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Compensation of Permanent Magnet Motors Torque Ripple by Means of Current Supply Waveshapes Control Determined by Finite Element Method

S.Clenet, Y.Lefèvre, N.Sadowski, S.Astier and M.Lajoie-Mazenc
LEEI/CNRS/ENSEEIHT 2, Rue Camichel 31071 TOULOUSE FRANCE

***Abstract*---A method to determine the supply current waveshapes for torque ripple compensation in permanent magnet motors without saliency and damping bars, by means of a finite element code, is presented. First, a theoretical analysis shows that current waveshapes producing constant torque can be determined. This analysis leads us to an expression for the current as a function of the motor emf and cogging torque. Several methods, based on finite element computation of the magnetic field, for computing the cogging torque and emf are proposed and compared. A special experimental bench for motor torque measurement is used to validate the theoretical results.**

I INTRODUCTION

Finite element codes for the computation of the electromagnetic fields are employed more and more by electrical motor designers. These codes enable designers to evaluate the performance and the operating qualities of motors during design. Several studies based on finite element code have shown that it is possible to minimize torque ripple by adapting the constructive parameters of the machine [1]. In this paper we show that finite element code can as well be used in order to define the input waveshapes of electrical motors.

Indeed, several works have shown that it is theoretically possible to reduce torque ripple by controlling the current input motors [2] [3]. However, these works only concern the elimination of electromagnetic torque ripple and disregard the cogging torque ripple. The taking the cogging torque into account leads to a more complex calculation since it cannot be determined by an analytical method; it requires the use of models based on a fine representation of the electromagnetic structure.

Our purpose is to describe a method to compensate for all torque ripple by feeding the motor with well fitted current waveshapes.

II THEORETICAL ANALYSIS OF THE TORQUE PRODUCED

The torque of an electric machine is given by the derivative of the magnetic coenergy with respect to the mechanical position of the rotor, keeping the currents constant

$$T = \left(\frac{dW^*}{d\theta}\right)_{i=Const} \tag{1}$$

If the laminations are not saturated (constant permeability) which is generally the case for permanent magnets machines without polar pieces, the energy and co-energy are equal:

$$W^* = W = W_a + \frac{1}{2}\left(\sum_{\substack{k=1\\j=1}}^{n} \left(L_{ij}\, i_k\, i_j\right)\right) + \sum_{j=1}^{n} \left(\Phi_{aj}\, i_j\right) \tag{2}$$

where n is the number of phases of the machine; Wa is the energy of the magnet in the iron circuit; L_{jk} is the mutual inductance between the phase j and the phase k, (self inductance of the phase j when j=k); Φ_{aj} is the magnet flux through the winding of the phase j and i_j is the current in the phase j.

Whence,

$$T = \left(\frac{dW}{d\theta}\right)_{i=Const} = \sum_{j=1}^{n} i_j \frac{d\Phi_{aj}}{d\theta} + \frac{1}{2}\sum_{\substack{j=1\\k=1}}^{n} \frac{L_{jk}}{d\theta}\, i_k\, i_j + \frac{dW_a}{d\theta} \tag{3}$$

The motor studied has no polar pieces and the magnet and the air permeabilities are practically equal, so the self and the mutual inductances both satisfy for every j,k:

$$\frac{dL_{jk}}{d\theta} = 0 \tag{4}$$

The emf e_j of the phase j is defined as the derivative with respect to time of the no-load flux passing through the phases j, so the emf e_j is given by:

$$e_j = \frac{d\Phi_{aj}}{dt} = \frac{d\Phi_{aj}}{d\theta}\,\Omega \tag{5}$$

where Ω represents the mechanical rotational speed of the motor.

Consequently, for a permanent magnets synchronous machine without saliency, the torque is given by:

$$T = T_{em} + T_d \tag{6}$$

where

$$T_d = \frac{dW_a}{d\theta} \tag{7}$$

is the cogging torque,

$$T_{em} = \sum_{j=1}^{n} \frac{i_j\, e_j}{\Omega} \tag{8}$$

is the electromagnetic torque and i_j the current in the phase j.

