MULTI CRITERIA DECISION MAKING METHODOLOGY FOR FUZZY RULE BASED SYSTEMS AND NETWORKS USING TOPSIS

by

ABDUL MALEK BIN YAAKOB

BSc (Hons) Mathematics, Universiti Teknologi MARA, Malaysia, 2008

MSc Mathematics, Universiti Teknologi Malaysia, Malaysia, 2011

AN ABSTRACT OF A DISSERTATION

submitted in partial fulfillment of the requirements for the degree

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Approved by:

Dr. Alexander Gegov

Abstract

Fuzzy systems and networks are vital within the armoury of fuzzy tools and applicable to real life decision making environments. Three types of fuzzy systems introduced in literatures which are systems with single rule base, systems with multiple rule bases and system with networked rule bases. This research introduces novel extension of the Technique of Ordering of Preference by Similarity to Ideal Solution (TOPSIS) methods and uses fuzzy systems and networks to solve multi-criteria decision making problems where both benefit and cost are presented as subsystems. In conjunction, the implementation of fuzzy sets type-1, type-2 and Z-number of proposed approaches is also presented. Furthermore, literatures have observed that tracking the performance of criteria is crucial by controlling the estimation of uncertainty of the criteria. Thus, the decision maker evaluates the performance of each alternative and further observes the performance for both benefit and cost criteria. This research improves significantly the transparency of the TOPSIS methods while ensuring higher effectiveness in comparison to established approaches. Ensuring the practicality and the effectiveness of proposed methods in a realistic scenario, the problem of ranking traded stock is studied. This case study is conducted based on stocks traded in a developing financial market such as Kuala Lumpur Stock Exchange. The ranking based on proposed methods is validated comparatively using performance indicators such as Spearman Rho correlation, Kendall Tau correlation, Root Mean Square Errror and Average Absolute Distance by assuming ranking based on return on investment as a benchmarking. Based on the case study, the proposed methods outperform the established TOPSIS methods in term of average rank position.

Declaration of Authorships

Whilst registred as a candidate for the above degree, I have not been registred for any other research award. The results and conclusions embodied in this thesis are the work of the named candidate and have not been submitted for any other academic award.

Signed: ---

Print Name: Abdul Malek Yaakob

Date: 25 July 2017

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Alternatives Level (AL), 87

Alternatives Systems (AS), 87

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Journal Articles

[P1] **Abdul Malek Yaakob**, Alexander Gegov, "*Interactive TOPSIS Based Group Decision Making Methodology using Z- Numbers"*, International Journal of Computational Intelligent System, 9(2), 311-324, 2016.

[P2] **Abdul Malek Yaakob**, Antoaneta Serguieva and Alexander Gegov, *"FN-TOPSIS: Fuzzy Networks for Ranking Traded Equities,"* IEEE Transaction on Fuzzy Systems, 25(2), pp. 315–332, 2017.

[P3] **Abdul Malek Yaakob**, Alexander Gegov, Siti Fatimah Abdul Rahman, "*Selection of Alternatives with Fuzzy Networks using Rule Base Aggregation*." Fuzzy Sets & Systems. 2017.

Conference Papers

[P4] **Abdul Malek Yaakob**, Alexander Gegov, "*Fuzzy Multi Criteria Decision Making Methodology for Selection of Alternatives using Z-Numbers",* IEEE Summer School on Computational Intelligence: Theory & Applications, Tunis, Tunisia, July 2014.

[P5] **Abdul Malek Yaakob**, Alexander Gegov, "*Fuzzy Rule Based Approach with Z-Numbers for Selection of Alternatives using TOPSIS*," IEEE International Conference on Fuzzy Systems, Istanbul, Turkey, August 2015, pp. 1-8.

[P6] **Abdul Malek Yaakob**, Ku Muhammad Naim Ku Khalif, Alexander Gegov and Siti Fatimah Abdul Rahman, *Interval Type 2- Fuzzy Rule Based System Approach for Selection of Alternatives using TOPSIS,* 7th International Joint Conference on Computational Intelligence, Lisbon, Portugal, Nov 2015, pp. 112-120.

[P7] Opeyemi Bello, Catalin Teodoriu, **Abdul Malek Yaakob**, Alexander Gegov, Joachim Oppelt, Javier Holzmann, Arash Asgharzadeh, *"Hybrid Intelligent Decision Support System for Drill Rig Performance Analysis* *and Selection During Well Construction,"* International Petroleum Technology Conference, Bangkok, Thailand, July 2016, pp. 1–17.

[P8] **Abdul Malek Yaakob**, Alexander Gegov, Mohamed Bader-El-Den, Siti Fatimah Abdul Rahman, *"Fuzzy systems with multiple rule bases for selection of alternatives using TOPSIS,"* IEEE International Conference on Fuzzy Systems, Vancouver, Canada, July 2016, pp. 2083–2090.

[P9] **Abdul Malek Yaakob**, Alexander Gegov, Siti Fatimah Abdul Rahman, *"Decision Making Problem using Fuzzy Networks with Rule Base Aggregation,"* IEEE International Conference on Fuzzy Systems, Naples, Italy, July 2017.

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Dedication

Dedicated to,

My mother,

Mek Mat Ali

For making me be who I am, and

My mother in law

Siti Samiah Dalil

My wife

Siti Fatimah Abdul Rahman

Sons and daugther

Muhammad Ihsan Wafiuddin (23 February 2011)

Ahmad Irfan Wabil (14 August 2012)

Ahmad Wafri Munawwar (01 Jan 2016)

Waailah ‘Irdhina (4 March 2017)

for supporting me all the way.

Last not least

All family members and friends

Thank you very much for the endless prayer and be blessed.

CHAPTER 1

1 INTRODUCTION

This chapter is described in the following sequences such that the first part is the background of the research followed by research questions and objectives. After that is research contribution and the final part is the thesis organization.

1.1 Background of Research

Decision making is the act of choosing between two or more courses of action and a thought process of selecting a logical choice from available options. It is regarded as a result of mental processes leading to a particular selection when surrounded by a number of alternatives, criteria, factors, variables [1]. Multi-criteria Decision Making (MCDM) method is one of the accustomed approaches to deal with decision making problems. MCDM aims at improving the quality of complex decisions by making the process more explicit, rational and efficient [2]. This approach often requires the Decision-Makers (DMs) to provide qualitative and quantitative measurements in determining the performance of each alternative with respect to attribute and the relative importance of evaluation attributed with respect to overall judgments [3]. MCDM has been found to adopt in real-life decision making situations. However, in the real world, due to complexity, vagueness and associated risks of decision processes may be considered difficult to solve [4]. The MCDM methods that are known as of current are The Simple Additive Weighting (SAW) which was introduced in 1954 by [5], Analytic Hierarchical Process (AHP) [6], ELimination and Choice Expressing REality (ELECTRE) [7] and Technique for Order Preference by Similarity to Ideal Solution

(TOPSIS) [8]. The method of TOPSIS was quoted as one of the most popular individual approaches in decision-making particularly in the alternative selection [9].

TOPSIS is chosen as the target for this research because of its simplicity and its ability to consider a non-limited number of alternative and criteria in the decision process [8]. In this case, TOPSIS is found to be more suitable for selection of alternative, particularly to support group decision making. Furthermore, a research conducted by [9] shows that TOPSIS method outperformed other MCDM methods, particularly the AHP methods, in terms of change of alternatives and criteria, agility and number of criteria and alternatives. TOPSIS has been successfully applied in MCDM problems as one of the most frequent methods used. The main advantage of the TOPSIS is that its easiness for computing and understanding, because the method is directly giving a definite value by experts to calculate their final results [10]. In addition, fuzzy set has also been extensively used for modeling decision processes based on vague and imprecise information such as jugdement of decision makers. Hence, fuzzy TOPSIS (FTOPSIS) is introduced to handle uncertainty in linguistic judgment.

Initial research on FTOPSIS was conducted by [11] who extended TOPSIS to type-1 fuzzy environments; this extended version used type-1 fuzzy linguistic value represented by type-1 fuzzy number. Overall, the type-1 fuzzy TOPSIS problem is to find the most desirable alternatives from a set of n feasible alternatives according to the decision information by decision makers about attribute weights and attribute values. There is no solution satisfying all attributes simultaneously due to the conflicting attribute problems. Thus, the solution is a set of non-inferior solutions or a compromise solution according to the Decision Makers (DMs) preferences [12]. The ability of providing results that are consistent with actual ranking remains the major concern in multi criteria decision making environment.

1.2 Research Questions

This section lists relevant research questions from the study which are shown in the following lists:

a) Are there any established TOPSIS methods that consider confident level of decision maker in their formulation that is capable to represent the reliability of decision maker?

b) Are there any established TOPSIS methods that consider influence degree of decision maker in their formulation, which is capable to represent the experience of decision maker?

c) Are there any established TOPSIS methods that allow decision maker to trace the performance of criteria such as benefit criteria and cost criteria in their formulations that is capable to represent the transparency of criteria?

d) Are there any established TOPSIS methods that integrate fuzzy systems and networks?

1.3 Research Objectives

This research embarks on the following objectives, which are in accordance to research questions in Section 1.2 and research contributions in Section 1.4. The objectives are:

a) To develop TOPSIS methodology based on fuzzy system with a single rule base using type-1, type-2 and Z-number implementation.
b) To develop TOPSIS methodology based on fuzzy system with multiple rule bases using type-1, type-2 and Z-number implementation.
c) To develop TOPSIS methodology based on fuzzy network with rule base aggregation using type-1, type-2 and Z-number implementation.
d) To develop TOPSIS methodology based on fuzzy network with rule base merging using type-1, type-2 and Z-number implementation.
e) To address stock selection problems using proposed methods and to compare the ranking based on proposed methods with established methods; thus, manifesting the applicability of the proposed methods.

f) To validate the proposed methods using Spearman rho correlation, Kendall tau correlation, RMSE and Average absolute distance and to illustrate the robustness of the methods.

1.4 Research Contributions

This section points out the main contributions of this research, especially in fuzzy decision making methodology. Four main contributions are represented by this research which will be mentioned in the following paragraphs.

The first contribution of this research is the development of TOPSIS methodology based on fuzzy systems with a single rule base is proposed which considers the influence degree of decision makers. Furthermore, the hybrid analysis of decision making processes that requires the use of human sensitivity to reflect influence degree of decision makers can be expressed by fuzzy rule base. In this case, the criteria or inputs of the system are joined together in a single rule base. Thus, the fuzzy TOPSIS based on fuzzy systems with multiple inputs and single output is introduced.

The second contribution of this research is the continuation of TOPSIS methods in the first contribution to fuzzy systems with multiple rule bases. This extension illustrates the capability of multiple rule bases in decision making analysis. In this case, the criteria or inputs of the system are categorised into two subsystems namely benefit rule base and cost rule base. Thus, the fuzzy TOPSIS based on fuzzy systems with multiple rule bases is introduced.

The third contribution of this research is another continuation of TOPSIS methods from the second contribution to fuzzy networks with rule bases aggregation. In this case, the criteria or inputs of the system are categorised into two subsystems namely benefit and cost rule bases. Then, the outputs from these two subsystems are used as inputs for the third subsystem namely alternatives rule bases. Hence, the fuzzy TOPSIS based on fuzzy network with rule base aggregation is introduced.

The fourth contribution of this research is a final continuation of TOPSIS methods from the third contribution to fuzzy networks with rule base merging. In this case, the fuzzy network operations involved are vertical merging and horizontal merging whereby benefit rule base and cost rule base are merged by vertical merging of rule base operation. This is further merged with alternatives rule base by horizontal merging operation. Consequently, the fuzzy TOPSIS based on fuzzy network with rule base merging is introduced.

In summary, some genuine contributions of this thesis can be pointed out:

1) This is the first study of TOPSIS decision making method that integrates fuzzy systems and networks. These approaches improve significantly the transparency of the TOPSIS methods while ensuring high effectiveness in comparison to established approaches. This study can contribute to help decision makers to track and be aware of the performance of benefit and cost criteria and to choose more profitable alternatives and achive problems solving objective.
2) The implementation of each approaches for 3 main type of fuzzy set namely type-1, type-2 and Z-number are proposed in this thesis. The comparative validation of fuzzy set based on case study considered is also included.
3) There are 12 novel methods proposed in this thesis, 4 approaches with 3 types of fuzzy sets.
4) Apart from the novel TOPSIS aproaches using fuzzy systems and networks, another contribution of this thesis is the real case study of stock selection problems; the objective of this case study is to rank several stocks listed on the financial market. The numerical examples of case study are comparatively validated the approaches using performace indicator such as Spearman rho, Kendall tau.
5) These novelties are underpinned by 9 publications preceding the thesis by the author including 1 IEEE Transactions article and 2

others journal articles, 3 IEEE Conference papers and 3 others conference papers.

1.5 Thesis Organisation

This section covers the organization of the thesis. There are altogether nine chapters in the thesis including this chapter where the remaining nine chapters are described as follow.

Chapter 2 discusses the literature review of the research. Chapter 3 outlines established research methodology of the thesis such that definitions and formulations used in this research are given.

In Chapter 4, development of fuzzy TOPSIS based on fuzzy systems with single rule base is thoroughly discussed. In Chapter 5, development of fuzzy TOPSIS based on multiple rule bases pointed out. Information provided in Chapter 5 underpins development of the methodology in Chapter 6 with some additional steps. Hence, Chapter 4, 5, and 6 covers the discussion on the novel of research methodology section of the thesis.

Chapter 7 focuses on the implementation of the proposed work in solving case studies of stock selection problem while validation and analysis of results are written in Chapter 8. The final chapter is the Chapter 9 where conclusion, contributions and recommendations for future work are highlighted.

1.6 Summary

In conclusion, a descriptive overview of the thesis in terms of its background research which led to the research questions and objectives were given. The penultimate part was the main contributions of this research and finally followed by the thesis organisation. The next chapter is Literature Review.

CHAPTER 2

2 LITERATURE REVIEW

2.1 Introduction

This chapter discussed in detail on the established works found in the literatures which are related to this research. The chapter starts with the description of basic notions of fuzzy sets which justify the applicability of fuzzy sets in human's decision making. Then, the evolution of fuzzy sets tools is chronologically highlighted where an overview of type–1 and its extensions, namely type–2 and Z–numbers are covered.

2.2 Fuzzy Sets

This section discusses the chronological development of fuzzy sets, specifically on tools used in decision making process. Fuzzy sets are pointed out as a suitable knowledge for human's decision making from the fact that basic notions of fuzzy sets are capable to appropriately represent the natural language. Even though fuzzy sets do represent the natural language quite well, distinguishing two or more natural languages used in a decision making problem is toilsome because they are defined qualitatively. Due to this qualitative measurement, [13] suggested a quantitative definition for fuzzy sets known as fuzzy numbers which is well–suited for natural languages.

In the literature of fuzzy sets, three kinds of fuzzy numbers were found namely type–1, type–2 and Z–numbers. All these fuzzy numbers are considered in this study. Among these three, type–1 is the most utilised in the literatures of fuzzy sets followed by type–2 and then Z–number. This ordered usage happens because the chronological development for type–1 was firstly developed in 1965 while type–2 in 1975 and Z– number in 2011; hence, affecting their utilisation frequency in the literature of fuzzy sets. Even though three types of fuzzy numbers were considered, they were not simultaneously utilised in representing the natural languages. This asynchronous practise was because they are theoretically and naturally different, indicating that only one type of fuzzy numbers can be used at a time. The details of the three fuzzy sets will be given in the following subsections.

2.2.1 Type-1 Fuzzy Sets

Type–1 fuzzy number or the classical fuzzy number is the first fuzzy sets introduced in literatures. In some established studies by [14]–[17], the term fuzzy number was originally used in their discussions. This term was later changed to type–1 as type–2 was then introduced. Both types are fuzzy numbers but the number assignment is due to their natural differences. According to [18], type–1 fuzzy numbers consist of both membership degree and the spread features which respectively correspond to confidence level and opinion of decision makers. Due to this feasible features, type–1 fuzzy numbers are oftenly applied in many decision making problems such as the evaluation of Taiwan's urban public transport system performance [19], the evaluation of engineering consultants' performances [20], fuzzy risk analysis[18], selection of beneficial project investment [21] and solution of air fighter selection problem [22].

2.2.1 Type-2 Fuzzy Sets

Type–2 fuzzy numbers were introduced in literatures of fuzzy sets in [23] as an extension of type–1 fuzzy numbers to model perceptions. This addendum is because the uncertainity representation of type–1 on natural language is scarce to model perception [24]. The uncertainity group of type-1 related to natural languages according to [25] are two, namely intra–personal uncertainty and inter–personal uncertainty. On the other hand, the studies that utilised type–2 in their decision making applications are mobile object based control tracking [26], doubly fed induction generator based wind energy systems in distribution networks [27], application on modelling words [28], images segmentation in medical imaging application [29], selection of best robot for production process [30] and selection of investment project evaluation [31], [32]. Even though type–2 is introduced to enhance type–1 in modelling perceptions, they are rarely used for decision making applications due to their natural complexity.

2.2.3 Z-Numbers

The Z–number is the latest presentation of fuzzy numbers introduced by [33] as an extension of type–1 but a completely different to type–2. Even though both Z –number and type–2 are the extensions of the first fuzzy number, the former is capable in measuring the reliability of the decision made as compared to the latter. Since fuzzy numbers are the medium of quantitative representation for natural languages [34], Z–number has enhanced the capability of both type–1 and type–2 [33]. According to [35], Z–number is represented by two embedded type–1 fuzzy numbers where one of them plays the role, while the other defines the reliability of the first one. Research on utilizing Z–numbers in decision making applications is inadequate as compared to other fuzzy numbers, as it is a recent fuzzy concept developed in the theory of fuzzy sets. Among the studies that utilized the concept of Z-number are risk analysis [36], stock selection problem [37],selection of software product in the market [38], selection of vehicle for journey [39],book selection process [40], evaluation of resilience engineering on health, safety, environment, and ergonomics factors [41] and machine-perception encapsulation problem [42].

2.3 Fuzzy Systems and Networks

This section discusses the development of fuzzy systems and networks, specifically on tools used in decision making process. Fuzzy systems, including fuzzy logic and fuzzy set theory provide a rich and meaningful addition to conventional logic [43]. The mathematics generated by these theories is consistent while fuzzy logic can be a generalization of conventional logic. Despite the concerns of classical logicians, the applications generated from or adapted to fuzzy logic are widely ranging and providing the opportunity for modeling of conditions that are inherently and imprecisely defined. Many systems have been modeled, simulated and even replicated with the help of fuzzy systems as well as the human's reasoning itself. The representation of human-originated information and the formalization of common sense reasoning have motivated different schools of research in artificial or computational intelligence in the second half of the twentieth century.

More than forty years of research have demonstrated that fuzzy system models are the most successful models to handle uncertainties in decision making. The major advantages of fuzzy system models are their robustness and transparency. Fuzzy systems achieve robustness by using fuzzy sets which incorporate imprecision in system models. In addition, unlike some other system models such as neural networks, the fuzzy system models are highly descriptive, in another word transparent.

In the last two decades, researchers have proposed several data-driven type-1 fuzzy system approaches that can extract the hidden rules of system behaviour automatically by using historical data. The fuzzy system proposed in [44]–[47] are among the most popular. Since these methods employ only the historical data, that is they do not involve expert knowledge, they are exactly data-driven techniques. Thus, in addition to being robust and transparent, these fuzzy system techniques can objectively recognise system structure.

In these former conventional fuzzy systems, the structure is characterized by type-1 fuzzy sets which define on a universe of discourse and map an element onto a precise number in the unit interval [0,1] [13]. Later, fuzzy system models formed with higher order fuzzy sets, such as type-2 fuzzy sets which firstly proposed by [23]. The historical review of type-2 fuzzy logic is discussed broadly in [48]. Even though type-2 is more complex to implement [49], a latter study by [50] shows a simple way to overcome the difficulties. Hence, type-2 fuzzy sets are frequently used to tackle the uncertainty associated with the membership functions such as [27], [50].

In addition, three types of fuzzy systems are discussed in the literatures namely system with single rule base, system with multiple rule bases and system with networked rule bases. The first system is defined as a black box nature, such that inputs are mapped directly to the output [51]. The second system is also known as term chained fuzzy system or hierarchical fuzzy system [52]. It is characterised with a white box nature, where the input is mapped to the outputs through an interval variable as connections. The third system is introduced as a theoretical concept in [53] and is characterised with white box nature a well, where the input is mapped to the outputs through intermediate variables. Although fuzzy network have been recently introduced, a significant volume of work have been done and dedicated to the theoretical development and application of fuzzy networks [53]–[58].

2.4 Decision Making Techniques

This section discusses the development of decision making techniques. Currently, several increasing interests in MCDM techniques are seen where a considerable amount of studies has been published on them. In about forty years since it was introduced, over seventy MCDM techniques have been developed to facilitate decision making practice [59]. MCDM is a practical tool for selecting and ranking a number of alternatives and its applications are numerous [60]–[63]. The most frequently used techniques are Simple Additive Weighting (SAW)[64], Analytical Hierarchy Process (AHP)[65], ELimination and Choice Expressing REality (ELECTRE) [66], Preference Ranking Organisation Method for Enrichment Evaluations (PROMETHEE) [67], [68], PROSPECT theory [69], [70] and Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) [71], [72].

SAW method is based on the weighted average. An assessment score is considered for all alternatives by multiplying the scaled importance given to the alternative of that element with the weights of relative importance directly assigned by decision maker. In the context of fuzzy environment, this method is called Fuzzy Simple Additive Weighted (FSAW) proposed by [73], aiming to improve the dimensionality of SAW method. SAW and FSAW were successfully applied in [74]–[76]. However, SAW only used for maximizing assessment criteria which should be transformed into the maximizing ones by the respective formulas prior to their relevance [74].

The AHP introduced by [6] is based on the decision maker assigning a relative value of weight for all of the criteria by pair-wise comparison. The shortcoming is that the exhaustive pair-wise comparison is tiresome and time consuming when there are a lot of alternatives to considered. In 1996, fuzzy AHP was introduced by [77] in order to handle vague information in AHP process. More than thousands of researchers published work on theory and application based on this approach [78]–[85]. However, recently, the fallacy of fuzzy AHP method have been revealed in [86]. Zhu highlighted five fallacies in his article; a) fuzzy numbers opposed the logic of fuzzy set theory, b) the operation rules of fuzzy numbers opposed the logic of the AHP, c) fuzzy AHP could not give an acceptable method to rank fuzzy numbers, d) the validity of the AHP/ANP in complex and uncertain environments and e) fuzzy ANP is a false proposition.

On the other hand, the ELECTRE which was introduced by [7] is categorised into three problems namely Choice problematic, ranking problematic and sorting problematic. For ranking problematic, ELECTRE II, ELECTRE III and ELECTRE IV are used. They are concerned with the ranking of all activities belong to a specified set of activities from the greatest to the worst. In 2011, [87] introduced fuzzy ELECTRE based on intuitionistic fuzzy set to describe uncertainty situation in decision making problems. A major problem with the ELECTRE methods is they used similar threshold values but provided different ranking towards alternatives [88].

The method called Preference Ranking Organisation Method for Enrichment Evaluations (PROMETHEE) was developed by [67] with the objective of identifying the pros and the cons of the alternatives and obtaining a ranking among them. With PROMETHEE as an outranking method, strong assumptions concerning the ‘true’ preference structure of the decision maker are avoided. One of the advantage of PROMETHEE method is dealing with uncertain and fuzzy information. Fuzzy PROMETHEE was introduced by [68] to enhance the weakness of preferences and incomparability of the method. However, the main drawback of this method is that PROMETHEE does not provide the possibility to the real structure of the decision problem. In the case of many criteria and options, difficulties may arise for the decision maker to obtain a clear view of the problems and to evaluate the results [89].

The PROSPECT theory created by [90] is a behavioral economic theory that describes the technique human choose among the probability of alternatives involving risks and the probabilities of outcomes are known [69]. The theory states that human make decisions based on the prospective value of losses and gains rather than the ultimate outcome. They evaluate these losses and gains using certain heuristics [91]. The model is descriptive that tries to model real-life choices, rather than optimal decisions, as normative models do. Fuzzy PROSPECT theory invented by [92] with main objective is to revisit the Kahneman and Tversky idea of PROSPECT theory and way of human thinking using fuzzy logic. PROSPECT theory has been well accepted in decision theory communities [50], [93], [94]. However, the original version of prospect theory gave rise to violations of first-order stochastic dominance and external efficacy belief in determining the outcome of the analysis [95]. Therefore, these techniques have limitations from one to another.

According to [96] and [97], TOPSIS has the following three advantages: (1) a sound logic that represents the rationale of individual choice, (2)a scalar value that record for both the best and worst alternatives concurrently and (3) a straightforward computation algorithm that can be easily programmed into a spreadsheet, i.e. Microsoft Excel. These advantages make TOPSIS a popular MCDM technique as compared to other related techniques such as AHP and ELECTRE [76]. In fact, TOPSIS is a value-based process that compares each alternative directly depending on information in the evaluation matrices and weights [98].

In recent years, TOPSIS has been effectively applied to the areas of human resources management [98], transportation [63], product design [99], manufacturing [100], water management [101], quality control [62], military

[60], tourism [102] and location analysis [103]. Thus, TOPSIS is chosen as the main body of expansion in this study. On the other hand, [104] has introduced the Fuzzy F-TOPSIS, which allows hybrid analysis between empirical knowledge of expert and the optimisation technique. They make use the capability of fuzzy rule base to represent the empirical knowledge of expert.

Based on [105], in decision making environment, tracking the performance of criteria is essential in order to take control and to not underestimate or overestimate uncertainty of the criteria. The proposed methods represent a systematic TOPSIS approach to estimate the strengths and weaknesses of alternatives that satisfy transactions, activities or functional requirements for a business. In addition, tracking of criteria allows decision makers to determine if a sound investment or decision and to provide a basis for comparing alternatives. This case involves comparing the total expected cost criteria of each alternative with the total expected benefits criteria; lest the benefits outweigh the costs and by how much. The inefficiencies described above bring the motivation of this study.

2.5 Technique for Order Preference by Similarity to Ideal Solution

This section discusses the development of Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS). The conventional TOPSIS which refers to non-fuzzy TOPSIS is a multi-criteria decision analysis method, which was originally developed by Hwang and Yoon in [8] with further developments in [106]. TOPSIS is based on the concept that the chosen alternative should have the shortest distance from the positive ideal solution and the longest distance from the negative ideal solution. It is a method of aggregation that compares the best score within a set of alternatives by identifying weights for each criterion, then normalising scores for each criterion and calculating the distance between each alternative and the ideal alternative.

Fuzzy TOPSIS or type-1 TOPSIS is introduced for the first time in 2000 based on type-1 fuzzy sets by Chen [11], with the reason that conventional one is inadequate to model real life situation [11]. By this condition, linguistic terms that can be expressed in triangular fuzzy number are used to describe the rating of each alternative and the weight of each criterion. Linguistic terms allows the possibility of dealing with linguistic terms in a precise way [107].

Among the studies that utilised type-1 TOPSIS is a case study on stock selection evaluating performance of securities listed on Bursa Malaysia [108] whose main objective is to apply a decision making model proposed by [109] to rank the stock and to determine the proportion to invest in particular stock based on stock financial data. The data is obtained from Shariah- Compliant securities listed on main board Bursa Malaysia. This method of decision making model could help investor to make the decision on constructing a profitable portfolio. The study found that Fuzzy TOPSIS could provide a good guideline to those who wish to involve in investment business. Other then that, Fuzzy TOPSIS has also been applied in robot selection [110], human health and safety risk management [111].

After a decade, the interval type-2 TOPSIS was introduced which provides additional degree of freedom [112], such that uncertainty and fuzziness in fuzzy multi criteria decision making problem can be handled with more flexiblity and more intelligent manner. This is because type-2 fuzzy sets are more suitable to represent uncertainty in type-1 fuzzy sets [113]. In this established fuzzy TOPSIS method, the evaluations and the weights of criteria are represented using type-2 fuzzy sets which have been successfully applied in numerous research areas such risk factor for chronic kidney disease [114], tool magazine [115] and supply chain risk management [116].

Most recently, the existing of the concept of Z-number by [33] leads the researchers to introduce the enhancement of fuzzy TOPSIS based on the idea of reliability of information discussed broadly by Zadeh called Z-TOPSIS [117]. This established method highlights that when dealing with real information, fuzziness is scanty and a degree of reliability of the information is very critical. Due to these reasons, the reliability of information have been take into account by using fuzzy expectation [35] resulting into low computational intricacy.

In addition, the fuzzy TOPSIS has been implemented in fuzzy rule based approach for the first time by [104]. In order to make the inference from fuzzy rules considering the different degrees of influence of decision makers, they have made some adaptations in the Meta rule [118] and centre of area defuzzification, then used them as a useful tool to create fuzzy rule base. Nonetheless, this method has high computational intricacy. As far this research concern, there is no research on fuzzy network for TOPSIS method is found in the literature of multi criteria decision making analysis. For that reason, this study introduces a methodology for ranking alternative using fuzzy network, which is proposed for the first time in Chapter 6.

2.6 Summary

All in all, this chapter digs the previous work related to this research and describes the foundation of the research based on the literatures. It covers the evolution of fuzzy sets, fuzzy system and networks, decision making techniques and TOPSIS. The next chapter is the research methodology.

CHAPTER 3

3 RESEARCH METHODOLOGY

3.1 Introduction

This chapter considers the establishment of methodologies of the thesis. It discusses fuzzy concepts and terminology, fuzzy systems, fuzzy networks, the established TOPSIS methods and performance indicator. Details on those aforementioned points are discussed in sections and subsections provided in this chapter.

3.2 Fuzzy Sets

Many research articles in the literatures of decision making indicated that the classical set theory serves a useful tool in solving decision making problems. It defines the membership degree of elements in a set using binary representation of 0 or 1 which respectively implying whether an element in a set is not a member or a member. For instance, consider the weather condition today; either ‘hot’ or ‘not hot’ by the classical sets. This consideration of only two binary terms by classical sets is limited as human perceptions are diversely vary, as different people employ different types of perceptions which are vague and fuzzy [119].

Due to the limitation of the classical sets, fuzzy set theory is therefore introduced in decision making environment as dealing with situations that are naturally fuzzy is important. Furthermore, fuzzy sets theory allows gradual assessments of an element degree of belongingness in the interval of 0 and 1 where these values indicate variety in terms of human perceptions about a situation perceived. Using definition by [119], definition of fuzzy sets is given as follows:

Definition 3.1 [119] *A fuzzy set A_i in a universe of discourse U is characterised by a membership function* $\mu_{A_i}(x)$ *which maps each element x in U such that x is real number in the interval [0, 1]. Membership function for A_i,* $\mu_{A_i}(x)$ *is given as*

$$\mu_{A_i}(x) : X \to [0,1]$$

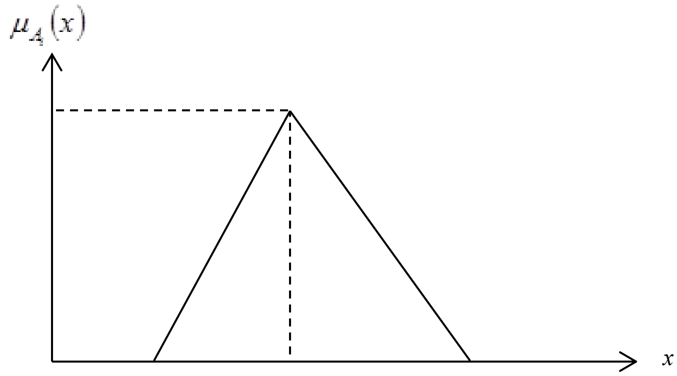

Fig. 3. 1: Membership function of a fuzzy set

The value of membership degree of a fuzzy set is defined in the interval of [0, 1] as presented in Fig.3.1. For instance, let $\mu_{hot}(x)$ be defined as the membership function of 'hot' for today's weather condition. If the membership value is approaching 0, then *x* is closer to 'not hot'. On the other hand, if the membership value is approaching 1, *x* is closer to 'hot'. The following Table 3.1 illustrates the differences between classical set theory and fuzzy set theory.

Table 3. 1: Differences between classical sets and fuzzy sets theories

Theory	Representation	Membership degree
Classical	Binary	{0 and 1}
Fuzzy	Gradual	[0, 1]

The literatures of fuzzy sets have defined three basic operations of fuzzy sets which are fuzzy union, fuzzy intersection and fuzzy complement. All of these operations are defined in [120] by the following definitions:

Let A_i and A_j be two fuzzy subsets of the universal interval U with membership functions for A_i and A_j are denoted by $\mu_{A_i}(x)$ and $\mu_{A_j}(x)$ respectively.

Definition 3.2 [120] *Fuzzy union, fuzzy intersection and fuzzy complement are define as*

a) *Fuzzy union of* A_i *and* A_j *is denoted by* $A_i \cup A_j$ *such that the membership function is defined as*

$$\mu_{A_i \cup A_j}(x) = max\left[\mu_{A_i}(x), \mu_{A_j}(x)\right], \text{ for all } x \in U$$

b) *Fuzzy intersection of* A_i *and* A_j *is denoted by* $A_i \cap A_j$ *such that the membership function is defined as*

$$\mu_{A_i \cap A_j}(x) = min\left[\mu_{A_i}(x), \mu_{A_j}(x)\right], \text{ for all } x \in U$$

c) *Fuzzy complement of* A_i *is denoted by* $\mu_{\overline{A}_i}(x)$ *such that the membership function is defined as*

$$\mu_{\overline{A}_i}(x) = 1 - \mu_{A_i}(x), \text{ for all } x \in U$$

3.3 Fuzzy Numbers

As discussed in Section 2.2, three types of fuzzy numbers are pointed out in the literatures of fuzzy sets namely type–1, type–2 and Z–number where all of them are defined chronologically as follows.

3.3.1 Type-1 Fuzzy numbers

Type-1 fuzzy number is the first fuzzy numbers established in the literatures of fuzzy sets [13]. As all fuzzy numbers are branching from type–1 fuzzy numbers, the definition of fuzzy number given by [121] reflects the definition of type-1 fuzzy number given as follows.

Definition 3.3 : [122] *A type-1 fuzzy number* A_i *is a fuzzy subset of the real line* $\mathcal{R}$ *that is both convex and normal and satisfies the following properties:*

i. μ_{A_i} *is a continuous mapping from* $\mathcal{R}$ *to the closed interval* $[0,w]$, $0 \le w \le 1$

ii. $\mu_{A_i}(x)=0$, *for all* $x \in [-\infty, a]$,

iii. μ_{A_i} *is strictly increasing on* $[a,b]$,

iv. $\mu_{A_i}(x)=w$, *for all* $x \in [b,c]$ *where* w *is a constant and* $0 \le w \le 1$,

v. μ_{A_i} *is strictly decreasing on* $[c,d]$,

vi. $\mu_{A_i}(x)=0$, *for all* $x \in [d,\infty]$,

where $a \le b \le c \le d$, a,b,c *and* d *are components of type – 1 fuzzy number and real while* w *represents its height.*

3.3.2 Type-2 Fuzzy numbers

Type-2 fuzzy numbers developed in the literatures of fuzzy sets as the extension of type-1 fuzzy numbers since the capability of type–1 to represent human perception is inadequate [25]. Therefore, the definition of type–2 fuzzy sets by [113] is given as follows.

Definition 3.4: [113] *A type – 2 fuzzy set* A_i *in a universe of discourse* U *is characterized by a type – 2 membership function* $\mu_{A_i}(x)$ *which maps each element* x *in* U *a real number in the interval [0,1].*

The membership function for A_i, $\mu_{A_i}(x)$ *is given as*

$$A_i = \{((x,u), \mu_{A_i}(x,u)) | \forall x \in U, \forall u \in J_x \subseteq [0,1], 0 \le \mu_{A_i}(x,u) \le 1\}$$

where $J_x \in [0,1]$ *represents an interval in [0, 1].*

According to [113], another representation of type–2 fuzzy set is given in the following equation depicted as

$$A_i = \int_{x \in U} \int_{u \in J_x} \mu_A(x,u)/(x,u)$$

where $J_x \in [0,1]$ and $\iint$ represent the union over all allowable *x* and *u*. Noted that, if $\mu_A(x,u) = 1$, then A_i is known as an interval type–2 fuzzy set.

Furthermore, this interval is a special case of type–2 fuzzy set according to [113] where it can be represented by the following equation

$$A_i = \int_{x \in U} \int_{u \in J_x} 1/(x,u)$$

where $J_x \subseteq [0, 1]$. The interval of type-2 fuzzy set is utilised in this research because this type is oftenly used in the literatures. According to [123], the representation of interval in type-2 fuzzy set using numbers is called as interval type-2 fuzzy numbers. The following Fig. 3.2 illustrated this interval.

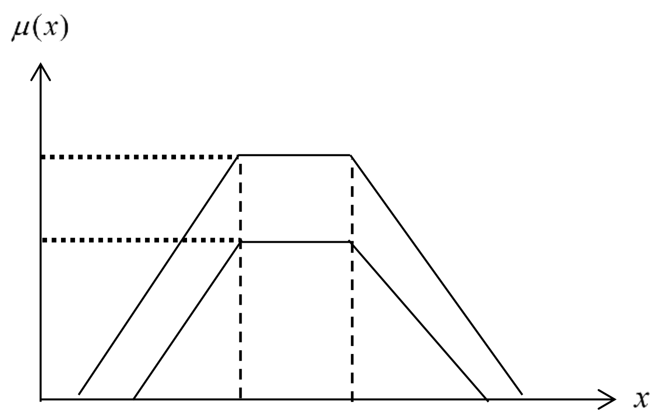

Fig. 3. 2: Interval type 2 membership function

Type-2 fuzzy number in Fig. 3.2 is noticeably more complex than type-1 in terms of its representation. This indicating that type-2 fuzzy number needs a more complicated computational technique than type-1 fuzzy number. According to [124], numerous defuzzification strategies are developed in the literatures of fuzzy sets which plan on converting type-2 into type-1. This strategy is intentionally introduced to reduce the complexity of type-2 without losing information on the computational results. Among those that considered this strategy are [125], [126], [127] and [124], [128]. Nevertheless, based on a thorough comparative analysis made by [124] on all the aforementioned methods, the reduction method by [126] outperformed other approaches on reducing type–2 into type–1. Therefore, without loss of generality in [126], the reduction method is as follows.

$$\mu_T(x_A) = \frac{1}{2}(\mu_L(x_A) + \mu_U(x_A))$$

where T is the resulting type –1 fuzzy numbers.

3.3.3 Z-Numbers

According to [33], Z–numbers are the newest type of fuzzy numbers introduced in the literatures of fuzzy sets. Definition of Z–numbers given by [35] is as follows.

Definition 3.5 [35] *A Z– number is an ordered pair of fuzzy number denoted as* $Z = (\tilde{A}, \tilde{B})$. *The first component* $\tilde{A}$ *is known as the restriction component where it is a real – valued uncertain on X whereas the second component* $\tilde{B}$ *is a measure of reliability for* $\tilde{A}$.

As mentioned in Chapter 2, Z-numbers are better in terms of their representation as compared to type-1 and type-2. This is due to the fact that Z-numbers is classified as the highest level in terms of generalised numbers than type -1 and type-2 whose level is 2 [33]. Therefore, [33] suggested the computational works involving Z-numbers needs to reduce the Z- numbers first into a certain level without losing the information from the computational results.

