the spelling corrections were specified by Dürer, many others reflected the taste for a Franconian spelling; Eser and Grebe, *Heilige und Hasen*, 56. Occasionally, the later printers of *Underweysung* corrected Dürer's spelling in his edits and additions to reflect the Franconian spelling.

43 – Dürer, *Underweysung der Messung*, Book IV, Diagram 51a. The diagram illustrating four cannonballs increasing in weight has been printed twice in the 1538 edition, with one print rotated 180 degrees. There is no indication that this duplication was specified in the Munich copy.

44 – Adjacent to Diagram 7, Dürer drew two new triangles with the note: "Die formen soll mann noch reissen dazu gihort ein andr daher" (One should still draw the [indicated] forms. Here belongs another below). These triangles were reprinted as part of Diagram 7 in the second edition, though the arrangement of the surfaces was reordered to make room for them, and two new triangles were added that do not appear in Dürer's copy. On the bottom of Diagram 8 in the Munich copy, Dürer has drawn three new diagrams and expanded the textual exposition. These three additional diagrams have been inserted into the 1538 edition, replacing Diagram "h" from the 1525 edition, and are accompanied by an explanatory text —which was originally inserted in the Munich copy. In the margins Dürer has also written: "Die figure sollen dartzu gemacht worden" (The figure should have been made with these [additions]).

45 – Faintly visible on the page are the words "den 2 formen manglen" (The two forms are missing). Given that there are three pattern diagrams,

the word "formen" may refer to the two pieces of paper upon which the patterns are drawn. In the print versions of the sketches, the middle drawing has been mirrored and the number of zigzag lines increased from the original three to four.

46 – "Fürbaβ wil ich ein fünf/sechs/siben/ un ein acht eck/ytlichs sunderlich zusammen setze/ doch eines anders dan/ das ander"; Dürer, *Underweysung der Messung*, Book II, Diagram 24.

47 - As an example, one sentence reads: "Ich reis eyn aufrechte lini.a.b. daran setz ich ein adre lini der end sey.c. un die lini.a.b. la β ich im a stet bleiben" (I draw a vertical line ab and I add another line to it ending in c. I leave the line ab in its position). With all the first-person pronouns removed from this sentence, as they are in Dürer's copy, the sentence would read "Draw a vertical line ab and add another line to it ending in c. Leave the line ab in its position"; Dürer, *Underweysung der Messung*, Book II, Diagram 40.

48 – There are provocative similarities between the styles of Alberti's and Dürer's treatises that deserve future contemplation. Both set out to transform painting as it was practiced by presenting detailed, coherent systems of art, though Alberti, critically and consciously, deployed the terminology of classical rhetoric in order to define and also valorize the creativity he witnessed taking place in artists' ateliers; Anthony Grafton, *Leon Battista Alberti: Master Builder of the Italian Renaissance* (Cambridge, MA: Harvard University Press, 2002), 119.

49 - Dürer is referring here to his Vier Bücher von Menschlicher Proportion.

50 – "Und damit gunstliger lieber Herz will ich meinem schreyben end geben/ und so mir Got genad verleycht die bucher so ich von menschlicher proportion un anderen darzu gehorend geschryben hab mit der zeyt in druck pringen/ und darpey meniglich gewarnet haben/ ob sich yemand understeen wurd mir diß außgangen buchlein wider nach zu drucken/ das ich das selb auch wider drucken will/ un außlassen geen mit meren und grosserem zusatz dann ietz beschehen ist/ darnach mag sich ein yetlicher richte ..."; Dürer, *Underweysung der Messung*, postscript.

51 – Agnes Dürer, Willibald Pirckheimer, and Hieronymous Andreae are the three parties mentioned in the subscript to the 1538 edition. Agnes was the holder of the imperial copyright of her late husband's published treatises, which had been secured directly from the emperor. As an imperial city owing its allegiance to the emperor, the council of Nuremberg is known to have attempted to enforce the copyright on her behalf, most notably in a case involving the artist Sebald Beham (1500–50); Alison Stewart, "The Artist's Lament in 1528. Exile, Printing, and the Reformation," in *Die Klage des Künstlers: Krise und Umbruch von der Reformation bis um 1800*, ed. Birgit Ulrike Münch, Andreas Tacke, Markwart Herzog, and Sylvia Heudecker (Petersburg: Michael Imhof, 2015), 70–81.