The origin of the torque ripple arises from the electromagnetic torque ripple and the cogging torque. The electromagnetic torque ripple are caused by the mismatch between the emf and the feeding currents; the cogging torque

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is the consequence of interaction between the magnets and the magnetic structure of the stator. The relation (6) shows that only the electromagnetic torque depends on the currents. Therefore, the only way to reduce torque ripple is that the electromagnetic torque ripple compensate the cogging torque.

III THE TORQUE CONTROL STRATEGY

In the case of variable speed control, synchronous motors are generally self-piloted and supplied by a current controlled voltage inverter. The self-piloting loop generates, from the position of the rotor versus stator, the current references, thus insuring the synchronism between the rotor and the stator magnetic fields. The voltage inverter control then imposes the motor currents equal to their references, acting on the supply voltage waveshapes. Generally, the current reference waveshapes are sinusoidal. The tuning variables are the current amplitude A and the phase difference ψ between the current reference and the emf. This phase difference ψ can be chosen to have a maximum average torque per ampere and the amplitude A, which does not depend on the position, imposes the average torque. But, instantaneous torque can have important ripple which are baneful for some applications like robotics and machine tools.

Our purpose is to show how torque ripple can be compensated by feeding the motor with well-fitted current waveshapes. Suppose that it will be achieved by acting on the instantaneous amplitude $A(\theta)$ of sinusoidal current references, considered as base functions of the polyphased feeding system. For a synchronous motor with n phases and 2p poles, figure 1 shows the principle of current references generation. The amplitude $A(\theta)$ sought depends on the motor characteristics.

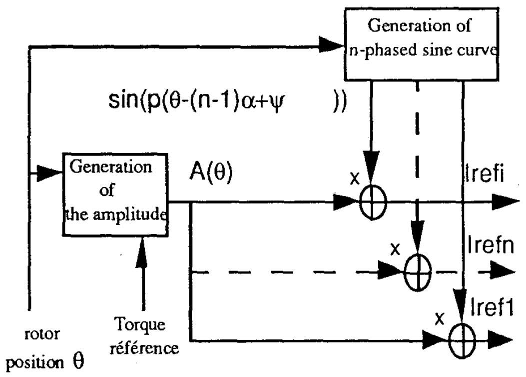

Figure 1: Principle of control and generation of current references; $\alpha=2\pi/p/n$, θ is the rotor position and ψ the phase difference angle between the currents and the emf.

IV COMPENSATING CURRENTS

With the chosen torque control, for a synchronous motor with n phases and 2p poles, the current references are given by the following relations:

$$\left.\begin{array}{l} Iref_1 = A(\theta)\sin[p(\theta+\psi)] \\ Iref_2 = A(\theta)\sin[p(\theta+\psi+\alpha)] \\ \quad\vdots \\ Iref_n = A(\theta)\sin[p(\theta+\psi+[n-1]\alpha)] \end{array}\right\} \quad (9)$$

where θ represents the position of the rotor with respect to stator, $\alpha=2\pi/p/n$ and ψ is the phase difference between currents and emf.

In the following study, the phase angle difference ψ will be fixed equal to zero, which corresponds to a maximun torque per ampere working for a smooth poles machine when the currents are sinusoidal. Considering the torque expression (6) and assuming that the actual currents perfectly follow their references, the electromagnetic torque becomes:

$$T_{em} = A(\theta)\sum_{i=1}^{n}\frac{e_i(p\theta)\sin[p(\theta-(i-1)\alpha)]}{\Omega} \quad (10)$$

which gives for the total torque:

$$T = A(\theta)\sum_{i=1}^{n}\frac{e_i(p\theta)\sin[p(\theta-(i-1)\alpha)]}{\Omega} + T_d(\theta) \quad (11)$$

With the chosen control method, the torque is constant and equal to T_{ref} if:

$$A(\theta) = \frac{(T_{ref} - T_d(\theta))\Omega}{\sum_{i=1}^{n} e_i(p\theta)\sin[p(\theta-[i-1]\alpha)]} \quad (12)$$

$$I_j = \frac{(T_{ref} - T_d(\theta))\Omega}{\sum_{i=1}^{n} e_i(p\theta)\sin[p(\theta-[i-1]\alpha)]}\sin[p(\theta-[j-1]\alpha)]$$

These results show that it is possible to control the instantaneous torque by acting on the amplitude of the current provided that the cogging torque and the emf are known.