3.4 Fuzzy Systems

Most fuzzy systems are systems with single rule base. They have either one rule base, e.g. a multiple output fuzzy system or a number of independent rule bases. In this sense, the most distinctive feature of a Single Rule Base system is the isolated nature of its rule bases. However, some processes can be better modelled by a system with Multiple Rule Bases, i.e. a system with some interconnections between its rule bases [129] .This is usually the case of multi-stage processes where the outputs from a particular stage are also the inputs to one or more subsequent stages. The systems with Multiple Rule Bases used for describing such processes are usually referred to as ‘chained fuzzy systems’.

A system with Multiple Rule Bases can be described by a network whereby all rule bases in a horizontal row represent a level and all rule bases in a vertical column represent a layer. The numbering of levels is from top to bottom whereas the numbering of layers is from left to right. Interconnections may exist between rule bases residing in the same layer as well as between rule bases, which are in different layers. Some of these interconnections can be a forward direction, i.e. from a particular layer to one or more subsequent layers. Other interconnections can be a backward direction, i.e. from a particular layer to the same layer or to preceding layers. The interconnections reflect the nature of the multi-stage processes being modelled, i.e. the outputs from each rule base which are also the inputs to the other rule bases in the same layer; either the preceding layers or the subsequent layers.

The layers in a multiple rule bases system represent a temporal hierarchy, i.e. processes that take place sequentially in time. As opposed to this, the levels in a multiple rule bases system represent a spatial hierarchy, i.e. processes that are subordinated to each other. Although this spatial subordination is relevant mainly within a particular layer, it often propagates across the whole network structures in the context of the interconnected rule bases.

The given two types of network hierarchy are often used to model systems with the purpose of reducing their quantitative and qualitative complexity. In this sense, the network structure of the fuzzy rule base is either a straightforward reflection of the system being modelled or a design decision aimed at achieving better effectiveness or higher efficiency.

3.5 Fuzzy Networks

A Fuzzy Network (FN) is a more recent type of fuzzy systems consisting of networked rule bases (nodes) and dealing with inputs sequentially and considering the connections and the structure of the systems. The rules for fuzzy networks are derived from expert’s knowledge. A networked fuzzy system is transparent and fairly accurate at the same time due to its hybrid nature, which facilitates the understanding and management of complex decision. As shown in Figure 3.4, $\{p_1, p_2, \ldots, p_m\}$ is the set of inputs and $\{z_1, z_2, \ldots, z_{m-2}\}$ is the set of connections, while the set of network nodes $\{N_{11}, N_{12}, \ldots, N_{1,m-1}\}$ and $\{I_{21}, I_{31}, I_{32} \ldots, I_{m-1,m-2}\}$ are identity nodes. Here q represents the output of the system.

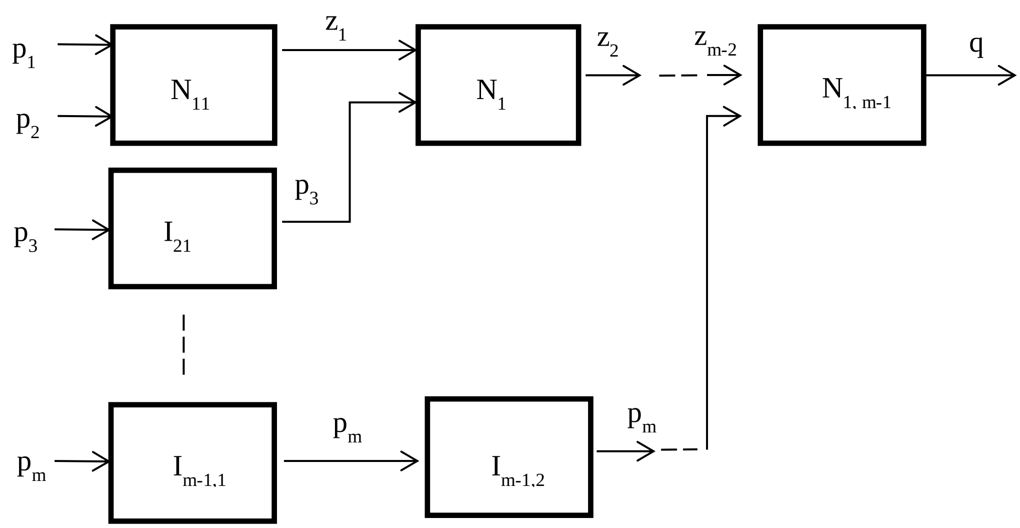

Fig. 3. 3: Fuzzy networks

Four formal models for fuzzy networks were characterized in [53], namely: (i) if-then rule and integer tables, (ii) block schemes and topological expressions, (iii) incidence and adjacency matrices, and (iv) Boolean matrices and binary relations. This thesis employ if-then rules and Boolean matrices to represent the fuzzy rules. Hence, the properties of such models are reviewed briefly. The choice is justified by the ability of these formal models to work with any number of nodes and to handle dynamics in fuzzy networks.

A fuzzy system with r rules, m inputs $p_1, \ldots, p_m$ taking the linguistic terms from the sets $\{S_{11}, \ldots, S_{1r}\}, \ldots, \{S_{m1}, \ldots, S_{mr}\}$, and n outputs $q_1, \cdots, q_n$ taking the linguistic terms from the output sets $\{T_{11}, \cdots, T_{1r}\}, \cdots, \{T_{n1}, \cdots, T_{nr}\}$, can be described by the following rule base:

Rule 1: If p_1 is S_{11} and $\cdots$and p_m is S_{m1} then q_1 is T_{11} and $\cdots$and q_l is T_{n1} (3.1)

$$\vdots \qquad \vdots \qquad \vdots$$

Rule r: If p_1 is S_{1r} and $\cdots$and p_m is S_{mr} then q_1 is T_{1r} and $\cdots$and q_{nl} is T_{nr}

A rule base is incorporated as a node within the fuzzy networks. A generalised Boolean matrix compresses information from a rule base represented by the node. The row and the column labels of the Boolean matrix are all possible permutation of linguistics terms of the inputs and outputs for this rule base. The elements of the Boolean matrix are either '0' s or '1's, where each '1' reflects the presented rule. The Boolean matrix representation of the rule base from Eq. (3.1) is given with Eq. (3.2).

$$\begin{array}{cccc} & T_{11}\cdots T_{n1} & \cdots & T_{1r}\cdots T_{nr} \\ S_{11}\cdots S_{m1} & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ S_{1r}\cdots S_{mr} & 0 & \cdots & 1 \end{array} \tag{3.2}$$

Boolean matrices are very suitable for formal representation of fuzzy network [56]. They describe fuzzy networks at lower level of abstraction with respect to the individual nodes. Boolean matrices also lend themselves easily to any manipulation for simplifying fuzzy networks to linguistically equivalent fuzzy systems by using the linguistics composition approach. In the next subsection, two basic Boolean matrix operations are briefly reviewed, as these two are involved in the development of Fuzzy Network-TOPSIS in Chapter 7.

3.4.1 Horizontal Merging of Rule Bases

Horizontal merging is a binary operation that can be applied to a pair of sequential nodes situated on the same level of a fuzzy network. This operation combines the operand nodes from the sequential nodes pair into a single product node. The operation can be applied when the output from the first node is fed forward as an input to the second node in the form of an intermediate variable. The product node has the output from the second operand node whereas the intermediate variable does not appear in the product node.

When Boolean matrices are used as formal models for the operand nodes, the horizontal merging operation is identical with Boolean matrix multiplication. The latter is similar to conventional matrix multiplication where each arithmetic multiplication is replaced by a 'minimum' operation and each arithmetic addition is replaced by a 'maximum' operation. The row labels of the product matrix are the same as the row labels of the second operand matrix and the column labels of the product matrix are the same as the column labels of the second operand matrix.

Therefore, if the first operand node is the rule base in Eq. (3.1), then its Boolean matrix is given by Eq. (3.2). Similarly, if the second operand node is the rule base in Eq. (3.3), its generalised Boolean matrix is expressed in Eq. (3.4).

Rule 1: If q_1 is T_{11} and $\cdots$ and q_m is T_{n1} then w_1 is R_{11} and $\cdots$ and W_g is R_{g1} (3.3)

$$\vdots \qquad \vdots \qquad \vdots$$

Rule r: If q_1 is T_{1r} and $\cdots$ and q_m is T_{nr} then w_1 is R_{1r} and $\cdots$ and W_g is R_{gr}

Then, the generalised Boolean matrix of Eq. (3.3) is described in Eq. (3.4) is as follows:

$$\begin{array}{cccc} & R_{11}\cdots R_{g1} & \cdots & R_{1r}\cdots R_{gr} \\ T_{11}\cdots T_{n1} & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ T_{1r}\cdots T_{nr} & 0 & \cdots & 1 \end{array} \tag{3.4}$$

If the product node is the rule base in Eq. (3.5):

Rule 1: If p_1 is S_{11} and $\cdots$ and p_m is S_{m1} then w_1 is R_{11} and $\cdots$ and W_g is R_{g1} (3.5)

$$\vdots \qquad \vdots \qquad \vdots$$

Rule r : If p_1 is S_{1r} and $\cdots$ and p_m is S_{mr} then w_1 is R_{1r} and $\cdots$ and w_g is R_{gr}

,

Therefore, its generalised Boolean matrix of Eq. (3.5) is constructed in Eq. (3.6) as follows:

$$\begin{array}{cccc} & R_{11}\cdots R_{g1} & \cdots & R_{1r}\cdots R_{gr} \\ S_{11}\cdots S_{m1} & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ S_{1r}\cdots S_{mr} & 0 & \cdots & 1 \end{array} \tag{3.6}$$

The fuzzy systems described by the rule base in Eq. (3.3) is with r rules, n inputs $q_1,\cdots,q_n$ taking the linguistic terms from the input sets $\{T_{11},\cdots,T_{1r}\},\cdots,\{T_{n1},\cdots,T_{nr}\}$, and g outputs $w_1,\cdots,w_g$ taking linguistic the terms from the set of outputs $\{R_{11},\cdots,R_{1r}\},\cdots,\{R_{g1},\cdots,R_{gr}\}$. Similarly, the fuzzy system described by the rule base in Eq. (5) is with r rules, m inputs $p_1,\cdots,p_m$ taking the linguistic terns from the input sets $\{S_{11},\cdots,S_{1r}\},\cdots,\{S_{m1},\cdots,S_{mr}\}$, and g outputs $w_1,\cdots,w_g$ taking the linguistic terms from the set of outputs $\{R_{11},\cdots,R_{1r}\},\cdots,\{R_{g1},\cdots,R_{gr}\}$. In general, the operand rule bases may have different number of rules but the number of rules in the product rule base are always equal to the number of rules in the first operand rule base. For simplicity, the notations used in Fig. 3.5 are in vector forms where the vectors x, y, v are of dimensions n, m, g, respectively.

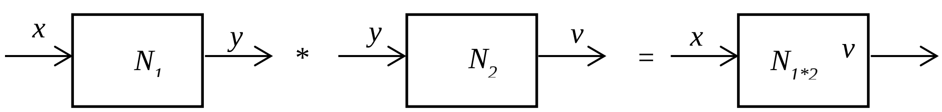

Fig. 3. 4: Horizontal merging of rule bases

When the property of associativity is related to the operation of horizontal merging, the latter is applied to three sequential nodes for merging them into a single node (see Figure 3.6). The product node $A*B*C$ has the same input to the first operand node A and the same output as the output from third operand node C , while the two connections do not appear in the product node.

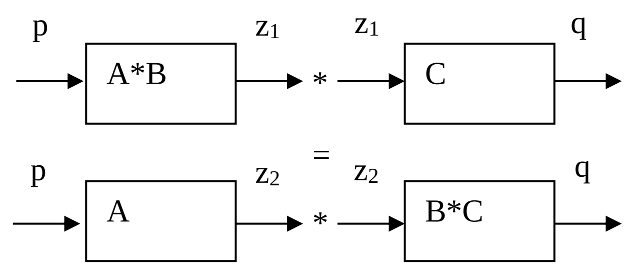

Fig. 3. 5: Associativity property of horizontal merging

Theorem 1[56]***:***

The operation of horizontal merging denoted by the symbol '' is associative in accordance with the following Eq. (3.7)*

$$(A * B) * C = A * (B * C) \quad , \tag{3.7}$$

where the horizontal merging of any three operands A , B and C left to right is equivalent to their horizontal merging from right to left as shown in Fig. 3.6.

3.4.2 Vertical Merging of Rule Bases

Vertical merging is a binary operation that can be applied to a pair of parallel nodes located in the same layer of a fuzzy network. The inputs of the product node represent the union of the inputs of the operand nodes and its outputs represent the union of the output from the operand nodes. When Boolean matrices are used as formal models for these nodes, the product matrix of the vertical merging operation is obtained by expanding each non-zero element from the first operand matrix to a block in the same as the second operand matrix as well as by expanding each zero element from the first operand matrix to a zero block in the same dimension as the second operand matrix. The row labels of the product matrix are all possible permutations of row labels of the operand matrices and the column labels of the product matrix are all permutations of column labels of the operand matrices.

Therefore, if the first operand node is the rule base in Eq. (3.1) is represented by the Boolean matrix in Eq. (3.2); hence, if the second operand is the rule base in Eq. (3.8), then the generalised Boolean matrix is expressed in Eq. (3.9).

$$
\begin{aligned}
&\text{Rule 1: If } w_1 \text{ is } R_{11} \text{ and } \cdots \text{and } w_g \text{ is } R_{g1} \text{ then } y_1 \text{ is } Q_{11} \text{ and } \cdots \text{and } w_h \text{ is } Q_{h1} \\
&\qquad\qquad\vdots \qquad\qquad\qquad \vdots \qquad\qquad\qquad \vdots \\
&\text{Rule } s\text{: If } w_1 \text{ is } R_{1s} \text{ and } \cdots \text{and } w_g \text{ is } R_{gs} \text{ then } y_1 \text{ is } Q_{1s} \text{ and } \cdots \text{and } w_h \text{ is } Q_{hs} \quad ,
\end{aligned}
\tag{3.8}
$$

The generalised Boolean matrix of Eq. (3.8) is described in Eq. (3.9):

$$
\begin{array}{c|ccc}
 & Q_{11}\cdots Q_{h1} & \cdots & Q_{1s}\cdots Q_{hs} \\
R_{11}\cdots R_{g1} & 1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
R_{1r}\cdots R_{gs} & 0 & \cdots & 1
\end{array}
\tag{3.9}
$$

If the product node is the rule given in Eq. (3.10).

Rule 1: If p_1 is S_{11} and $\cdots$and p_m is S_{m1} and w_1 is R_{11} and $\cdots$and w_g is R_{g1} (3.10)

then q_1 is T_{11} and $\cdots$and q_n is T_{n1} and y_1 is Q_{11} and $\cdots$and y_h is Q_{h1}

$$\vdots \qquad\qquad \vdots \qquad\qquad \vdots$$

Rule $r \cdot s$: If p_1 is S_{1r} and $\cdots$and p_m is S_{mr} and w_1 is R_{1s} and $\cdots$and w_g is R_{gs}

then q_1 is T_{1r} and $\cdots$and q_n is T_{nr} and y_1 is Q_{1s} and $\cdots$and y_h is Q_{hs}

Thus, the generalised Boolean matrix of Eq. (3.10) is constructed in Eq. (3.11) as follows:

$$\begin{array}{cccc} & T_{11}\cdots T_{n1}\ Q_{11}\cdots Q_{h1} & \cdots & T_{1r}\cdots T_{nr}\ Q_{1s}\cdots Q_{hs} \\ S_{11}\cdots S_{m1}\ R_{11}\cdots R_{g1} & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ S_{1r}\cdots S_{mr}\ R_{gr}\cdots R_{gs} & 0 & \cdots & 1 \end{array} \tag{3.11}$$

In this case, the fuzzy system described by the rule base in Eq. (3.8) has s rules, g inputs $w_1 \cdots w_g$ taking the linguistic terms from the input sets $\{R_{11}\cdots R_{1s}\},\cdots,\{R_{g1}\cdots R_{gs}\}$, and h outputs $y_1 \cdots y_h$ taking the linguistic terms from the output sets $\{Q_{11}\cdots Q_{1s}\},\cdots,\{Q_{h1}\cdots Q_{hs}\}$. However, the fuzzy system described by the rule base in Eq. (10) is with $r \cdot s$ rules, $m+g$ inputs $x_1 \cdots x_m, w_1 \cdots w_g$ taking the linguistic terms from the input sets $\{S_{11},\cdots,S_{1r}\},\cdots,\{S_{m1},\cdots,S_{mr}\},\{R_{11},\cdots,R_{1s}\},\cdots,\{R_{g1}\cdots R_{gs}\}$, and $n+h$ outputs $q_1,\cdots,q_g, y_1,\cdots,y_h$ taking the linguistic terms from the output sets $\{T_{11},\cdots,T_{1r}\},\cdots,\{T_{n1},\cdots,T_{nr}\},\{Q_{11},\cdots,Q_{1s}\},\cdots,\{Q_{h1}\cdots Q_{hs}\}$. The number of rules in the product rule base is equal to the product of the number of rules in the operand rule bases. For simplicity, the notations used in Fig. 3.7 are expressed in vector forms where the vectors x, y, v, w have dimensions n, m, g, h, respectively.

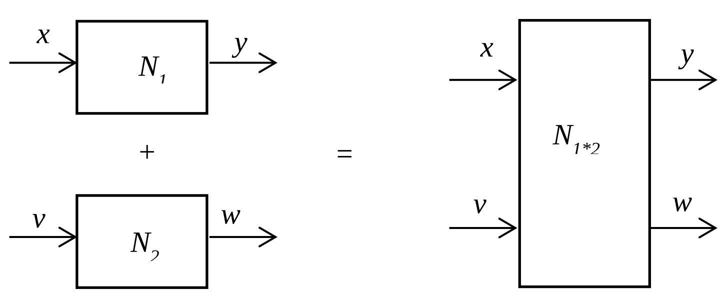

Fig. 3. 6: Vertical merging of rule base

When the property of associativity is related to the operation of vertical merging '+', the latter is applied to three parallel nodes for merging them into a single node. The property can be applied when none of the outputs from the three nodes A, B and C are fed to any of the inputs to those three nodes. In this case, the input set to the node $A+B+C$ is the union of the inputs to the operand nodes A, B and C. Whereas, the output set from the product node is the union of the outputs from the operand nodes.

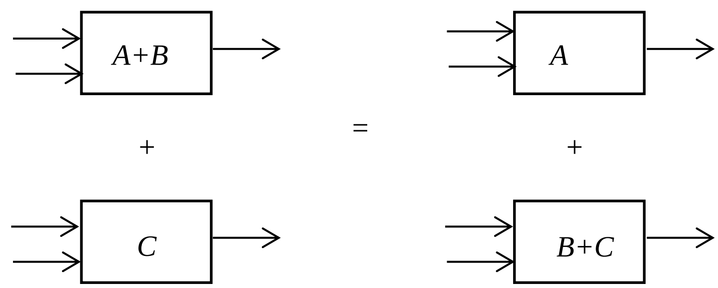

Fig. 3. 7: Associativity property of vertical merging

Theorem 2 [56]***:***

The operation of vertical merging denoted by the symbol '+' is associative in accordance with the following Eq. (3.12)

$$(A+B)+C = A+(B+C) \tag{3.12}$$

Where the vertical merging of any three operands A, B and C, top to bottom, is equivalent to their vertical merging from bottom to top, as shown in Fig. 3.8.

3.6 Established TOPSIS Methods

In this subsection, non- fuzzy TOPSIS and established fuzzy TOPSIS will be reviewed.

3.6.1 Conventional TOPSIS

The conventional TOPSIS introduced in 1981 which is also known as non-fuzzy approach has achieved a prominent acknowledgment in the decision making developments and has been successfully applied in many real problems. In this section, the algorithm taken from [8] does not consider the uncertainty of the information. Nonetheless, the algorithms within conventional TOPSIS are given in the following paragraphs.

TOPSIS assumes that there are *m* alternatives with *n* criteria and score of each alternative with respect to each criterion. Let x_{ij} score of option i with respect to criterion j. Hence, there is a matrix *X* such that $X = (x_{ij})_{mxn}$. Then, let B be the set of benefit attributes or criteria (more is better) and C be the set of negative attributes or criteria (less is better). The essence of the method is now presented as follows:

Step 1: *Construct normalized decision matrix.*

This step transforms various attribute dimensions into non-dimensional attributes, which allows comparisons across criteria. Normalize scores or data as follows:

$$r_{ij} = \frac{x_{ij}}{\sum x_{ij}^{2}} \text{ for } i = 1, \cdots, m; \quad j = 1, \cdots, n$$

Step 2: *Construct the weighted normalized decision matrix.*

If we have a set of weights for each criterion such that w_j for $j = 1, \cdots, n$. Multiply each column of the normalized decision matrix by its associated weight. An element of the new matrix is:

$$v_{ij} = w_{ij}(\cdot)r_{ij}$$

Step 3: *Determine the positive ideal and negative ideal solutions.*

Positive ideal solution.

$$A^{+} = \{v_1^{+}, \cdots, v_n^{+}\} \text{ where}$$

$$v_j^{+} = \{\max_i(v_{ij}) \text{ if } j \in B, \ \min_i(v_{ij}) \text{ if } j \in C\}$$

Negative ideal solution.

$$A^- = \left\{v_1^-, \cdots, v_n^-\right\} \text{ where}$$

$$v_j^- = \{\min_i(v_{ij}) \text{ if } j \in B, \ \max_i(v_{ij}) \text{ if } j \in C\}$$

Step 4: *Calculate the separation measures for each alternative.*

The separation from the ideal alternative is:

$$S_i^+ = \sqrt{(v_j^+ - v_{ij})^2} \text{ for } i = 1, \cdots, m$$

Similarly, the separation from the negative ideal alternative is:

$$S_i^- = \sqrt{(v_j^- - v_{ij})^2} \text{ for } i = 1, \cdots, m$$

Step 5*: Calculate the relative closeness to the ideal solution CCi*

$$CC_i = \frac{S_i^-}{S_i^+ + S_i^-} \text{ for } 0 \le CC_i \le 1$$

Select the option with CC_i closest to 1.

3.6.2 Non-Rule Based Type-1 Fuzzy TOPSIS

Type-1 TOPSIS was introduced for the first time in fuzzy environment in 2000 by Chen [11]. The flow of the method is now presented as follows:

Step 1*: Construct Decision Matrix* $\tilde{D}$ *and Weight Matrix* $\tilde{W}$

Assume that a decision group has K persons and the importance of the criteria and the rating of alternatives with respect to each criterion can be calculated as

$$\tilde{x}_{ij} = \frac{1}{K}[\tilde{x}_{ij}^1 (+) \tilde{x}_{ij}^2 (+) \cdots (+) \tilde{x}_{ij}^K]$$

$$\tilde{w}_j = \frac{1}{K}[\tilde{w}_j^1 (+) \tilde{w}_j^2 (+) \cdots (+) \tilde{w}_j^K]$$

where $\tilde{x}_{ij}^K$ and $\tilde{w}_j^K$ are the rating and the importance weight of the K^{th} decision maker. Multi-criteria decision making problem can be easily expressed in a matrix format as

$$\tilde{D} = \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \cdots & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & \cdots & \tilde{x}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{x}_{m1} & \tilde{x}_{m2} & \cdots & \tilde{x}_{mn} \end{bmatrix}$$

$$\tilde{W} = [\tilde{w}_1 \quad \tilde{w}_2 \quad \cdots \quad \tilde{w}_n]$$

where $\tilde{x}_{ij}$ and $\tilde{w}_j$ are the linguistic terms. These linguistic terms can be described by fuzzy numbers of $\tilde{x}_{ij} = (a_{ij}, b_{ij}, c_{ij})$ and $\tilde{w}_j = (w_{j1}, w_{j2}, w_{j3})$

Step 2: *Construct Normalized Fuzzy Decision Matrix,* $\tilde{R}$

For making various scales comparable, the linear scale transformation is used to construct normalized fuzzy decision matrix as

$$\tilde{R} = [\tilde{r}_{ij}]_{mxn} \text{ where}$$

$$\tilde{r}_{ij} = \left(\frac{a_{ij}}{c_j^*}, \frac{b_{ij}}{c_j^*}, \frac{c_{ij}}{c_j^*} \right), \quad j \in B;$$

$$\tilde{r}_{ij} = \left(\frac{a_j^-}{c_{ij}}, \frac{a_j^-}{b_{ij}}, \frac{a_j^-}{a_{ij}}, \right), \; j \in C;$$

$$c_j^* = \max_i c_{ij} \text{ if } j \in B;$$

$$a_j^- = \min_i a_{ij} \text{ if } j \in C;$$

where B and C are the sets of benefit criteria and cost criteria, respectively, and the technique mentioned above is to preserve the property so that the ranges of normalized fuzzy numbers belong to $[0,1]$.

Step 3: *Construct the Weighted Normalized Fuzzy Decision Matrix,* $\tilde{V}$

By considering the different importance of each criterion, we can construct the weighted normalized fuzzy decision matrix as

$$\tilde{V} = [\tilde{v}_{ij}]_{m \times n} \; i = 1,2,\ldots,m \text{ and } j = 1,2,\ldots,n$$

where $\tilde{v}_{ij} = \tilde{r}_{ij}(\cdot)\tilde{w}_j$.

Step 4: *Find Fuzzy Positive-Ideal Solution,* A^* *and Fuzzy Negative-Ideal Solution,* A^-

Based on the weighted normalized fuzzy decision matrix, the elements $\tilde{v}_{ij}$, for all i and j are normalized positive triangular fuzzy numbers and their ranges belong to the closed interval $[0,1]$. Then, we can define the fuzzy positive-ideal solution and fuzzy negative-ideal solution as

$$A^* = (\tilde{v}_1^*, \tilde{v}_2^*, \ldots, \tilde{v}_n^*),$$

$$A^- = (\tilde{v}_1^-, \tilde{v}_2^-, \ldots, \tilde{v}_n^-),$$

where $\tilde{v}_j^* = (1,1,1)$ and, $\tilde{v}_j^- = (0,0,0)$ for $j = 1,2,\ldots,n$.

Step 5*: Find Distance of Each Alternative from* A^+ *and* A^-

The distance of each alternative from A^+ and A^- can be simultaenously calculated as

$$d_i^+ = \sum_{j=1}^{n} d(\tilde{v}_{ij}, \tilde{v}_j^+),\ i = 1,2,\ldots,m,$$

$$d_i^- = \sum_{j=1}^{n} d(\tilde{v}_{ij}, \tilde{v}_j^-),\ i = 1,2,\ldots,m,$$

where $d(\cdot,\cdot)$ is the distance measurement between two fuzzy numbers.

Step 6: *Find Closeness Coefficient,* CC_i

A closeness coefficient is defined to determine the ranking order of all alternatives once the d_i^+ and d_i^- of each alternative A_i for $i = 1,2,\ldots,m$ have been calculated. The closeness coefficient of each alternative is calculated as

$$CC_i = \frac{d_i^-}{d_i^+ + d_i^-},\ i = 1,2,\ldots,m$$

Obviously, the alternative A_i is closer to A^+ and farther from A^- as CC_i approaches 1. Therefore, according to the closeness coefficient, we can determine the ranking order of all alternatives and select the best one from among a set of feasible alternatives.

3.6.3 Non-Rule Based Interval Type-2 Fuzzy TOPSIS

Interval type-2 TOPSIS is introduced for the first time in [112].The essence of the method is presented as follows:

Step 1: *Construct Fuzzy Decision Matrix,* (D_K) *and Fuzzy Weight of Alternative* (W_K) *as*

$$(D_K) = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix} \text{ and}$$

$$W_K = [w_1 \quad w_2 \quad \cdots \quad w_n]$$

where x_{ij} and w_i are interval type-2 fuzzy set based from Table 1-2 respectively. These intervals represent the rating and the important weights of the K^{th} decision maker of alternative A_i with respect to criterion C_j $(j = 1, \cdots, n)$ respectively.

Step 2: Weighted fuzzy decision matrix (V_K)

The weighted fuzzy decision matrix (V_K) is as follows:

$$V_K = \lfloor v_{ij} \rfloor_{m \times n}$$

for $i = 1, \cdots, m$ and $j = 1, \cdots, n$

where $v_{ij} = x_{ij}(\cdot)w_{ij}$ is a multiplication of interval type-2 fuzzy set.

Step 3: *Construct the ranking weighted decision matrix*

Calculate the ranking value $Rank_K(A_i)$ [130]. Finding this value, the maximum number S of edges in the upper membership function v_{ij}^U and the lower membership function v_{ij}^L of interval type–2 fuzzy set v_{ij} are defined, where $1 \le i \le n$ and $1 \le j \le m$. If S is odd number and $s \ge 3$, then $r = s+1$. If S is even number and $s \ge 4$, then $r = s$.

The $Rank(A_i)$ of interval type-2 fuzzy set is shown below.

$$Rank(A_i) = \sum_{j\in\{U,L\}} M_1\left(A_i^U\right) + \sum_{j\in\{U,L\}} M_2\left(A_i^j\right) + \cdots + \sum_{j\in\{U,L\}} M_{r-1}\left(A_i^j\right)$$
$$-\frac{1}{r}\left(\sum_{j\in\{U,L\}} S_1\left(A_i^j\right) + \sum_{j\in\{U,L\}} S_2\left(A_i^j\right) + \cdots + \sum_{j\in\{U,L\}} S_r\left(A_i^j\right)\right)$$
$$+\sum_{j\in\{U,L\}} H_1\left(A_i^j\right) + \sum_{j\in\{U,L\}} H_2\left(A_i^j\right) + \cdots + \sum_{j\in\{U,L\}} H_{r-2}\left(A_i^j\right)$$

where $M_p\left(A_i^j\right)$ denotes the average of the elements a_{ip}^j and $a_{i(p+1)}^j$

$$M_p\left(A_i^j\right) = \left(a_{ip}^j + a_{i(p+1)}^j\right) \Big/ 2 \qquad \text{for } 1 \le p \le r-1$$

$$S_q\left(A_i^j\right) = \sqrt{\frac{1}{2}\sum_{k=q}^{q+1}\left(a_{ik}^j - \frac{1}{2}\sum_{k=q}^{q+1} a_{ik}^j\right)^2} \qquad \text{for } 1 \le p \le r-1$$

The $S_r\left(A_i^j\right)$ denotes the standard deviation of the elements $a_{i1}^j, a_{i2}^j, \cdots, a_{ir}^j$

$$S_r\left(A_i^j\right) = \sqrt{\frac{1}{r}\sum_{k=1}^{r}\left(a_{ik}^j - \frac{1}{r}\sum_{k=1}^{r} a_{ik}^j\right)^2}$$

The $H_p\left(A_i^j\right)$ denotes the membership value of the element $a_{i(p+1)}^j$, $1 \le p \le r-2$, $j \in \{U, L\}$ and r is even number.

Step 4: *The fuzzy positive ideal solution* $\left(A^+\right)$ *and the fuzzy negative ideal solution* $\left(A^-\right)$ is as shown below:

$$A^+ = \left(v_1^+, v_2^+, \cdots, v_n^+\right) \quad \text{and} \quad A^- = \left(v_1^-, v_2^-, \cdots, v_n^-\right)$$

where

$$v_i^+ = \begin{cases} \max\limits_{1\le j\le n}\left\{Rank\left(v_{ij}\right)\right\}, x_i \in B \\ \min\limits_{1\le j\le n}\left\{Rank\left(v_{ij}\right)\right\}, x_i \in C \end{cases}$$

and

$$v_i^- = \begin{cases} \min_{1\le j\le n}\{Rank(v_{ij})\}, x_i \in B \\ \max_{1\le j\le n}\{Rank(v_{ij})\}, x_i \in C \end{cases}$$

where B denotes the set of benefit attribute and C denotes the set of cost attribute and $1 \le i \le n$

Calculate the distance $d^+(A_i)$ between each alternative A_i and the fuzzy positive ideal solution A^+.

$$d^+(A_i) = \sqrt{\sum_{i=1}^{m}\left(Rank(v_{ij}) - v_i^+\right)^2} \text{ for}$$

$1 \le j \le n$

Calculate the distance $d^-(A_i)$ between each alternative A_i and the fuzzy negative ideal solution A^-.

$$d^-(A_i) = \sqrt{\sum_{i=1}^{m}\left(Rank(v_{ij}) - v_i^-\right)^2} \text{ for } 1 \le j \le n$$

Step 5: *The closeness coefficient* (CC_i)

Calculate the relative degree of closeness (CC_i) of A_i calculated as

$$CC_i = \frac{d_i^-}{d_i^+ + d_i^-} \text{ for } i = 1, \cdots, m$$

Therefore, from the value of Γ, the ranking order of all alternatives can be determined. The best alternative has higher value of Γ.

3.6.4 Non-Rule Based Z-Fuzzy TOPSIS

Most recently, Z-TOPSIS was introduced by [117]. The flow of the method is presented as follows:

Step 1: *Use the information from Table 3 to derive Component B, and then convert z-number to type-1 fuzzy number*

Assume a Z-number, $Z=(\tilde{A},\tilde{B})$, Let $\{\tilde{A}=(x,\mu_{\tilde{A}})\,|\,x\in[0,1]\},\{\tilde{B}=(x,\mu_{\tilde{B}})\,|\,x\in[0,1]\}$, where $\mu_{\tilde{A}}$ and $\mu_{\tilde{B}}$ are triangular membership functions. The second part (reliability) needs to be converted into crisp number using fuzzy expectation as

$$\alpha=\frac{\int x\mu_{\tilde{B}}dx}{\int \mu_{\tilde{B}}dx}$$

where $\int$ denotes an algebraic integration. Then, add the weight of the second part (reliability) to the first part (restriction). The weighted Z-number can be denoted as

$$\tilde{Z}^{\alpha}=\{(x,\mu_{\tilde{A}^{\alpha}})\,|\,\mu_{\tilde{A}^{\alpha}}(x)=\alpha\mu_{\tilde{A}}(x),x\in[0,1]\}$$

These numbers can be type-1 fuzzy number such that

$$\tilde{Z}^{'}=\{<x,\mu_{\tilde{Z}^{\alpha}}(x)>|\,\mu_{\tilde{Z}^{\alpha}}(x)=\mu_{\tilde{A}}(\frac{x}{\sqrt{\alpha}}),x\in[0,1]\}$$

[35] has proved that $\tilde{Z}^{'}$ has the same Fuzzy Expectation with $\tilde{Z}^{\alpha}$.

Step 2: *Construct Decision Matrix* $\tilde{D}$ *and Weight Matrix* $\tilde{W}$

If a decision group has K persons, then the importance of the criteria and the rating of alternatives with respect to each criterion can be calculated as

$$\tilde{x}_{ij}=\frac{1}{K}[\tilde{x}_{ij}^{1}(+)\tilde{x}_{ij}^{2}(+)\cdots(+)\tilde{x}_{ij}^{K}]$$
$$\tilde{w}_{j}=\frac{1}{K}[\tilde{w}_{j}^{1}(+)\tilde{w}_{j}^{2}(+)\cdots(+)\tilde{w}_{j}^{K}]$$

where $\tilde{x}_{ij}^{K}$ and $\tilde{w}_{j}^{K}$ are the rating and the importance weight of the K^{th} decision maker. Multi-criteria decision making problem can be easily expressed in matrix format as

$$\tilde{D}=\begin{bmatrix}\tilde{x}_{11} & \tilde{x}_{12} & \cdots & \tilde{x}_{1n}\\ \tilde{x}_{21} & \tilde{x}_{22} & \cdots & \tilde{x}_{2n}\\ \vdots & \vdots & \ddots & \vdots\\ \tilde{x}_{m1} & \tilde{x}_{m2} & \cdots & \tilde{x}_{mn}\end{bmatrix}$$

$$\tilde{W}=\begin{bmatrix}\tilde{w}_{1} & \tilde{w}_{2} & \cdots & \tilde{w}_{n}\end{bmatrix}$$

where $\tilde{x}_{ij}$ for all i, j and $\tilde{w}_j$, $j = 1,2,\cdots,n$ are the linguistic terms. These terms can be described by fuzzy numbers of $\tilde{x}_{ij} = (a_{ij}, b_{ij}, c_{ij})$ and $\tilde{w}_j = (w_{j1}, w_{j2}, w_{j3})$

Step 3: *Construct Normalized Fuzzy Decision Matrix* $\tilde{R}$,

For making various scales comparable, the linear scale transformation is used to construct normalized fuzzy decision matrix as

$$\tilde{R} = [\tilde{r}_{ij}]_{mxn}$$

where B and C are the set of benefit criteria and cost criteria, respectively, and

$$\tilde{r}_{ij} = \left(\frac{a_{ij}}{c_j^*}, \frac{b_{ij}}{c_j^*}, \frac{c_{ij}}{c_j^*} \right), \quad j \in B;$$

$$\tilde{r}_{ij} = \left(\frac{a_j^-}{c_{ij}}, \frac{a_j^-}{b_{ij}}, \frac{a_j^-}{a_{ij}}, \right), \; j \in C;$$

$$c_j^* = \max_i d_{ij} \text{ if } j \in B; \; a_j^- = \min_i a_{ij} \text{ if } j \in C;$$

The normalization technique is to preserve the property so that the ranges of normalized fuzzy numbers belong to $[0,1]$.

Step 4: *Construct the Weighted Normalized Fuzzy Decision Matrix,* $\tilde{V}$

By considering the different importance of each criterion, we can construct the weighted normalized fuzzy decision matrix as

$$\tilde{V} = [\tilde{v}_{ij}]_{m \times n} \; i = 1,2,\ldots,m \text{ and } j = 1,2,\ldots,n$$

where $\tilde{v}_{ij} = \tilde{r}_{ij}(\cdot)\tilde{w}_j$.

Step 5: *Find Fuzzy Positive-Ideal Solution,* A^* *and Fuzzy Negative-Ideal Solution,* A^-

Based on the weighted normalized fuzzy decision matrix, the elements $\tilde{v}_{ij}$, for all i and j are normalized positive triangular fuzzy numbers and their ranges belong to the closed interval $[0,1]$. Then, we can define the fuzzy positive-ideal solution and fuzzy negative-ideal solution as

$A^* = (\tilde{v}_1^*, \tilde{v}_2^*, \ldots, \tilde{v}_n^*)$,

$A^- = (\tilde{v}_1^-, \tilde{v}_2^-, \ldots, \tilde{v}_n^-)$,

where $\tilde{v}_j^* = (1,1,1)$ and, $\tilde{v}_j^- = (0,0,0)$ for $j = 1,2,\ldots,n$.

Step 6: *Find Distance of Each Alternative from A^* and A^-*

The distance of each alternative from A^* and A^- can be calculated as

$$d_i^* = \sum_{j=1}^{n} d(\tilde{v}_{ij}, \tilde{v}_j^*),\ i = 1,2,\ldots,m,$$

$$d_i^- = \sum_{j=1}^{n} d(\tilde{v}_{ij}, \tilde{v}_j^-),\ i = 1,2,\ldots,m,$$

where $d(\cdot,\cdot)$ is the distance measurement between two fuzzy numbers.

Step 7: *Find Closeness Coefficient, CC_i*

A closeness coefficient is defined to determine the ranking order of all alternatives once the d_i^* and d_i^- of each alternative A_i for $i = 1,2,\ldots,m$ have been calculated. The closeness coefficient of each alternative is calculated as

$$CC_i = \frac{d_i^-}{d_i^* + d_i^-},\ i = 1,2,\ldots,m$$

Obviously, the alternative A_i is closer to A^* and farther from A^- as CC_i approaches 1. Therefore, according to the closeness coefficient, we can determine the ranking order of all alternatives and select the best one from among a set of feasible alternatives.

3.7 Assessing Ranking Performance

In science and technology researches, validating or assessing the performance of proposed methods is very important. In this research, the author has considered four established performance indicators to validate and to measure the accuracy of the proposed methods, namely Spearman rho correlation coefficient, Kendall'Tau correlation, RMSE and Average Absolute

Distance coefficient. The detail explanations on these indicators will be discussed in the following subsections.