52 - The 1525 edition of the Underweysung der Messung did not include an imperial privilege explicitly protecting the rights of the author and printer. Instead, beneath the errata list on the last page is a line "Keyserliche freyheit wirt in dem nechsten büchlein der Proporcion so ich zu drucken for hab eyngeleybet wirt"-alluding to the privilege covering the Underweysung being located in Dürer's forthcoming book on proportion, Vier Bücher der menschlicher Proportion (1528), which indeed it was. This sentence, alongside a similar one on the title page, has been crossed out in the Munich copy, possibly to insure that they would not be carried over to the second edition. It would appear that the existing privilege covering the Underweysung was deemed to extend to the 1538 edition as well, and that the changes that had been made were not considered to have constituted a substantially altered edition requiring, or deserving, of a new privilege. On the revisions required of new privileges, see George Hoffmann, "The Montaigne Monopoly: Revising the Essais under the French Privilege System," PMLA, 108, no. 2 (1993): 308-19.

53 – "Underweysung der Messung/ mit dem Zirckel und richtscheyt/ in Linien Eben un ganzen Corporen/ durch Albrecht Dürer zusammen gezogen/ un durch in selbs als er noch auff erden war an vil orten gebessert/ in sonderheyt mit xxii figure gemert/ die selbigen auch mit eygner handt auffgerissen/ wie es dann eyn yder werckman erkenen wirdt/ Nun aber zu nutz allen kunst liebhabenden in truck geben. 1538."

54 – "Auch sind noch vill hubscher corpora zumachen/ die auch in einer holen kugel mit all jren ecken an ruren/aber sie haben ungleyche felder/ der selben wil ich eins teyls hernach auf reyssen/ und gantz aufgethan/ auf das sie ein yetlicher selbs zamen mug legen/ welicher sie aber machen will der reyß sie grosser auf ein zwifach gepabt papier/ un schneyd mit einem scharpfen messer auf der einen seyten all ryß durch den einen pogen papiers/ und so dan all ding auß dem ubrige papier geledigt wirt/ als dan lege man das corpus zusamen/ so lest es sich geren in den risen piegen/ darumb nym des nachfolgeten auf reyssens acht/ dan soliche ding sind zu vill sachen nutz"; Dürer, *Underweysung der Messung*, Book IV, Diagram 34a. In the 1538 edition, this explanatory text has been moved to the following page above Book IV, Diagram 35 (a truncated tetrahedron): "Das Erst corpus/ das nit ganz mit seinen planen gleych an einander ist" (The first body that has surfaces that are not completely identical to each other).

55 – The definition beneath Diagram 29 relates to the next solid in Dürer's geometrical sequence (Diagram 30) and continues onto the following page. Beneath Diagram 30 is the textual description of the unfolded solid on the following page (Diagram 31: a solid with "twenty equal triangular sides").

56 - These additional solids are the truncated cube, the cuboctahedron, the truncated octahedron, the rhombicuboctahedron, the cubus sinus, and the truncated cuboctahedron. There is also a non-Archimedean polyhedron (the snub cube) composed of six dodecagons and thirty-two triangles, though like the Archimedean solids represented by Dürer it was probably derived by trimming one of the Platonic solids-in this case the cube-and unfolding the resulting geometry. Dürer/Strauss, Painter's Manual, 345. The 1538 edition is the first presentation in the Renaissance of the truncated cuboctahedron and the snub cube. See http://www.georgehart.com/virtual-polyhedra/durer.html/. In the 1525 edition, the truncated tetrahedron is framed by its own definition above and the definition of the truncated cube below. Diagram 36 of the truncated cube is above the definition of the cuboctahedron; Diagram 37 of the cuboctahedron is above the definition of the truncated octahedron; Diagram 38 of the truncated octahedron is above the definition of the rhombicuboctahedron; Diagram 39 of the rhombicuboctahedron is alone on its own page. Diagram 40-the cubus sinusis flanked by its own definition above and the definition of the truncated cuboctahedron below; Diagram 41-the truncated cuboctahedron -is on its own page. The final unfolded polyhedron, Diagram 42, is below its own definition. The six-sided prism (erroneously labeled Diagram 34 in the 1525 edition and corrected to Diagram 43 in the 1538 edition) hovers above the next section of the treatise, which then shifts to investigating the Delic Problem.

57 – The drawing shows that Dürer changed his mind about the opening lines of the top paragraph. He has crossed out the words "Noch sind zwey corpora zu machen das ist" and replaced them with "Ein anders das mach." Also, unlike in the other descriptions of nets, Dürer is careful to emphasize that when folded up the edges of these new solids will all touch the surface of a hollow sphere: "Dis Corpus rüret in einer holen kugeln mit allen seinen acken an."

58 - Diagram 43c is labeled Bx in Dürer's drawing.