V. APPLICATION OF FINITE ELEMENTS METHOD

A field calculation code called EFCAD [4] based on the finite element method has been used to compute the cogging torque and the emf of the motor. The rotor motion is taken into account by means of the moving band technique with special quadrilateral elements. Different torque computation methods have been systematically studied and validated by comparison with experiments. The Maxwell stress tensor has been chosen to compute the torque produced because it is now the most used method, easy to implement, well fitted to the moving band technique and gives results in good agreement with experiments [4].

A permanent magnet synchronous machine with 3 phases, 6 poles and 36 slots is studied as an example. Figure 2 shows the mesh used and the elements of the moving band during the rotation. Several methods for computing cogging torque and emf have been used.

V-1-. Computation of the cogging torque

The cogging torque has been calculated by means of EFCAD, simulating the unexited rotating motor at a constant speed. This simulation has been worked out with a rotation step equal to the discretization step of the moving band. The result is presented in figure 3.

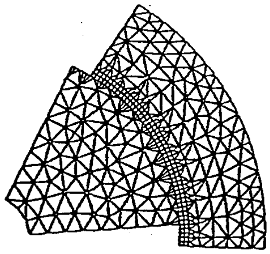

Figure 2 : Mesh of the motor with the moving band.

V-2-. Computation of the emf

Two methods which enable us to compute the emf are presented hereunder.

The first is by the computation of the emf by deriving the no-load flux. Owing to the definition, the emf of one phase is the derivative of the off-circuit flux in this phase with respect to time:

$$e_i = \frac{d\Phi_i}{dt} \qquad (13)$$

where e_i is the emf in the phase i and Φ_{ai} is the flux in the phase i.

The simulation of the unfed working of the motor gives simultaneously the cogging torque and the flux Φi in each phase i as a fonction of the position (Figure 3). To compute the derivative, the simple Euler formula has been first used:

$$e_i\,(t) = \frac{\Phi_i(n)-\Phi_i(n-1t)}{T} \qquad (14)$$

where T represents the sampling period, so thus:

$$T = \Omega\,\Delta\theta \qquad (15)$$

$\Delta\theta$ is the rotation step used and Ω is the chosen rotation speed. This formula introduces some errors and particularly a phase lag with respect to actual emf approximatively equal to half the rotation step. This small phase lag is enough to introduce some errors in the computation of the compensation currents by means of (12) which generates torque ripple as shown in figure 4. This latter result disagrees with the theoretical analysis. Furthermore, we found out that a denser mesh does not bring improvement.

So, to improve the calculation of emf from the flux, another formula has been used. For a given position, the emf per phase can be calculated, using the two former and the two latter positions, thus:

$$e_i\,(n) = \frac{\Phi_i(n-2)-8\Phi_i(n-1)+8\Phi_i(n+1)-\Phi_i\,(n+2)}{12\,T} \qquad (16)$$

Consequently, only the emf corresponding to the two steps behind can be computed, at each step of calculation. So, to get the emf at n points, n+4 points must be computed.

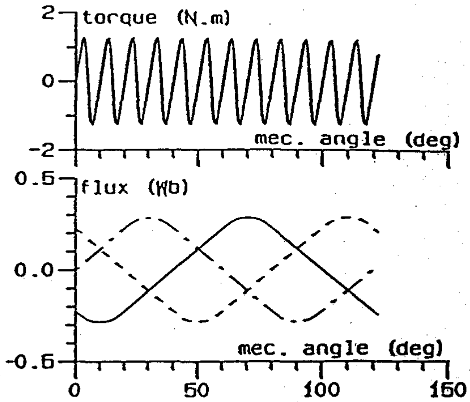

Figure 3: Simulated cogging torque and no-load flux for the three phases.

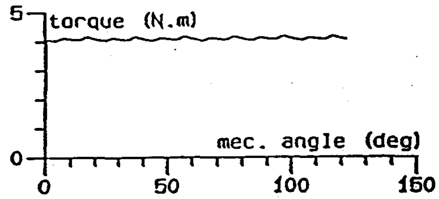

Figure 4: Simulated torque when the motor is supplied by currents computed from the emf calculated by the Euler formula.