3.7.1 *Spearman's Rho Correlation*

The rankings are compared descriptively using Spearman rho correlation. The advantages of this correlation method are its easy algebraic structure and intuitively simple interpretation. Moreover, the method is less sensitive to bias due to the effect of outliers and can be used to reduce the weight of outliers (large distances get treated as a one-rank difference) [30], [100], [114], [131]–[134]. In general, the coefficient of rho (ρ) measures the strength of association between two ranked variables. The formula used to calculate Spearman's Rank is shown below.

$$\rho = 1 - \frac{6\sum \partial_i^2}{n^3 - n}$$

Where ∂_i represents the difference between the ranks and n donated number of alternatives considered. The Spearman correlation coefficient, ρ can take values between +1 to -1. A perfect relation, a zero relation and a negative relation to the ranks are indicated by $\rho = 1$, $\rho = 0$ and $\rho = -1$, respectively. Therefore, the closer ρ is to zero, the weaker the relationship between the ranks.

3.7.2 *Kendall'Tau Correlation*

The rankings are compared descriptively using Kendall'Tau rank correlation (τ)[135]. The advantages of Kendall tau correlation are its easy algebraic structure and intuitively simple interpretation. In general, the coefficient of tau shows the degree of concordances between two columns of ranking data. The Tau Coefficient can be determined by

$$\tau = \frac{\sum G_{ij} - \sum J_{ij}}{\sum G_{ij} + \sum J_{ij}}$$

where G_{ij} and J_{ij} represent the concordance pair and the discordances pair, respectively. In particular, the concordance pair interprets the number of observed ranks below a particular rank which are larger than that particular rank, whereas the discordance is the number of observed ranks below a particular rank which are smaller than that particular rank [76], [135]–[139]. Testing the significant of the rank, the statistical z-score defined by [140] is as follows.

$$z = \frac{3 * \tau * \sqrt{n(n-1)}}{\sqrt{2(2n+5)}}$$

Obviously, the statistical z-score shows how far that data is from the mean. The distance from the mean is measured in term of a standard deviation. The bigger the z- score value, the more significant the ranking to the actual ranking.

3.7.3 Root Mean Square Errors

The rankings are also compared descriptively using root mean square error (RMSE). RMSE is frequenly used to measure the diference between prediction ranking and actual ranking as benchmark. The RMSE value can be derived as follows.

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(X_{obs,i} - X_{model,i})^2}{n}}$$

Where X_{obs} is the observed values and X_{model} is the modelled values at time or place of i. RMSE serves to aggregate the ranking into a single measure of predictive power and its values can act as an indicator to evaluate a method's performance. A small *RMSE* value is better and closer to the ranking of benchmark rank.

3.7.4 Average Absolute Distance

The performance measure is the average absolute distance (AAD)[141]. The accuracy of ranking is the similarity distance between every two rankings expressed as follow.

$$AAD_M = \frac{1}{n}\sum_{i=1}^{n} \left|\Re_i - \aleph_i\right|$$

$\Re_i$ represents the ranking of alternative i^{th} for $i = 0, \cdots, n$ based on the tested ranking method, *M*, whereas $\aleph_i$ represents the ranking of benchmarking. The smaller ADD_M , the better and closer to the ranking of benchmark rank [142]–[144].

3.8 Summary

In conclusion, the theoretical preliminaries of this thesis are presented which covers definitions, terminology and fuzzy concepts including the reviews of the basic types of fuzzy systems and fuzzy networks and performance indicators. In Chapter 4, the thesis will discuss the development of TOPSIS methodology based on fuzzy systems with single rule base.

CHAPTER 4

4 FUZZY SYSTEM APPROACH WITH SINGLE RULE BASE

4.1 Introduction

In this chapter, the novel version of fuzzy TOPSIS is presented using fuzzy system with single rule base. This approach has capability of treating conflict in MCDM by representing decision maker opinion as fuzzy rules. In this approach, different influence degrees for each decision maker and various importance degrees assigned to each variable are analysed. In order to generate a fuzzy rule base, some adaptations of established fuzzy TOPSIS methods [11], [112], [117] are made. Along that, their extensions to type-2 and Z-numbers implementation are presented.

A fuzzy system consists of a single rule base where inputs are simultaneously processed is shown in Fig. 4.1, where $\{CR_1 \cdots CR_n\}$ is the set of inputs. In this case, the multiple inputs consist of benefit and cost criteria are joined together in a system called Alternatives Rule Base which has multiple inputs and single output. In addition, the rules for this system are derived based on the experts' knowledge about the decision making process.

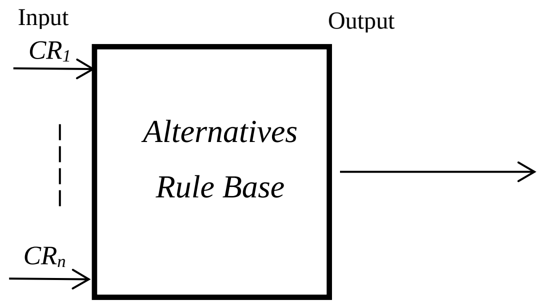

Fig. 4. 1: Fuzzy system model using single rule base

The use of techniques associated with the empirical knowledge of experts allowing a hybrid analysis of the decision making problems where the process of decision making requires the use of human sensitivity, which often can be expressed by a fuzzy rules base. The influence level of this rule is defined by influence degree that the criteria will receive in the analysis of the problem. The authors have adopted the methods described in [104] for the knowledge of the influence degree of each decision maker. In a case where one decision maker has more knowledge regarding the domain, consequently the opinion of this expert may have more importance than the other decision makers in analysing the problem. Thus, the proposed method can identify and aggregate the different opinions of decision makers with varying influence degrees to suggest the final solution.

This chapter is organized in four sections; Section 4.2 is the formulation and the explanation of fuzzy TOPSIS using fuzzy system with single rule base approach followed by Section 4.3 and 4.4 where the extended implementation of type-2 and Z-number are constructed respectively. The last section is the summary of this chapter.

4.2 Type-1 Fuzzy Set Implementation

In this section, the author has introduced some modifications to the established fuzzy flexi TOPSIS in [104] since it is too complex for computation and understanding. However, the idea of representing the influence degree of decision makers as fuzzy rule in decision process is adopted in this research.

The decision makers ask to evaluate their influence degree or knowledge in the domain themselves.

Basically, the concepts of fuzzy rules have a capability to represent the uncertainty of information into decision making evaluation. Enhancing this capability to deal with vagueness and effectively representing decision information, this research proposes additional element that represents the evaluation of system with single rule base in established T1-TOPSIS called T1-SFS TOPSIS. The main objective of this modification is to introduce the ability of fuzzy system with single rule base in the established T1-TOPSIS [11]. Thus, the evaluation by this proposed method allows the empirical knowledge of the expert, represented by fuzzy rule, being considered in the decision making process.

The established fuzzy TOPSIS method from [13] is displayed in Table 4.1 and Table 4.2 representing the importance of criteria and the rating of the alternative. On the other hand, Table 4.3 is proposed in this section is used to identify the alternative level for the consequent part of the fuzzy rule. The 4th parameter in triangular type 1 fuzzy number represent the height of membership degree is 1.

Table 4. 1: Linguistic terms for importance weight of each criterion

Linguistic Terms		Triangular Type 1 Fuzzy Number
Very Low (VL)	1	(0.00,0.00,0.10,1)
Low (L)	2	(0.00,0.10,0.25,1)
Medium Low (ML)	3	(0.15,0.30,0.45,1)
Medium (M)	4	(0.35,0.50,0.65,1)
Medium High (MH)	5	(0.55,0.70,0.85,1)
High (H)	6	(0.80,0.90,1.00,1)
Very High (VH)	7	(0.90,1.00,1.00,1)

Table 4. 2: Linguistic terms for rating of all alternative

Linguistic Terms		Triangular Type 1 Fuzzy Number
Very Poor (VP)	1	(0,0,1,1)
Poor (P)	2	(0,1,3,1)
Medium Poor (MP)	3	(1,3,5,1)
Fair (F)	4	(3,5,7,1)
Medium Good (MG)	5	(5,7,9,1)
Good (G)	6	(7,9,10,1)
Very Good (VG)	7	(9,10,10,1)

The linguistic terms that represents the consequents of rules was named "Alternative Level" and is represented by fuzzy sets "Very bad", "Bad", "Regular", "Good" and "Very Good".

Table 4. 3: Linguistic terms for alternative level

Linguistic Terms		Triangular Type 1Fuzzy Number
Very Bad(VB)	1	(0.00,0.00,0.25,1)
Bad (B)	2	(0.00,0.25,0.50,1)
Regular (R)	3	(0.25,0.50,0.75,1)
Good (G)	4	(0.5,0.75,1.00,1)
Very Good (VG)	5	(0.75,1.00,1.00,1)

The following algorithm is conducted to obtain the ranking of alternatives, where step 1-5 are adopted from [11]. Dealing with the influence degree of decision maker, step 6-11 are introduced benefitting the rule-based approach.

T1-SFS TOPSIS algorithm

Step 1: Construct decision matrix where each decision maker opinion is evaluated independently

In the decision matrices D_k and weight matrices W_k $(k = 1, \cdots, K)$, n is assumed to be the number of criteria as shown in Eq. (4.1).

$$D_k = \begin{array}{c} \\ CR_1 \\ CR_2 \\ \vdots \\ CR_n \end{array} \begin{array}{c} \begin{array}{cccc} A_1 & A_2 & \cdots & A_j \end{array} \\ \begin{bmatrix} x_{11,k} & x_{12,k} & \cdots & x_{1m,k} \\ x_{21,k} & x_{22,k} & \cdots & x_{2m,k} \\ \vdots & \vdots & \ddots & \vdots \\ x_{e1,k} & x_{e2,k} & \cdots & x_{em,k} \end{bmatrix} \end{array} ;\ \text{for } i = 1,\cdots,n \text{ and } j = 1,\cdots,m \tag{4.1}$$

$$W_k = \begin{array}{c} \begin{array}{cccc} CR_1 & CR_2 & \cdots & CRn \end{array} \\ \begin{bmatrix} g_{1,k} & g_{2,k} & \cdots & g_{e,k} \end{bmatrix} \end{array} \quad , \text{ for } k = 1,\cdots,K.$$

$x_{ij,k}$ is type-1 fuzzy sets representing the rating of alternatives A_j $(j = 1,\cdots,m)$ with respect to criteria CR_i $(i = 1,\cdots,n)$ according to the k^{th} decision maker. $g_{i,k}$ is type-1 fuzzy sets representing the weights of criteria CR_i $(i = 1,\cdots,n)$ according to the k^{th} decision maker, where $k = 1,\cdots,K$.

Step 2: Construct the weighted and the normalized decision matrices

The fuzzy rating and weight of each criterion are the linguistic terms described with type-1 triangular fuzzy numbers. The rating of alternatives A_j $(j = 1,\cdots,m)$ is described with type-1 triangular fuzzy numbers $x_{ij,k} = \left(a_{ij,k}^{x}, b_{ij,k}^{x}, c_{ij,k}^{x}\right)$, while the importance of criteria CR_i $(i = 1,\cdots,n)$ is represented by $g_{i,k} = \left(a_{i,k}^{g}, b_{i,k}^{g}, c_{i,k}^{g}\right)$, for $k = 1,\cdots,K$. The normalized fuzzy decision matrices R_k and weighted normalized fuzzy decision matrices V_k are calculated as shown in Eq. (4.2) where B and C are the sets of benefit criteria and cost criteria, respectively.

$$R_k = \left[r_{ij,k}\right]_{e\times m} \ , \tag{4.2}$$

$$r_{ij,k} = \begin{cases} r_{ij.k}^{Benefit} = \left(\dfrac{a_{ij,k}^{x}}{c_{i,k}^{x*}}, \dfrac{b_{ij,k}^{x}}{c_{i,k}^{x*}}, \dfrac{c_{ij,k}^{x}}{c_{i,k}^{x*}}\right), for\, Benefit_i \in B \\ \\ r_{ij.k}^{Cost} = \left(\dfrac{a_{i,k}^{x*}}{c_{ij,k}^{x}}, \dfrac{a_{i,k}^{x*}}{b_{ij,k}^{x}}, \dfrac{a_{i,k}^{x*}}{a_{ij,k}^{x}}\right), for\, Cost_i \in C \end{cases}$$

$$c_{i,k}^{x*} = \max_j c_{ij,k}^x, \ (i=1,\cdots,n), \ (j=1,\cdots,m)$$

$$a_{i,k}^{x*} = \min_j a_{ij,k}^x, \ (i=1,\cdots,n), \ (j=1,\cdots,m)$$

Next, the weighted matrix, V_k is constructed as follow

$$V_k = [v_{ij,k}]_{n\times m} \ ,$$

$$V_k = \begin{array}{c c} & \begin{array}{cccc} A_1 & A_2 & \cdots & A_j \end{array} \\ \begin{array}{c} CR_1 \\ CR_2 \\ \vdots \\ CR_n \end{array} & \begin{bmatrix} v_{11,k} & v_{12,k} & \cdots & v_{1m,k} \\ v_{21,k} & v_{22,k} & \cdots & v_{2m,k} \\ \vdots & \vdots & \ddots & \vdots \\ v_{e1,k} & v_{e2,k} & \cdots & v_{em,k} \end{bmatrix} \end{array} \text{; for } i=1,\cdots,n \text{ and } j=1,\cdots,m$$

where

$$v_{ij.k} = r_{ij,k}(\cdot) g_{i,k}$$

and

$v_{ij,k} = (a_{ij,k}^v, b_{ij,k}^v, c_{ij,k}^v)$ are type-1 fuzzy sets, for $k=1,\cdots,K$.

Step 3: Find the Fuzzy Positive Ideal Solution (FPIS) and Fuzzy Negative Ideal Solution (FNIS) for each alternative.

The FPIS and FNIS solutions are correspondingly $A_k^+ = (v_{1,k}^+, v_{2,k}^+, \cdots, v_{n,k}^+)$ and $A_k^- = (v_{1,k}^-, v_{2,k}^-, \cdots, v_{n,k}^-)$, where $v_{i,k}^+ = (1 \quad 1 \quad 1 \quad)$ and $v_{i,k}^- = (0 \quad 0 \quad 0 \quad)$ are Type-1 fuzzy sets, for $k=1,\cdots,K$ and $i=1,\cdots,n$

Step 4: Find the distance between each alternative to FPIS and FNIS.

The distance for criteria of each alternative *j*, A_j from A_k^+ is $\Delta_{j,k}^+$, is calculated in Eq. (4.3).

$$\Delta_{j,k}^+ = \sum_{i=1}^{n} \Delta_k (v_{ij,k}, v_{i,k}^+) \tag{4.3}$$

where

$$\Delta_k (v_{ij,k}, v_{i,k}^+) = \sqrt{\frac{1}{3}\left[(a_{ij,k}^v - 1)^2 + (b_{ij,k}^v - 1)^2 + (c_{ij,k}^v - 1)^2\right]} \ ,$$

for $j = 1, \cdots, m$, and $k = 1, \cdots, K$.

The distance for criteria of each alternative j, A_j, from A_k^- is $\Delta_{j,k}^-$, is calculated in Eq. (4.4).

$$\Delta_{j,k}^- = \sum_{i=1}^{n} \Delta_k \left(v_{ij,k}, v_{i,k}^- \right) \text{, where} \tag{4.4}$$

$$\Delta_k \left(v_{ij,k}, v_{i,k}^- \right) = \sqrt{\frac{1}{3} \left[\left(a_{ij,k}^v - 0 \right)^2 + \left(b_{ij,k}^v - 0 \right)^2 + \left(c_{ij,k}^v - 0 \right)^2 \right]} \text{ ,}$$

for $j = 1, \cdots, m$, and $k = 1, \cdots, K$.

Step 5: Find the Closeness Coefficients (CC).

The closeness coefficients $CC_{j,k}$ for the systems is calculated in Eq. (4.5):

$$CC_{j,k} = \frac{\Delta_{j,k}^-}{\Delta_{j,k}^+ + \Delta_{j,k}^-} \tag{4.5}$$

for $j = 1, \cdots, m$ and $k = 1, \cdots, K$.

Step 6: Derive the Influenced Closeness Coefficient (ICC) by applying the influence degree of each decision maker, then find Normalized ICC (NICC), divide it by its maximum value.

Let θ_k denotes the influence degree, between 0 (un-influential) and 10 (very influential) of decision maker k, where $k = 1, \cdots, K$. Next, let σ_k stands for the normalized influence degree of the k^{th} decision maker, $k = 1, \cdots, K$, which evaluated using Eq. (4.6):

$$\sigma_k = \theta_k \Big/ \sum_{l=1}^{K} \theta_l \text{ , for } k = 1, \cdots, K \text{ . and } l = 1, \cdots, K \tag{4.6}$$

Eq. (4.7) evaluates the influence closeness coefficients $ICC_{j,k}^B$ for each DM k, respectively along the criteria.

$$ICC_{j,k} = \sigma_k \times CC_{j,k} \tag{4.7}$$

for $j = 1,\cdots,m$ and $k = 1,\cdots,K$.

Normalising the coefficients is necessary to ensure that their values vary between 0 to 1. Accordingly, the Eq. (4.8) evaluates the normalized coefficients, where $NICC_{j,k}$ is the normalized influence closeness coefficients for the systems, as related to the k^{th} decision maker.

$$NICC_{j,k} = {ICC_{j,k}}\Big/{\max_{j} ICC_{j,k}} \tag{4.8}$$

for $j = 1,\cdots,m$ and $k = 1,\cdots,K$.

The $NICC_{j,k}$ will take the linguistic terms from Table 4.3 for the level of alternatives performance.

Step 7: Construct the antecedent matrices for the single system based on DMs opinions

Having the opinions D_k of all DMs $(k = 1,\cdots,K)$ on each alternative j $(j = 1,\cdots,m)$ with respect to each criterion $i\,(i = 1,\cdots,n)$, we can define the antecedent matrix X_k for each DM k, as given in Eq. (4.9) and $\hat{x}_{ij,k}$ is the linguistic terms describing decision makers' opinion.

$$X_k = \begin{array}{c} \\ CR_1 \\ CR_2 \\ \vdots \\ CR_n \end{array} \begin{array}{c} \begin{array}{cccc} A_1 & A_2 & \cdots & A_j \end{array} \\ \begin{bmatrix} \hat{x}_{11,k} & \hat{x}_{12,k} & \cdots & \hat{x}_{1m,k} \\ \hat{x}_{21,k} & \hat{x}_{22,} & \cdots & \hat{x}_{2m,k} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{x}_{n1,k} & \hat{x}_{n2,k} & \cdots & \hat{x}_{nm,k} \end{bmatrix} \end{array} \text{ for } k = 1,\cdots,K\,, \tag{4.9}$$

Step 8: Construct the consequent matrices for the single system based on the value of the NICC coefficients.

After the $NICC_j^k$ coefficients for all decision makers $(k = 1,\cdots,K)$ are determined, the consequent matrix Λ_k is then defined in Eq. (4.10).

$$\Lambda_k = \begin{array}{c} \begin{matrix} A_1 & A_2 & \cdots & A_j \end{matrix} \\ \begin{bmatrix} \lambda_{1,k} & \lambda_{2,k} & \cdots & \lambda_{m,k} \end{bmatrix} \end{array} \text{ for } k = 1,\cdots,K \text{ and } j = 1,\cdots,m \tag{4.10}$$

where $\lambda_{j,k}$ is the linguistic terms representing the output of the system, based respectively on the values of $NICC_j^k$

Step 9: Derive rule bases for each alternative

The rule base for each DM is constructed using Eq. (4.9) and Eq. (4.10) is expressed in Eq. (4.11) such that,

$$\text{if} \quad X_k = \begin{array}{c|c} & \begin{matrix} A_1 & A_2 & \cdots & A_j \end{matrix} \\ \begin{matrix} CR_1 \\ CR_2 \\ \vdots \\ CR_n \end{matrix} & \begin{bmatrix} \hat{x}_{11,k} & \hat{x}_{12,k} & \cdots & \hat{x}_{1m,k} \\ \hat{x}_{21,k} & \hat{x}_{22}, & \cdots & \hat{x}_{2m,k} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{x}_{n1,k} & \hat{x}_{n2,k} & \cdots & \hat{x}_{nm,k} \end{bmatrix} \end{array} \tag{4.11}$$

$$\text{then} \quad \Lambda_k = \begin{array}{c} \begin{matrix} A_1 & A_2 & \cdots & A_j \end{matrix} \\ \begin{bmatrix} \lambda_{1,k} & \lambda_{2,k} & \cdots & \lambda_{m,k} \end{bmatrix} \end{array}$$

Rule 1: If CR_1 is $\hat{x}_{11,k}$ and $\cdots$and CR_n is $\hat{x}_{n1,,k}$ then A_1 is $\lambda_{1,k}$

$\vdots \qquad \vdots \qquad \vdots$

Rule n_j : If CR_1 is $\hat{x}_{1m,K}$ and $\cdots$and CR_n is $\hat{x}_{nm,K}$ then A_m is $\lambda_{m,k}$

Step 10: Derive final score for each alternative.

Producing a final score Γ_j for each alternative A_j , is by averaging the aggregate membership value of the consequent part of the rules as in Eq. (4.11). Then multiply with the influence multiplier based on the *K* DMs average influence degree for alternative *j* as shown in Eq. (4.12) below.

$$\Gamma_j = \frac{\sum_{Rule=1}^{n} \sum_{k=1}^{K} \hat{\lambda}_{j,k} \times NICC_{j,k}}{\beta_j(K)} \quad , \text{for } j = 1,\cdots,m. \tag{4.12}$$

Where $\hat{\lambda}_{j,k}$ is the aggregate membership value of the consequent part of the rules and β_j is the number of active rules for each alternative.

Step 11: Finally rank alternative base on final score value, the higher final value the better the alternative performance.

Thus, the ranking order of all alternatives can be determined such that better alternatives j have higher values of Γ_j.

4.3 Type-2 Fuzzy Set Implementation

In this section, the author extends the established T2-TOPSIS method from [112] using a single fuzzy rule based approach for handling multi criteria decision making process called T2-SFS TOPSIS. The main purpose of this modification is to extend the capabilities of the established method with the fuzzy rules based approach. Thus, the implementation by the proposed method allows the empirical knowledge of the expert, represented by fuzzy rules, also to be considered in the decision making process.

In implementing type-2 fuzzy set of fuzzy TOPSIS using single rule base, the linguistics terms are given in Table 4.4, Table 4.5 and Table 4.6, the rating of alternatives and weighting the importance of criteria are presented in interval type-2 fuzzy set. All linguistic terms are written in the form of triangular type-2 fuzzy numbers, where 4th parameter of fuzzy number represents the height of membership degree.

Table 4. 4: Interval type 2 linguistic terms for the importance weight

Linguistic Terms		Triangular Interval Type 2 Fuzzy Number
Very Low (VL)	1	(0.00,0.00,0.10,1)(0.00,0.00,0.05,0.9)
Low (L)	2	(0.00,0.10,0.30,1)(0.05,0.10,0.20,0.9)
Medium Low (ML)	3	(0.10,0.30,0.50,1)(0.20,0.30,0.40,0.9)
Medium (M)	4	(0.30,0.50,0.70,1)(0.40,0.50,0.60,0.9)
Medium High (MH)	5	(0.50,0.70,0.90,1)(0.60,0.70,0.80,0.9)
High (H)	6	(0.70,0.90,1.00,1)(0.80,0.90,0.95,0.9)
Very High (VH)	7	(0.90,1.00,1.00,1,)(0.95,1.00,1.00,0.9)

Table 4. 5: Interval type 2 linguistic terms for rating

Linguistic Terms		Triangular Interval Type 2 Fuzzy Number
Very Poor (VP)	1	(0, 0, 1, 1)(0, 0, 0.5, 0.9)
Poor (P)	2	(0, 1, 3, 1)(0.5, 1, 2, 0.9)
Medium Poor (MP)	3	(1, 3, 5, 1)(2, 3, 4, 0.9)
Fair (F)	4	(3, 5, 7, 1)(4, 5, 6, 0.9)
Medium Good (MG)	5	(5, 7, 9, 1)(6, 7, 8, 0.9)
Good (G)	6	(7, 9, 10, 1)(8, 9, 9.5, 0.9)
Very Good (VG)	7	(9, 10, 1,1)(9.5, 10, 0.9)

Table 4. 6: Interval type 2 linguistic terms for alternatives level

Linguistic Terms		Triangular Type 2 Fuzzy Number
Very Bad (VB)	1	(0.00,0.00,0.25, 1)(0.00,0.00,0.25,0.9)
Bad (B)	2	(0.00,0.25,0.50, 1)(0.00,0.25,0.50,0.9)
Regular (R)	3	(0.25,0.50,0.75, 1)(0.25,0.50,0.75,0.9)
Good (G)	4	(0.50,0.75,1.00, 1)(0.50,0.75,1.00,0.9)
Very Good (VG)	5	(0.75,1.00,1.00, 1)(0.75,1.00,1.00,0.9)

In terms of steps involved in the implementation of type-2 fuzzy sets in fuzzy system with single rule base, the concept of ranking triangular interval type-2 fuzzy sets is relevant to step 3-5 prior to find the ranking distance of alternatives from positive ideal solutions and negative ideal solutions. The other steps are the same as type-1 fuzzy sets implementation discussed in Section 4.2.

<u>T2-SFS TOPSIS algorithm</u>

Step 3:

Find the Fuzzy Positive Ideal Solution (FPIS) and Fuzzy Negative Ideal Solution (FNIS) for each alternative and the distance between each alternative to FPIS and FNIS.

In order to construct the ranking weighted decision matrices, for $j=1,\ldots,m$ and $k=1,\ldots,m$, the ranking value of each type-2 fuzzy set $v_{ij,k}$, i.e. $Rank(v_{ij,k})$ needs to be calculated. The maximum number n of edges in the upper membership function $v^U_{ij,k}$ and the lower membership function $v^L_{ij,k}$ are firstly defined, where $i=1,\ldots,e+f$ and $j=1,\ldots,m$. If n is an odd number and $n\geq 3$, then $r=n+1$. If n is an even number and $n\geq 4$, then $r=n$. The $Rank(v_{ij,k})$ of an interval type-2 fuzzy set is presented in Eq. (4.13).

$$Rank(v_{ij,k}) = \sum_{l\in\{U,L\}} M_1(v_{ij,k}^l) + \sum_{l\in\{U,L\}} M_2(v_{ij,k}^l) + \cdots + \sum_{l\in\{U,L\}} M_{r-1}(v_{ij,k}^l) \tag{4.13}$$
$$-\frac{1}{r}\left(\sum_{l\in\{U,L\}} S_1(v_{ij,k}^l) + \sum_{l\in\{U,L\}} S_2(v_{ij,k}^l) + \cdots + \sum_{l\in\{U,L\}} S_r(v_{ij,k}^l)\right) +$$
$$+ \sum_{l\in\{U,L\}} H_1(v_{ij,k}^l) + \sum_{l\in\{U,L\}} H_2(v_{ij,k}^l) + \cdots + \sum_{l\in\{U,L\}} H_{r-2}(v_{ij,k}^l)$$

Where $M_p(v_{ij,k}^l)$ denotes the average of the elements $a_{ij,k,p}^{v,l}$ and $a_{ij,k,(p+1)}^{v,l}$, i.e. $M_p(v_{ij,k}^l) = \left(a_{ij,k,p}^{v,l} + a_{ij,k,(p+1)}^{v,l}\right)\big/2$, for $p = 1,\ldots,r-1$. While $S_p(v_{ij,k}^l)$ denotes the standard deviation of elements $a_{ij,k,1}^{v,l}, a_{ij,k,2}^{v,l}, \cdots, a_{ij,k,p}^{v,l}$, i.e. $S_p(v_{ij,k}^l) = \sqrt{\frac{1}{p}\sum_{t=1}^{p}\left(a_{ij,k,t}^{v,l} - \frac{1}{p}\sum_{t=1}^{p} a_{ij,k,t}^{v,l}\right)^2}$, for $p = 1,\ldots,r$. Finally, $H_p(v_{ij,k}^l)$ denotes the membership value of the element $a_{ij,k,(p+1)}^{v,l}$ for $p = 1,\ldots,r-2$, where $l \in \{U,L\}$ and r is an even number.

Step 4:

Determine the fuzzy positive ideal solution and the fuzzy negative ideal solution.

The fuzzy positive ideal solution $A_k^+ = \left(v_{1,k}^+, v_{2,k}^+, \cdots, v_{(n),k}^+\right)$ and the fuzzy negative ideal solution $A_k^- = \left(v_{1,k}^-, v_{2,k}^-, \cdots, v_{(n),k}^-\right)$ are defined in Eq. (4.14) for n number of criteria.

$$A_k^+ = \left(v_{1,k}^+, v_{2,k}^+, \cdots, v_{(n),k}^+\right) \text{and } A_k^- = \left(v_{1,k}^-, v_{2,k}^-, \cdots, v_{(n),k}^-\right), \tag{4.14}$$

where

$$v_{i,k}^+ = \begin{cases} \max\limits_{1\le j\le n}\left\{Rank\left(v_{ij,k}^B\right)\right\}, & B_i \in B \\ \min\limits_{1\le j\le n}\left\{Rank\left(v_{ij,k}^C\right)\right\}, & C_i \in C \end{cases}$$

and

$$v_{i,k}^- = \begin{cases} \min\limits_{1\le j\le n}\left\{Rank\left(v_{ij,k}^B\right)\right\}, & B_i \in B \\ \max\limits_{1\le j\le n}\left\{Rank\left(v_{ij,k}^C\right)\right\}, & C_i \in C \end{cases}$$

Where B denotes the set of benefit criteria and C denotes the set of cost criteria, for $i = 1,\ldots,m$ and n number of criteria.

Step 5:

Find the distances of each alternative from fuzzy positive ideal solution and fuzzy negative ideal solution.

The distance $\Delta^{+}_{j,k}$ between each alternative $A_{j,k}$ and the fuzzy positive ideal solution A^{+}_{k} is calculated using Eq. (4.15).

$$\Delta^{+}_{j,k} = \sqrt{\sum_{i=1}^{n}\left(Rank\left(v_{ij,k}\right) - v^{+}_{i,k}\right)^{2}} \text{ for } j = 1,\ldots,m \text{ and } k = 1,\ldots,K \tag{4.15}$$

Consequently, the distance $\Delta^{-}_{j,k}$ between each alternative $A_{j,k}$ and the fuzzy negative ideal solution A^{-}_{k} is calculated is calculated using Eq. (4.16).

$$\Delta^{-}_{j,k} = \sqrt{\sum_{i=1}^{n}\left(Rank\left(v_{ij,k}\right) - v^{-}_{i,k}\right)^{2}} \text{ for } j = 1,\ldots,m \text{ and } k = 1,\ldots,K \tag{4.16}$$

4.4 Z-Number Implementation

In this section, the author have modified the established Z-TOPSIS method introduced in [117] using fuzzy system with single rule base. Basically, the concepts of Z-Numbers are capable to represent the reliability of decision maker into decision making evaluation. In order to enhance the capability to deal with vagueness and to represent the decision information more effectively, the author proposes additional elements representing the evaluation of a fuzzy systems in Z-TOPSIS. The main objective of this modification is to introduce the ability of fuzzy rule based system in established Z-TOPSIS. Thus, the evaluation by the proposed method allows the empirical knowledge of the expert, represented by fuzzy rule, and the reliability of decision maker being considered in the decision making process.

The Z-numbers implementation of fuzzy TOPSIS with single rule base are presented in Table 4.1, Table 4.2 and Table 4.3 from Section 4.2 with an additional Table 4.7 for the linguistic terms representing decision maker reliability. The 4th parameter in triangular type 1 fuzzy number represent the height of membership degree is 1.

Table 4. 7: Linguistic terms for expert's reliability

Linguistic Terms	Triangular Fuzzy Number
Strongly Unlikely (SU)	(0.00, 0.00, 0.10,1)
Unlikely (U)	(0.00, 0.10, 0.25,1)
Somewhat Unlikely (SWU)	(0.15, 0.30, 0.45,1)
Neutral (N)	(0.35, 0.50, 0.65,1)
Somewhat Likely (SWL)	(0.55, 0.70, 0.85,1)
Likely (L)	(0.80, 0.90, 1.00,1)
Strongly Likely (SL)	(0.90, 1.00, 1.00,1)

Here, the reliability of experts is taken into consideration during the decision making process. The experts are advised to use the linguistic terms in Table 4.7 to evaluate the confidence in their decision. Decision makers are not supposed to use negative weight to represent their opinion. Otherwise, this would imply the use of unreliable information, which is undesirable. This applies at the start of step 1 of the algorithm described in type-1 fuzzy sets implementation of fuzzy TOPSIS with single rule base. The other steps are the same as the implementation discussed in Section 4.2.

<u>Z-SFS TOPSIS algorithm</u>

Step 1:

Use the information from Table 4.7 to derive the second component B of the Z-numbers, and then convert the Z-numbers to Type-1 fuzzy numbers.

Let $Z_{ij,k} = (x_1, x_2)$ be the Z-number for the system, where $\left\{x_1 = \left(q, \mu_{x_1}\right) \middle| q \in [0,1]\right\}$ is the rating of alternative and $\left\{x_2 = \left(r, \mu_{x_2}\right) \middle| r \in [0,1]\right\}$ is the experts reliabilities with respect to each criteria, also μ_{x_1} and μ_{x_2} are triangular membership functions. The second part (x_2) needs to be converted into a crisp number using fuzzy expectation, as shown in Eq. (4.17).

$$\alpha = \frac{\int q\mu_{x_2} dq}{\int \mu_{x_2} dx} \tag{4.17}$$

Where $\int$ denotes an algebraic integration. Then add the weight of the second part

(x_2) to the first part (x_1). The weighted Z-numbers can be denoted as

$$Z_{ij,k}^{\alpha} = \left\{ \left(q, \mu_{x_1^{\alpha}} \right) \middle| \mu_{x_1^{\alpha}}(q) = \alpha\mu_{x_1}(q), q \in [0,1] \right\}$$

These can be represented with type-1 fuzzy numbers as:

$$Z_{ij,k}^{\alpha} = \left\{ \left\langle q, \mu_{Z_{ij,k}^{\alpha}}(q) \right\rangle \middle| \mu_{Z_{ij,k}^{\alpha}}(q) = \mu_{x_1}\left(\frac{q}{\sqrt{\alpha}} \right), q \in [0,1] \right\}$$

[35] has proven that $Z_{ij,k}$ has the same Fuzzy Expectation as $Z_{ij,k}^{B,a}$. The remaining steps of the algorithm are the same as for the type-1 fuzzy sets implementation.

4.5 Summary

In summary, this chapter extended TOPSIS algorithm using the capabilities of fuzzy system with single rule bases. In Section 4.1 a brief review and introduction about the system are written. Section 4.2 presents the algorithm to implement type-1 fuzzy number whereas Section 4.3 presents the algorithm of interval type-2 implementation. The algorithm for the implementation of Z numbers are presented in Section 4.4. The proposed methods allow hybrid analysis of empirical knowledge of experts in the process of decision making. Ensuring the applicability and the practicality of the proposed methods in this chapter, the case study of stock selection is will be carried out in Chapter 8. In the next chapter, the capabilities of fuzzy system using multiple rule bases in TOPSIS formulation will be presented.

CHAPTER 5

5 FUZZY SYSTEM APPROACH WITH MULTIPLE RULE BASES

5.1 Introduction

In this chapter, the TOPSIS approach with single rule base proposed in Chapter 4 is extended using multiple rule bases. The main aims of the extension are to apply the ability of multiple rule bases in decision processes and to improve the level of transparency for each criterion. In this approach, the criteria are divided into two categories; benefit and cost, as shown in Fig. 5.1. Therefore, by using the division in the beginning of the TOPSIS analysis, a decision maker can trace the performance of both criteria. The next section will describe the formulation of type-1 TOPSIS using fuzzy system with multiple rule bases, namely T1-MFS TOPSIS. The detail explanations of extensions to type -2 and Z implementation are discussed in Section 5.3 and 5.4 respectively.

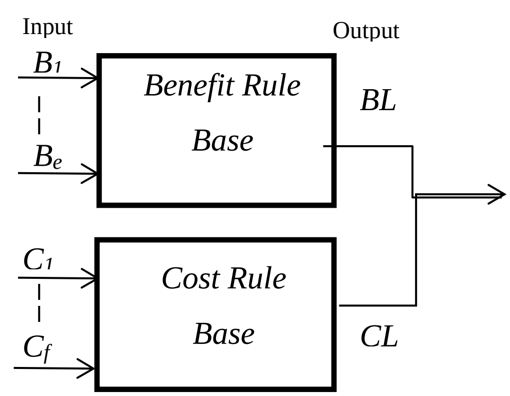

Fig. 5. 1: Fuzzy system model using multiple rule bases

5.2 Type-1 Fuzzy Set Implementation

The following algorithm is conducted to obtain the ranking base of alternatives, whereby the linguistic term in Table 4.1- 4.3 are used to represent the importance of criteria and the rating of each alternative.

T1-MFS TOPSIS algorithm

Step 1: Construct a decision matrix where each decision maker opinion is evaluated independently and categorised into two Criteria Categories as Benefit Criteria and Cost Criteria defined through a Benefit system (BS) and a Cost system (CS)

The decision matrices are denoted by D_k^B, D_k^C and the weight matrices are given as W_k^B, W_k^C, for $(k = 1, \cdots, K)$. e is defined as the number of benefit criteria and f is the number of cost criteria, as shown in Eq. (5.1) below.

$$D_k^B = \begin{array}{c} \\ B_1 \\ B_2 \\ \vdots \\ B_e \end{array} \begin{array}{c} \begin{array}{cccc} A_1 & A_2 & \cdots & A_m \end{array} \\ \begin{bmatrix} x_{11,k} & x_{12,k} & \cdots & x_{1m,k} \\ x_{21,k} & x_{22,k} & \cdots & x_{2m,k} \\ \vdots & \vdots & \ddots & \vdots \\ x_{e1,k} & x_{e2,k} & \cdots & x_{em,k} \end{bmatrix} \end{array} \text{ and } \tag{5.1}$$

$$D_k^C = \begin{array}{c} \\ C_1 \\ C_2 \\ \vdots \\ C_f \end{array} \begin{array}{c} \begin{array}{cccc} A_1 & A_2 & \cdots & A_m \end{array} \\ \begin{bmatrix} y_{11,k} & y_{12,k} & \cdots & y_{1m,k} \\ y_{21,k} & y_{22,k} & \cdots & y_{2m,k} \\ \vdots & \vdots & \ddots & \vdots \\ y_{f1,k} & y_{f2,k} & \cdots & y_{fm,k} \end{bmatrix} \end{array}$$

$$W_k^B = \begin{array}{c} \begin{array}{cccc} B_1 & B_2 & \cdots & B_e \end{array} \\ \begin{bmatrix} g_{1,k} & g_{2,k} & \cdots & g_{e,k} \end{bmatrix} \end{array} \quad \text{and} \quad W_k^C = \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_e \end{array} \\ \begin{bmatrix} h_{1,k} & h_{2,k} & \cdots & h_{f,k} \end{bmatrix} \end{array}$$

The variables $x_{ij,k}$ and $y_{ij,k}$ in the decision matrices represent the rating of alternatives A_j $(j = 1, \cdots, m)$ with respect to benefit criteria B_i $(i = 1, \cdots, e)$ and cost criteria C_i $(i = 1, \cdots, f)$, respectively. On the other hand, the variables $g_{i,k}$

and $h_{i,k}$ in the weight matrices represent the weights of benefit criteria $B_{i,k}$ $(i = 1,\cdots,e)$ and the weights of cost criteria C_i $(i = 1,\cdots, f)$, respectively. All four variables are type-1 fuzzy sets evaluated according to the k^{th} decision maker, where $k = 1,\cdots,K$.