59 – Dürer/Strauss, *Painter's Manual*, 465. Strauss also notes that the drawing of the circumference of a circle has been copied from the anonymous fifteenthcentury *Geometria Deutsch*, which contains a nearly identical diagram. The text describing the British Museum version of the drawing has been crossed out and the word "mechonice" added after "zircellini." London, British Museum, Sloane 5229/123r. In the Munich drawing, "mechonice" has been added seamlessly into the last sentence of the definition, "so ist a.c. so lanck also die zircellini mechonice/ wie das hernach ist aufgerissen," emphasizing that the line ac to which Dürer is referring has been drawn by mechanical means. 60 – For Dürer's literary remains concerning *Underweysung*, with the exception of the Munich copy which is not mentioned, see Rupprich, *Dürer: Schriftlicher Nachlass*, III: 307–67. On the phenomenon of "second-order notes"—notes that have recopied and compiled by the author—see Ann Blair, "Note Taking as an Art of Transmission," *Critical Inquiry* 31, no. 1 (2004): 85–107.

61 – The beginning of the description reads: "Noch sind zwey schöne corpora zu machen, das erst aus zwentzig sex ecketter flachen feldern, gleich seitig vnd wincklich"; repr. in Rupprich, *Dürer: Schriftlicher Nachlass*, III: 351. In the Munich copy, the text has been slightly altered: "Ein anders, das mach auß zweintzig sechsecketer flachen feldern, gleichseitig vnd wincklich" This second sentence was directly translated into print in the 1538 edition.

62 - In the 1538 edition, Diagram 36 has been reprinted on the same page as its description; Diagram 37 has been reprinted with two different descriptions; Diagram 39 has been reprinted with the description of the cubus simus below it; Diagram 40—the cubus simus, which in the 1525 edition is flanked by two definitions—only displays the definition of the following truncated cubocatahedron.

63 – No commentary on *Underweysung* has drawn attention to how the images and descriptions of the polyhedral nets are or are not aligned.

64 – The full title is "Ein eigentliche unf grundtliche anweysung/ in die Geometria/ sonderlich aber/ wie alle Regulierte/ und Unregulierte Corpora/ in der grundt gelegt/ und in das Perspecktiff gebracht/ auch mit iren Linien auffzogen sollen warden."

65 – The perspectival solids 35 and 36 are referred to as "auff der fierung stenent" and "auff der den Driangel stenent." In other words, each describes a view centered on either the square or the triangle of the rhombicuboctahedron.

66 – The instructions for the construction of the perspective device are contained in Dürer/Strauss, *Painter's Manual*, 430–31.

67 - R-147, ff. 178^r, 179^r. Dresden Sächsische Landesbibliothek.

68 – As for how the *Skizzenbuch* ended up in Dresden, the provenance can only be definitively traced back to the book collection of the Sächsischer Premierminister, Graf Heinrich von Brühl (1700–63), though there are hints that the book may have previously been in the possession of Rudolf II in Prague; Thomas Haffner, "Die Dresdener Dürerhandschrift: ein bedeutendes Dokument der Kunst-, Wissenschafts- und Sammlungsgeschichte," *Wissenschaftliche Zeitschrift der Technischen Universität Dresden* 55, nos 1–2 (2006): 156–58.

69 – Allen Dunn, "The Pleasures of the Text: VOLATILE VISUALITY," Soundings: An Interdisciplinary Journal 85, no. 3/4 (2002): 221–223; Christy Anderson, "The Secrets of Vision in Renaissance England," Studies in the History of Art 59 [Symposium Papers XXXVI: The Treatise on Perspective: Published and Unpublished] (2003): 322–347.

70 – A representative example of the text describing the perspectival setup, written above the image in Dürer's notes and carried over into the 1538 edition, reads as follows, and makes no mention of the nude subject: "Darzu soll ma haben ein ram mit einem gitter von starckem schwartzen zwirn gemacht/ die lucken oder fierungen eine ungeferlich zweyer finger breyt/ Darnach soll man haben ein absehen obsen zugespitzt/ also gemacht/ das man es höher oder niderer richten mag/ das bedeut das aug mit dem o."

71 – Schauerte suspects that the later Dresden drawings may have been augmented and developed for print by Hieronymous Andreae on the basis of Dürer's initial drawing in Munich; Schauerte, "Dürer und Spranger," 39.

72 – Jeffrey Chipps Smith, "Dürer in the Service of Princes and the Emperor," in *Albrecht Dürer—His Art in Context*, ed. Jochen Sander (Munich: Prestel, 2014), 308–313.