From (16), the emf has been computed with EFCAD (Figure 5), then the compensating currents and, at last, the torque produced with these currents as shown in the figure 6. The torque shape reveals no apparent ripple, in agreement with the theory. Such a process reduces the errors but does not enable to compute the emf at each rotation step since the flux in the two following steps must be known.

We describe now the second method of computation of the emf from the static torque. The formula (6) shows that the emf can as well be computed from the torque produced by well known currents. Thus, if only the phase i is supplied by a constant current I, then the static torque ($Tstat_i$) satisfies:

$$Tsat_i = Td + \frac{e_i\,I}{\Omega} \qquad (17)$$

which then gives for the emf,

$$e_i = \frac{Tsat_i - Td}{I}\,\Omega \qquad (18)$$

Thus, computing the static and cogging torques, it is possible to infer the emf of the three phases. At each rotation step to calculate simultaneously the three electromotive forces, the equations of the fields corresponding to the succesive supply of the three phases are solved simultaneously. This is possible since the motor is not saturated and the field equations are linear.

The simulated static torque corresponding to the supply of one phase by one Ampere and the inferred emf by (18) are

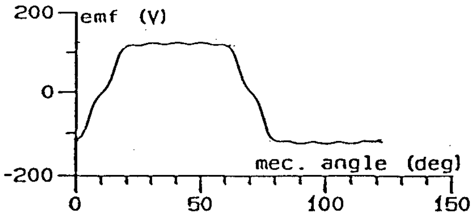

Figure 5 : Calculated emf by the derivative of the no-load flux

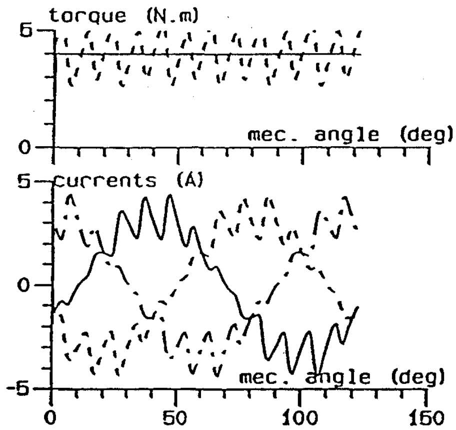

Figure 6 : Upper figure : compensated torque (continuous curve) and *uncompensated torque wit sinusoidal current* (dashed curve). Lower figure : the three compensation currents.

presented in the figure 7. Few differences can be observed with the emf calculated by derivation. The currents computed by means of (12) are similar to those presented in the figure 5.

Meanwhile, if the compensated torque computed by means of the two methods of emf calculation are compared in details as in figure 8, it can be observed that the torque produced by the currents computed from the static torque has fewer ripple than the torque produced by the currents computed from the derivative of the no-load flux.

This second method of emf calculation seems better than the first one and furthermore an emf computation can be done at each rotation step.

VI EXPERIMENTAL VERIFICATION

We have at one's disposal a special torque measurement bench on which a torque control loop has been realised to control the instantaneous amplitude A(θ) as a way of producing a constant torque. The torque and the current according to the position can be recorded. The compensated torque is presented in figure 9 where it can be compared with the torque measured in the case of a sinusoidal supply. The ripples have been divided by 20. The currents are presented in figure 10 and they can be compared with the simulated currents (figure 5). We find out few differences which indirectly validates the proposed method of torque ripple compensation.

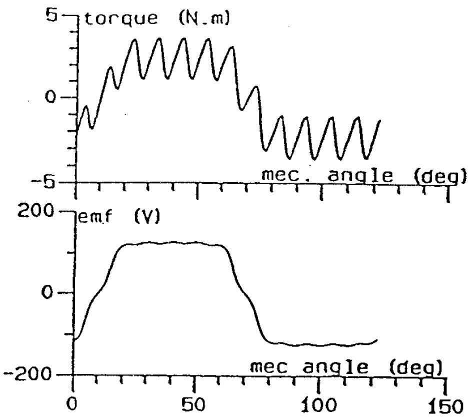

Figure 7: upper figure: simulated static torque.
lower figure: calculated emf from the simulated static torque

CONCLUSION

In this paper, we have shown that it is theoretically possible, knowing the cogging torque and the emf of a permanent magnets synchronous machine without saliency and damping bars, to compute the currents which enable us to obtain a constant torque.