Step 2: Construct the weighted and the normalized decision matrices

The fuzzy rating and weight of each criterion are the linguistic terms described with type-1 triangular fuzzy numbers. The ratings of alternatives A_j $(j = 1,\cdots,m)$ are described with type-1 triangular fuzzy numbers $x_{ij,k} = \left(a^x_{ij,k}, b^x_{ij,k}, c^x_{ij,k}\right)$ and $y_{ij,k} = \left(a^y_{ij,k}, b^y_{ij,k}, c^y_{ij,k}\right)$, while the importance of benefit criteria B_i $(i = 1,\cdots,e)$ and cost criteria C_i $(i = 1,\cdots, f)$ are respectively represented by $g_{i,k} = \left(a^g_{i,k}, b^g_{i,k}, c^g_{i,k}\right)$ and $h_{i,k} = \left(a^h_{i,k}, b^h_{i,k}, c^h_{i,k}\right)$, for $k = 1,\cdots,K$. The normalised fuzzy decision matrices R_k and the weighted normalised fuzzy decision matrices V_k are calculated as shown in Eq. (5.2).

$$R_k = \left[r_{ij,k}\right]_{(e+f)\times m} , \tag{5.2}$$

where

$$r_{ij,k} = \begin{cases} r^B_{ij.k} = \left(\dfrac{a^x_{ij,k}}{c^{x*}_{i,k}}, \dfrac{b^x_{ij,k}}{c^{x*}_{i,k}}, \dfrac{c^x_{ij,k}}{c^{x*}_{i,k}},\right), \ for \ B_i \in B \\ \\ r^C_{ij.k} = \left(\dfrac{a^{y*}_{i,k}}{c^y_{ij,k}}, \dfrac{a^{y*}_{i,k}}{b^y_{ij,k}}, \dfrac{a^{y*}_{i,k}}{a^y_{ij,k}}\right), \ for \ C_i \in C \end{cases}$$

$$c^{x*}_{i,k} = \max_j c^x_{ij,k} , \ (i = 1,\cdots,e), \ (j = 1,\cdots,m)$$

$$a^{y*}_{i,k} = \min_j a^y_{ij,k} , \ (i = 1,\cdots, f), \ (j = 1,\cdots,m)$$

B and C are the sets of benefit criteria and cost criteria respectively.

$$V_k = \left[v_{ij,k}\right]_{(e+f)\times m} ,$$

where

$$v_{ij,k}=\begin{cases} v^{B}_{ij.k}=r_{ij,k}(\cdot)g_{i,k} & ,\ for\ B_i\in B \\ v^{C}_{ij.k}=r_{ij,k}(\cdot)h_{i,k} & ,\ for\ C_i\in C \end{cases}$$

and

$v_{ij,k}=\left(a^{v}_{ij,k},b^{v}_{ij,k},c^{v}_{ij,k}\right)$ are type-1 fuzzy sets, for $k=1,\cdots,K$.

Step 3: Find the Fuzzy Positive Ideal Solution (FPIS) and Fuzzy Negative Ideal Solution (FNIS) for each alternative.

The FPIS and FNIS solutions are correspondingly $A_k^{+}=\left(v^{+}_{1,k},v^{+}_{2,k},\cdots,v^{+}_{(e+f),k}\right)$ and $A_k^{-}=\left(v^{-}_{1,k},v^{-}_{2,k},\cdots,v^{-}_{(e+f),k}\right)$, where $v^{+}_{i,k}=(1\quad 1\quad 1\quad)$ and $v^{-}_{i,k}=(0\quad 0\quad 0\quad)$ are Type-1 fuzzy sets, for $k=1,\cdots,K$.

Step 4: Find the distance between each alternative to FPIS and FNIS.

The distance for benefit criteria of each alternative Aj from A_k^{+} is $\Delta^{B+}_{j,k}$, calculated as shown in Eq. (5.3).

$$\Delta^{B+}_{j,k}=\sum_{i=1}^{e}\Delta^{B}_{k}\left(v^{B}_{ij,k},v^{+}_{i,k}\right), \tag{5.3}$$

where

$$\Delta^{B}_{k}\left(v^{B}_{ij,k},v^{+}_{i,k}\right)=\sqrt{\frac{1}{3}\left[\left(a^{v,B}_{ij,k}-1\right)^2+\left(b^{v,B}_{ij,k}-1\right)^2+\left(c^{v,B}_{ij,k}-1\right)^2+\left(d^{v,B}_{ij,k}-1\right)^2\right]}$$

,

for $j=1,\cdots,m$, and $B_i\in B$, and $k=1,\cdots,K$.

The distance for benefit criteria of each alternative A_j from $A^{-}_{j,k}$ is $\Delta^{B-}_{j,k}$, calculated as shown in Eq. (5.4).

$$\Delta^{B-}_{j,k}=\sum_{i=1}^{e}\Delta^{B}_{k}\left(v^{B}_{ij,k},v^{-}_{i,k}\right), \tag{5.4}$$

where

$$\Delta_k^B\left(v_{ij,k}^B, v_{i,k}^-\right) = \sqrt{\frac{1}{3}\left[\left(a_{ij,k}^{v,B} - 0\right)^2 + \left(b_{ij,k}^{v,B} - 0\right)^2 + \left(c_{ij,k}^{v,B} - 0\right)^2\right]}$$

for $j = 1, \cdots, m$, and $B_i \in B$, and $k = 1, \cdots, K$.

The distance for cost criteria of each alternative A_j from A_k^+ is $\Delta_{j,k}^{C+}$, calculated as shown in Eq. (5.5).

$$\Delta_{j,k}^{C+} = \sum_{i=1}^{f} \Delta_k^C\left(v_{ij,k}^C, v_{i,k}^+\right), \tag{5.5}$$

where

$$\Delta_k^C\left(v_{ij,k}^C, v_{i,k}^+\right) = \sqrt{\frac{1}{3}\left[\left(a_{ij,k}^{v,C} - 1\right)^2 + \left(b_{ij,k}^{v,C} - 1\right)^2 + \left(c_{ij,k}^{v,C} - 1\right)^2\right]}$$

for $j = 1, \cdots, m$, and $C_i \in C$, and $k = 1, \cdots, K$.

Finally, the distance for cost criteria of each alternative A_j from A_k^- is $\Delta_{j,k}^{C-}$, calculated as shown in Eq. (5.6).

$$\Delta_{j,k}^{C-} = \sum_{i=1}^{f} \Delta_k^C\left(v_{ij,k}^C, v_{i,k}^-\right), \tag{5.6}$$

where

$$\Delta_k^C\left(v_{ij,k}^C, v_{i,k}^-\right) = \sqrt{\frac{1}{3}\left[\left(a_{ij,k}^{v,C} - 0\right)^2 + \left(b_{ij,k}^{v,C} - 0\right)^2 + \left(c_{ij,k}^{v,C} - 0\right)^2\right]}$$

for $j = 1, \cdots, m$, and $C_i \in C$, and $k = 1, \cdots, K$.

Step 5: Find the Closeness Coefficients (CC) for the benefit and cost systems.

The closeness coefficients denoted by $CC_{j,k}^B$ and $CC_{j,k}^C$ for the benefit systems and the cost systems, respectively, are calculated in Eq. (5.7):

$$CC_{j,k}^B = \frac{\Delta_{j,k}^{B-}}{\Delta_{j,k}^{B+} + \Delta_{j,k}^{B-}} \quad , \quad CC_{j,k}^C = \frac{\Delta_{j,k}^{C-}}{\Delta_{j,k}^{C+} + \Delta_{j,k}^{C-}} \tag{5.7}$$

for $j = 1, \cdots, m$ and $k = 1, \cdots, K$.

Step 6: Derive the Influenced Closeness Coefficient (ICC) by applying the influence degree of each decision maker, then find Normalised ICC (NICC), dividing NICC by maximum value of NICC.

Let θ_k be the influence degree, between 0 (un-influential) and 10 (very influential), of decision maker k, where $k=1,\cdots,K$. Next, let σ_k stands for the normalized influence degree of the k^{th} decision maker, $k=1,\cdots,K$, evaluated using Eq. (5.8):

$$\sigma_k = \theta_k \Big/ \sum_{l=1}^{K}\theta_l \qquad , \text{ for } k=1,\cdots,K. \tag{5.8}$$

Eq. (5.9) then evaluates the influence closeness coefficients $ICC^B_{j,k}$ and $ICC^C_{j,k}$ for each DM *k*, respectively along the benefit and cost criteria.

$$ICC^B_{j,k} = \sigma_k \times CC^B_{j,k} \qquad \text{and} \qquad ICC^C_{j,k} = \sigma_k \times CC^C_{j,k} \tag{5.9}$$

for $j=1,\cdots,m$ and $k=1,\cdots,K$.

The coefficients are then normalised so that their values vary between 0 to 1. Eq. (5.10) evaluates the normalised coefficients, where $NICC^B_{j,k}$ and $NICC^C_{j,k}$ are respectively the normalised influence closeness coefficients for the benefit and cost systems as related to the k^{th} decision maker.

$$NICC^B_{j,k} = ICC^B_{j,k} \Big/ \max_j ICC^B_{j,k} \qquad \text{and} \tag{5.10}$$

$$NICC^C_{j,k} = ICC^C_{j,k} \Big/ \max_j ICC^C_{j,k}$$

for $j=1,\cdots,m$ and $k=1,\cdots,K$.

Both $NICC^B_{j,k}$ and $NICC^C_{j,k}$ will take the linguistic terms from Table 4.3 for the level of alternatives performance.

Step 7: Construct the antecedent matrices for the BS and CS based on DMs opinions

Having the opinions D_k^B and D_k^C of all DMs $(k=1,\cdots,K)$ on each alternative j $(j=1,\cdots,m)$ with respect to each benefit criterion $i\,(i=1,\cdots,e)$ and each cost criterion $i\,(i=1,\cdots,f)$, we can define the BS antecedent matrix X_k and the CS antecedent matrix Y_k for each DM k, as given by Eq. (5.11) below.

$$
X_k = \begin{array}{c} \\ B_1 \\ B_2 \\ \vdots \\ B_e \end{array}
\begin{array}{c} \begin{array}{cccc} A_1 & A_2 & \cdots & A_m \end{array} \\
\begin{bmatrix} \hat{x}_{11,k} & \hat{x}_{12,k} & \cdots & \hat{x}_{1m,k} \\ \hat{x}_{21,k} & \hat{x}_{22,} & \cdots & \hat{x}_{2m,k} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{x}_{e1,k} & \hat{x}_{e2,k} & \cdots & \hat{x}_{em,k} \end{bmatrix} \end{array} \text{ and}
\tag{5.11}
$$

$$
Y_k = \begin{array}{c} \\ C_1 \\ C_2 \\ \vdots \\ C_f \end{array}
\begin{array}{c} \begin{array}{cccc} A_1 & A_2 & \cdots & A_m \end{array} \\
\begin{bmatrix} \hat{y}_{11,k} & \hat{y}_{12,k} & \cdots & \hat{y}_{1m,k} \\ \hat{y}_{21,k} & \hat{y}_{22,k} & \cdots & \hat{y}_{2m,k} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{y}_{f1,k} & \hat{y}_{f2,k} & \cdots & \hat{y}_{fm,k} \end{bmatrix} \end{array} \text{ for } k=1,\cdots,K,
$$

The entries $\hat{x}_{ij,k}$ and $\hat{y}_{ij,k}$ are the linguistic terms describing decision makers' opinions on benefit and cost criteria respectively.

Step 8: Construct the consequent matrices for the BS and CS systems based on the value of the NICC coefficients.

After the $NICC_j^{B,k}$ and $NICC_j^{C,k}$ coefficients are determined for all decision maker $(k=1,\cdots,K)$, the benefit consequent matrix Λ_k and the cost consequent matrix Ψ_k are then defined as shown in Eq. (5.12).

$$
\Lambda_k = BL \begin{array}{c} \begin{array}{cccc} A_1 & A_2 & \cdots & A_m \end{array} \\ \begin{bmatrix} \lambda_{1,k} & \lambda_{2,k} & \cdots & \lambda_{m,k} \end{bmatrix} \end{array} ,
\tag{5.12}
$$

$$
\Psi_k = CL \begin{array}{c} \begin{array}{cccc} A_1 & A_2 & \cdots & A_m \end{array} \\ \begin{bmatrix} \psi_{1,k} & \psi_{2,k} & \cdots & \psi_{m,k} \end{bmatrix} \end{array} \quad \text{for } k=1,\cdots,K ,
$$

The entries of $\lambda_{i,k}$ and $\psi_{i,k}$ are the linguistic terms representing the output of the BS and CS systems, respectively based on the values of $NICC_j^{B,k}$ and $NICC_j^{C,k}$

Step 9: Derive rules for each alternative for benefit and cost system

The rule base of benefit system for DM1 is constructed using Eq. (5.11) and Eq. (5.12) demonstrated in Eq. (5.13), such that:

$$\text{If } X_k = \begin{array}{c} \\ B_1 \\ B_2 \\ \vdots \\ B_e \end{array} \begin{array}{c} \begin{array}{cccc} A_1 & A_2 & \cdots & A_m \end{array} \\ \begin{bmatrix} \hat{x}_{11,k} & \hat{x}_{12,k} & \cdots & \hat{x}_{1m,k} \\ \hat{x}_{21,k} & \hat{x}_{22}, & \cdots & \hat{x}_{2m,k} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{x}_{e1,k} & \hat{x}_{e2,k} & \cdots & \hat{x}_{em,k} \end{bmatrix} \end{array}, \text{ then} \tag{5.13}$$

$$\Lambda_k = BL \begin{array}{c} \begin{array}{cccc} A_1 & A_2 & \cdots & A_m \end{array} \\ \begin{bmatrix} \lambda_{1,k} & \lambda_{2,k} & \cdots & \lambda_{m,k} \end{bmatrix} \end{array}$$

Rule 1: If B_1 is $\hat{x}_{11,k}$ and $\cdots$and B_e is $\hat{x}_{e1,k}$ then BL is $\lambda_{1,k}$

$\vdots$ $\vdots$ $\vdots$

Rule n_j : If B_1 is $\hat{x}_{1m,k}$ and $\cdots$and B_e is $\hat{x}_{em,k}$ then BL is $\lambda_{m,k}$

Furthermore, the rule bases for cost system are constructed as follow:

$$\text{If } Y_k = \begin{array}{c} \\ C_1 \\ C_2 \\ \vdots \\ C_f \end{array} \begin{array}{c} \begin{array}{cccc} A_1 & A_2 & \cdots & A_m \end{array} \\ \begin{bmatrix} \hat{y}_{11,k} & \hat{y}_{12,k} & \cdots & \hat{y}_{1m,k} \\ \hat{y}_{21,k} & \hat{y}_{22,k} & \cdots & \hat{y}_{2m,k} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{y}_{f1,k} & \hat{y}_{f2,k} & \cdots & \hat{y}_{fm,k} \end{bmatrix} \end{array}, \text{ then } \quad \Psi_k = CL \begin{array}{c} \begin{array}{cccc} A_1 & A_2 & \cdots & A_m \end{array} \\ \begin{bmatrix} \psi_{1,k} & \psi_{2,k} & \cdots & \psi_{m,k} \end{bmatrix} \end{array}$$

Rule 1: if C_1 is $\hat{y}_{11,k}$ and $\cdots$and C_f is $\hat{y}_{f1,k}$ then CL is $\psi_{1,k}$

Rule n_j : If C_1 is $\hat{y}_{fm,k}$ and $\cdots$and C_f is $\hat{y}_{fm,k}$ then CL is $\psi_{m,k}$

Step 10:

Derive final score for each alternative,

The final score Γ_j for each alternative *j*, is obtained by averaging the aggregate membership value of the consequent part of the n_j rules in Eq. (5.13). Then, multiply the averaged value with the influence multiplier based on the *K* DMs average influence degree for alternative *j* as shown in Eq. (5.14).

$$\Gamma_j = \frac{\sum_{rule=1}^{n}\sum_{k=1}^{K}\left(\left[\hat{\lambda}_{j,k}^{B}\left(\frac{e}{e+f}\right)\times NICC_{j,k}^{B}\right]+\left[\hat{\lambda}_{j,k}^{C}\left(\frac{f}{e+f}\right)\times NICC_{j,k}^{C}\right]\right)}{2(\beta_{j,k})K} \tag{5.14}$$

The variables $\hat{\lambda}_{j,k}^{B}$ and $\hat{\lambda}_{j,k}^{C}$ represent the aggregate membership value of benefit subsystem and cost subsystem respectively for each alternative $j = 1,\cdots,25$ and k decision makers.

Step 11: Finally, rank alternative base on final score value, the higher final value the better the alternative performance.

Thus, the ranking order of all alternatives can be determined such that higher values of Γ_j mean better alternatives of j.

5.3 Type-2 Fuzzy Set Implementation

In this section, the author extends the established T2-TOPSIS methods from [112] by using multiple rule bases approach, called T2-MFS TOPSIS, for handling the multi criteria decision making processes. The main purpose of this modification is to extend the capabilities of the established TOPSIS method with the fuzzy rules based approach. Thus, the implementation by the proposed method allows the empirical knowledge of the expert, represented by fuzzy rules to be considered in the decision making process.

For implementing type-2 fuzzy sets of this approach, the concept of ranking triangular interval type-2 fuzzy sets is relevant to the steps 3-5 before finding the distance of alternatives from positive ideal solutions and negative ideal solutions. The linguistic terms and other steps are the same as type-1 fuzzy sets implementation in Section 5.2.

T2-MFS TOPSIS algorithm

Step 3:

Find the Fuzzy Positive Ideal Solution (FPIS) and Fuzzy Negative Ideal Solution (FNIS) for each alternative and the distance between each alternative to FPIS and FNIS.

The ranking weighted decision matrices for benefit subsystem for $j=1,\ldots,m$ and $k=1,\ldots,m$, is constructed by calculating the ranking value of each interval type-2 fuzzy numbers $v_{ij,k}^{B}$, i.e. $Rank(v_{ij,k}^{B})$. Firstly, the maximum number n of edges in the upper membership function, $v_{ij,k}^{B,U}$ and the lower membership function, $v_{ij,k}^{B,L}$, are defined for $i=1,\ldots,e$ and $j=1,\ldots,m$. If n is an odd number and $n \geq 3$, then $r=n+1$. If n is an even number and $n \geq 4$, then $r=n$. Then, the $Rank(v_{ij,k}^{B})$ of an interval type-2 is presented in Eq. (42).

$$\begin{aligned} Rank\left(v_{ij,k}^{B}\right) = {} & \sum_{l\in\{U,L\}} M_1\left(v_{ij,k}^{B,l}\right) + \sum_{l\in\{U,L\}} M_2\left(v_{ij,k}^{B,l}\right) + \cdots + \sum_{l\in\{U,L\}} M_{r-1}\left(v_{ij,k}^{B,l}\right) \\ & - \frac{1}{r}\left(\sum_{l\in\{U,L\}} S_1\left(v_{ij,k}^{B,l}\right) + \sum_{l\in\{U,L\}} S_2\left(v_{ij,k}^{B,l}\right) + \cdots + \sum_{l\in\{U,L\}} S_r\left(v_{ij,k}^{B,l}\right) \right) \\ & + \sum_{l\in\{U,L\}} \mu_1\left(v_{ij,k}^{B,l}\right) + \sum_{l\in\{U,L\}} \mu_2\left(v_{ij,k}^{B,l}\right) + \cdots + \sum_{l\in\{U,L\}} \mu_{r-2}\left(v_{ij,k}^{B,l}\right) \end{aligned} \tag{5.16}$$

The first 3 entries of $M_p\left(v_{ij,k}^{B,l}\right)$ are the average of the elements $a_{ij,k,p}^{B,l}$ and $a_{ij,k,(p+1)}^{B,l}$, i.e. $M_p\left(v_{ij,k}^{l}\right) = \left(a_{ij,k,p}^{B,l} + a_{ij,k,(p+1)}^{B,l}\right)\big/2$, for $p=1,\ldots,r-1$. The second entries, $S_p\left(v_{ij,k}^{B,l}\right)$ are the standard deviation of elements from benefit subsystem

$a_{ij,k,1}^{B,l}, a_{ij,k,2}^{B,l}, \cdots, a_{ij,k,p}^{B,l}$, i.e. $S_p\left(v_{ij,k}^{B,l}\right) = \sqrt{\frac{1}{2}\sum_{p=1}^{p+1}\left(a_{ij,k,p}^{B,l} - \frac{1}{2}\sum_{P=1}^{p+1} a_{ij,k,p}^{B,l}\right)^2}$, for $p = 1, \ldots, r$. Lastly, $\mu_p\left(v_{ij,k}^{B,l}\right)$ denotes the membership value of the element $a_{ij,k,(p+1)}^{B,l}$ for $p = 1, \ldots, r-2$, where $l \in \{U, L\}$ and r are even numbers. Similarly, the construction of the ranking weighted decision matrices for cost criteria, for $j = 1, \ldots, m$ and $k = 1, \ldots, m$, is by calculating the ranking value of each interval type-2, that is $v_{ij,k}^{C}$ elements of cost subsystem, i.e. $Rank(v_{ij,k}^{C})$. Firstly, the maximum number n of edges in the upper membership function $v_{ij,k}^{C,U}$ and the lower membership function $v_{ij,k}^{C,L}$ are defined for $i = 1, \ldots, f$ and $j = 1, \ldots, m$. If n is an odd number and $n \geq 3$, then $r = n+1$. If n is an even number and $n \geq 4$, then $r = n$. Therefore, the $Rank(v_{ij,k}^{C})$ of an interval type-2 is presented in Eq. (5.17).

$$
\begin{aligned}
Rank\left(v_{ij,k}^{C}\right) = & \sum_{l\in\{U,L\}} M_1\left(v_{ij,k}^{C,l}\right) + \sum_{l\in\{U,L\}} M_2\left(v_{ij,k}^{C,l}\right) + \cdots + \sum_{l\in\{U,L\}} M_{r-1}\left(v_{ij,k}^{C,l}\right) \\
& - \frac{1}{r}\left(\sum_{l\in\{U,L\}} S_1\left(v_{ij,k}^{C,l}\right) + \sum_{l\in\{U,L\}} S_2\left(v_{ij,k}^{C,l}\right) + \cdots + \sum_{l\in\{U,L\}} S_r\left(v_{ij,k}^{C,l}\right)\right) + \\
& + \sum_{l\in\{U,L\}} \mu_1\left(v_{ij,k}^{C,l}\right) + \sum_{l\in\{U,L\}} \mu_2\left(v_{ij,k}^{C,l}\right) + \cdots + \sum_{l\in\{U,L\}} \mu_{r-2}\left(v_{ij,k}^{C,l}\right)
\end{aligned}
\tag{5.17}
$$

The variables $M_p\left(v_{ij,k}^{C,l}\right)$ are the average of the elements $a_{ij,k,p}^{C,l}$ and $a_{ij,k,(p+1)}^{C,l}$, i.e. $M_p\left(v_{ij,k}^{l}\right) = \left(a_{ij,k,p}^{C,l} + a_{ij,k,(p+1)}^{C,l}\right)/2$, for $p = 1, \ldots, r-1$. The second variables $S_p\left(v_{ij,k}^{C,l}\right)$ are the standard deviation of elements from cost sub system $a_{ij,k,1}^{C,l}, a_{ij,k,2}^{C,l}, \cdots, a_{ij,k,p}^{C,l}$, i.e. $S_p\left(v_{ij,k}^{C,l}\right) = \sqrt{\frac{1}{2}\sum_{p=1}^{p+1}\left(a_{ij,k,p}^{C,l} - \frac{1}{2}\sum_{P=1}^{p+1} a_{ij,k,p}^{C,l}\right)^2}$, for $p = 1, \ldots, r$. Finally, $\mu_p\left(v_{ij,k}^{C,l}\right)$ is the membership value of the element $a_{ij,k,(p+1)}^{C,l}$ for $p = 1, \ldots, r-2$, where $l \in \{U, L\}$ and r are even numbers.

Step 4:

Define fuzzy positive ideal solution and fuzzy negative ideal solution

The fuzzy positive ideal solution $A_k^+ = \left(v_{1,k}^+, v_{2,k}^+, \cdots, v_{(e+f),k}^+\right)$ and the fuzzy negative ideal solution $A_k^- = \left(v_{1,k}^-, v_{2,k}^-, \cdots, v_{(e+f),k}^-\right)$ are defined in Eq. (5.18) such that:

$$A_k^+ = \left(v_{1,k}^+, v_{2,k}^+, \cdots, v_{e+f,k}^+\right) \text{and}$$
$$A_k^- = \left(v_{1,k}^-, v_{2,k}^-, \cdots, v_{e+f,k}^-\right) \tag{5.18}$$

where

$$v_{i,k}^+ = \begin{cases} \max\limits_{1 \le j \le e+j} \left\{Rank\left(v_{ij,k}^B\right)\right\}, & B_i \in B \\ \min\limits_{1 \le j \le e+f} \left\{Rank\left(v_{ij,k}^C\right)\right\}, & C_i \in C \end{cases}$$

and

$$v_{i,k}^- = \begin{cases} \min\limits_{1 \le j \le e+f} \left\{Rank\left(v_{ij,k}^B\right)\right\}, & B_i \in B \\ \max\limits_{1 \le j \le e+f} \left\{Rank\left(v_{ij,k}^C\right)\right\}, & C_i \in C \end{cases}$$

The superscripts B and C denotes the set of benefit criteria and the set of cost criteria, respectively for $i = 1, \ldots, m$.

Step 5:

Find the distances for benefit criteria of each alternative from

The distance $\Delta_{j,k}^{B+}$ between each alternative $A_{j,k}$ and the fuzzy positive ideal solution A_k^+ is calculated using Eq. (5.19).

$$\Delta_{j,k}^{B+} = \sqrt{\sum_{i=1}^{e} \left(Rank\left(v_{ij,k}^B\right) - v_{i,k}^{B+}\right)^2} \tag{5.19}$$

for $j = 1, \ldots, m$ and $k = 1, \ldots, K$

The distance $\Delta_{j,k}^{B-}$ between each alternative $A_{j,k}$ and the fuzzy negative ideal solution A_k^- is calculated is calculated using Eq. (5.20).

$$\Delta_{j,k}^{B-} = \sqrt{\sum_{i=1}^{e}\left(Rank\left(v_{ij,k}^{B}\right) - v_{i,k}^{B-}\right)^2} \tag{5.20}$$

for $j = 1,\ldots,m$ and $k = 1,\ldots,K$

The distance $\Delta_{j,k}^{C+}$ between each alternative $A_{j,k}$ and the fuzzy positive ideal solution A_k^+ is calculated with Eq. (5.21).

$$\Delta_{j,k}^{C+} = \sqrt{\sum_{i=1}^{f}\left(Rank\left(v_{ij,k}^{C}\right) - v_{i,k}^{C+}\right)^2} \tag{5.21}$$

for $j = 1,\ldots,m$ and $k = 1,\ldots,K$

The distance $\Delta_{j,k}^{C-}$ between each alternative $A_{j,k}$ and the fuzzy negative ideal solution A_k^- is calculated using Eq. (5.22).

$$\Delta_{j,k}^{C-} = \sqrt{\sum_{i=1}^{f}\left(Rank\left(v_{ij,k}^{C}\right) - v_{i,k}^{C-}\right)^2} \tag{5.22}$$

for $j = 1,\ldots,m$ and $k = 1,\ldots,K$

The remaining steps of the algorithm are the same as for the type-1 fuzzy sets implementation in Section 5.2.

5.4 Z-Number Implementation

The Z-number implementation of TOPSIS using fuzzy system with multiple rule bases, namely Z-MFS TOPSIS, the Table 4.1, Table 4.2- 4.3 from Chapter 4 are used with an additional Table 4.7 for the linguistic terms representing decision maker reliability. This implementation is applied at the start of step 1 of the algorithm described in type-1 fuzzy number implementation of fuzzy TOPSIS with multiple rule bases. The other steps are the same as discussed in Section 5.2

Z-MFS TOPSIS algorithm

Step 1:

Use the information from Table 7 to derive the second component B of the Z-numbers and then convert the Z-numbers to type-1 fuzzy numbers.

Let $Z^B_{ij,k} = (x^B_1, x^B_2)$ be a Z-number for a benefit subsystem where $\left\{x^B_1 = \left(q, \mu_{x^B_1}\right) \middle| q \in [0,1]\right\}$ is the rating of alternative and $\left\{x^B_2 = \left(r, \mu_{x^B_2}\right) \middle| r \in [0,1]\right\}$ is the experts' reliabilities with respect to benefit criteria. $\mu_{x^B_1}$ and $\mu_{x^B_2}$ are triangular membership functions.

The second part, which is (x^B_2), needs to be converted into a crisp number using fuzzy expectation as shown in Eq. (5.23).

$$\alpha = \frac{\int q\mu_{x^B_2} dq}{\int \mu_{x^B_2} dx} \tag{5.23}$$

The symbol $\int$ denotes an algebraic integration. Then, add the weight of the second part, (x^B_2), to the first part, (x^B_1). The weighted Z-numbers can be denoted as:

$$Z^{B,\alpha}_{ij,k} = \left\{\left(q, \mu_{x^{B,\alpha}_1}\right) \middle| \mu_{x^{B,\alpha}_1}(q) = \alpha\mu_{x^B_1}(q), q \in [0,1]\right\}$$

These can be represented with type-1 fuzzy numbers as:

$$Z^{B,\alpha}_{ij,k} = \left\{\left\langle q, \mu_{Z^{B,\alpha}_{ij,k}}(q)\right\rangle \middle| \mu_{Z^{B,\alpha}_{ij,k}}(q) = \mu_{x^B_1}\left(\frac{q}{\sqrt{\alpha}}\right), q \in [0,1]\right\}$$

Let $Z_{ij,k}^{C} = (x_1^C, x_2^C)$ be a Z-number for cost subsystem, where $\left\{x_1^C = \left(q, \mu_{x_1^B}\right) \middle| q \in [0,1]\right\}$ is the rating of alternative and $\left\{x_2^C = \left(r, \mu_{x_2^C}\right) \middle| r \in [0,1]\right\}$ is the experts' reliabilities with respect to cost criteria. $\mu_{x_1^C}$ and $\mu_{x_2^C}$ are triangular membership functions. The second part, which is (x_2^C), representing the reliability of decision maker needs to convert into a crisp number using fuzzy expectation shown in Eq. (46).

$$\alpha = \frac{\int q\mu_{x_2^C} dq}{\int \mu_{x_2^C} dx} \tag{5.24}$$

Similarly, $\int$ denotes an algebraic integration. Then, add the weight of the second part, (x_2^C), to the first part, (x_1^C). The weighted Z-numbers can be denoted as:

$$Z_{ij,k}^{C,\alpha} = \left\{\left(q, \mu_{x_1^{C,\alpha}}\right) \middle| \mu_{x_1^{C,\alpha}}(q) = \alpha\mu_{x_1^C}(q), q \in [0,1]\right\}$$

These can be represented with type-1 fuzzy numbers as:

$$Z_{ij,k}^{C,\alpha} = \left\{\left\langle q, \mu_{Z_{ij,k}^{C,\alpha}}(q)\right\rangle \middle| \mu_{Z_{ij,k}^{C,\alpha}}(q) = \mu_{x_1^C}\left(\frac{q}{\sqrt{\alpha}}\right), q \in [0,1]\right\}$$

[35] proved that $Z_{ij,k}^{B}$ and $Z_{ij,k}^{C}$ have the same Fuzzy Expectation as $Z_{ij,k}^{B,\alpha}$ and $Z_{ij,k}^{C,\alpha}$ respectively. The remaining steps of the algorithm are the same as for the type-1 fuzzy sets implementation.

5.5 Summary

In summary, this chapter extended TOPSIS algorithm using the capabilities of multiple rule bases. Section 5.1 introduces the multiple rule bases approaches. Section 5.2 then presents the algorithm to implement type-1 fuzzy number, whereas Section 5.3 presents the algorithm of interval type-2 implementation. The algorithm for the implementation of Z-numbers is presented in Section 5.4. The proposed methods allow hybrid analysis of empirical knowledge of experts in the process of decision making. Moreover, they also improve the level of transparency for criteria of methods proposed in Chapter 4, which are based on fuzzy system with single rule base. The applicability and practicality of proposed methods in this chapter will be verified using the case study of stock selection in Chapter 7. The next chapter will present the capabilities of fuzzy networks in TOPSIS formulations.

CHAPTER 6

6 FUZZY NETWORK APPROACH

6.1 Introduction

The existed literatures on decision analysis have no studies on fuzzy networks using TOPSIS methods. For that reason, this chapter introduces a novel approach for ranking alternatives using fuzzy network. In this contex, the rule based aggregation and rule based merging operation of fuzzy network are used in the following sections.

6.2 Fuzzy Network Approach with Rule Base Aggregation

In this approach, the decision makers' opinions are evaluated independently since they may have different influence degrees, depending on their experience. Furthermore, criteria are categorised either into benefit criteria or cost criteria. Each category will correspondingly generate either the benefit fuzzy systems or the cost fuzzy systems. The outputs of each system are Benefit Levels (BL) and Cost Levels (CL), respectively. Figure 6.1 illustrates the proposed Generalised Fuzzy Network Model using rule base aggregation, where Benefit Systems (BS), Cost Systems (CS) and Alternatives Systems (AS) are incorporated in the form of fuzzy network nodes. The inputs are the benefit criteria$B_1,\dots,B_e$ and the cost criteria $C_1,\dots,C_f$. At the end of the process, Alternatives Level (AL) are determined.

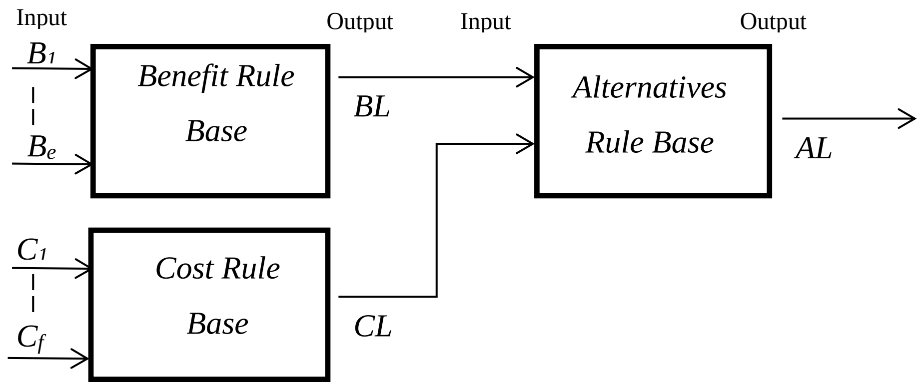

Fig. 6. 1: Fuzzy network model using rule base aggregation

The next sections discusse the implementation of type-1, type-2 and Z-fuzzy sets implementation of TOPSIS using fuzzy network with rule base aggregation. For type-1 fuzzy set implementation, Step 1-8 and Step 13 are identical to the steps proposed in Section 5.2, whereby Step 9 – 12 are additionally introduced in this chapter.

6.2.1 Type-1 Fuzzy Set Implementation

<u>T1-AFN TOPSIS algorithm</u>

Step 1: Construct decision matrix where each decision maker's opinion is evaluated independently and categorised into two Criteria Categories as Benefit Criteria and Cost Criteria defined through a Benefit system (BS) and a Cost system (CS)

- Identical to step 1 in Section 5.2.

Step 2: Construct the weighted and the normalized decision matrices

- Identical to step 2 in Section 5.2.

Step 3: Find the Fuzzy Positive Ideal Solution (FPIS) and Fuzzy Negative Ideal Solution (FNIS) for each alternative.

- Identical to step 3 in Section 5.2.

Step 4: Find the distance between each alternative to FPIS and FNIS.

- Identical to step 4 in Section 5.2.

Step 5: Find the Closeness Coefficients (CC) for the benefit and cost.

- Identical to step 5 in Section 5.2.

- *Step 6: Derive the Influenced Closeness Coefficient (ICC) by applying the influence degree of each decision maker, then find the Normalised ICC (NICC) and divide it by its maximum value.*
- Identical to step 6 in Section 5.2.

Step 7: Construct the antecedent matrices for BS and CS based on DMs opinions
- Identical to step 1 in Section 5.2.

Step 8: Construct the consequent matrices for BS and CS systems based on the value of the NICC coefficients.
- Identical to step 1 in Section 5.2.

**Step 9: Construct the antecedent matrices for the alternatives system (AS)*

The AS antecedent matrices M_k are based on the Benefit Levels Λ_k and Cost Levels Ψ_k, which are the outputs of the BS and CS systems, correspondingly. The antecedent matrix of a system with two inputs, i.e. BL and CL, each taking m possible values, is usually of size $2\times(m\cdot m)$, as presented in Eq. (6.1).

$$M_k = \begin{array}{c} BL \\ CL \end{array}\begin{bmatrix} \lambda_{1,k} & \cdots & \lambda_{1,k} & \cdots & \lambda_{m,k} & \cdots & \lambda_{m,k} \\ \psi_{1,k} & \cdots & \psi_{m,k} & \cdots & \psi_{1,k} & \cdots & \psi_{m,k} \end{bmatrix} \text{for} \tag{6.1}$$

$k = 1, \cdots, K$.

However, in this case, each tuple of inputs $\left(\lambda_{j,k}, \psi_{j,k}\right)$ stands for the assessed levels of the same alternative j through two types of criteria – benefits and costs. Therefore, the AS antecedent matrices M_k are of size $2\times m$, as constructed in Eq. (6.2).

$$M_k = \begin{array}{c} \\ BL \\ CL \end{array} \begin{array}{c} \begin{array}{ccccc} A_1 & A_2 & A_3 & \cdots & A_m \end{array} \\ \begin{bmatrix} \lambda_{1,k} & \lambda_{2,k} & \lambda_{3,k} & \cdots & \lambda_{m,k} \\ \psi_{1,k} & \psi_{2,k} & \psi_{3,k} & \cdots & \psi_{m,k} \end{bmatrix} \end{array} \quad \text{for} \tag{6.2}$$

$k = 1, \cdots, K$.

**Step 10: Construct the consequent matrices for the alternative system (AS)*

The AS consequent matrices are derived as shown in Eq. (6.3-6.5). Then, the aggregation $\xi_{j,k}$ of weighted $NICC^B_{j,k}$ and $NICC^C_{j,k}$, is calculated as shown in Eq. (6.3).

$$\xi_{j,k} = \frac{NICC^B_{j,k} \times \left(\frac{e}{e+f} \right) + NICC^C_{j,k} \times \left(\frac{e}{e+f} \right)}{2} \tag{6.3}$$

for $j = 1, \cdots, m$ and $k = 1, \cdots, K$.

After that, the values of $\xi_{j,k}$ are normalized to ensure they lie within $[0,1]$, as calculated in Eq. (6.4):

$$N\xi_{j,k} = \xi_{j,k} \Big/ \max_j \xi_{j,k} \qquad \text{for } j = 1, \cdots, m \text{ and} \tag{6.4}$$

$k = 1, \cdots, K$.

For $N\xi_{j,k}$, the linguistic terms is taken from Table 4.3 for the alternatives levels. The K AS consequent matrix, in this case, is of size $1 \times m$ rather than $1 \times m \cdot m$, which is described in Eq. (6.5).

$$N_k = AL \begin{array}{c} \begin{array}{cccc} A_1 & A_2 & \cdots & A_m \end{array} \\ \begin{bmatrix} N\xi_{1,k} & N\xi_{2,k} & \cdots & N\xi_{m,k} \end{bmatrix} \end{array} \quad , \tag{6.5}$$

for $k = 1, \cdots, K$ and AL is the level of alternatives.

Step 11: Derive rule bases for the subsystems

The alternative system is presented with K matrix decision rules is constructed in Eq. (6.6) such that,

$$\text{If} \quad M_k = \begin{array}{c} \\ BL \\ CL \end{array} \begin{array}{c} \begin{array}{cccc} A_1 & A_2 & \cdots & A_m \end{array} \\ \begin{bmatrix} \lambda_{1,k} & \lambda_{1,k} & \cdots & \lambda_{m,k} \\ \psi_{1,k} & \psi_{2,k} & \cdots & \psi_{m,k} \end{bmatrix} \end{array} \tag{6.6}$$

then $N_k = AL\begin{array}{cccc} A_1 & A_2 & \cdots & A_m \\ \left[N\xi_{1,k}\right. & N\xi_{2,k} & \cdots & \left.N\xi_{m,k}\right] \end{array}$, for $k = 1,\cdots,K$.

This system is described using the rule bases in Eq. (6.7) such that:

Rule 1 : If BL is $\lambda_{1,k}$ and CL is $\psi_{1,k}$ then AL is $N\xi_{1,k}$ (6.7)

$\vdots \qquad \vdots \qquad \vdots$

Rule m : If BL is $\lambda_{m,k}$ and CL is $\psi_{m,k}$ then AL is $N\xi_{m,k}$, for $k = 1,\cdots,K$;

and BL, CL and AL are respectively the level of benefits, costs and alternatives.

Step 12: Derive final score for each alternative.

The final score Γ_j for each alternative *j*, is calculated by averaging the aggregate membership value of the consequent part of the n_j rules in Eq. (6.7) and multiplied by the influence multiplier based on the *K* DMs average influence degree for alternative *j* as shown in Eq. (6.8).

$$\Gamma_j = \frac{\sum_{rule=1}^{n}\sum_{k=1}^{K}\left(N\hat{\xi}_{j,k} \times \left[NICC_{j,k}^{B} + NICC_{j,k}^{C}\right]\right)}{\left(\beta_{j,k}\right)K} , \tag{6.8}$$

for $j = 1\cdots m$ and $k = 1\cdots K$.

The denominator $\beta_{j,k}$ is the number of rules for each alternative A_j and $N\hat{\xi}_{j,k}$ represents the aggregate membership value of rules of each alternative.

Step 13: Finally, rank alternative base on final score value, the higher final value the better the alternative performance.

Thus, the ranking order of all alternatives can be determined such that high values of Γ_j means better alternatives j.

6.2.2 *Type-2 Fuzzy Set Implementation*

The formulation is identical to Section 5.3 for the interval type-2 implementation of fuzzy system with multiple rule bases. This implimentation is now applied to formulation in Section 6.2.1.

6.2.3 *Z-Number Implementation*

The formulation is also identical to Section 5.4 for the Z-number implementation of fuzzy system with multiple rule bases and now it is applied to the formulation in Section 6.2.1.

6.3 Fuzzy Network Approach with Rule Base Merging

In this section, the TOPSIS approach proposed in section 6.2 is extended by using the fuzzy network with rule base merging aiming to apply its ability in TOPSIS decision processes.

The decision maker opinions in this approach are independently evaluated since they may have different influence degrees, depending on their experience in an area. Furthermore, the criteria are categorised either into benefit criteria or cost criteria. Each category correspondingly generates the benefit fuzzy system or the cost fuzzy system, where the outputs of the systems are Benefit Levels (BL) or Cost Levels (CL), representing the performance of each category. Fig. 6.2 illustrates the proposed Generalised Fuzzy Network Model for TOPSIS, where Benefit Systems (BS), Cost Systems (CS) and Alternatives Systems (AS) are incorporated in the form of fuzzy network nodes. The inputs are the benefit criteria $B_1,\ldots,B_e$ and the cost criteria $C_1,\ldots,C_f$. At the end of the processes, Alternatives Level (AL) are determined. The dotted frame represents the vertical merging of rule bases and the dashed frame illustrates the horizontal merging of rule bases.

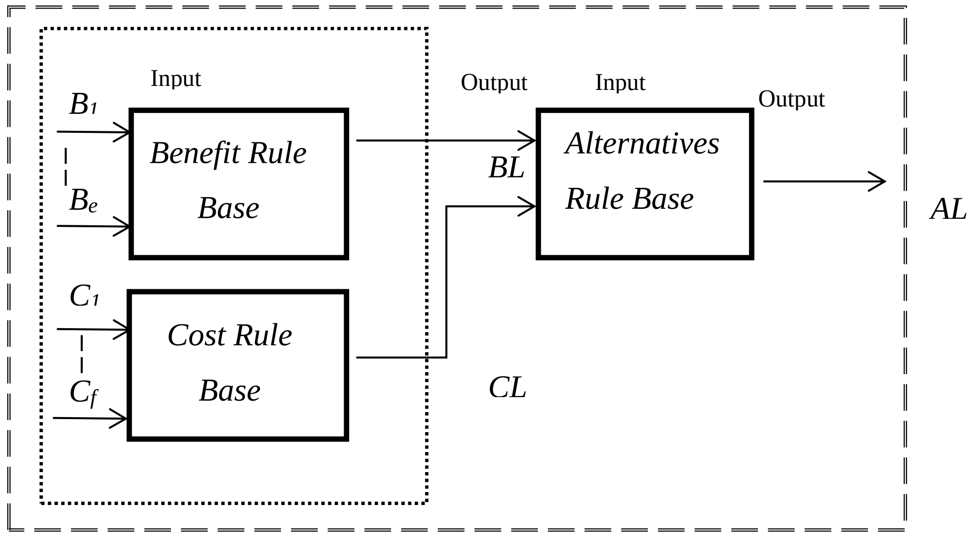

Fig. 6. 2: Fuzzy network model using rule base merging

The next sections will discuss the implementation of type-1, type-2 and Z-fuzzy sets to fuzzy network with rule base merging. For type-1 fuzzy set implementation of this approach, namely T1-MFN TOPSIS, Step 1-8 and Step 12-13 are identical to the steps discussed in Section 5.2 and step 9-10 are identical to the step discussed in Section 6.2.1. In addition, step 11 – 14 are introduced as part of the proposed algorithm in this subsection.

6.3.1 Type-1 Fuzzy Set Implementation

T1-MFN TOPSIS algorithm

Step 1: Construct decision matrix where each decision maker opinion is evaluated independently and categorised into two Criteria Categories which are Benefit Criteria and Cost Criteria defined respectively through a Benefit system (BS) and a Cost system (CS).

- Identical to step 1 in Section 5.2

Step 2: Construct the weighted and the normalised decision matrices

- Identical to step 2 in Section 5.2

Step 3: Find the Fuzzy Positive Ideal Solution (FPIS) and Fuzzy Negative Ideal Solution (FNIS) for each alternative.

- Identical to step 3 in Section 5.2

Step 4: Find the distance between each alternative to FPIS and FNIS.

-Identical to step 4 in Section 5.2

Step 5: Find the Closeness Coefficients (CC) for the benefit and cost systems.

-Identical to step 5 in Section 5.2

Step 6: Derive the Influenced Closeness Coefficient (ICC) by applying the influence degree of each decision maker, then find the Normalised ICC (NICC), divide it by its maximum value.

-Identical to step 6 in Section 5.2

Step 7: Construct the antecedent matrices for the BS and CS based on DMs opinions

-Identical to step 7 in Section 5.2

Step 8: Construct the consequent matrices for the BS and CS systems based on the value of the NICC coefficients.

-Identical to step 8 in Section 5.2

Step 9: Construct the antecedent matrices for the alternative system

-Identical to step 9 in Section 6.2.1

Step 10: Construct the consequent matrices for the alternative system

-Identical to step 10 in Section 6.2.1

**Step 11: Construct the generalised Boolean matrices representing BS, CS and AS systems.*

After deriving the rules for the three systems - BS, CS and AS – we can now translate these rules into Boolean matrix forms. Firstly, the generalised BS Boolean matrix for each alternative j is constructed in Eq. (6.9), based on the opinions of all K decision makers.

$$
\begin{array}{c|ccc}
 & \lambda_{j,1} & \cdots & \lambda_{j,K} \\
\hline
x_{1j,1}\cdots x_{ej,1} & 1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
x_{1j,K}\cdots x_{ej,K} & 0 & \cdots & 1
\end{array} \tag{6.9}
$$

for $j = 1, \cdots, m$

The rows and the columns of the Boolean matrix are all possible permutation for the BS rule base of the linguistics terms for the input (1-7) as in Tables 4.1 and 4.2, and of the linguistic terms for the output (1-5) as in Table 4.3.

Secondly, the generalised CS Boolean matrix for each alternative j is constructed in Eq. (6.10) based on the opinions of all K decision makers.

$$
\begin{array}{c|ccc}
 & \psi_{j,1} & \cdots & \psi_{j,K} \\
\hline
y_{1j,1}\cdots y_{fj,1} & 1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
y_{1j,K}\cdots y_{fj,K} & 0 & \cdots & 1
\end{array} \tag{6.10}
$$

, for $j = 1, \cdots, m$

Similarly, the rows and the columns of the Boolean matrix are all possible permutation for the CS rule base of the linguistic terms for the input (1-7) as in Tables 4.1 and 4.2, and of the linguistic terms for the output (1-5) as in Table 4.3.

Finally, the AS generalised Boolean matrix for each alternative j is introduced in Eq. (6.11) based on the opinions of all K decision makers.

$$
\begin{array}{c|ccc}
 & N\xi_{j,1} & \cdots & N\xi_{j,K} \\
\hline
\lambda_{j,1}\quad \psi_{j,1} & 1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
\lambda_{j,K}\quad \psi_{j,K} & 0 & \cdots & 1
\end{array}
\tag{6.11}
$$

$$, \text{for } j = 1, \cdots, m$$

**Step 12: Perform vertical merging to merge Boolean of the BS and CS.*

The vertical merging of the BS and CS generalised Boolean matrices will produce the generalised Boolean matrix constructed in Eq. (6.12).

$$
\begin{array}{c|ccc}
 & \begin{matrix}\lambda_{j,1} \\ \psi_{j,1}\end{matrix} & \cdots & \begin{matrix}\lambda_{j,K} \\ \psi_{j,K}\end{matrix} \\
\hline
\begin{matrix}x_{1j,1}\cdots x_{ej,1} \\ y_{1j,1}\cdots y_{fj,1}\end{matrix} & 1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
\begin{matrix}x_{1j,K}\cdots x_{ej,K} \\ y_{1j,K}\cdots y_{fj,K}\end{matrix} & 0 & \cdots & 1
\end{array}
\tag{6.12}
$$

$$\text{for } j = 1, \cdots, m$$

Step 13: Perform horizontal merging to merge the resultant matrix from step 12 with the Boolean matrix of the AS.

The result of the generalised Boolean matrix for the overall system of each alternative j is produced in Eq. (6.13) based on the opinions of all K decision makers.

$$
\begin{array}{cccc}
 & N\xi_{j,1} & \cdots & N\xi_{j,K} \\
x_{1j,1}\cdots x_{ej,1} & 1 & \cdots & 0 \\
y_{1j,1}\cdots y_{fj,1} & & & \\
\vdots & \vdots & \ddots & \vdots \\
x_{1j,K}\cdots x_{ej,K} & 0 & \cdots & 1 \\
y_{1m,k}\cdots y_{fm,k} & & &
\end{array}
\quad (6.13)
$$

$$\text{for } j = 1,\cdots,m$$

Step 14: Derive rules for each stock based on horizontal merging of Boolean matrix.

Derive the rules for the alternatives based on the generalised Boolean matrix from Eq. (6.13), as shown in Eq. (6.14) for $j = 1,\cdots,m$.

Rule 1: If B_1 is $x_{1j,1}$ and $\cdots$and B_e is $x_{ej,1}$ and C_1 is $y_{1j,1}$ and $\cdots$and C_f is $y_{fj,1}$ then AL is $N\xi_{j,1}$ (6.14)

$\vdots \qquad \vdots \qquad \vdots$

Rule n_j: If B_1 is $x_{1j,K}$ and $\cdots$and B_e is $x_{ej,K}$ and C_1 is $y_{1j,K}$ and $\cdots$and C_f is $y_{fj,K}$ then AL is $N\xi_{j,K}$

Step 15: Derive final score for each alternative,

- Identical to step 12 in Section 6.2.1

Step 16: Finally rank the alternative base on final score value, the higher final value the better the alternative performance.

- Identical to step 13 in Chapter 6.2.1

6.3.2 *Type-2 Fuzzy Set Implementation*

The formulation is identical to Section 5.3 for the interval type-2 implementation of fuzzy system with multiple rule bases but now it is applied to formulation in Section 6.3.1

6.3.3 Z-Number Implementation

The formulation is identical to Section 5.4 for the Z-number implementation of fuzzy system with multiple rule bases but now it is applied to formulation in Section 6.3.1.

6.4 Summary

In summary, this chapter extends the TOPSIS methods by using the properties of fuzzy network. Section 6.1 presents introduction of the chapter. Section 6.2 reviews some introduction of fuzzy network approach and 6.2.1 presents the algorithm to implement type-1 fuzzy number, whereas Section 6.2.2 presents the algorithm of interval type-2 implementation. The algorithm for the implementation of Z-numbers is presented in Section 6.2.3.

Furthermore, Section 6.3 briefly reviews fuzzy networks with merging of rule base. Section 6.3.1 presents the algorithm to implement type-1 fuzzy number, whereas Section 6.3.2 presents the algorithm of interval type-2. The algorithm for the implementation of Z-numbers is presented in Section 6.3.3. The proposed methods allow hybrid analysis of empirical knowledge of experts in the process of decision making as well improve the level transparency for criteria of method proposed in Chapter 5 that is based on fuzzy system with multiple rule bases. The applicability and the practicality of proposed method in this chapter will be verified by the case study of stock selection in Chapter 7.

CHAPTER 7

7 CASE STUDY

7.1 Introduction to Stock Selection

A stock market plays an importance role in the economic development of any country. It is regarded as a mechanism for effective mobilization of domestic fund to support economic development and also to efficiently allocate resources [96]. The stock market is one of the most important sources for companies to raise money allowing businesses to go public or raise additional capital for expansion. In Malaysia, the Bursa Malaysia or Kuala Lumpur Stock Exchange (KLSE) is the only stock market in the country. Its importance has been acknowledged by government with the Securities Commission as the role to oversee the sound development of stock industry in Malaysia. Securities Commission Malaysia (SCM) is a self-funding statutory body to investigative and to enforce powers. The SCM's many regulatory functions include supervising exchanges, approving authority for corporate bond issues, regulating all matters relating to stock and futures contracts, and ensuring proper conduct of market institutions and licensed persons. All these functions are the SCM's ultimate responsibility to protect investors. One of the many things people always want to know about the stock market is, "How do I make money grows in the stock market?" Therefore, researches regarding the stock market problem were studied and classified as either fundamental analysis or technical analysis with several approaches. Market Modern portfolio analysis was pioneered by Markowitz in the year 1952. The stock selection model was first formulated by Markowitz called mean–variance model. Based on this model, absolute deviation portfolio optimization and semi-absolute deviation portfolio optimization models were proposed.

The literatures had mentioned several approaches to construct a portfolio. For example, [145] introduced multi agent model for multi period portfolio selection. In this, model they managed an equal share of initial investment and divided profit and loss at the end of investment. This system gives a better average performance than the single system. Another instance is when [146] made a comparison between Stochastic Programming and fuzzy mathematical programming through a portfolio problem, they came up with fuzzy solution. In 2004, the Mean-Gini Analysis Model proposed by [147] argued that Mean-Gini analysis model is efficient by the criteria of stochastic dominance (SD). This approach is applicable to all risk adverse decision makers. Based from [148], their research paper has developed a minimax regret approach based on regret function, which treats the expected return rates of stock as fuzzy or possibility variables. In order to solve a portfolio management problem, [149] developed stochastic soft constraints fuzzy model for portfolio selection problem which captures both uncertainty and imprecision. It is based on stochastic and possibility programming. By applying some parametric and non-parametric method, [150] used scenario generation techniques to solve portfolio selection problem. In Sharpe's single-index model, proposed by [151], the return of each asset is related to variations in their turn on a market index. In short, these studies show the various methods of constructing a portfolio to optimize their results.

One of the problems in stock market is allocating one's capital to appropriate stocks so that the investment can bring the most profitable return. Deciding which assets is challenging because of the uncertainty on their returns. Most investors choose stocks based on a company's financial data. They want to buy stocks among major stocks traded on the KLSE but which stock they should buy and the priority of stocks to invest. Consequently, this research is conducted to fulfil their expectation by using fuzzy approaches to produce ranking of stock traded in the market to investors.

The indicators of stock performance and company's reputation are represented by the ratios of benefit criteria and cost criteria. The first benefit criteria ratio is one of the most importance ratio considered in investment is market value of firm (B1) defined as market value of firm-to-earnings before amortization, interest and taxes ratio. This ratio is one of the most frequently used financial indicators and the higher the ratio the better the stock [109]. The second ratio is return on equity (B2) which is used to examine the company's earning from the investment of its shareholders. Portfolio managers examine this ratio when deciding when to trade (buy or sale) stocks. High values of the ratio indicating a healthy company[152]. Furthermore, the current ratio (B3) is one of the methods to measure the liquidity of a company. The higher the ratio, the more liquidity of the company; hence, a better position [153]. Finally, the market value or net sales (B4) is market value ratios of interest to the investor such as earnings per common share, the price-to-earnings ratio, market value-to book value ratio, earning-to-price ratio. The lower this ratio, the better the stock[109]. On the other hand, the first cost criteria ratio is debt or equity ratio (C1) which belongs to long term solvency ratios that are intended to address the firm's long run ability to meet its obligations. DMs consider a better performance if its value is low. Price or earnings ratio (C2) measure the ratio of market price of each share of common stock to the earnings per share, the lower the ratio, the better the stock [154]. All in all, the benefit criteria and the cost criteria are respectively labelled as B1, B2, B3, B4 and C1, C2.

This research study the problem of ranking traded stock in developing financial markets within a crisis period. The applicability and the validity of the proposed methods are described in Chapter 4-6 in a realistic scenario. Decision makers with different levels of experience evaluate 30 stocks listed on the Main Board of the Kuala Lumpur Stock Exchange (KLSE). The list of stocks include AMMB Holdings (S1), Astro Malaysia Holdings (S2), Axiata Group Bhd (S3), British American Tobacco (Malaysia) (S4), CIMB Group Holdings (S5), Digi.com (S6), Genting (S7), Genting Malaysia BHD (S8), Hong Leong Bank (S9), Hong Leong Financial (S10), IHH Healthcare (S11),

IOI (S12), KLCC PROP & KLCC REITS - STAPLED SC (S13), Kuala Lumpur Kepong (S14), Malayan Banking (S15), Maxis Bhd (S16), MISC (S17), PETRONAS Chemicals Group Bhd (S18), Petronas Dagangan Bhd (S19), Petronas Gas (S20), PPB Group (S21), Public Bank BHD (S22), RHB Capital (S23), Sapura Kencana Petroleum (S24), Sime Darby Bhd (S25), Telekom Malaysia (S26), Tenaga Nasional (S27), UMW Holdings (S28), YTL Corp (S29), Westports Holdings Bhd (S30). The access to all data used in this case study was ethically approved in advance as can be seen in Appendix 1.

The linguistic terms in Tables 7.1- 7.4 are converted by using the fuzzy numbers in Tables 4.1- 4.3 respectively, where IC represents important criteria and CL is confident level of decision maker about their decision. The rating (R) of criterion for each stock, the importance of criteria and the influence weight of each decision maker are based on decision maker opinions presented in Table 7.1-7.4. The experts opinion in Table 7.1 presented in linguistic terms with respect to 6 criteria considered in this study which consist of 4 benefit criteria and 2 cost criteria.

Table 7. 1: Importance of benefit and cost criteria based on DMs opinions

	MVF (B1)		**ROE (B2)**		**D/E (C1)**		**CR (B3)**		**MV/NS (B4)**		**P/E (C2)**	
	IC	CL	IC	CL	IC	CL	IC	CL	IC	CL	IC	CL
DM1	H	L	M	SL	H	SWL	H	SL	VH	SL	H	SL
DM2	VH	L	MH	SL	H	SWL	M	SL	H	SWL	M	SL
DM3	VH	SL	MH	L	VH	SL	M	SWL	H	SL	MH	SWL

Table 7. 2: Rating of each criterion for each stock based on DM1 opinions

Stocks	MVF (B1)		ROE (B2)		D/E (C1)		CR (B3)		MV/NS (B4)		P/E (C1)	
	R	CL	R	CL	R	CL	R	CL	R	CL	R	CL
S1	VG	SL	P	SWL	F	SWL	P	SWL	F	SWL	VG	L
S2	VG	L	MG	SL	MP	L	MP	SL	F	SL	F	SWL
S3	MP	SL	MP	L	MG	SL	MP	L	F	L	MG	SL
S4	G	SWL	VG	SWL	G	L	MP	SWL	MG	SWL	MG	SL
S5	MP	SL	P	L	MP	SL	P	SL	F	L	G	L
S6	MP	L	MG	SL	F	SWL	MP	L	MG	SL	MG	SWL
S7	F	SL	P	SWL	G	L	MG	SWL	G	L	G	SL
S8	MG	L	MP	SL	G	L	MG	L	MG	L	G	SWL
S9	MG	SWL	MP	L	MG	SL	P	L	P	SWL	G	L
S10	VG	L	MP	SL	MG	SL	P	SL	G	L	VG	L
S11	MP	SWL	MP	L	VG	SWL	MG	L	P	SL	P	SL
S12	F	L	F	SWL	F	SWL	G	SWL	MG	L	MG	L
S13	VG	SL	MP	L	VG	L	MG	SL	P	SL	G	SWL
S14	MG	SL	F	SWL	G	SL	MG	L	G	SWL	MG	SL
S15	P	SWL	F	L	MG	L	P	SL	VP	SL	G	L
S16	F	L	MG	SL	MP	SWL	MP	L	F	SL	MG	SWL
S17	MP	SL	MP	L	G	SWL	F	SWL	P	L	G	SL
S18	P	L	F	L	VG	SL	VG	L	MG	SL	MG	L
S19	G	SWL	F	SL	G	L	MP	SL	VG	SWL	MP	SL
S20	MG	L	MG	SWL	VG	SL	F	SWL	VP	L	MG	L
S21	VG	SL	MP	SWL	VG	SWL	MG	SL	MP	SL	G	SWL
S22	MP	SL	MG	L	MG	SWL	P	SL	P	SWL	G	SL
S23	G	SWL	F	SL	MG	SL	P	L	MG	L	VG	L
S24	VG	L	F	SWL	F	L	MG	SWL	G	SL	G	SL
S25	MP	L	F	L	G	SWL	G	SL	G	L	G	L
S26	MP	L	F	SWL	MG	SWL	MG	SWL	F	SWL	MG	SWL
S27	P	SWL	MG	L	MG	SL	MG	L	G	SWL	VG	L
S28	G	L	MG	SWL	MG	L	G	SL	VG	SL	G	SL
S29	VG	SL	MG	SL	MP	SWL	MP	L	VG	L	VG	L
S30	G	L	G	L	F	L	F	L	VP	SL	MG	L

Table 7.2 shows the DM1 opinion on rating of stock and the reliability of their opinion based on financial data considered in this study.

Table 7. 3: Rating of each criterion for each stock based on DM2 opinion

Stocks	MVF (B1)		ROE (B2)		D/E (C1)		CR (B3)		MV/NS (B4)		P/E (C2)	
	R	CL	R	CL	R	CL	R	CL	R	CL	R	CL
S1	MG	SL	VP	SL	MP	N	VP	L	MP	SWL	MG	SL
S2	VG	N	G	L	F	SWL	F	L	MG	L	MG	L
S3	MP	SWL	P	N	MG	SL	MG	SL	MP	SL	F	SL
S4	VG	SWL	VG	SWL	VG	SWL	G	SL	G	SWL	G	SWL
S5	P	SL	P	SWL	P	SL	P	L	MP	L	MG	L
S6	MP	N	F	SL	F	SWL	MP	SWL	MP	L	F	SL
S7	MG	L	MP	L	G	L	MG	SWL	F	SL	G	SL
S8	MG	SL	F	L	G	SWL	MG	SL	MG	SWL	G	SL
S9	MG	L	F	SWL	G	SL	MG	L	F	SL	G	SWL
S10	VG	SWL	F	SL	G	SL	MG	SL	G	L	VG	SWL
S11	G	L	MG	L	VG	N	G	SWL	VG	SWL	G	SWL
S12	F	SL	MG	SL	MG	L	G	L	MG	SL	G	L
S13	VG	SWL	F	SWL	G	L	G	SL	F	L	G	L
S14	MG	L	MG	N	G	SWL	G	SWL	VG	L	G	SL
S15	P	SL	MG	L	F	SL	MP	SL	P	SWL	MG	SL
S16	F	N	G	SWL	MG	SWL	MG	SWL	MG	SWL	G	SWL
S17	P	SL	P	SL	F	SL	MP	SL	VP	L	MG	SWL
S18	P	SWL	MG	SWL	G	L	F	SL	G	L	G	SWL
S19	G	SL	G	SWL	VG	N	F	SWL	VG	SL	MG	SL
S20	G	L	G	SL	VG	SL	MG	L	MP	SWL	G	SWL
S21	G	SWL	F	L	VG	L	MG	L	F	L	MG	SL
S22	F	SWL	G	L	G	SWL	MP	SWL	MP	L	MG	SWL
S23	F	L	MP	N	F	SL	P	SL	F	SL	G	L
S24	G	SL	MG	SL	MG	SWL	MG	L	MG	SWL	G	SWL
S25	F	SWL	MG	L	MG	L	G	SL	MG	L	G	SL
S26	P	L	MG	SWL	MP	N	F	SWL	MP	SWL	F	SL
S27	P	N	F	SL	MP	SWL	F	L	F	L	G	SWL
S28	MG	SL	F	N	MP	SL	MG	SWL	MG	SL	MG	L
S29	VG	SL	G	SWL	MG	SWL	F	SL	VG	L	VG	SL
S30	MG	SWL	MG	SL	P	N	MP	L	P	L	F	L

Table 7.3 shows the DM2 opinion on rating of stock and the reliability of their opinion based on financial data considered in this study.

Table 7. 4: Rating of each criterion for each stock based on DM3 opinion

Stocks	MVF (B1)		ROE (B2)		D/E (C1)		CR (B3)		MV/NS (B4)		P/E (C2)	
	R	CL	R	CL	R	CL	R	CL	R	CL	R	CL
S1	MG	SWL	VP	L	MP	SL	VP	SWL	P	SWL	G	SWL
S2	VG	L	G	SL	MG	SWL	MG	SWL	MG	L	MG	L
S3	MP	SL	P	L	F	L	MP	L	F	SL	MG	L
S4	VG	SL	VG	SL	VG	SL	MG	L	G	SL	G	SL
S5	P	L	VP	SWL	P	L	MP	SWL	MP	L	MG	L
S6	F	SWL	F	L	MP	SL	F	SL	F	SL	F	SWL
S7	MG	SL	MP	L	MG	SWL	G	SL	MG	SL	G	SWL
S8	G	L	F	SL	MG	SL	G	L	MG	SWL	G	SL
S9	G	L	F	SL	G	L	F	L	MG	SL	G	L
S10	VG	SL	MG	L	VG	SL	MG	SL	G	SL	VG	SL
S11	G	SWL	G	SL	VG	SWL	G	SL	G	L	VG	SL
S12	MG	SL	MG	SWL	MG	SL	G	SWL	G	L	MG	SWL
S13	G	SWL	MG	SWL	G	SWL	MG	SWL	F	SWL	G	SL
S14	G	L	G	L	G	L	MG	L	G	SWL	G	L
S15	MP	SL	MG	SWL	F	L	P	L	P	L	MG	SWL
S16	G	SL	G	L	MG	SWL	MG	SL	MG	SL	G	L
S17	P	SWL	P	SL	F	SL	MP	SL	P	SL	MG	L
S18	MP	L	MG	SWL	G	L	G	SWL	MG	SWL	MG	SWL
S19	VG	SWL	G	SWL	VG	SWL	MG	L	VG	SL	G	SL
S20	G	SWL	G	L	VG	SL	MG	L	F	L	G	SL
S21	VG	L	MG	SL	G	SWL	MG	SL	MG	SL	G	L
S22	F	SL	G	SL	G	SL	F	SL	F	SWL	MG	SL
S23	MG	L	F	SWL	F	SL	P	SL	F	SWL	G	SWL
S24	VG	SWL	MG	SWL	G	SWL	G	SWL	MG	L	G	SWL
S25	F	L	MG	SWL	MG	SL	G	L	MG	L	MG	L
S26	P	SWL	F	SL	F	L	F	L	MP	SL	F	L
S27	P	SL	F	SL	F	SWL	F	SL	MG	SWL	G	SL
S28	MG	L	F	L	MG	L	MG	SWL	G	SL	MG	SWL
S29	VG	SWL	G	L	MG	SL	MG	SL	VG	SWL	VG	SL
S30	F	SL	F	SL	F	SL	MP	SL	VP	SL	MG	L

Table 7.4 shows the DM3 opinion on rating of stock and the reliability of their opinion based on financial data considered in this study.

7.2 Conventional Approach

In this section, the ranking of 30 stocks based on established conventional TOPSIS approach in the case study is presented in Table 7.5. In this approach, the weighting and the rating of each alternative are assumed as crisp value and no uncertainty, no reliability as well as no influence degree are considered. Table 7.5 shows the final ranking of 30 stocks based on conventional approach which indicate S29 is the most preferable where as S26 is the worst for investment. The comparative analysis of approaches has been done in more details in Chapter 8.

Table 7. 5: Ranking based on conventional TOPSIS

Stock	CC	Rank
S1	0.2885	5
S2	0.2867	27
S3	0.2869	26
S4	0.2880	13
S5	0.2877	21
S6	0.2864	29
S7	0.2884	10
S8	0.2884	8
S9	0.2884	9
S10	0.2894	2
S11	0.2869	24
S12	0.2876	22
S13	0.2884	7
S14	0.2880	14
S15	0.2877	19
S16	0.2880	16
S17	0.2877	20
S18	0.2876	23
S19	0.2866	28
S20	0.2880	15
S21	0.2881	11
S22	0.2877	18
S23	0.2888	3
S24	0.2884	6
S25	0.2880	12
S26	0.2864	30
S27	0.2888	4
S28	0.2877	17
S29	0.2894	1
S30	0.2869	25

7.3 Non-Rule Based Fuzzy Approach

The 30 stocks are ranked by considering the established TOPSIS methods- namely T1, T2 and Z-TOPSIS which correspond to the methodology discussed in Section 3.6. These approaches have considered uncertainty at certain levels without the expert's experience. These rankings of 30 stocks based on three established TOPSIS methods of non-rule based fuzzy approach are provided in Table 7.6. Table 7.6 shows the final ranking of 30 stocks based on TOPSIS non rule bases fuzzy approach for T1, T2 and Z implementation which indicate S4 is most preferable for T1 , T2 implemtation and S26 be the best for Z implementation, where as S5 is the worst option for T1, T2, and S27 is the worst for Z to invest. The comparative analysis of approaches has been done in more details in Chapter 8.

Table 7. 6: Ranking based on established non-rule based fuzzy TOPSIS

	T1- TOPSIS		T2-TOPSIS		Z- TOPSIS	
Stocks	Final Score	Rank	Final Score	Rank	Final Score	Rank
S1	0.409	27	0.3911	27	0.453	14
S2	0.602	16	0.6782	13	0.547	2
S3	0.435	26	0.4095	25	0.243	25
S4	0.714	1	0.8758	1	0.529	6
S5	0.316	30	0.2219	30	0.274	24
S6	0.454	23	0.4443	23	0.300	23
S7	0.606	15	0.6597	16	0.414	19
S8	0.643	8	0.7389	7	0.484	12
S9	0.570	19	0.6161	18	0.447	15
S10	0.677	6	0.7812	5	0.538	3
S11	0.634	11	0.7307	8	0.422	18
S12	0.622	14	0.7002	11	0.427	17
S13	0.636	9	0.6772	14	0.486	10
S14	0.692	2	0.8460	2	0.490	9
S15	0.391	28	0.3271	28	0.239	26
S16	0.592	17	0.6636	15	0.433	16
S17	0.371	29	0.2920	29	0.195	29
S18	0.591	18	0.6055	19	0.221	28
S19	0.681	5	0.8058	3	0.502	7
S20	0.623	13	0.6522	17	0.501	8
S21	0.650	7	0.7180	10	0.529	5
S22	0.523	20	0.5278	21	0.370	21
S23	0.521	21	0.5385	20	0.408	20
S24	0.685	3	0.7979	4	0.535	4
S25	0.625	12	0.6851	12	0.332	22
S26	0.443	25	0.4067	26	0.234	27
S27	0.510	22	0.4923	22	0.194	30
S28	0.634	10	0.7270	9	0.486	11
S29	0.685	4	0.7713	6	0.578	1
S30	0.443	24	0.4188	24	0.475	13

7.4 Fuzzy System Approach with Single Rule Base

In this section, the case study of stock selection based on the proposed methods in Chapter 4 is described in step by step manner.

T1-SFS TOPSIS algorithm

Step 1: Construct decision matrix where each decision maker opinion is evaluated independently

Based on the information provided by the experts in Tables 7.1-7.4 and using Eq. (4.1), the decision matrices for the system can be constructed. The linguistic terms in Tables 7.1-7.4 can be converted by using the fuzzy numbers in Tables 4.1- 4.3, respectively. The rating of each criterion for each stock and the importance of criteria are based on decision maker's opinions.

Step 2: Construct the weighted and the normalised decision matrices

The normalised decision matrix R_k and the weighted normalised decision matrix V_k can be constructed for each k, using equations Eq. (4.2) correspondingly.

For example, the calculations for S1 using the opinion of DM1 is as follows:

$$g_{1,1} = (0.8, 0.9, 1)$$

$$x_{1\,1,1} = (9, 10, 10)$$

$$c_{1,1}^{x*} = 10$$

$$r_{1\,1,1} = (9/10, 10/10, 10/10) = (0.9, 1, 1)$$

$$v_{1\,1,1} = (0.8 \times 0.9, , 0.9 \times 1, 1 \times 1) = (0.72, , 0.9, 1)$$

Step 3: Find the Fuzzy Positive Ideal Solution and Fuzzy Negative Ideal Solution for each alternative.

The Fuzzy Positive Ideal Solution (FPIS) and the Fuzzy Negative Ideal Solution (FNIS) for each stock based on the derived system and the distances between the rating of criteria for each stock and the FPIS and FNIS can be evaluated as follows.

FPIS and FNIS are determined as:

$$A_k^+ = \left[(1,1,1)_{1,k}, (1,1,1)_{2,k}, \ldots, (1,1,1)_{30,k}\right]$$
$$A_k^- = \left[(0,0,0)_{1,k}, (0,0,0)_{2,k}, \ldots, (0,0,0)_{30,k}\right]$$

Step 4: Find the distance between each alternative to FPIS and FNIS.

The distances $\Delta^+_{j,k}$ or $\Delta^-_{j,k}$ between the rating according to DM k of benefit criteria for $i = 1, \ldots, 4$ for each stock j ($j = 1, \ldots, 30$) and the FPIS A_k^+ or FNIS A_k^- are calculated using Eq. (4.3) and Eq. (4.4). For example, the distance between the first stock S1 according to DM1 and the FPIS A_1^+ is calculated using Eq. (4.3) for $j = 1$ and $k = 1$, as follows:

$$\Delta_k^+\left(v_{ij,k}, v_{i,k}^+\right) = \Delta_1^+\left(v_{11,1}, v_{1,1}^+\right)$$
$$= \sqrt{\frac{1}{3}\left[(0.72-1)^2 + (0.9-1)^2 + (1-1)^2\right]} = 0.1811$$

and similarly

$$D_k^+\left(v_{ij,k}, v_{i,k}^+\right) = D_1^+\left(v_{21,1}, v_{2,1}^+\right) = 1.2582$$

$$D_1^+\left(v_{31,1}, v_{3,1}^+\right) = 0.6958$$

$$D_1^+\left(v_{41,1}, v_{4,1}^+\right) = 01.1728$$

$$D_1^+\left(v_{51,1}, v_{5,1}^+\right) = 0.6590$$

$$D_1^+\left(v_{61,1}, v_{6,1}^+\right) = 0.1811$$

producing:

$$\Delta^+_{j,k} = \sum_{i=1}^{i} \Delta_k\left(v_{1j,k}, v_{1,k}^+\right)$$

$$D_{1,1}^+ = \sum_{i=1}^{6} D_1\left(v_{i1,1}, v_{i,1}^+\right)$$
$$= 0.1811 + 1.2582 + 0.6958 + 0.1.1728 + 0.6590 + 0.1811 = 4.1480$$

Next, using Eq. (4.4) for $j = 1$ and $k = 1$, the distance between S1 according to

DM1 and the FPIS A_1^- is calculated as:

$$\Delta_k^-\left(v_{ij,k}, v_{i,k}^-\right) = \Delta_1^-\left(v_{11,1}, v_{1,1}^-\right)$$
$$= \sqrt{\frac{1}{3}\left[(0.72-0)^2 + (0.9-0)^2 + (1-0)^2\right]} = 1.3087$$

and similarly

$$D_k^-\left(v_{ij,k}, v_{i,k}^-\right) = D_1^-\left(v_{21,1}, v_{2,1}^-\right) = 0.1992$$

$$D_1^-\left(v_{31,1}, v_{3,1}^-\right) = 0.8026$$

$$D_1^-\left(v_{41,1}, v_{4,1}^-\right) = 0.3089$$

$$D_1^-\left(v_{51,1}, v_{5,1}^-\right) = 0.8252$$

$$D_1^-\left(v_{61,1}, v_{6,1}^-\right) = 1.3087$$

producing:

$$D_{j,k}^- = \sum_{i=1}^{i} D_k\left(v_{1j,k}, v_{1,k}^-\right)$$
$$= D_{1,1}^- = \sum_{i=1}^{4} D_1\left(v_{i1,1}, v_{i,1}^-\right)$$
$$= 1.3087 + 0.1992 + 0.8026 + 0.3089 + 0.8252 + 1.3087 = 4.7534$$

Step 5: Find the Closeness Coefficients (CC).

The closeness coefficients for the benefit system $CC_{j,k}$ is found using Eq. (5) for each stock S*j*, $j = 1,\ldots,25$.

For example, the closeness coefficient for S1 in the benefit system under the first decision maker $k = 1$ is calculated using Eq. (4.5) as follows:

$$CC_{j,k} = \frac{D_{j,k}^-}{D_{j,k}^+ + D_{j,k}^-} = CC_{1,1} = \frac{D_{1,1}^-}{D_{1,1}^+ + D_{1,1}^-} = \frac{4.7534}{4.1480 + 4.7534} = 0.5340$$

Step 6: Derive the Influenced Closeness Coefficient (ICC) by applying the influence degree of each decision maker, then find Normalised ICC (NICC), dividing NICC by maximum value of NICC.

The Influenced Closeness Coefficient $ICC_{j,k}$ for each DM *k* is derived by applying the influence degree q_k of each decision maker by using Eq. (4.6) and Eq. (4.7). Then, the normalized coefficient $NICC_{j,k}$ is calculated using Eq. (4.8).