To compute the cogging torque and the emf, a field calculation software using finite elements has been used. The movement is taken into account by means of the moving band technique associated with the Maxwell stress tensor method to compute the torque.

Two methods of calculation of compensation currents have been tested. The first one is based on the emf calculation from the derivative of the flux. It gives good results provided a well-fitted method of derivation is used but the emf cannot be computed at each rotation step. The second method is based on the emf calculation from the static torque. This latter method gives better results and furthermore enables the emf computation at each step of calculation and rotation.

The proposed method of torque compensation has been validated using a special experimental bench.

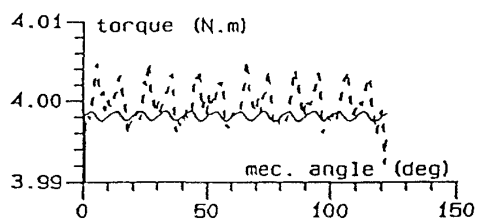

Figure. 8 : Simulated torque produced by currents computed by means of the derivative of the no-load flux (dashed curve) and by means of the static torque.

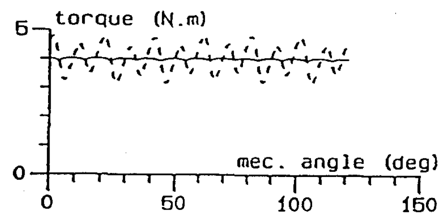

Figure 9 : Compensated torque (continuous curve) compared to torque obtained with sinusoidal current (dashed curve) measured on a torque bench.

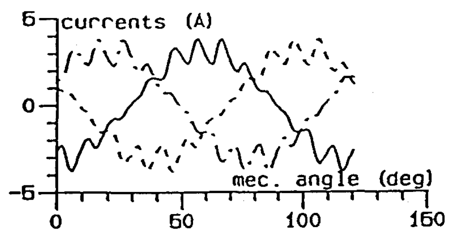

Figure 10 : Three phase measured currents and compensating torque ripple

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Stephane Clenet was born in Nantes, France, in 1967. He received the Engineer Diploma from the Enseeiht, Toulouse, France, in 1990. He is currently at the Leei Toulouse as a doctoral student.

Yvan Lefevre was born in Madagascar in 1959. He received the Engineering degree from the Enseeiht, Toulouse, France in 1983 and the Doctor degree from the INPT of Toulouse in 1988. He presently works in this laboratory as Chargé de Recherche at the CNRS. More particulary he works with modeling of Electrical Machines.

Nelson Sadowski was born in Sao Bento do Sul, Brazil, in 1959. He received the Engineering degree from the Universidade Federal de Santa Catarina (UFSC), Florianopolis, Brazil in 1982 and the M.Sc. Degree from the same University in 1985. He is an Assistent Professor in the Department of Electrical Engineering of the UFSC and a research member in the GRUCAD/UFSC laboratory. In 1989 he joined the LEEI, Toulouse as a doctoral student. His research interests include the modeling of Electrical Machines and Applied Electromagnetic.

Stéphan Astier was born in Marseille, France, in 1953. He received the Engineer Diploma from the Enseeiht, Toulouse in 1977 and the Eng. Doctor Degree from the INPT of Toulouse in 1979 after doing research on permanent magnet machines. Then, he joined the ENSUT of Dakar, Senegal, as a researcher-teacher working on renewable energy applications for the Sahelian countries. He joined the LEEI Toulouse in 1989 as a "Maître de Conférences" and presently works on electric actuator design.

Michel Lajoie-Mazenc was born in Decazeville, France, in 1938. He received the Doctorat d'Etat degree from the University of Toulouse in 1969. He joined the LEEI de Toulouse in 1965. He presently works in this Laboratory as Directeur de Recherche at the CNRS. He coordinates the research of the group "Machines and Actuators with Electronic Commutation" of the Laboratory. More particularly, he studies electronically commutated permanent magnet synchronous machines with the aim of an optimum machine and converter adaptation. He is also the Manager of the LEEI.