For example, the influence degree of DM1 is $q_1 = 5$, as given in Table 8.1, and using Eq. (4.6), this normalised expertise is:

$$\sigma_k = \frac{\theta_k}{\sum_{l=1}^{K} \theta_l} = \sigma_1 = \frac{\theta_1}{\sum_{l=1}^{3} \theta_l} = \frac{5}{5+6+7} = 0.2778$$

The Influenced Closeness Coefficient $ICC_{1,1}$ for the benefit system for stock S1 according to DM1 is calculated using Eq. (4.7) as:

$$ICC_{j,k} = \sigma_k \times CC_{j,k} = ICC_{1,1} = \sigma_1 \times CC_{1,1} = 0.2778 \times 0.5340 = 0.1483 \quad ,$$

Next, the influenced closeness coefficients are normalized prior to matching the coefficients to the linguistic terms in Table 4.3. Using Eq. (4.8), $NICC_{1,1}$ is calculated as:

$$NICC_{j,k} = {ICC_{j,k}} \Big/ {\max_j ICC_{j,k}}$$

$$= NICC_{1,1} = {ICC_{1,1}} \Big/ {\max_j ICC_{j,k}} = \frac{0.1483}{0.2078} = 0.7137$$

Finally, the normalised coefficients are matched to the linguistic terms in Table 4.3:

$$NICC_{1,1} = 0.7137 \cong G$$

Step 7: Construct the antecedent matrices for the single system based on DMs opinions

The antecedent matrix X_k for the system is constructed using Eq. (4.9) for $k = 1, \cdots, K$, based on DM *k* opinions given in Tables 8.2-8.4. Each decision maker has a separate antecedent matrix. For example, by using Eq. (4.9) and the first decision maker $k = 1$ as detailed in Tables 8.1- 8.4, the antecedent matrix X_1 for the system is:

$$X_k = \begin{array}{c} \\ CR_1 \\ CR_2 \\ \vdots \\ CR_e \end{array} \begin{array}{c} \begin{array}{cccc} S_1 & S_2 & \cdots & S_m \end{array} \\ \begin{bmatrix} x_{11,k} & x_{12,k} & \cdots & x_{1m,k} \\ x_{21,k} & x_{22}, & \cdots & x_{2m,k} \\ \vdots & \vdots & \ddots & \vdots \\ x_{e1,k} & x_{e2,k} & \cdots & x_{em,k} \end{bmatrix} \end{array} = X_1 = \begin{array}{c} \\ CR_1 \\ CR_2 \\ \vdots \\ CR_6 \end{array} \begin{array}{c} \begin{array}{cccc} S_1 & S_2 & \cdots & S_{30} \end{array} \\ \begin{bmatrix} x_{1,1,1} & x_{1,2,1} & \cdots & x_{1,30,1} \\ x_{2,1,1} & x_{2,2,1} & \cdots & x_{2,30,1} \\ \vdots & \vdots & \ddots & \vdots \\ x_{6,1,1} & x_{6,2,1} & \cdots & x_{6,30,1} \end{bmatrix} \end{array}$$

$$= \begin{array}{c} \\ CR_1 \\ CR_2 \\ \vdots \\ CR_6 \end{array} \begin{array}{c} \begin{array}{cccc} S_1 & S_2 & \cdots & S_{30} \end{array} \\ \begin{bmatrix} VG & VG & \cdots & G \\ P & MG & \cdots & G \\ \vdots & \vdots & \ddots & \vdots \\ VG & F & \cdots & MG \end{bmatrix} \end{array} ,$$

where CR_e are the benefit criteria and the cost criteria.

Step 8: Construct the consequent matrices for the single system based on the value of the NICC coefficients.

The consequent matrices Λ_k for the system are constructed using Eq. (4.10) for $k = 1, \cdots, K$, based on the values of $NICC_{j,k}$ calculated in step 6 before and matched to the linguistic terms in Table 4.3. Each decision maker has a separate consequent matrix. Then, using Eq. (4.10), the consequent matrix Λ_1 is:

$$\Lambda_k = AL \begin{array}{c} \begin{array}{cccc} S_1 & S_2 & \cdots & S_m \end{array} \\ \begin{bmatrix} \lambda_{1,k} & \lambda_{2,k} & \cdots & \lambda_{m,k} \end{bmatrix} \end{array} = \Lambda_1 = AL \begin{array}{c} \begin{array}{cccc} S_1 & S_2 & \cdots & S_{30} \end{array} \\ \begin{bmatrix} \lambda_{1,1} & \lambda_{2,1} & \cdots & \lambda_{30,1} \end{bmatrix} \end{array} =$$

$$AL \begin{array}{c} \begin{array}{cccc} S_1 & S_2 & \cdots & S_{30} \end{array} \\ \begin{bmatrix} G & G & \cdots & G \end{bmatrix} \end{array}$$

where AL is the alternative level.

Step 9: Derive rule bases for each alternative

The rule base of the single system for DM1 is constructed using Eq. (4.11) as follows:

If $X_1 = \begin{array}{c} \\ CR_1 \\ CR_2 \\ \vdots \\ CR_6 \end{array} \begin{array}{c} \begin{array}{cccc} S_1 & S_2 & \cdots & S_{30} \end{array} \\ \begin{bmatrix} VG & VG & \cdots & G \\ P & MG & \cdots & G \\ \vdots & \vdots & \ddots & \vdots \\ VG & F & \cdots & MG \end{bmatrix} \end{array}$, then $\Lambda_1 = AL \begin{array}{c} \begin{array}{cccc} S_1 & S_2 & \cdots & S_{30} \end{array} \\ \begin{bmatrix} G & G & \cdots & G \end{bmatrix} \end{array}$

Rule 1: If CR_1 is *VG*, CR_2 is *P*, $\cdots$ and CR_6 is V*G*. Then, the output *AL* is *G*.

Rule 2: If CR_1 is *VG*, CR_2 is M*G*, $\cdots$ and CR_6 is *F*. Then, the output *AL* is *G*.

Rule 3: If CR_1 is *G*, CR_2 is *G*, $\cdots$ and CR_6 is M*G*. Then, the output *AL* is *G*.

Step 10: Derive final score for each alternative.

The final score for each alternative $j = 1, \cdots, 30$ is derived using Eq. (4.12) by taking the average of the aggregate membership value of the consequent part of all active rules in the overall system for stock *j*, and then multiply with the influence multiplier based on the average influence degree across all *K* decision makers DMs for each stock *j*. For example, S1 generated from step 9 has three active rules and Eq. (4.12) is used in order to obtained final score for S1. The average aggregate membership value for the output of the three rules is calculated and then multiplied with the influence multiplier for S1 across all DMs.

$$\Gamma_j = \frac{\sum_{Rule=1}^{n} \sum_{k=1}^{K} \hat{\lambda}_{j,k} \cdot NICC_{j,k}}{n \cdot K} = \Gamma_1 = \frac{\sum_{Rule=1}^{3} \sum_{k=1}^{3} \hat{\lambda}_{j,k} \cdot NICC_{j,k}}{3 \cdot 3}$$

$$= [0.5667(0.7140) + 0.5667(0.4661) + 0.5667(0.4694)] / 9$$
$$+ [0.5667(0.7140) + 0.5667(0.4661) + 0.5667(0.4694)] / 9$$
$$+ [0.5667(0.7140) + 0.5667(0.4661) + 0.5667(0.4694)] / 9$$
$$= 0.3116$$

Step 11: Finally rank alternative base on final score value, the higher final value the better the alternative performance.

Thus, the ranking order of all alternatives can be determined such that better alternatives j have higher values of Γ_j. The ranking based on type-1, type-2 and Z-number fuzzy set implementation of the proposed TOPSIS methods using fuzzy system with single rule base approach are provided in Table 7.7

Table 7. 7: Ranking based on proposed methods with single rule base

	Type-1 Implementation		Type-2 implementation		Z implementation	
Stocks	Final Score	Rank	Final Score	Rank	Final Score	Rank
S1	0.3116	27	0.1957	27	0.2491	28
S2	0.6737	15	0.5152	14	0.6380	13
S3	0.3310	26	0.2336	26	0.3317	24
S4	0.8631	1	0.7696	3	0.7429	5
S5	0.1558	30	0.1111	30	0.1532	30
S6	0.3453	23	0.2851	23	0.3210	25
S7	0.6762	14	0.5076	15	0.6593	11
S8	0.7762	8	0.6179	7	0.7557	4
S9	0.5866	19	0.4754	18	0.5764	17
S10	0.8172	6	0.6981	6	0.8270	1
S11	0.6497	17	0.6094	9	0.5800	16
S12	0.7499	11	0.5898	11	0.6256	14
S13	0.7679	9	0.5741	13	0.6017	15
S14	0.8359	2	0.8186	1	0.7877	3
S15	0.2629	28	0.1757	28	0.2565	27
S16	0.6617	16	0.5035	16	0.5503	19
S17	0.2483	29	0.1301	29	0.2355	29
S18	0.6084	18	0.4716	19	0.5671	18
S19	0.8227	5	0.7213	4	0.6927	8
S20	0.6965	13	0.5001	17	0.6806	10
S21	0.7840	7	0.6082	10	0.6970	7
S22	0.4919	20	0.3785	20	0.4234	20
S23	0.4890	21	0.3350	21	0.4184	21
S24	0.8267	4	0.7770	2	0.7086	6
S25	0.7543	10	0.5793	12	0.6918	9
S26	0.3362	25	0.2419	24	0.2581	26
S27	0.4771	22	0.3155	22	0.3593	22
S28	0.7066	12	0.6126	8	0.6426	12
S29	0.8272	3	0.7041	5	0.8142	2
S30	0.3369	24	0.4119	13	0.5038	12

Table 7.5 shows the final ranking of 30 stocks based on fuzzy systems with single rule base approach for T1, T2 and Z implemention which indicate S4, S14 and S10 respectively are most preferable where as S5 is the worst to investment for all implementation. The comparative analysis of approaches has been done in more details in Chapter 8.

7.5 Fuzzy System Approach with Multiple Rule Bases

In this section, the case study of stock selection based on proposed method in Chapter 5 is described in step by step manner.

T1-MFS TOPSIS algorithm

In this study, the processes of ranking stocks follow the proposed methods of fuzzy system using multiple rule bases in Chapter 5. Figure 5.1 illustrates the fuzzy system includes 4 benefit criteria and 2 cost criteria.

Step 1: Construct decision matrix where each decision maker opinion is evaluated independently and categorised into two Criteria Category as Benefit Criteria and Cost Criteria define through a Benefit system (BS) and a Cost system (CS)

Based on the information provided by experts in Tables 7.1-7.4 and using Eq. (5.1), the decision matrices for the benefit and cost systems can be constructed. The linguistic terms in Tables 7.1-7.4 can be converted by using the fuzzy numbers in Tables 4.1- 4.3 respectively.

Step 2: Construct the weighted and the normalised decision matrices

Considering the benefit system, the normalized decision matrix R_k^B and the weighted normalised decision matrix V_k^B can be constructed for each *k*, using equations Eq. (5.2) correspondingly. For example, the calculations for S1 using the opinion of DM1 is as follows:

$$g_{1,1} = (0.8, 0.9, 1)$$

$$x_{11,1} = (9, 10, 10)$$

$$c_{1,1}^{x*} = 10$$

$$r_{11,1}^B = (9/10, 10/10, 10/10) = (0.9, 1, 1)$$

$$v_{11,1}^B = (0.8 \times 0.9, ,0.9 \times 1, 1 \times 1) = (0.72, ,0.9, 1)$$

This step is repeated for the cost system to calculate the normalised decision matrix R_k^C and the weighted normalised decision matrix V_k^C.

Step 3: Find the Fuzzy Positive Ideal Solution (FPIS) and Fuzzy Negative Ideal Solution (FNIS) for each alternative.

The Fuzzy Positive Ideal Solution (FPIS) and the Fuzzy Negative Ideal Solution (FNIS) for each stock based on both systems as well as the distances between the rating of criteria for each stock and the FPIS and FNIS can be evaluated as follows.

FPIS and FNIS are determined as:

$$A_k^+ = \left[(1,1,1)_{1,k}, (1,1,1)_{2,k}, \ldots, (1,1,1)_{30,k}\right]$$
$$A_k^- = \left[(0,0,0)_{1,k}, (0,0,0)_{2,k}, \ldots, (0,0,0)_{30,k}\right]$$

Step 4: Find the distance between each alternative to FPIS and FNIS

The distances $\Delta_{j,k}^{B+}$ and $\Delta_{j,k}^{B+}$, between the rating according to DM k of benefit criteria $i=1,\ldots,4$ for each stock j ($j=1,\ldots,30$) and the FPIS A_k^+ or FNIS A_k^- are calculated using Eq. (5.3) and Eq. (5.4). For example, the distance between the first stock S1 according to DM1 and the FPIS A_1^+ is calculated using Eq. (5.3) for $j=1$ and $k=1$, as follows.

$$\Delta_k^{B+}\left(v_{ij,k}, v_{i,k}^+\right) = \Delta_1^{B+}\left(v_{11,1}, v_{1,1}^+\right)$$
$$= \sqrt{\frac{1}{3}\left[(0.72-1)^2 + (0.9-1)^2 + (1-1)^2\right]} = 0.1811$$

And similarly:

$$D_k^{B+}\left(v_{ij,k}\,,\, v_{i,k}^+\right) = D_1^{B+}\left(v_{21,1}\,,\, v_{2,1}^+\right) = 0.409$$

$$D_1^{B+}\left(v_{31,1}\,,\, v_{3,1}^+\right) = 1.1728$$

$$D_1^{B+}\left(v_{31,1}\,,\, v_{3,1}^+\right) = 0.6590$$

producing:

$$D_{j,k}^{B+} = \sum_{i=1}^{i} D_k^{B+}\left(v_{1j,k}, v_{1,k}^+\right) = D_{1,1}^{B+} = \sum_{i=1}^{4} D_1^{B+}\left(v_{i1,1}, v_{i,1}^+\right)$$
$$= 0.1811 + 1.2582 + 1.1728 + 0.6590 = 3.2711$$

Next, using Eq. (5.4) for $j=1$ and $k=1$, the distance between S1 according to DM1 and the FNIS A_1^- is calculated as:

$$\Delta_k^{B-}\left(v_{ij,k}, v_{i,k}^{-}\right) = \Delta_1^{B-}\left(v_{11,1}, v_{1,1}^{-}\right)$$

$$= \sqrt{\frac{1}{3}\left[(0.72-0)^2 + (0.9-0)^2 + (1-0)^2\right]} = 1.3087$$

and similarly

$$\Delta_k^{B-}\left(v_{ij,k}, v_{i,k}^{-}\right) = \Delta_1^{B-}\left(v_{21,1}, v_{2,1}^{-}\right) = 0.1992$$

$$\Delta_1^{B-}\left(v_{31,1}, v_{3,1}^{-}\right) = 0.3089$$

$$\Delta_1^{B-}\left(v_{31,1}, v_{3,1}^{-}\right) = 0.8252$$

producing:

$$\Delta_{j,k}^{B-} = \sum_{i=1}^{i} \Delta_k^{B-}\left(v_{1j,k}, v_{1,k}^{-}\right) = \Delta_{1,1}^{B-} = \sum_{i=1}^{4} \Delta_1^{B-}\left(v_{i1,1}, v_{i,1}^{-}\right)$$
$$= 1.3087 + 0.1992 + 0.3089 + 0.8252 = 2.6420$$

Now, the distances $\Delta_{j,k}^{C+}$ and $\Delta_{j,k}^{C-}$, between the rating according to DM k of cost criteria $i = 1,\ldots,2$ for each stock j $(j = 1,\cdots,30)$ and the FPIS A_k^{+} or FNIS A_k^{-} are calculated using Eq. (5.5) and Eq. (5.6). For example, the distance between the first stocks S1 according to DM1 and the FPIS A_1^{+} is calculated using Eq. (5.3) for $j = 1$ and $k = 1$, as follows:

$$\Delta_k^{C+}\left(v_{ij,k}, v_{i,k}^{+}\right) = \Delta_1^{C+}\left(v_{11,1}, v_{1,1}^{+}\right)$$

$$= \sqrt{\frac{1}{3}\left[(0.24-1)^2 + (0.45-1)^2 + (0.7-1)^2\right]} = 0.6960$$

and similarly:

$$\Delta_k^{C+}\left(v_{ij,k}, v_{i,k}^{+}\right) = \Delta_1^{C+}\left(v_{21,1}, v_{2,1}^{+}\right) = 0.181$$

giving:

$$\Delta_{j,k}^{C+} = \sum_{i=1}^{i} \Delta_k^{C}\left(v_{1j,k}, v_{1,k}^{+}\right) = \Delta_{1,1}^{C+} = \sum_{i=1}^{2} \Delta_1^{C}\left(v_{i1,1}, v_{i,1}^{+}\right) = 0.696 + 0.181 = 0.8770$$

Next, using Eq. (5.6) for $j = 1$ and $k = 1$, the distance between S1 according to DM1 and the FNIS A_1^{-} is calculated as:

$$\Delta_k^{C-}\left(v_{ij,k},v_{i,k}^{-}\right)=\Delta_1^{C-}\left(v_{11,1},v_{1,1}^{-}\right)$$
$$=\sqrt{\frac{1}{3}\left[(0.24-0)^2+(0.45-0)^2+(0.7-0)^2\right]}=0.803$$

and similarly

$$\Delta_k^{C-}\left(v_{ij,k}\ ,\ v_{i,k}^{-}\right)=\Delta_1^{C-}\left(v_{21,1}\ ,\ v_{2,1}^{-}\right)=0.339$$

resulting into:

$$\Delta_{j,k}^{C-}=\sum_{i=1}^{i}\Delta_k^{C-}\left(v_{1j,k},v_{1,k}^{-}\right)=\Delta_{1,1}^{C-}=\sum_{i=1}^{2}\Delta_1^{C-}\left(v_{i1,1},v_{i,1}^{-}\right)=0.803+1.309=2.112$$

Step 5: Find the Closeness Coefficients (CC) for the benefit and cost systems.

The closeness coefficients for the benefit system $CC_{j,k}^{B}$ and for the cost system $CC_{j,k}^{C}$ is calculated using Eq. (5.7) for each stock S*j*, $j=1,\cdots,30$. For example, the closeness coefficient for S1 in the benefit system under the first decision maker $k=1$ is calculated using Eq. (5.7) as follows

$$CC_{j,k}^{B}=\frac{\Delta_{j,k}^{B-}}{\Delta_{j,k}^{B+}+\Delta_{j,k}^{B-}}=CC_{1,1}^{B}=\frac{\Delta_{1,1}^{B-}}{\Delta_{1,1}^{B+}+\Delta_{1,1}^{B-}}=\frac{2.6420}{3.2711+2.6420}=0.4468$$

and the closeness coefficient in the cost system

$$CC_{j,k}^{C}=\frac{\Delta_{j,k}^{C-}}{\Delta_{j,k}^{C+}+\Delta_{j,k}^{C-}}=CC_{1,1}^{C}=\frac{\Delta_{1,1}^{C-}}{\Delta_{1,1}^{C+}+\Delta_{1,1}^{C-}}=\frac{2.112}{0.877+2.112}=0.707$$

Step 6: Derive the Influenced Closeness Coefficient (ICC) by applying the influence degree of each decision maker, then find Normalised ICC (NICC), dividing NICC by maximum value of NICC.

The Influenced Closeness Coefficients $ICC_{j,k}^{B}$ and $ICC_{j,k}^{C}$ for each DM *k* are derived by applying the influence degree θ_k of each decision maker by using Eq. (5.8) and Eq. (5.9). Then, the normalized coefficients $ICC_{j,k}^{B}$ and $ICC_{j,k}^{C}$ are calculated using Eq. (5.10). For example, the influence degree of DM1 is $q_1=5$, as given in Table 8.1, and using Eq. (5.8) decision maker normalised expertise is:

$$\sigma_k = \frac{\theta_k}{\sum_{l=1}^{K}\theta_l} = \sigma_1 = \frac{\theta_1}{\sum_{l=1}^{3}\theta_l} = \frac{5}{5+6+7} = 0.2778$$

Then, the Influenced Closeness Coefficient $ICC_{1,1}^{B}$ for the benefit system for stock S1 according to DM1 is calculated using Eq. (5.9) such that:

$$ICC_{j,k}^{B} = \sigma_k \times CC_{j,k}^{B} = ICC_{1,1}^{B} = \sigma_1 \times CC_{1,1}^{B} = 0.2778 \times 0.4468 = 0.1241 \quad ,$$

and similarly, the corresponding Influenced Closeness Coefficient for the cost system $ICC_{1,1}^{C}$ resulted into:

$$\begin{aligned} ICC_{j,k}^{C} &= \sigma_k \times CC_{j,k}^{C} = ICC_{1,1}^{C} \\ \sigma_1 \times CC_{1,1}^{C} &= 0.2778 \times 0.707 = 0.1963 \end{aligned} \; .$$

Next, the influenced closeness coefficients are normalized prior to matching the coefficients to the linguistic terms in Table 4.3. Using Eq. (5.10), $NICC_{1,1}^{B}$ and $NICC_{1,1}^{C}$ are calculated as:

$$NICC_{j,k}^{B} = {ICC_{j,k}^{B}}\Big/{\max_j ICC_{j,k}^{B}} = NICC_{1,1}^{B} = {ICC_{1,1}^{B}}\Big/{\max_j ICC_{j,k}^{B}} = \frac{0.1241}{0.2075} = 0.5982$$

and

$$NICC_{j,k}^{C} = {ICC_{j,k}^{C}}\Big/{\max_j ICC_{j,k}^{C}} = NICC_{1,1}^{C} = {ICC_{1,1}^{C}}\Big/{\max_j ICC_{1,1}^{C}} = \frac{0.1963}{0.2339} = 0.8392$$

Finally, the normalised coefficients are matched to the linguistic term in Table 4.3:

$$NICC_{1,1}^{B} = 0.5982 \; @R$$
$$NICC_{1,1}^{C} = 0.8392 \; @VG$$

Step 7: Construct the antecedent matrices for the BS and CS based on DMs opinions

The antecedent matrices X_k for the benefit system are constructed using Eq. (5.11) for $k = 1, \cdots, K$, based on DM *k* opinions detailed in Tables 8.1-8.4. Each decision maker has a separate benefit antecedent matrix. Similarly, the antecedent matrix Y_k is produced for the cost system. Thus, the antecedent for the benefit and cost rule bases are also generated in this step. For example, using Eq. (5.11) and the first decision maker $k=1$ as detailed in Tables 8.2 and 8.4, the antecedent matrix X_1 for the benefit system is:

$$X_k = \begin{array}{c} \\ B_1 \\ B_2 \\ \vdots \\ B_e \end{array} \begin{array}{c} \begin{array}{cccc} S_1 & S_2 & \cdots & S_m \end{array} \\ \begin{bmatrix} \hat{x}_{11,k} & \hat{x}_{12,k} & \cdots & \hat{x}_{1m,k} \\ \hat{x}_{21,k} & \hat{x}_{22}, & \cdots & \hat{x}_{2m,k} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{x}_{e1,k} & \hat{x}_{e2,k} & \cdots & \hat{x}_{em,k} \end{bmatrix} \end{array} = X_1 = \begin{array}{c} \\ B_1 \\ B_2 \\ B_3 \\ B_4 \end{array} \begin{array}{c} \begin{array}{cccc} S_1 & S_2 & \cdots & S_{30} \end{array} \\ \begin{bmatrix} \hat{x}_{1,1,1} & \hat{x}_{1,2,1} & \cdots & \hat{x}_{1,30,1} \\ \hat{x}_{2,1,1} & \hat{x}_{2,2,1} & \cdots & \hat{x}_{2,30,1} \\ \hat{x}_{3,1,1} & \hat{x}_{3,2,1} & \ddots & \hat{x}_{3,30,1} \\ \hat{x}_{4,1,1} & \hat{x}_{4,2,1} & \cdots & \hat{x}_{4,30,1} \end{bmatrix} \end{array} =$$

$$\begin{array}{c} \\ B_1 \\ B_2 \\ B_3 \\ B_4 \end{array} \begin{array}{c} \begin{array}{cccc} S_1 & S_2 & \cdots & S_{30} \end{array} \\ \begin{bmatrix} VG & VG & \cdots & G \\ P & MG & \cdots & G \\ P & MP & \ddots & F \\ F & F & \cdots & VP \end{bmatrix} \end{array},$$

where B_i is the four benefit criteria.

then, using Eq. (5.11) and the first decision maker $k=1$ as detailed in Tables 8.1 and 8.2, the antecedent matrix Y_1 for the cost system is:

$$Y_k = \begin{array}{c} \\ C_1 \\ C_2 \\ \vdots \\ C_e \end{array} \begin{array}{c} \begin{array}{cccc} S_1 & S_2 & \cdots & S_m \end{array} \\ \begin{bmatrix} \hat{y}_{11,k} & \hat{y}_{12,k} & \cdots & \hat{y}_{1m,k} \\ \hat{y}_{21,k} & \hat{y}_{22}, & \cdots & \hat{y}_{2m,k} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{y}_{e1,k} & \hat{y}_{e2,k} & \cdots & \hat{y}_{em,k} \end{bmatrix} \end{array} = Y_1 = \begin{array}{c} \\ C_1 \\ C_2 \end{array} \begin{array}{c} \begin{array}{cccc} S_1 & S_2 & \cdots & S_{30} \end{array} \\ \begin{bmatrix} \hat{y}_{1,1,1} & \hat{y}_{1,2,1} & \cdots & \hat{y}_{1,30,1} \\ \hat{y}_{2,1,1} & \hat{y}_{2,2,1} & \cdots & \hat{y}_{2,30,1} \end{bmatrix} \end{array} =$$

$$\begin{array}{c} \\ C_1 \\ C_2 \end{array} \begin{array}{c} \begin{array}{cccc} S_1 & S_2 & \cdots & S_{30} \end{array} \\ \begin{bmatrix} F & MP & \cdots & F \\ VG & F & \cdots & MG \end{bmatrix} \end{array}$$

, and C_i is the two cost criteria.

Step 8: Construct the consequent matrices for the BS and CS systems based on the value of the NICC coefficients.

The consequent matrix for the benefit and cost rule bases are generated in this step. The consequent matrices Λ_k for the benefit system are constructed using

Eq. (5.12) for $k=1,\cdots,K$, based on the values of $NICC_{j,k}^{B}$ calculated at step 6 and matched to the linguistic terms in Table 4.3. Similarly, the consequent matrices Ψ_k are calculated for the cost system. After determining the $NICC_j^{B,k}$ and $NICC_j^{C,k}$ coefficients for all decision makers $(k=1,\cdots,K)$, the benefit consequent matrix Λ_k and the cost consequent matrix Ψ_k are then defined using Eq. (5.12). First is the consequent matrix Λ_1 which is:

$$\Lambda_k = BL\begin{array}{c} \begin{array}{cccc} S_1 & S_2 & \cdots & S_m \end{array} \\ \left[\begin{array}{cccc} \lambda_{1,k} & \lambda_{2,k} & \cdots & \lambda_{m,k} \end{array}\right] \end{array} = \Lambda_1 = BL\begin{array}{c} \begin{array}{cccc} S_1 & S_2 & \cdots & S_{30} \end{array} \\ \left[\begin{array}{cccc} \lambda_{1,1} & \lambda_{2,1} & \cdots & \lambda_{30} \end{array}\right] \end{array}$$

$$= BL\begin{array}{c} \begin{array}{cccc} S_1 & S_2 & \cdots & S_{30} \end{array} \\ \left[\begin{array}{cccc} R & G & \cdots & G \end{array}\right] \end{array}$$

and BL is the benefit level.

Then, the consequent matrix Ψ_1 is:

$$\Psi_k = CL\begin{array}{c} \begin{array}{cccc} S_1 & S_2 & \cdots & S_m \end{array} \\ \left[\begin{array}{cccc} \psi_{1,k} & \psi_{2,k} & \cdots & \psi_{m,k} \end{array}\right] \end{array} = \Psi_1 = CL\begin{array}{c} \begin{array}{cccc} S_1 & S_2 & \cdots & S_{25} \end{array} \\ \left[\begin{array}{cccc} \psi_{1,1} & \psi_{2,1} & \cdots & \psi_{25,1} \end{array}\right] \end{array}$$

$$= CL\begin{array}{c} \begin{array}{cccc} S_1 & S_2 & \cdots & S_{30} \end{array} \\ \left[\begin{array}{cccc} VG & R & \cdots & G \end{array}\right] \end{array}$$

and CL is the cost level.

Step 9: Derive rules for each alternative

The rule base of the benefit system for DM1 is constructed using Eq. (5.13), as follows.

If $X_1 = \begin{array}{c} \\ B_1 \\ B_2 \\ B_3 \\ B_4 \end{array}\begin{array}{c} \begin{array}{cccc} S_1 & S_2 & \cdots & S_{30} \end{array} \\ \left[\begin{array}{cccc} VG & VG & \cdots & G \\ P & MG & \cdots & G \\ P & MP & \ddots & F \\ F & F & \cdots & VP \end{array}\right] \end{array}$, then $\Lambda_1 = BL\begin{array}{c} \begin{array}{cccc} S_1 & S_2 & \cdots & S_{30} \end{array} \\ \left[\begin{array}{cccc} R & G & \cdots & G \end{array}\right] \end{array}$

Rule 1: If B_1 is *VG*, B_2 is *P*, B_3 is *P* and B_4 is *F*, then the output *BL* is *R*.

Rule 2: If B_1 is *VG*, B_2 is *MG*, B_3 is *MP* and B_4 is *F*, then the output *BL* is *G*,

$\vdots \qquad \vdots \qquad \vdots$

Rule 3: If B_1 is *G*, B_2 is *G*, B_3 is *F* and B_4 is *VP*, then the output *BL* is *G*.

The rule base for the cost system is constructed using the same analogy.

Step 10: Derive the weighted benefit level (WBL) and the weighted cost level (WCL)

The WBL and WCL are derived by taking the average of the aggregate membership value of consequent part of all active rules multiplied with the weight of systems based on number of input for each system. The average is then multiplied with the influence multiplier based on the average as shown in Eq. (5.14). For example, the WBL and WCL of S1 are calculated as follow:

$$WBL_{j,k} = \left[\frac{1}{n}\sum_{rule=1}^{n}\hat{\lambda}_{j,k}^{B} \times \left(\frac{e}{e+f}\right)\right] \times \frac{1}{k}\sum_{k=1}^{k} NICC_{j,k}^{B}$$

$$= \left[\frac{1}{3}\sum_{rule=1}^{3}\hat{\lambda}_{j,k}^{B} \times \left(\frac{4}{4+2}\right)\right] \times \frac{1}{3}\sum_{k=1}^{3} NICC_{j,k}^{B}$$

$$= \left[\frac{1}{3}(0.5+0.5+0.3) \times \left(\frac{2}{3}\right)\right] \times \frac{1}{3}(0.5982+0.4092+0.363) = 0.1320$$

Whereas, for the cost system:

$$WCL_{j,k} = \left[\frac{1}{n}\sum_{rule=1}^{n}\hat{\lambda}_{j,k}^{C} \times \left(\frac{f}{e+f}\right)\right] \times \frac{1}{k}\sum_{k=1}^{k} NICC_{j,k}^{C}$$

$$WCL_{j,k} = \left[\frac{1}{3}\sum_{rule=1}^{3}\hat{\lambda}_{j,k}^{C} \times \left(\frac{2}{4+2}\right)\right] \times \frac{1}{3}\sum_{k=1}^{3} NICC_{j,k}^{C}$$

$$= \left[\frac{1}{3}(0.9+0.5+0.7) \times \left(\frac{1}{3}\right)\right] \times \frac{1}{3}(0.8392+0.5821+0.6398) = 0.1603$$

$\hat{\lambda}_{j,k}^{B}$ and $\hat{\lambda}_{j,k}^{C}$ represent the aggregate membership value of benefit subsystem and cost subsystem respectively for each alternative $j = 1, \cdots, 30$, k decision maker.

Step 11: Derive final score for each alternative,

The final score for each alternative $j = 1, \ldots, 30$ is derived using Eq. (5.15) by taking the average of weighted benefit level and the weighted cost level as shown Eq. (5.14).

$$\Gamma_j = \left(WBL_{j,k} + WCL_{j,k}\right)\Big/2 = \frac{0.1320 + 0.1603}{2} = 0.1461$$

Step 12: Finally rank alternative base on final score value such that the higher final value the better the alternative performance.

Thus, the ranking order of all alternatives can be determined such that better alternatives j have higher values of Γ_j. The final score and ranking positions for all 30 stocks considered in this case study based on type-1, type-2 and z-number fuzzy set of the proposed fuzzy system that is TOPSIS, using multiple rule bases approach are provided in Table 7.8. Table 7.8 shows the final ranking based on proposed methods in Chapter 5, which indicate that S4, S14 and S29 are the best stock for T1, T2 and Z implementation respectively where as S5 is the worst stock to invest for all three methods.

Table 7. 8: Ranking based on proposed methods with multiple rule bases

	Type-1 Implementation		Type-2 implementation		Z implementation	
Stocks	Final Score	Rank	Final Score	Rank	Final Score	Rank
S1	0.1461	28	0.0850	27	0.1258	29
S2	0.3097	15	0.2659	16	0.2911	13
S3	0.1755	25	0.1149	24	0.1773	23
S4	0.4239	1	0.3896	2	0.3617	4
S5	0.0938	30	0.0359	30	0.0936	30
S6	0.1895	23	0.1213	23	0.1513	25
S7	0.3087	16	0.2801	12	0.2905	14
S8	0.3447	9	0.3196	8	0.3330	7
S9	0.2924	19	0.2489	18	0.2583	17
S10	0.3818	6	0.3634	3	0.3843	2
S11	0.3296	11	0.3024	9	0.2742	16
S12	0.3331	10	0.2803	11	0.3228	9
S13	0.3584	7	0.2890	10	0.2875	15
S14	0.4106	2	0.4025	1	0.3658	3
S15	0.1522	27	0.0845	28	0.1336	27
S16	0.3085	17	0.2395	19	0.2549	19
S17	0.1397	29	0.0765	29	0.1342	26
S18	0.3025	18	0.2610	17	0.2576	18
S19	0.3955	5	0.3437	5	0.3284	8
S20	0.3187	14	0.2767	14	0.3102	12
S21	0.3484	8	0.3296	6	0.3137	10
S22	0.2563	20	0.2091	20	0.2337	20
S23	0.2341	21	0.1734	21	0.2191	21
S24	0.4059	3	0.3632	4	0.3354	6
S25	0.3245	13	0.2728	15	0.3373	5
S26	0.1781	24	0.1060	25	0.1319	28
S27	0.2141	22	0.1549	22	0.1910	22
S28	0.3296	12	0.2794	13	0.3132	11
S29	0.3981	4	0.3288	7	0.3866	1
S30	0.1637	26	0.0924	26	0.1763	24

7.6 Fuzzy Network Approach with Rule Base Aggregation

In this section, the case study of stock selection based on proposed method in Chapter 6.2 is described using step by step manner.

<u>T1-AFN TOPSIS algorithm</u>

Step 1: Construct decision matrix where each decision maker opinion is evaluated independently and categorised into two Criteria Categories: Benefit Criteria and Cost Criteria that are defined through a Benefit system (BS) and a Cost system (CS)

- Identical to step 1 in Section 5.2.

Step 2: Construct the weighted and the normalised decision matrices

- Identical to step 2 in Section 5.2.

Step 3: Find the Fuzzy Positive Ideal Solution (FPIS) and Fuzzy Negative Ideal Solution(FNIS) for each alternative.

- Identical to step 3 in Section 5.2.

Step 4: Find the distance between each alternative to FPIS and FNIS.

- Identical to step 4 in Section 5.2.

Step 5: Find the Closeness Coefficients (CC) for the benefit and cost systems.

- Identical to step 5 in Section 5.2.

Step 6: Derive the Influenced Closeness Coefficient (ICC) by applying the influence degree of each decision maker, then find the Normalised ICC (NICC), divide it by its maximum value.

- Identical to step 6 in Section 5.2.

Step 7: Construct the antecedent matrices for the BS and CS based on DMs opinions

- Identical to step 1 in Section 5.2.

Step 8: Construct the consequent matrices for the BS and CS systems based on the value of the NICC coefficients.

- Identical to step 1 in Section 5.2.

Step 9: Construct the antecedent matrices for the alternative system (AS)

The Alternatives System (AS) in this application is the Equity System (ES) and the antecedent matrices M_k of each DM k for ES are constructed using Eq. (6.1) based on the Benefit Level (BL) and Cost Level (CL), which are the outputs of the benefit system BS and cost system CS, respectively. Each decision maker has a separate stock antecedent matrix M_k. Next, the ES consequent matrices N_k are derived using Eq. (6.3) - (6.5), while calculating the aggregations $\xi_{j,k}$ of weighted coefficients $NICC^B_{j,k}$ and $NICC^C_{j,k}$ for each stock j $(j = 1,\cdots,30)$, then producing the normalised aggregations $N\xi_{j,k}$, and constructing the AS consequent matrices N_k based on $N\xi_{j,k}$. Each decision maker k has a separate stock consequent matrix N_k.

For example, based on the benefit and cost levels BL and CL evaluated in step 1-8 and using Eq. (6.1), the AS antecedent matrix M_1 according to DM1 is evaluated as:

$$M_k = \begin{array}{c} \\ BL \\ CL \end{array}\!\!\begin{array}{c} \begin{array}{ccccc} S_1 & S_2 & S_3 & \cdots & S_m \end{array} \\ \begin{bmatrix} \lambda_{1,k} & \lambda_{2,k} & \lambda_{3,k} & \cdots & \lambda_{m,k} \\ \psi_{1,k} & \psi_{2,k} & \psi_{3,k} & \cdots & \psi_{m,k} \end{bmatrix} \end{array} =$$

$$M_1 = \begin{array}{c} \\ BL \\ CL \end{array}\!\!\begin{array}{c} \begin{array}{cccc} S_1 & S_2 & \cdots & S_{30} \end{array} \\ \begin{bmatrix} \lambda_{1,1} & \lambda_{2,1} & \cdots & \lambda_{30,1} \\ \psi_{1,1} & \psi_{2,1} & \cdots & \psi_{30,1} \end{bmatrix} \end{array} = \begin{array}{c} \\ BL \\ CL \end{array}\!\!\begin{array}{c} \begin{array}{cccc} S_1 & S_2 & \cdots & S_{30} \end{array} \\ \begin{bmatrix} R & G & \cdots & G \\ VG & R & \cdots & G \end{bmatrix} \end{array}$$

Step 10: Construct the consequent matrices for the alternative system (AS)

The AS consequent matrix N_1 according to DM1 is derived such that:

the aggregated closeness coefficient $\xi_{j,1}$ for each stock $j = 1,\ldots,30$ is calculated using Eq. (6.3) and based on the normalised closeness coefficients $NICC^{B}_{j,1}$ and $NICC^{C}_{j,1}$ according to DM1. For example, if $j = 1$:

$$\xi_{j,k} = \frac{NICC^{B}_{j,k} \times \left(\frac{e}{e+f}\right) + NICC^{C}_{j,k} \times \left(\frac{f}{e+f}\right)}{2}$$

$$= \xi_{1,1} = \frac{NICC^{B}_{1,1} \times \left(\frac{4}{4+2}\right) + NICC^{C}_{1,1} \times \left(\frac{1}{4+2}\right)}{2} =$$

$$= \xi_{1,1} = \frac{0.598 \times \left(\frac{2}{3}\right) + 0.8822\left(\frac{1}{3}\right)}{2} = 0.3794$$

The normalised aggregated closeness coefficients $N\xi_{j,1}$ for each stock $j = 1,\ldots,30$ is calculated using Eq. (6.4) and based on the values $\xi_{j,1}$ produced above. For example, if $j = 1$:

$$N\xi_{j,k} = \xi_{j,k} \Big/ \max_{j} \xi_{j,k} = Nx_{1,1} = x_{1,1} \Big/ \max_{j} x_{j,1} = \frac{0.3794}{0.4992} = 0.7601$$

and the value of $N\xi_{1,1}$ is matched to the linguistic terms for stock level in Table 3:

$$N\xi_{1,1} = 0.7601 \cong G$$

The AS consequent matrix N_1 for DM1 is constructed using Eq. (6.5) and based on the values $N\xi_{j,1}$ for each stock *j* produced above; e.g. for $j = 1$:

$$N_k = AL \begin{array}{c} \begin{array}{cccc} S_1 & S_2 & \cdots & S_m \end{array} \\ \begin{bmatrix} N\xi_{1,k} & N\xi_{2,k} & \cdots & N\xi_{m,k} \end{bmatrix} \end{array}$$

$$= N_1 = AL \begin{array}{c} \begin{array}{cccc} S_1 & S_2 & \cdots & S_{30} \end{array} \\ \begin{bmatrix} N\xi_{1,1} & N\xi_{2,1} & \cdots & N\xi_{30,1} \end{bmatrix} \end{array} = \begin{array}{c} \begin{array}{cccc} S_1 & S_2 & \cdots & S_{30} \end{array} \\ \begin{bmatrix} G & G & \cdots & G \end{bmatrix} \end{array}$$

where *AL* is the alternative level.

Step 11: Derive rules for each alternative

Therefore, the stock system rule base according to DM1 is evaluated using Eq. (6.6) as:

If $M_1 = \begin{array}{c c} & \begin{array}{cccc} S_1 & S_2 & \cdots & S_{30} \end{array} \\ \begin{array}{c} BL \\ CL \end{array} & \begin{bmatrix} R & G & \cdots & G \\ VG & R & \cdots & G \end{bmatrix} \end{array}$, then $N_1 = \begin{array}{c c} & \begin{array}{cccc} S_1 & S_2 & \cdots & S_{30} \end{array} \\ AL & \begin{bmatrix} G & G & \cdots & G \end{bmatrix} \end{array}$

Rule 1: If *BL* is *R* and *CL* is *VG* then *AL* is *G*,

Rule 2: If *BL* is *G* and *CL* is *R* then *AL* is *G*.

$\vdots \qquad \vdots \qquad \vdots$

Rule 3: If *BL* is *G* and *CL* is *G* then *AL* is *G*.

Step 12: Derive the final score for each alternative.

The final score for each alternative $j = 1,\ldots,30$ is derived using Eq. (6.8) by averaging the aggregate membership value of the consequent part of all active rules in the overall system for stock *j*, and then multiplying with the influence multiplier based on the average influence degree across all *K* decision makers DMs for each stock *j*.

For example, S1 generated from the case has 3 active rules. Then, Eq. (6.8) is used to obtained the final score of S1. The average aggregate membership value for the output of the 3 rules is calculated and then multiplied with the influence multiplier for S1 across all DMs.

$$\Gamma_j = \frac{\sum_{Rule=1}^{n} N\xi_{j,k}}{n} \bullet \frac{\sum_{k=1}^{K}\left(NICC_{j,k}^{B} + NICC_{j,k}^{C}\right)}{K}$$

$$= \Gamma_1 = \frac{\sum_{Rule=1}^{3} N\xi_{1,k}}{3} \bullet \frac{\sum_{k=1}^{3}\left(NICC_{j,k}^{B} + NICC_{j,k}^{C}\right)}{3} =$$

$$= \frac{0.7601 + 0.5253 + 0.5484}{3} \bullet \frac{0.7 + 0.5 + 0.5}{3} = 0.3464$$

Step 13: Finally rank the alternative base on final score value such that the higher final value the better the alternative performance.

The final score and ranking positions for all 30 stocks can be determined. The ranking based on type-1, type-2 and Z-number implementations of the proposed Fuzzy Network-TOPSIS with rule base aggregation approaches are provided in Table 7.9. Table 7.9 shows the final ranking of 30 stocks based on fuzzy networks with aggregation of rule base approach for T1, T2 and Z implemention which indicate S4, S14 and S10 respectively are most preferable where as S5 is the worst to investment for all implementation. The comparative analysis of approaches has been done in more details in Chapter 8.

Table 7. 9: Ranking based on proposed methods with rule base aggregation

	Type-1 Implementation		Type-2 implementation		Z implementation	
Stocks	Final Score	Rank	Final Score	Rank	Final Score	Rank
S1	0.3464	28	0.1599	28	0.2444	28
S2	0.5662	21	0.4955	16	0.5759	15
S3	0.4631	23	0.2368	24	0.3269	24
S4	0.8544	1	0.7585	3	0.7286	5
S5	0.2731	30	0.0753	30	0.1508	30
S6	0.3459	29	0.2427	23	0.3166	25
S7	0.7727	9	0.5205	14	0.6499	11
S8	0.7959	8	0.6732	7	0.7422	4
S9	0.6970	15	0.4909	19	0.5666	17
S10	0.8377	2	0.7058	4	0.8127	1
S11	0.7164	13	0.6048	10	0.5672	16
S12	0.6752	16	0.5758	12	0.6139	13
S13	0.8084	6	0.6483	8	0.5911	14
S14	0.8272	3	0.8049	1	0.7740	3
S15	0.4447	24	0.1748	27	0.2524	27
S16	0.6535	19	0.4940	17	0.5421	19
S17	0.3577	26	0.1324	29	0.2011	29
S18	0.7635	10	0.4912	18	0.5579	18
S19	0.7403	11	0.7050	5	0.6782	9
S20	0.8049	7	0.5201	15	0.6673	10
S21	0.8165	4	0.6174	9	0.6869	7
S22	0.6559	18	0.3871	20	0.4157	20
S23	0.5733	20	0.3325	21	0.4127	21
S24	0.8099	5	0.7617	2	0.6930	6
S25	0.7070	14	0.5778	11	0.6794	8
S26	0.3475	27	0.2326	25	0.2533	26
S27	0.5032	22	0.3099	22	0.3529	22
S28	0.6620	17	0.5347	13	0.6310	12
S29	0.7266	12	0.6893	6	0.7972	2
S30	0.3751	25	0.1888	26	0.4907	12

7.7 Fuzzy Network Approach with Rule Base Merging

In this section, the case study of stock selection based on proposed method in Chapter 6.3 is described in step by step manner.

<u>T1-MFN TOPSIS algorithm</u>

Step 1: Construct the decision matrix where each decision maker opinion is evaluated independently and categorised into two Criteria Category: Benefit Criteria and Cost Criteria defined through a Benefit system (BS) and a Cost system (CS)

- Identical to step 1 in Section 5.2

Step 2: Construct the weighted and the normalised decision matrices

- identical to step 2 in Section 5.2

Step 3: Find the Fuzzy Positive Ideal Solution (FPIS) and Fuzzy Negative Ideal Solution (FNIS) for each alternative.

- identical to step 3 in Section 5.2

Step 4: Find the distance between each alternative to FPIS and FNIS.

- identical to step 4 in Section 5.2

Step 5: Find the Closeness Coefficients (CC) for the benefit and cost systems.

- identical to step 5 in Section 5.2

Step 6: Derive the Influenced Closeness Coefficient (ICC) by applying the influence degree of each decision maker, then find Normalised ICC (NICC) by dividing it by its maximum value.

- identical to step 6 in Section 5.2

Step 7: Construct the antecedent matrices for the BS and CS based on DMs opinions

- identical to step 7 in Section 5.2

Step 8: Construct the consequent matrices for the BS and CS systems based on the value of the NICC coefficients.

- identical to step 8 in Section 5.2

Step 9: Construct the antecedent matrices for the alternative system

- identical to step 9 in Section 6.2

Step 10: Construct the consequent matrices for the alternative system

- identical to step 10 in Section 6.2

Step 11: Construct the generalised Boolean matrices representing BS, CS and AS systems.

Having listed the rules for 3 systems – BS, CS, AS – we now present these rules in Boolean matrix form. The Boolean matrices for each stock are constructed based on the opinions from all DMs. For example, using Eq. (6.1), the row and column labels of the Boolean matrix are all possible permutations of linguistics terms for the input (1-7) as in Table 4.1. The linguistic terms for the output (1-5) as in Table 4.3 are for the benefit rule base. The Boolean matrix of the benefit system for S1 is produced as shown in Eq. (7.1).

$$
\begin{array}{c|cccc}
 & 2 & 3 & 4 & 5 \\
\hline
1111 & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
5112 & 1 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
5113 & 0 & 1 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
7224 & 0 & 1 & 0 & 0 \\
7777 & 0 & 0 & 0 & 0
\end{array}
\tag{7.1}
$$

Next, using Eq. (6.10), the Boolean matrix of the cost system for S1 is defined as shown in Eq. (7.2).

$$
\begin{array}{c|cccc}
 & 2 & 3 & 4 & 5 \\
\hline
11 & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
35 & 0 & 1 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
36 & 0 & 0 & 1 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
47 & 0 & 0 & 0 & 1 \\
77 & 0 & 0 & 0 & 0
\end{array}
\tag{7.2}
$$

The system ES Boolean matrix for S1 is evaluated in Eq. (7.3) below.

$$
\begin{array}{cccccc}
 & 1 & 2 & 3 & 4 & 5 \\
11 & 0 & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
24 & 0 & 0 & 1 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
33 & 0 & 0 & 1 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
35 & 0 & 0 & 1 & 0 & 0 \\
55 & 0 & 0 & 0 & 0 & 1
\end{array}
\qquad (7.3)
$$

Step 12: Perform vertical merging to merge Boolean of the BS and CS.

Vertical merging is performed to merge the Boolean matrices of BS in Eq. (7.1) and CS in Eq. (7.2) for each stock. Then the horizontal merging is performed to merge the Boolean matrix obtained from the vertical merging operation with the AS Boolean matrix for each stock. For example, applying vertical merging of the BS and CS Boolean matrices for S1, the resultant Boolean matrix constructed using Eq. (6.13) is as shown in Eq. (7.4).

$$
\begin{array}{cccccccccccc}
 & 11 & \cdots & 23 & 24 & 25 & \cdots & 33 & 34 & 35 & \cdots & 55 \\
1111/11 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
5\,1\,1\,2/35 & 0 & \cdots & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
5\,1\,1\,2/36 & 0 & \cdots & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
5\,1\,1\,2/47 & 0 & \cdots & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
5\,1\,1\,3/35 & 0 & \cdots & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
5\,1\,1\,3/36 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
5\,1\,1\,3/47 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
7\,2\,2\,4/35 & 0 & \cdots & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
7\,2\,2\,4/36 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 1 & 0 & \cdots & 0 \\
7\,2\,2\,4/47 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
7777/77 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}
\qquad (7.4)
$$

Step 13: Perform horizontal merging to merge the resultant matrix from step 12 with the Boolean matrix of the AS.

Next, the resultant Boolean matrix for the overall system is produced as shown in Eq. (7.5) through horizontal merging between the Boolean matrices in Eq. (7.3) and Eq. (7.4).

	1	2	3	4	5
1111/11	0	0	0	0	0
⋮	⋮	⋮	⋮	⋮	⋮
5 1 1 2/35	0	0	0	0	0
5 1 1 2/36/3	0	0	1	0	0
5 1 1 2/47	0	0	0	0	0
⋮	⋮	⋮	⋮	⋮	⋮
5 1 1 3/35/3	0	0	1	0	0
5 1 1 3/36	0	0	0	0	0
5 1 1 3/47/3	0	0	1	0	0
⋮	⋮	⋮	⋮	⋮	⋮
7 2 2 4/35	0	0	0	0	0
7 2 2 4/36	0	0	0	0	0
7 2 2 4/47/3	0	0	1	0	0
⋮	⋮	⋮	⋮	⋮	⋮
7777/77	0	0	0	0	0

(7.5)

Only the rows containing 1 are shown, along with the first and last rows.

Step 14: Derive rules for each stock based on horizontal merging of Boolean matrix.

From the Boolean matrix in Eq. (7.5) above, the rule bases for stock S1 is derived as described in Eq. (7.6).

(7.6)

Rule 1: 5 1 1 2/36/3	5	1	1	2	3	6	3
Rule 2: 5 1 1 3/35/3	5	1	1	3	3	5	3
Rule 3: 5 1 1 3/47/3	5	1	1	3	4	7	3
Rule 4: 7 2 2 4/47/3	7	2	2	4	4	7	3

The rules in Eq. (7.6) with 6 inputs and 1 output can be represented in linguistic terms stated in Eq. (7.7) on the next page.

Rule 1: If B1 is MG, B2 is VP, B3 is VP, B4 is P, C1 is MP and C2 is G, then S1 is G. (7.7)

Rule 2: If B1 is MG, B2 is VP, B3 is VP, B4 is MP, C1 is MP and C2 is MG, then S1 is G.

Rule 3: If B1 is MG, B2 is VP, B3 is VP, B4 is MP, C1 is F and C2 is VG, then S1 is G.

Rule 4: If B1 is VG, B2 is P, B3 is P, B4 is F, C1 is F and C2 is VG, then S1 is G.

Step 15: Derive final score for each alternative,

The final score for each alternative $j = 1, \cdots, 30$ is derived by averaging the aggregate membership value of the consequent part of all active rules in the overall system for stock *j*, and then multiplying with the influence multiplier based on the average influence degree across all *K* decision makers DMs for each stock *j*.

For example, S1 generated from the Boolean matrix operation has 4 active rules. Eq (6.8) is used to obtain the final score of S1. The average aggregate membership value for the output of the 6 rules is calculated and then multiplied with the influence multiplier for S1 across all DMs.

$$\Gamma_j = \frac{\sum_{Rule=1}^{n} \sum_{k=1}^{K} N\xi_{j,k} \cdot \left(NICC_{j,k}^{B} + NICC_{j,k}^{C} \right)}{n \cdot K}$$

$$= \Gamma_1 = \frac{\sum_{Rule=1}^{4} \sum_{k=1}^{3} N\xi_{1,k} \cdot \left(NICC_{j,k}^{B} + NICC_{j,k}^{C} \right)}{4 \cdot 3}$$

$$= {}^{0.9(0.94)+0.9(0.79)+0.9(0.91)}\!\Big/_{18} + {}^{0.9(0.94)+0.9(0.79)+0.9(0.91)}\!\Big/_{18}$$
$$+ {}^{0.9(0.94)+0.9(0.79)+0.9(0.91)}\!\Big/_{18} + {}^{0.9(0.94)+0.9(0.79)+0.9(0.91)}\!\Big/_{18}$$

$$= 0.7901$$

Step 16: Finally, rank the alternative base on final score value such that the higher final value the better the alternative performance.

The final score and ranking positions for all 30 stocks can be determined. The ranking based on type-1, type-2 and Z-number implementation of the proposed Fuzzy Network-TOPSIS with rule base merging approach are provided in Table 7.10. Table 7.10 shows the final ranking of 30 stocks based on fuzzy networks with merging of rule base approach for T1, T2 and Z implemention which indicate S4 is the most preferable stock for T1 and Z, and S14 is the best for T2 implementation where as S5 is the worst to investment for T1 and T2 implementation, and S1 is the worst stock for Z. The comparative analysis of approaches has been done in more details in Chapter 8.

Table 7. 10: Ranking based on proposed methods with rule base merging

	Type-1 Implementation		Type-2 implementation		Z implementation	
Stocks	Final Score	Rank	Final Score	Rank	Final Score	Rank
S1	0.2860	29	0.1319	29	0.1319	30
S2	0.6054	20	0.4897	18	0.5221	20
S3	0.4300	23	0.2566	23	0.3592	22
S4	0.8504	1	0.7621	3	0.8231	1
S5	0.2530	30	0.0759	30	0.1612	29
S6	0.3624	26	0.2362	24	0.2835	25
S7	0.7432	10	0.5466	15	0.7028	10
S8	0.7779	7	0.6909	8	0.7461	6
S9	0.6594	17	0.6017	12	0.6267	15
S10	0.8189	3	0.7835	2	0.7835	3
S11	0.7278	12	0.6529	10	0.6847	12
S12	0.6608	16	0.5640	14	0.5967	16
S13	0.7791	6	0.6585	9	0.7408	7
S14	0.8235	2	0.8066	1	0.8066	2
S15	0.3996	24	0.1974	26	0.3188	24
S16	0.6417	18	0.4420	19	0.5722	17
S17	0.3137	28	0.1595	28	0.2465	27
S18	0.7307	11	0.5212	16	0.6701	13
S19	0.7447	9	0.7155	6	0.6933	11
S20	0.7701	8	0.6481	11	0.7113	9
S21	0.7916	5	0.7547	4	0.7547	4
S22	0.6127	19	0.4239	20	0.5578	18
S23	0.5573	21	0.3406	21	0.4688	21
S24	0.8101	4	0.7496	5	0.7496	5
S25	0.7173	13	0.5891	13	0.6608	14
S26	0.3213	27	0.2334	25	0.2521	26
S27	0.4378	22	0.2992	22	0.3509	23
S28	0.6985	15	0.5067	17	0.5357	19
S29	0.7104	14	0.6919	7	0.7185	8
S30	0.3624	25	0.1755	27	0.2424	28

7.8 Summary

In summary, the established and novel TOPSIS methods are applied to the case studies of stock selection problems. Section 7.1 briefly introduces stock selection problems, whereas Section 7.2 describes the case study based on the conventional approach. The non-rule based fuzzy approach is described in Section 7.3. The application of proposed approach of fuzzy system with single rule bases and multiple rule bases are described in Section 7.4 and 7.5 respectively. After that, the application of proposed approach of fuzzy network with rule base aggregation and rule base merging are described in Section 7.6 and 7.7 respectively. In the next chapter, for validation purposes, the results of this case study from this chapter is compared descriptively with actual return on investment by using three established rank performance such as spearman rho correlation, Kendal tau, RMSE and average absolute distance.

CHAPTER 8

8 VALIDATION AND ANALYSIS OF RESULTS

8.1 Introduction

For validating the proposed fuzzy system and network on TOPSIS, the author considers the established TOPSIS methods, namely conventional TOPSIS [8] and the non-rule based fuzzy TOPSIS approaches - T1-TOPSIS [11], T2-TOPSIS[112], Z-TOPSIS [117]. All these established methods are applied to evaluate the score and the final ranking of the stocks from the case study as discussed in Chapter 7, and then compare them with the ranking produce based on proposed approaches introduced in Chapter 4-6. The actual price of stocks are used for benchmarking as shown in Table 8.1 based on trading shares of the 30 companies for a month in the Kuala Lumpur Stock Exchange (KLSE).

8.2 Return on Investment

Return on Investment (ROI) is a ratio that measures the amount of return on an investment relative to the investment cost. It is calculated by dividing the benefit (or return) of an investment to the cost of the investment. The result is expressed either as a percentage or a ratio.

$$ROI = \frac{(Gain \quad from \quad Investment \quad - Cost \quad of \quad Investment)}{Cost \quad of \quad Investment}$$

In the above formula, "Gain from Investment” refers to the proceeds obtained from the sale of the investment of interest. In this case, it refers to stock selling price, whereas cost of investment refers to stock buying cost, in the period investment as shown in Table 8.1. ROI is measured as a percentage so that it can be easily compared with returns from other investments, allowing investor to measure a variety of types of investments against one another. The ranking of 30 stocks considered based on ROI is shown in Table 8.1 as well.

Table 8. 1: Stock Price based on investment period

No.	Stock	Buy	Sell	ROI (%)	Rank
1	AMMB Holdings	6.05	5.05	-16.53	30
2	Astro Malaysia Holdings	3	3.07	2.33	9
3	Axiata Group Bhd	6.46	6.37	-1.39	21
4	British American Tobacco (Malaysia)	63.12	67.5	6.94	2
5	CIMB Group Holdings	5.54	5.38	-2.89	24
6	Digi.com	5.53	5.4	-2.35	23
7	Genting	8.1	8.14	0.49	18
8	Genting Malaysia BHD	4.23	4.27	0.95	14
9	Hong Leong Bank	13.42	13.58	1.19	12
10	Hong Leong Financial	15.4	15.86	2.99	4
11	IHH Healthcare	5.58	6.01	7.71	1
12	IOI	4.14	4.24	2.42	8
13	KLCC Prop & KLCC Reits - Staples SC	7.02	7.09	1.00	13
14	Kuala Lumpur Kepong	21.74	22	1.20	11
15	Malayan Banking	9.17	9.2	0.33	20
16	Maxis Bhd	6.49	6.67	2.77	5
17	MISC	8.13	7.8	-4.06	27
18	PETRONAS Chemicals Group Bhd	6.36	6.39	0.47	19
19	Petronas Dagangan Bhd	20.58	21.08	2.43	7
20	Petronas Gas	21.54	22.08	2.51	6
21	PPB Group	15.1	15.44	2.25	10
22	Public Bank BHD	18.84	19	0.85	15
23	RHB Capital	7.59	7.43	-2.11	22
24	SapuraKencana Petroleum	2.43	2.45	0.82	16
25	Sime Darby Bhd	8.67	8.72	0.58	17
26	Telekom Malaysia	6.78	6.55	-3.39	26
27	Tenaga Nasional	12.74	12.2	-4.24	28
28	UMW Holdings	10.32	10	-3.10	25
29	YTL Corp	1.53	1.58	3.27	3
30	Westports Holdings Bhd	4.23	4.01	-5.20	29

Table 8.1 show actual ranking of 30 stock considered based on return on investment for short term period. Based on the percentage of price change, S11 is the most profitable and S1 is the worst stock to invest. The comparative analysis of approaches has been done in more details in Chapter 8.

8.3 Spearman's rho Correlation

This section discusses the validation based on Spearman rho correlation. Table 8.2 presents the ranking of 30 stocks considered in this study based on 4 established fuzzy TOPSIS methods and 12 proposed methods. Additionally, Tables 8.3- 8.4 present the computation of rho value to illustrate the closeness of proposed methods to the actual ranking of 30 stocks by assuming return on investment as benchmarking. Considering the case study and criteria set used, i.e. B1, B2, B3, B4, C1 and C2 of stock selection described in Section 7.1, the proposed method T2-MFN TOPSIS (Rank 1) outperforms all established methods followed by the proposed methods Z-MFN TOPSIS (Rank 2) and T1-MFS TOPSIS (Rank 3), T2-MFS TOPSIS (Rank 4) as shown in the last row of Tables 8.2- 8.3.

Table 8. 2: Ranking for all methods considered for Spearman Rho analysis

STOCK	Bench-Mark	Conv.	Non-rule based fuzzy			Fuzzy system with Single rule base			Fuzzy system with Multiple rule bases			Fuzzy networks with Rule base aggregation			Fuzzy system with Rule base merging		
	ROI	C	T1	T2	Z	T1 SFS	T2 SFS	Z SFS	T1-MFS	T2-MFS	Z-MFS	T1 AFN	T2 AFN	Z AFN	T1 MFN	T2 MFN	Z- MFN
S1	30	5	27	27	14	27	27	28	28	27	29	28	28	28	29	29	30
S2	9	27	16	13	2	15	14	13	15	16	13	21	16	15	20	18	20
S3	21	26	26	25	25	26	26	24	25	24	23	23	24	24	23	23	22
S4	2	13	1	1	6	1	3	5	1	2	4	1	3	5	1	3	1
S5	24	21	30	30	24	30	30	30	30	30	30	30	30	30	30	30	29
S6	23	29	23	23	23	23	23	25	23	23	25	29	23	25	26	24	25
S7	18	10	15	16	19	14	15	11	16	12	14	9	14	11	10	15	10
S8	14	8	8	7	12	8	7	4	9	8	7	8	7	4	7	8	6
S9	12	9	19	18	15	19	18	17	19	18	17	15	19	17	17	12	15
S10	4	2	6	5	3	6	6	1	6	3	2	2	4	1	3	2	3
S11	1	24	11	8	18	17	9	16	11	9	16	13	10	16	12	10	12
S12	8	22	14	11	17	11	11	14	10	11	9	16	12	13	16	14	16
S13	13	7	9	14	10	9	13	15	7	10	15	6	8	14	6	9	7
S14	11	14	2	2	9	2	1	3	2	1	3	3	1	3	2	1	2
S15	20	19	28	28	26	28	28	27	27	28	27	24	27	27	24	26	24
S16	5	16	17	15	16	16	16	19	17	19	19	19	17	19	18	19	17
S17	27	20	29	29	29	29	29	29	29	29	26	26	29	29	28	28	27
S18	19	23	18	19	28	18	19	18	18	17	18	10	18	18	11	16	13
S19	7	28	5	3	7	5	4	8	5	5	8	11	5	9	9	6	11
S20	6	15	13	17	8	13	17	10	14	14	12	7	15	10	8	11	9
S21	10	11	7	10	5	7	10	7	8	6	10	4	9	7	5	4	4
S22	15	18	20	21	21	20	20	20	20	20	20	18	20	20	19	20	18
S23	22	3	21	20	20	21	21	21	21	21	21	20	21	21	21	21	21
S24	16	6	3	4	4	4	2	6	3	4	6	5	2	6	4	5	5
S25	17	12	12	12	22	10	12	9	13	15	5	14	11	8	13	13	14
S26	26	30	25	26	27	25	24	26	24	25	28	27	25	26	27	25	26
S27	28	4	22	22	30	22	22	22	22	22	22	22	22	22	22	22	23
S28	25	17	10	9	11	12	8	12	12	13	11	17	13	12	15	17	19
S29	3	1	4	6	1	3	5	2	4	7	1	12	6	2	14	7	8
S30	29	25	24	24	13	24	25	23	26	26	24	25	26	23	25	27	28

Table 8. 3: Spearman rho Correlation coefficient

Stock	Conventional		Non- rule based system fuzzy approach						Fuzzy system with single rule base						Fuzzy system with multiple rule bases					
	(EM)		T1 (EM)		T2 (EM)		Z (EM)		T1-SFS (NM)		T2-SFS (NM)		Z-SFS (NM)		T1 MFS (NM)		T2 MFS (NM)		Z MFS (NM)	
S1	25	625	3	9	3	9	16	256	3	9	3	9	2	4	2	4	3	9	1	1
S2	-18	324	-7	49	-4	16	7	49	-6	36	-5	25	-4	16	-6	36	-7	49	-4	16
S3	-5	25	-5	25	-4	16	-4	16	-5	25	-5	25	-3	9	-4	16	-3	9	-2	4
S4	-11	121	1	1	1	1	-4	16	1	1	-1	1	-3	9	1	1	0	0	-2	4
S5	3	9	-6	36	-6	36	0	0	-6	36	-6	36	-6	36	-6	36	-6	36	-6	36
S6	-6	36	0	0	0	0	0	0	0	0	0	0	-2	4	0	0	0	0	-2	4
S7	8	64	3	9	2	4	-1	1	4	16	3	9	7	49	2	4	6	36	4	16
S8	6	36	6	36	7	49	2	4	6	36	7	49	10	100	5	25	6	36	7	49
S9	3	9	-7	49	-6	36	-3	9	-7	49	-6	36	-5	25	-7	49	-6	36	-5	25
S10	2	4	-2	4	-1	1	1	1	-2	4	-2	4	3	9	-2	4	1	1	2	4
S11	-23	529	-10	100	-7	49	-17	289	-16	256	-8	64	-15	225	-10	100	-8	64	-15	225
S12	-14	196	-6	36	-3	9	-9	81	-3	9	-3	9	-6	36	-2	4	-3	9	-1	1
S13	6	36	4	16	-1	1	3	9	4	16	0	0	-2	4	6	36	3	9	-2	4
S14	-3	9	9	81	9	81	2	4	9	81	10	100	8	64	9	81	10	100	8	64
S15	1	1	-8	64	-8	64	-6	36	-8	64	-8	64	-7	49	-7	49	-8	64	-7	49
S16	-11	121	-12	144	-10	100	-11	121	-11	121	-11	121	-14	196	-12	144	-14	196	-14	196
S17	7	49	-2	4	-2	4	-2	4	-2	4	-2	4	-2	4	-2	4	-2	4	1	1
S18	-4	16	1	1	0	0	-9	81	1	1	0	0	1	1	1	1	2	4	1	1
S19	-21	441	2	4	4	16	0	0	2	4	3	9	-1	1	2	4	2	4	-1	1
S20	-9	81	-7	49	-11	121	-2	4	-7	49	-11	121	-4	16	-8	64	-8	64	-6	36
S21	-1	1	3	9	0	0	5	25	3	9	0	0	3	9	2	4	4	16	0	0
S22	-3	9	-5	25	-6	36	-6	36	-5	25	-5	25	-5	25	-5	25	-5	25	-5	25
S23	19	361	1	1	2	4	2	4	1	1	1	1	1	1	1	1	1	1	1	1
S24	10	100	13	169	12	144	12	144	12	144	14	196	10	100	13	169	12	144	10	100
S25	5	25	5	25	5	25	-5	25	7	49	5	25	8	64	4	16	2	4	12	144
S26	-4	16	1	1	0	0	-1	1	1	1	2	4	0	0	2	4	1	1	-2	4
S27	24	576	6	36	6	36	-2	4	6	36	6	36	6	36	6	36	6	36	6	36
S28	8	64	15	225	16	256	14	196	13	169	17	289	13	169	13	169	12	144	14	196
S29	2	4	-1	1	-3	9	2	4	0	0	-2	4	1	1	-1	1	-4	16	2	4
S30	4	16	5	25	5	25	16	256	5	25	4	16	6	36	3	9	3	9	5	25
	0	3904	0	1234	0	1148	0	1676	0	1276	0	1282	0	1298	0	1096	0	1126	0	1272
ρ		0.131		0.725		0.745		0.627		0.716		0.715		0.711		0.756		0.749		0.717
Rank		16		7		5	15			9		10		11		3		4		8

Table 8. 4: Spearman rho correlation coefficient (Cont.)

Stock	Fuzzy Network with rule base aggregation						Fuzzy Network with rule base merging					
	T1-AFN (NM)		T2-AFN (NM)		Z- AFN (NM)		T1-MFN (NM)		T2-MFN (NM)		Z-MFN (NM)	
	∂_i	∂_i^2	∂_i	∂_i^2	∂_i	∂_i^2	∂_i	∂_i^2	∂_i	∂_i^2	∂_i	∂_i^2
S1	2	4	2	4	2	4	1	1	1	1	0	0
S2	-12	144	-7	49	-6	36	-11	121	-9	81	-11	121
S3	-2	4	-3	9	-3	9	-2	4	-2	4	-1	1
S4	1	1	-1	1	-3	9	1	1	-1	1	1	1
S5	-6	36	-6	36	-6	36	-6	36	-6	36	-5	25
S6	-6	36	0	0	-2	4	-3	9	-1	1	-2	4
S7	9	81	4	16	7	49	8	64	3	9	8	64
S8	6	36	7	49	10	100	7	49	6	36	8	64
S9	-3	9	-7	49	-5	25	-5	25	0	0	-3	9
S10	2	4	0	0	3	9	1	1	2	4	1	1
S11	-12	144	-9	81	-15	225	-11	121	-9	81	-11	121
S12	-8	64	-4	16	-5	25	-8	64	-6	36	-8	64
S13	7	49	5	25	-1	1	7	49	4	16	6	36
S14	8	64	10	100	8	64	9	81	10	100	9	81
S15	-4	16	-7	49	-7	49	-4	16	-6	36	-4	16
S16	-14	196	-12	144	-14	196	-13	169	-14	196	-12	144
S17	1	1	-2	4	-2	4	-1	1	-1	1	0	0
S18	9	81	1	1	1	1	8	64	3	9	6	36
S19	-4	16	2	4	-2	4	-2	4	1	1	-4	16
S20	-1	1	-9	81	-4	16	-2	4	-5	25	-3	9
S21	6	36	1	1	3	9	5	25	6	36	6	36
S22	-3	9	-5	25	-5	25	-4	16	-5	25	-3	9
S23	2	4	1	1	1	1	1	1	1	1	1	1
S24	11	121	14	196	10	100	12	144	11	121	11	121
S25	3	9	6	36	9	81	4	16	4	16	3	9
S26	-1	1	1	1	0	0	-1	1	1	1	0	0
S27	6	36	6	36	6	36	6	36	6	36	5	25
S28	8	64	12	144	13	169	10	100	8	64	6	36
S29	-9	81	-3	9	1	1	-11	121	-4	16	-5	25
S30	4	16	3	9	6	36	4	16	2	4	1	1
	0	1364	0	1176	0	1324	0	1360	0	994	0	1076
ρ		0.697		0.7384		0.7055		0.697		0.7789		0.7606
Rank		14		6		12		13		1		2

8.4 Kendall'Tau Correlation

This section discusses the validation based on Kendall' Tau correlation. Table 8.5 presents the ranking of 30 stocks considered in this case study based on 4 established fuzzy TOPSIS methods and 12 proposed methods. In addition, Tables 8.6- 8.7 present the computation of tau value to illustrate closeness of ranking produce by proposed method to the actual ranking of 30 stocks. Considering the case criteria set used, i.e. B1, B2, B3, B4, C1 and C2 of stock selection described in Section 7.1, the proposed method T2-MFN TOPSIS (Rank 1) outperforms all established methods followed by the proposed Z-MFN TOPSIS (Rank 2) and T1-MFS TOPSIS (Rank3), as shown in the last row of Tables 8.6 - 8.7.

Table 8. 5: Ranking for all methods considered for Kendall' Tau analysis

	Actual	Conv.	Non-rule based Fuzzy approach			Fuzzy system with Single rule base			Fuzzy system with Multiple rule bases			Fuzzy network with Rule base aggregation			Fuzzy network with Rule base merging		
STOCK			T1	T2	Z	T1-SFS	T2--SFS	Z--SFS	T- MFS	T2-MFS	Z-MFS	T1-AFN	T2-AFN	Z-AFN	T1-MFN	T2-MFN	Z-MFN
S11	1	24	11	8	18	17	9	16	11	9	16	13	10	16	12	10	12
S4	2	13	1	1	6	1	3	5	1	2	4	1	3	5	1	3	1
S29	3	1	4	6	1	3	5	2	4	7	1	12	6	2	14	7	8
S10	4	2	6	5	3	6	6	1	6	3	2	2	4	1	3	2	3
S16	5	16	17	15	16	16	16	19	17	19	19	19	17	19	18	19	17
S20	6	15	13	17	8	13	17	10	14	14	12	7	15	10	8	11	9
S19	7	28	5	3	7	5	4	8	5	5	8	11	5	9	9	6	11
S12	8	22	14	11	17	11	11	14	10	11	9	16	12	13	16	14	16
S2	9	27	16	13	2	15	14	13	15	16	13	21	16	15	20	18	20
S21	10	11	7	10	5	7	10	7	8	6	10	4	9	7	5	4	4
S14	11	14	2	2	9	2	1	3	2	1	3	3	1	3	2	1	2
S9	12	9	19	18	15	19	18	17	19	18	17	15	19	17	17	12	15
S13	13	7	9	14	10	9	13	15	7	10	15	6	8	14	6	9	7
S8	14	8	8	7	12	8	7	4	9	8	7	8	7	4	7	8	6
S22	15	18	20	21	21	20	20	20	20	20	20	18	20	20	19	20	18
S24	16	6	3	4	4	4	2	6	3	4	6	5	2	6	4	5	5
S25	17	12	12	12	22	10	12	9	13	15	5	14	11	8	13	13	14
S7	18	10	15	16	19	14	15	11	16	12	14	9	14	11	10	15	10
S18	19	23	18	19	28	18	19	18	18	17	18	10	18	18	11	16	13
S15	20	19	28	28	26	28	28	27	27	28	27	24	27	27	24	26	24
S3	21	26	26	25	25	26	26	24	25	24	23	23	24	24	23	23	22
S23	22	3	21	20	20	21	21	21	21	21	21	20	21	21	21	21	21
S6	23	29	23	23	23	23	23	25	23	23	25	29	23	25	26	24	25
S5	24	21	30	30	24	30	30	30	30	30	30	30	30	30	30	30	29
S28	25	17	10	9	11	12	8	12	12	13	11	17	13	12	15	17	19
S26	26	30	25	26	27	25	24	26	24	25	28	27	25	26	27	25	26
S17	27	20	29	29	29	29	29	29	29	29	26	26	29	29	28	28	27
S27	28	4	22	22	30	22	22	22	22	22	22	22	22	22	22	22	23
S30	29	25	24	24	13	24	25	23	26	26	24	25	26	23	25	27	28
S1	30	5	27	27	14	27	27	28	28	27	29	28	28	28	29	29	30

Table 8. 6: Kendal Tau coefficient correlation

	Conventional		Non-Rule Based System Fuzzy Approach						Fuzzy System Approach with single rule base						Fuzzy System approach with multiple rule bases					
	Conv.		T1		T2		Z		T1 SFS		T2 SFS		Z SFS		T1-MFS		T2-MFS		Z-MFS	
Stock	C	D	C	D	C	D	C	D	C	D	C	D	C	D	C	D	C	D	C	D
S11	6	23	19	10	22	7	12	17	13	16	21	8	14	15	19	10	21	8	14	15
S4	16	12	28	0	28	0	23	5	28	0	26	2	24	4	28	0	27	1	25	3
S29	27	0	25	2	23	4	27	0	26	1	24	3	26	1	25	2	22	5	27	0
S10	26	0	23	3	23	3	25	1	23	3	23	3	26	0	23	3	25	1	26	0
S16	13	12	13	12	15	10	13	12	13	12	14	11	11	14	13	12	11	14	11	14
S20	13	11	16	8	13	11	20	4	15	9	13	11	18	6	15	9	15	9	16	8
S19	2	21	21	2	22	1	20	3	21	2	21	2	19	4	21	2	21	2	19	4
S12	6	16	15	7	17	5	12	10	16	6	17	5	14	8	17	5	17	5	18	4
S2	2	19	13	8	15	6	21	0	13	8	14	7	14	7	14	7	13	8	15	6
S21	12	8	18	2	16	4	19	1	18	2	16	4	17	3	17	3	18	2	16	4
S14	10	9	19	0	19	0	18	1	19	0	19	0	19	0	19	0	19	0	19	0
S9	12	6	11	7	12	6	12	6	11	7	12	6	12	6	11	7	11	7	12	6
S13	13	4	15	2	13	4	16	1	15	2	13	4	12	5	16	1	15	2	12	5
S8	12	4	15	1	15	1	14	2	15	1	15	1	16	0	15	1	15	1	14	2
S22	8	7	10	5	9	6	9	6	10	5	10	5	10	5	10	5	10	5	10	5
S24	11	3	14	0	14	0	14	0	14	0	14	0	14	0	14	0	14	0	13	1
S25	9	4	12	1	12	1	8	5	13	0	12	1	13	0	12	1	11	2	13	0
S7	9	3	11	1	11	1	9	3	11	1	11	1	12	0	11	1	12	0	11	1
S18	4	7	10	1	10	1	2	9	10	1	10	1	10	1	10	1	10	1	10	1
S15	6	4	2	8	2	8	3	7	2	8	2	8	3	7	3	7	2	8	3	7
S3	2	7	3	6	4	5	3	6	3	6	3	6	5	4	4	5	5	4	6	3
S23	8	0	7	1	7	1	5	3	7	1	7	1	7	1	7	1	7	1	7	1
S6	1	6	5	2	5	2	4	3	5	2	5	2	4	3	5	2	5	2	4	3
S5	2	4	0	6	0	6	3	3	0	6	0	6	0	6	0	6	0	6	0	6
S28	3	2	5	0	5	0	5	0	5	0	5	0	5	0	5	0	5	0	5	0
S26	0	4	2	2	2	2	2	2	2	2	3	1	2	2	3	1	3	1	1	3
S17	1	2	0	3	0	3	1	2	0	3	0	3	0	3	0	3	0	3	1	2
S27	2	0	2	0	2	0	0	2	2	0	2	0	2	0	2	0	2	0	2	0
S30	0	1	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
S1																				
	236	199	335	100	337	98	321	114	331	104	333	102	330	105	340	95	337	98	331	104
		0.0851		0.5402		0.5494		0.4759		0.5218		0.5310		0.5172		0.5632		0.5494		0.5218
τ		0.6601		4.1926		4.2640		3.6931		4.0499		4.1213		4.0142		4.3711		4.2640		4.0499
Rank		16		6		4		15		9		8		11		3		4		9

Table 8. 7: Kendal Tau coefficient correlation (Cont.)

	Fuzzy Network with rule base aggregation						Fuzzy Network with rule base merging					
	T1 AFN		T2 AFN		Z AFN		T1 MFN		T2 MFN		Z MFN	
Stock	C	D	C	D	C	D	C	D	C	D	C	D
S11	17	12	20	9	14	15	18	11	20	9	18	11
S4	28	0	26	2	24	4	28	0	26	2	28	0
S29	17	10	23	4	26	1	16	11	22	5	21	6
S10	26	0	24	2	26	0	25	1	25	1	25	1
S16	11	14	13	12	11	14	12	13	11	14	13	12
S20	20	4	14	10	18	6	19	5	18	6	19	5
S19	16	7	21	2	18	5	18	5	20	3	17	6
S12	13	9	16	6	15	7	13	9	15	7	13	9
S2	9	12	13	8	13	8	10	11	11	10	10	11
S21	19	1	16	4	17	3	18	2	19	1	19	1
S14	19	0	19	0	19	0	19	0	19	0	19	0
S9	12	6	11	7	12	6	11	7	15	3	12	6
S13	16	1	15	2	12	5	16	1	15	2	15	2
S8	15	1	15	1	16	0	15	1	15	1	15	1
S22	10	5	10	5	10	5	10	5	10	5	11	4
S24	14	0	14	0	14	0	14	0	14	0	14	0
S25	11	2	13	0	13	0	11	2	13	0	11	2
S7	12	0	11	1	12	0	12	0	12	0	12	0
S18	11	0	10	1	10	1	11	0	11	0	11	0
S15	6	4	3	7	3	7	6	4	4	6	6	4
S3	6	3	5	4	5	4	6	3	6	3	7	2
S23	7	1	7	1	7	1	7	1	7	1	7	1
S6	1	6	5	2	4	3	4	3	5	2	5	2
S5	0	6	0	6	0	6	0	6	0	6	1	5
S28	5	0	5	0	5	0	5	0	5	0	5	0
S26	1	3	3	1	2	2	2	2	3	1	3	1
S17	1	2	0	3	0	3	1	2	1	2	2	1
S27	2	0	2	0	2	0	2	0	2	0	2	0
S30	1	0	1	0	1	0	1	0	1	0	1	0
S1												
	326	109	335	100	329	106	330	105	345	90	342	93
		0.4989		0.5402		0.5126		0.5172		0.5862		0.5724
τ		3.8715		4.1926		3.9785		4.0142		4.5495		4.4424
Rank		14		6		13		11		1		2

8.5 Root Mean Square Error

This section discusses the validation based on root mean square error. Tables 8.8- 8.9 present the computation of RMSE values to illustrate the closeness of ranking produced by the proposed method to the actual ranking of 30 stocks. Considering the case criteria set used, i.e. B1, B2, B3, B4, C1 and C2 of stock selection described in Section 7.1, the proposed method T2-MFN TOPSIS (Rank 1) outperforms all established methods followed by the proposed Z-MFN TOPSIS (Rank 2) and T1-MFS TOPSIS (Rank3), as shown in the last row of Tables 8.8-8.9.

Table 8. 8: Root Mean Square Error Value

	Non-F	Fuzzy non -rule based			Fuzzy system with single rule base			Fuzzy system with multiple rule bases		
Stock	Conv.	T1	T2	Z	T1-FS	T2-FS	Z-FS	T1-MFS	T2-MFS	Z-MFS
S1	625	9	9	256	9	9	4	4	9	1
S2	324	49	16	49	36	25	16	36	49	16
S3	25	25	16	16	25	25	9	16	9	4
S4	121	1	1	16	1	1	9	1	0	4
S5	9	36	36	0	36	36	36	36	36	36
S6	36	0	0	0	0	0	4	0	0	4
S7	64	9	4	1	16	9	49	4	36	16
S8	36	36	49	4	36	49	100	25	36	49
S9	9	49	36	9	49	36	25	49	36	25
S10	4	4	1	1	4	4	9	4	1	4
S11	529	100	49	289	256	64	225	100	64	225
S12	196	36	9	81	9	9	36	4	9	1
S13	36	16	1	9	16	0	4	36	9	4
S14	9	81	81	4	81	100	64	81	100	64
S15	1	64	64	36	64	64	49	49	64	49
S16	121	144	100	121	121	121	196	144	196	196
S17	49	4	4	4	4	4	4	4	4	1
S18	16	1	0	81	1	0	1	1	4	1
S19	441	4	16	0	4	9	1	4	4	1
S20	81	49	121	4	49	121	16	64	64	36
S21	1	9	0	25	9	0	9	4	16	0
S22	9	25	36	36	25	25	25	25	25	25
S23	361	1	4	4	1	1	1	1	1	1
S24	100	169	144	144	144	196	100	169	144	100
S25	25	25	25	25	49	25	64	16	4	144
S26	16	1	0	1	1	4	0	4	1	4
S27	576	36	36	4	36	36	36	36	36	36
S28	64	225	256	196	169	289	169	169	144	196
S29	4	1	9	4	0	4	1	1	16	4
S30	16	25	25	256	25	16	36	9	9	25
RMSE	11.408	6.414	6.186	7.474	6.522	6.537	6.578	6.044	6.126	6.512
Rank	16	7	5	15	9	10	11	3	4	8

Table 8. 9: Root Mean Square Error (Cont.)

Stock	Fuzzy network with rule base aggregation			Fuzzy network with rule base merging		
	T1-AFN	T2-AFN	Z-AFN	T1-MFN	T2-MFN	Z-MFN
S1	4	4	4	1	1	0
S2	144	49	36	121	81	121
S3	4	9	9	4	4	1
S4	1	1	9	1	1	1
S5	36	36	36	36	36	25
S6	36	0	4	9	1	4
S7	81	16	49	64	9	64
S8	36	49	100	49	36	64
S9	9	49	25	25	0	9
S10	4	0	9	1	4	1
S11	144	81	225	121	81	121
S12	64	16	25	64	36	64
S13	49	25	1	49	16	36
S14	64	100	64	81	100	81
S15	16	49	49	16	36	16
S16	196	144	196	169	196	144
S17	1	4	4	1	1	0
S18	81	1	1	64	9	36
S19	16	4	4	4	1	16
S20	1	81	16	4	25	9
S21	36	1	9	25	36	36
S22	9	25	25	16	25	9
S23	4	1	1	1	1	1
S24	121	196	100	144	121	121
S25	9	36	81	16	16	9
S26	1	1	0	1	1	0
S27	36	36	36	36	36	25
S28	64	144	169	100	64	36
S29	81	9	1	121	16	25
S30	16	9	36	16	4	1
RMSE	6.743	6.261	6.643	6.733	5.756	5.989
Rank	14	6	12	13	1	2

8.6 Average Absolute Distances

This section discusses the validation of proposed methods based on average absolute distance. Table 8.10 - 8.11 presents the computation of absolute distance value to illustrate the closeness of ranking produce by the proposed method to the actual ranking of 30 stocks. Considering the case criteria set used, i.e. B1, B2, B3, B4, C1 and C2 of stock selection described in Section 7.1, the proposed method T2-MFN TOPSIS (Rank 1) outperforms all established methods followed by proposed methods Z-MFN TOPSIS (Rank 2) and T2-MFS TOPSIS (Rank 3), as shown in the last row of Tables 8.10 - 8.11.

Table 8. 10: Average absolute distance coefficient

Stock	Non-F Conv.	Fuzzy non -rule based T1	T2	Z	Fuzzy system with single rule base T1-FS	T2-FS	Z-FS	Fuzzy system with multiple rule bases T1-MFS	T2-MFS	Z-MFS
S1	25	3	3	16	3	3	2	2	3	1
S2	18	7	4	7	6	5	4	6	7	4
S3	5	5	4	4	5	5	3	4	3	2
S4	11	1	1	4	1	1	3	1	0	2
S5	3	6	6	0	6	6	6	6	6	6
S6	6	0	0	0	0	0	2	0	0	2
S7	8	3	2	1	4	3	7	2	6	4
S8	6	6	7	2	6	7	10	5	6	7
S9	3	7	6	3	7	6	5	7	6	5
S10	2	2	1	1	2	2	3	2	1	2
S11	23	10	7	17	16	8	15	10	8	15
S12	14	6	3	9	3	3	6	2	3	1
S13	6	4	1	3	4	0	2	6	3	2
S14	3	9	9	2	9	10	8	9	10	8
S15	1	8	8	6	8	8	7	7	8	7
S16	11	12	10	11	11	11	14	12	14	14
S17	7	2	2	2	2	2	2	2	2	1
S18	4	1	0	9	1	0	1	1	2	1
S19	21	2	4	0	2	3	1	2	2	1
S20	9	7	11	2	7	11	4	8	8	6
S21	1	3	0	5	3	0	3	2	4	0
S22	3	5	6	6	5	5	5	5	5	5
S23	19	1	2	2	1	1	1	1	1	1
S24	10	13	12	12	12	14	10	13	12	10
S25	5	5	5	5	7	5	8	4	2	12
S26	4	1	0	1	1	2	0	2	1	2
S27	24	6	6	2	6	6	6	6	6	6
S28	8	15	16	14	13	17	13	13	12	14
S29	2	1	3	2	0	2	1	1	4	2
S30	4	5	5	16	5	4	6	3	3	5
δ	**3.520**	**3.200**	**3.840**	**3.280**	5.200	5.000	5.267	4.800	4.933	4.933
Rank	15	10	16	13	9	7	11	2	5	5

Table 8. 11: Average absolute distance (cont.)

Stock	Fuzzy network with rule base aggregation			Fuzzy network with rule base merging		
	T1-AFN	T2-AFN	Z-AFN	T1-MFN	T2-MFN	Z-MFN
S1	2	2	2	1	1	0
S2	12	7	6	11	9	11
S3	2	3	3	2	2	1
S4	1	1	3	1	1	1
S5	6	6	6	6	6	5
S6	6	0	2	3	1	2
S7	9	4	7	8	3	8
S8	6	7	10	7	6	8
S9	3	7	5	5	0	3
S10	2	0	3	1	2	1
S11	12	9	15	11	9	11
S12	8	4	5	8	6	8
S13	7	5	1	7	4	6
S14	8	10	8	9	10	9
S15	4	7	7	4	6	4
S16	14	12	14	13	14	12
S17	1	2	2	1	1	0
S18	9	1	1	8	3	6
S19	4	2	2	2	1	4
S20	1	9	4	2	5	3
S21	6	1	3	5	6	6
S22	3	5	5	4	5	3
S23	2	1	1	1	1	1
S24	11	14	10	12	11	11
S25	3	6	9	4	4	3
S26	1	1	0	1	1	0
S27	6	6	6	6	6	5
S28	8	12	13	10	8	6
S29	9	3	1	11	4	5
S30	4	3	6	4	2	1
δ	5.667	5.000	5.333	5.600	4.600	4.800
Rank	15	7	12	14	1	2

8.7 Analysis of results

In this section, the results are analyzed based on three aspects – firstly is the performance of the established methods (EM) and novel methods (NM), followed by the performance of approaches and finally, the performance of type-1, type-2 and Z-number.

8.7.1 Comparison of established and novel methods

In this subsection, the established and novel methods are compared based on average ranking position by using four different performance indicators - namely, spearman rho, Kendall tau, RMSE and average absolute distance. Four established and twelve novel methods are proposed in this research as shown in Table 8.10.

Based on the case study considered, Table 8.12 shows the average ranking position of established and novel methods of all four performance indicators. Derived from Spearman rho, Kendall tau, RMSE and average absolute distance, the novel methods average rank positions are 7.75, 7.58, 7.55 and 7.50, respectively; outperforming the established methods with average rank positions 10.75, 10.25, 10.75 and 10, respectively.

Table 8. 12: Comparison of established (EM) and novel methods (PM)

Established methods (EM)	Spearman rho		Kendal tau		RMSE		AAD	
	Rho	Rank position	Tau	Rank position	Coef	Rank Position	Coeff	Rank Position
Conv. TOPSIS	0.1315	16	0.0851	16	11.41	16	8.87	16
T1-TOPSIS	0.7255	7	0.5402	6	6.41	7	5.20	9
T2-TOPSIS	0.7446	5	0.5494	4	6.19	5	4.80	2
Z-TOPSIS	0.6271	15	0.4759	15	7.47	15	5.47	13
Average rank for EM		**10.75**		**10.25**		**10.75**		**10.00**
Novel methods (NM)								
T1-SFS TOPSIS	0.7161	9	0.5218	9	6.52	9	5.20	9
T2-SFS TOPSIS	0.7148	10	0.5310	8	6.54	10	5.00	7
Z-SFS TOPSIS	0.7112	11	0.5172	11	6.58	11	5.27	11
T1-MFS TOPSIS	0.7562	3	0.5632	3	6.04	3	4.80	2
T2-MFS TOPSIS	0.7495	4	0.5494	4	6.13	4	4.93	5
Z-MFS TOPSIS	0.7170	8	0.5218	9	6.51	8	4.93	5
T1-AFN TOPSIS	0.6966	14	0.4989	14	6.74	14	5.67	15

T2-AFN TOPSIS	0.7384	6	0.5402	6	6.26	6	5.00	7
Z-AFN TOPSIS	0.7055	12	0.5126	13	6.64	12	5.33	12
T1-MFN TOPSIS	0.6974	13	0.5172	11	6.73	13	5.60	14
T2-MFN TOPSIS	0.7789	1	0.5862	1	5.76	1	4.60	1
Z-MFN TOPSIS	0.7606	2	0.5724	2	5.99	2	4.80	2
Average rank for NM		**7.75**		**7.58**		**7.75**		**7.50**

8.7.2 Comparison of Approaches

In this subsection, six approaches – namely, conventional approach, fuzzy non rule base approach, fuzzy system with single rule base (SFS), fuzzy system with multiple rule bases (MFS), fuzzy network with rule base aggregation (AFN) and fuzzy network with rule base merging (MFN) are compared based on average rank positions of each approach as shown in Table 8.13. The average rank positions for the six approaches considered in this research are shown in Table 8.13. MFS, consists of three novel methods, is the best approach using spearman rho and RMSE with average ranking position 5.00. MFN, consists of three novel methods, is the best approach using Kendal tau with average rank position 4.67. MFS and AFN, consist of six novel methods, are the best approaches using average absolute distance with average rank position 4.00.

Table 8. 13: Average performance of each approach

Approaches	Spearman Rho	Rank Position	Kendall Tau	Rank Position	RMSE	Rank Position	AAD	Rank Position
Conventional	16.00	6	16.00	6	16.00	6	16.00	6
Fuzzy Non-RBS	9.00	3	8.33	3	9.00	3	8.00	3
SFS	10.00	4	9.33	4	10.00	4	9.00	4
MFS	5.00	1	5.33	2	5.00	1	4.00	1
AFN	10.67	5	11.00	5	10.67	5	11.33	5
MFN	5.33	2	4.67	1	5.33	2	5.67	2

8.7.3 Comparison of Fuzzy sets

Three types of fuzzy sets- namely, type-1, type-2 and Z-number implementation are compared based on average rank position. As seen in Table 8.14, the implementation of type-2 fuzzy set outperforms the others with average rank positions of 4.3, 4.6, 5.2 and 4.4. Type-1 is better than Z-number and conventional except when using AAD where Z-number is better. The conventional fuzzy number stays the lowest for all rank performances.

Table 8. 14: Average performance of each fuzzy set

Fuzzy sets	Spearman Rho	Rank Position	Kendall Tau	Rank Position	RMSE	Rank Position	AAD	Rank Position
Conventional	16	4	16	4	16	4	16	4
Type 1	9.2	2	8.6	2	9.2	2	9.8	3
Type 2	4.3	1	4.6	1	5.2	1	4.4	1
Z	9.6	3	10	3	9.6	3	8.6	2

8.8 Summary

In this chapter, the ranking of 30 stocks in this study has been validated comparatively using four established performance indicators such as Spearman rho correlation, Kendall tau, RMSE and average absolute distance by assuming return on investment as benchmarking. The first section is Section 8.1 where a concise summary of steps used to validate the proposed methods is described. Section 8.2 then presents the ranking of 30 stocks based on return on investment for the short investment period. The subsequent sections are the validation of results based on spearman rho correlation, Kendall tau correlation, RMSE and average absolute distance. The final section is Section 8.7 where results are analyzed by comparing them with the established and novel methods, approaches and fuzzy sets. The next chapter will conclude the thesis and a possible future research of this study will be mentioned as well.

CHAPTER 9

9 CONCLUSION

9.1 Introduction

This research has four main objectives. The first two objectives are the development of fuzzy TOPSIS from fuzzy system which consists of single rule base and multiple rule bases as discussed in Chapter 4 and 5. The last two objectives are the development of fuzzy TOPSIS based on fuzzy networks which consist of rule base aggregation and rule base merging as discussed in Chapter 6. Additionally, the fuzzy number implementation of type-1, type-2 and Z-number for the proposed methods are formulated accordingly. Consequently, the work of this research includes constructing a proposed fuzzy TOPSIS model as well as applying the stock selection problems in the Kuala Lumpur Stock Exchange. Moreover, this research consists of identifying several criteria contributing to the best stock selection involved where the opinions from three decision makers are considered. Lastly, the validation analysis such spearman rho correlation, Kendal tau correlation and average absolute distance are thoroughly tested by considering actual return on investment as benchmark ranking to prove the practicality and effectiveness of proposed methods.

This chapter illustrates the contributions of this research, scope of this research and recommendations for future works. It summaries all the work contributed to knowledge in every chapter of the thesis and suggests some significant recommendations for improving the knowledge of fuzzy sets and decision making. Therefore, with no loss of generality of all chapters in the thesis, the details on those points will be reinstated in the ensuing sections.

9.2 Contributions

As far as this research is concerned, four main contributions to knowledge are presented in Chapter 4-6 of the thesis. These contributions are underpinned by publications [P1-P9] indicating the strength and the novelty of the research in improving and enhancing the theory of fuzzy sets, particularly in fuzzy decision making environment. Some of the contributions in the thesis have been highlighted in an article on PhD success stories published in the University of Portsmouth Research Newsletter and recognised by a prize awarded at the Faculty of Technology Research Conference for the best journal paper authored by a PhD student (see Appendix 2).

The first contribution of this research is the development of fuzzy TOPSIS methodology formulation as highlighted in Chapter 4. This methodology is based on fuzzy systems with single rule base, in which decision makers' opinion and knowledge are represented as fuzzy rules. Instead of calculating the average of opinions as in the established methods, this research evaluates the opinion of each decision makers independently. In developing these methods, the linguistic term of alternatives level such "Very Good", "Good", "Regular", "Bad", and "Very Bad" are proposed in this research. Later, the implementation of this formulation using fuzzy numbers of type-1, type-2 and Z-number is developed.

As highlighted in Chapter 5, fuzzy TOPSIS methodology proposed in Chapter 4 is modified for fuzzy systems with multiple rule bases. In this novel approach, the sets of criteria are categorised into two subsystems - namely benefit rule base and cost rule base. In this way, the decision maker can assess the performance of benefit and cost for each alternative. Later, the implementation of this formulation using type-1, type-2 and Z-number is developed.

In Section 6.2 of Chapter 6, the proposed fuzzy TOPSIS are extended to fuzzy network with rule base aggregation. In this approach, three subsystems are involved, two of which are the benefit and the cost rule base from Chapter 5 and the third subsystem is called the alternatives rule base added to makes use of fuzzy network approach. The outputs of benefit and cost rule base, namely benefit level and cost level, are the inputs for this additional subsystem. The aggregation of rule base implementation in this method is to find the final scores for each rule. Later, the implementation of this formulation using type-1, type-2 and Z-number is developed.

The contribution from Chapter 6.3 is the development of fuzzy TOPSIS based on fuzzy networks with rule base merging. Here, the extension of proposed formulation in Chapter 5 is carried to implement the fuzzy network using rule base merging. Vertical rule base merging is used as a connection between benefit rule base and cost rule base, then those connected with alternative rule base via horizontal rule base merging. Finally, the implementation of this formulation using type-1, type-2 and Z-number is developed.

In summary, contributions to knowledge from this research using newly proposed methods are succinctly described. The scope of this research is provided in the following section.

9.3 Scope of the Research

This research has made significant contributions with positive implications in decision making environment. Nonetheless, several limitations are needed to be conferred as well and they are:

a. The newly proposed MCDM model is limited to the TOPSIS methods only; thus, excluding other MCDM method such as PROMETHEE, AHP, and ELECTRE.

b. The validation of the proposed methods is solely based on the local case study in selection of stock traded in KLSE, Malaysia and limited to 30 stocks, 6 financial criteria and 3 decision maker’s opinions; hence, excluding other financial markets such as London Stock exchange.

9.4 Recommendation for Future Research

This research has triggered many questions for future investigation. Future research may explore many different areas, cases and methods and they are:

a. To use diverse models for example using AHP model within the fuzzy systems and networks.

b. To use a different way of calculating the criteria weight for decision making method so that the difference of using various weights can be observed.

c. To test the system with a larger number of decision makers. This is a very crucial factor in real world decision problems, which have handled distinct human's behavior.

d. To implement other shapes of fuzzy numbers evaluated in various world applications such as triangular, Gaussian and so on. Different shapes of membership functions will provide different outcomes.

e. To explore on the stock selection in developed countries such as London Stock Exchange to widen the case studies.

f. To develop a decision making software from this method using either Matlab, JAVA or Visual Basic.

g. To used the proposed methods in other decision making problems, particularly the selection problems such as tourism, finance, control and economy.

9.5 Summary

The groundwork of this research that leads to its contributions to knowledge, its scope and some recommendation of future work is thoroughly annotated. Thus, the thesis shall end its discussion by citing all the references used throughout the thesis, which are provided in the next page after this chapter.

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APPENDIX 1

Appendix 1.1: Ethics Opinion Letter

University of Portsmouth
Faculty of Technology

Technology Faculty Ethics Committee

ethics-tech@port.ac.uk

Date 28/11/16

Abdul Yaakob

School of Computing

Dear Abdul,

Study Title:	**MULTI CRITERIA DECISION MAKING METHODOLOGY FOR FUZZY RULE BASED SYSTEMS AND NETWORKS USING TOPSIS**
Ethics Committee reference:	AY1

The Ethics Committee reviewed the above application by an email discussion forum between the dates of 18/11/16 and 28/11/16.

Ethical opinion

The members of the Committee present gave a favourable ethical opinion of the above research on the basis described in the application form, protocol and supporting documentation.

Conditions of the favourable opinion: Please confirm that you have made these changes to your School Ethics Coordinator before collecting data.

That participants consent is recorded in a more formal manner than just assumed by the return of the form.

Section 11.4 needs amending to 'received a favourable opinion' as opposed to 'approved' by the ethics committee: note we do not “approve” projects.

Further details are needed on how the data is to be used, stored and presented in published work.

Add version numbers and dates to all research instruments to allow version control and cross reference.

Recommendations: (You should give these due consideration but there is no obligation to comply or respond)

The robustness of the methodology was questioned by reviewers specifically the small sample size and how it would be established that the respondents were experts.

The need for anonymous responses should also be considered.

The favourable opinion of the EC does not grant permission or approval to undertake the research. Management permission or approval must be obtained from any host organisation, including University of Portsmouth, prior to the start of the study.

Summary of discussion on Moodle

There we discussions about the appropriateness of the methodology. The low risk was noted. Overall the resubmitted application was given a favourable opinion by the three reviewers.

Documents reviewed

The documents reviewed at the meeting were:

Document	*Version*	*Date*
Application Form	1	14/11/16
Questionnaire	Absent	Absent

Statement of compliance

The Committee is constituted in accordance with the Governance Arrangements set out by the University of Portsmouth

After ethical review

Reporting requirements

The attached document acts as a reminder that research should be conducted with integrity and gives detailed guidance on reporting requirements for studies with a favourable opinion, including:

- Notifying substantial amendments
- Notification of serious breaches of the protocol
- Progress reports
- Notifying the end of the study

Feedback

You are invited to give your view of the service that you have received from the Faculty Ethics Committee. If you wish to make your views known please contact the administrator ethics-tech@port.ac.uk

Please quote this number on all correspondence: AY1

University of Portsmouth

Faculty of Technology

Yours sincerely and wishing you every success in your research

John Williams

Chair Technology FEC

Email: ethics-tech@port.ac.uk

Appendix 1.2: Ethics Application Form

University of Portsmouth

Faculty of Technology

Application for Ethical Review – Staff and Postgraduate Research Students

1. Study Title and Key Dates

1.1 Title: Multi criteria decision making methodology for fuzzy rule based systems and networks using TOPSIS
1.2 Date of submission: 14 Nov 2016 **Version Number: 1** **Ethics Committee Reference Number:** AY1 **1.3 Date of study commencement: 1 Feb 2017** **Projected date of study completion (fully written up):**

2. Applicant Details: Please complete either 2.1 or 2.2 as appropriate

2.1 Principal Investigator (Member of staff –personally or as a supervisor of a taught student)
Name: Alexander Gegov Title /Role: Reader Department: School of Computing **Telephone: 02392421367 Email: alexander.gegov@port.ac.uk**
2.2 Principal Investigator (PGRS)
Name: Abdul Malek Yaakob Title /Role: Research Student Department: School of Computing **Course of study: Computing** **Telephone: 02392846460 Email: abdul.yaakob@port.ac.uk** **First Supervisor's Name: Alexander Gegov Telephone: 02392421367 Email: alexander.gegov@port.ac.uk** **Names and contact details of any other supervisors (if relevant)** Supervisors will have to confirm to ethics-tech@port.ac.uk that this proposal is ready for ethical review, either by submitting the application on behalf of the student, or by sending the student or ethics-tech@port.ac.uk, a separate email confirming that this protocol (version and date) is ready to be submitted to Technology Faculty Ethics Committee for ethical review.
2.3 Co-Researchers / Collaborators
-
2.4 Independent or Peer Reviewer
-

3. Funding Details

Fully funded by the Universiti Utara Malaysia, Malaysia

4. Research Sites

This research will involve financial experts within Universiti Utara Malaysia. Also, will take place at Universiti Utara Malaysia. There is no risk involve and no consent is require regarding health, safety and welfare of both researcher and participants.

5. Insurance Arrangements

This research does not require an insurance arrangements.

6. Study Summary

6.1 Study Summary
Fuzzy systems and networks are vital within the armoury of fuzzy tool and applicable to real life decision-making environments. There are three types of fuzzy systems introduced in literature- systems with single rule base, systems with multiple rule bases and system with networked rule bases. This research introduces novel extension of the Technique of Ordering of Preference by Similarity to Ideal Solution (TOPSIS) method and uses fuzzy system and networks to solve multi-criteria decision-making problem where both benefit and cost criteria are presented as subsystems. Along that, the implementation for type 1, type 2 and Z fuzzy sets of proposed approaches is also presented. Furthermore, the literature is observed that is essential to track the performance of criteria, in order to take control and not underestimate or overestimate uncertainty of the criteria. Thus the decision maker evaluates the performance of each alternative and further observes the performance for both benefit and cost criteria. This research improves significantly the transparency of the TOPSIS methods while ensuring higher effectiveness in comparison to established approaches. To ensure practicality and effectiveness of proposed methods in a realistic scenario, the problem of ranking traded stock is studied. This case study is conducted based on stocks traded in a developing financial market such as Kuala Lumpur Stock Exchange. In this study, the human participant are experts on stock market research from Universiti Utara Malaysia. The ranking based on proposed methods is validated comparatively using performance indicators such as Spearman Rho correlation, Kendall Tau correlation and Average absolute distance by assuming ranking based on return on investment as a benchmarking.
6.2 Main Ethical Issues
There are no ethical issues because there is no collection of sensitive data from participant; no vulnerable participants; no sensitive data, no risks of disclosing unprosecuted crimes; no risk of disclosing professional malpractice; no risks of accidental disclosure of personal and/or sensitive data; no safeguarding concerns; no use of deception etc.
6.3 Other Risks or Concerns
There are no risks to the University’s reputation; no conflicts of interest – no financial conflicts; no personal relationships with other researchers or participants; no expectations of employers if conducting research in the place of work.

7. Compliance With Codes, Guidance, Policies and Procedures

The study reflect the University's adherence to the commitments set out in the Concordat to Support Research Integrity and the RCUK Policy and Guidelines on Governance of Good Research Conduct.

8. Study Aims and Objectives

8.1 Main Aim / Research Question/Hypothesis
The main aim of this study is to use expert knowledge for ranking of stock on financial market.
8.2 Primary Objective
The primary objective of this study is to collect expert opinion on importance of criteria for evaluation of the stocks and level of confident on this evaluation.
8.3 Secondary Objective(s)
The secondary objective of this study is to collect expert opinion on rating of stocks performance and level of confident on this rating.

9. Research Methods

9.1 Research Method(s)
A survey will be conducted and intended medium for this survey is paper. In order to collect expert opinion, questionnaires will be distributed to experts. The draft survey is attached for the Ethics Committee to review.

10. Recruitment of Participants

10.1 General Considerations
This research considers financial expert as potential participants.

10.2 The Research Population
The potential participants are experts in financial markets. They are Lecturers and Professors in finance at Universiti Utara Malaysia.
10.3 Sampling Strategy
Sample size is 3. Sample population is expert in financial market. The sampling methodology is based on the level of expertise in financial market.

10.4 Recruitment Strategy – Invitations to Potential Participants
Letter of Invitation is attached as cover letter for questionnaire.

10.5 Obtaining Consent
The participants will be asked to respond by email to confirm their consent formally.

10.6 Organisational Consent
This research does not need the consent of any organisation.
10.7 Participant Withdrawal
The participants are free to withdraw from the study or refuse to answer the questionnaire at any time.

11. Research Data Management

11.1 General
The data from this survey will be used only for this research.

11.2 Data Collection and Analysis
This survey will only collecting information from expert based on their experience. Type of information collected is 1) Level of expert experience on the area. 2) Expert opinion on the importance of criteria. 3) Expert rating of stocks based on their performance.

11.3 Data Storage
The data from the questionnaire will be used as an input to the algorithm that will be apply for the research case study. **The data from the survey will be store in the electronic secure excel file. and all hard copy completed survey paper will be kept in locked filling cabinet.** **The data from survey maybe presented in research publication if require by the review.**
11.4 Destruction, Retention and Reuse of Data
The data from this survey will only use for research that received a favourable opinion from the ethics committee.

11.5 Personal Data – Confidentiality and Anonymisation
This research does not collect personal data.
11.6 Organisational Data
This research does not collect any organizational data or no personal data.

11.7 Security Sensitive Data
This research does not required access to security sensitive data.

12. Risks

12.1 Risks to Participants
There is no risk to participants.
12.2 Risks to Researchers
There is no risk to researchers.

13. Publication Plans

The results from this research will be submitted to relevant journals and conferences.

14. References

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15. Appendices

Study Title: Multi criteria decision making methodology for fuzzy rule based systems and networks using TOPSIS		
Document	**Date**	**Version No.**
Application Form	14/11/2016	1
Invitation Letter	14/11/2016	1
Supervisor Email Confirming Application	14/11/2016	1
Questionnaire	14/11/2016	1

16. Declaration

Declaration by Principal Investigator, and, if necessary, the Supervisor

1. The information in this form is accurate to the best of my/our knowledge and belief and I/we take full responsibility for it.
2. I/we undertake to conduct the research in compliance with the University of Portsmouth Ethics Policy, UUK Concordat to Support Research Integrity, the UKRIO Code of Practice and any other guidance I/we have referred to in this application.
3. If the research is given a favourable opinion I/we undertake to adhere to the study protocol, the terms of the full application as approved and any conditions set out by the Ethics Committee in giving its favourable opinion.
4. I/we undertake to notify the Ethics Committee of substantial amendments to the protocol or the terms of the approved application, and to seek a favourable opinion before implementing the amendment.
5. I/we undertake to submit annual progress reports (if the study is of more than a year's duration) setting out the progress of the research, as required by the Ethics Committee.
6. I/we undertake to inform the Ethics Committee when the study is complete and provide a declaration accordingly.
7. I/we am/are aware of my/our responsibility to be up to date and comply with the requirements of the law and relevant guidelines relating to security and confidentiality of personal data, including the need to register, when necessary, with the appropriate Data Protection Officer. I/we understand that I/we am/are not permitted to disclose identifiable data to third parties unless the disclosure has the consent of the data subject.
8. I/we undertake to comply with the University of Portsmouth Research Data Management Policy.
9. I /we understand that research records/data may be subject to inspection by internal and external bodies for audit purposes if required.
10. I/we understand that any personal data in this application will be held by the Ethics Committee, its Administrator and its operational managers and that this will be managed according to the principles established in the Data Protection Act 1998.
11. I understand that the information contained in this application, any supporting documentation and all
correspondence with the Ethics Committee and its Administrator relating to the application:

- Will be held by the Ethics Committee until at least 3 years after the end of the study
- Will be subject to the provisions of the Freedom of Information Acts and may be disclosed in response to requests made under the Acts except where statutory exemptions apply.
- May be sent by email or other electronic distribution to Ethics Committee members.

Principal Investigator………………. **Date……………………** 11 Nov 2016

(Abdul Malek Yaakob)

Supervisor……………………………….. **Date……………………** 11 Nov 2016

(Dr Alexander Gegov)

Appendix 1.3: Cover Letter and Questionnaire

School of Computing
University of Portsmouth
Buckingham Building
Lion Terrace
Portsmouth PO1 3HE
United Kingdom

T: +44 (0)23 9284 6363
F: +44 (0)23 9284 2181

Study Title: Multi criteria decision-making methodology for fuzzy rule based systems and networks using TOPSIS.

Name of researcher and supervisor: Abdul Malek Yaakob and Dr. Alexander Gegov

Invitation

Thank you for reading this. I would like to invite you to take part in my research study by completing this questionnaire. It is entirely up to you whether you participate but your responses would be valued. You have been identified as a potential respondent by your capacity as financial expert**.** My study is to develop decision-making methodology for fuzzy rule based systems and networks using TOPSIS, basically this methodology is to rank alternative based on experts' opinion on alternative performance. For the validation purposes of the proposed methods a case study of stock selection is conducted. I need your opinion in order to rank 30 stocks listed on Kuala Lumpur Stock Exchange based on 6 criteria considered in the study. I neither need your name nor any identifying details; the questionnaire can be completed anonymously and all reasonable steps will be taken to ensure confidentiality. Responses from completed questionnaires will be collated for analysis; once this is complete the original questionnaires will be retain until successful completion of my PhD**.** Up to this stage, completed questionnaires will be stored in locked filing cabinet. I do believe there is no risks or benefits associated with participant. If you wish to learn more about the results of the research please contact my supervisor or me. Thank you for agreeing to provide information regarding your thoughts in this study.

Kind Regards

Abdul Malek Yaakob
Research Student, School of Computing
University of Portsmouth, United Kingdom

PART A: EXPERTISE ON THE AREA

Instruction: Using the scale 1- 7, to indicate your expertise on the area, where 1 represent not expert and 7 represent highly expert.

1	2	3	4	5	6	7

PART B: IMPORTANCE OF CRITERIA

Table 1: Linguistic Terms for Importance of Criteria (**IC**)

Abbrev	Stands for:
VL	Very Low
L	Low
ML	Medium Low
M	Medium
MH	Medium High
H	High
VH	Very High

Table 2: Linguistic Terms for Stock Evaluation (**R**)

Abbrev	Stands for:
VP	Very Poor
P	Poor
MP	Medium Poor
F	Fair
MG	Medium Good
G	Good
VG	Very Good

Table 3: Linguistic Terms for Reliability of Decision (**CL**)

Abbrev	Stands for:
SUL	Strongly Unlikely
UL	Unlikely
SWU	Somewhat Unlikely
N	Neutral
SWL	Somewhat likely
L	Likely
SL	Strongly Likely

Instruction: Please fill in the blank based on importance of criteria (IC) and confidence level of the decision (CL) (refer Table 1 and Table 3)

Criteria	IC	CL
1) Market Value Firm/Earning Before Amortization, Interest and Taxes **(MVF/EBAIT)**		
2) Return On Equity **(ROE)**		
3) Dept/Equity Ratio **(D/E)**		
4) Current Ratio **(CR)**		
5) Market Value/ Net Sales **(MV/NS)**		
6) Price-To-Earnings Ratio **(P/E)**		

PART C: STOCK EVALUATION

Instruction: Please give rating of each stock based on the performance of stocks given in Appendix A and give confident level of the decision (CL) [refer Table 2 and Table 3]

Example:

	Stock	*MVF*		*ROE*		*D/E*		*CR*		*MV/NS*		*P/E*	
		R	*CL*	*R*	*CL*	*R*	*CL*	*R*	*CL*	*R*	*CL*	*R*	*CL*
0	*MYX: XXXX*	*VG*	*SL*	*G*	*VL*	*VG*	*L*	*G*	*N*	*VG*	*L*	*VG*	*SWL*

	Stock	**MVF**		**ROE**		**D/E**		**CR**		**MV/NS**		**P/E**	
		R	**CL**	**R**	**CL**	**R**	**CL**	**R**	**CL**	**R**	**CL**	**R**	**CL**
1	MYX: 1015												
2	MYX: 6399												
3	MYX: 6888												
4	MYX: 4162												
5	MYX: 1023												
6	MYX: 6947												
7	MYX: 3182												
8	MYX: 4715												

9	MYX: 5819												
10	MYX: 1082												
11	MYX: 5225												
12	MYX: 1961												
13	MYX: 5235SS												
14	MYX: 2445												
15	MYX: 1155												
16	MYX: 6012												
17	MYX: 3816												
18	MYX: 5183												
19	MYX: 5681												
20	MYX: 6033												
21	MYX: 4065												
22	MYX: 1295												

23	MYX: 1066						
24	MYX: 5218						
25	MYX: 4197						
26	MYX: 4863						
27	MYX: 5347						
28	MYX: 4588						
29	MYX: 4677						
30	MYX: 5246						

Thank you for completing the questionnaire please return it to:

Abdul Malek Yaakob, School of Quantitative Sciences, Universiti Utara Malaysia, 06010 Sintok Kedah, Malaysia.

If you have any concerns regarding this research please contact my supervisor or me in the first instance. If you are not entirely happy with a response please contact head of department and /or the University Complaints Officer.

APPENDIX A: STOCK PERFORMANCE

BIL	STOCKS	MVF	ROE	D/E	CR	MV/NS	P/E
1	MYX: 1015	16969.86	13.90	109.66	0.00	3.56	10.65
2	MYX: 6399	15761.24	79.49	504.98	1.04	3.01	30.07
3	MYX: 6888	55348.59	11.64	69.79	0.79	2.96	23.36
4	MYX: 4162	18445.24	174.73	70.67	0.72	3.85	20.46
5	MYX: 1023	46747.86	9.19	103.95	0.00	3.08	14.42
6	MYX: 6947	41829.50	301.54	152.69	0.46	5.96	20.65
7	MYX: 3182	30956.23	4.68	38.12	3.67	1.70	15.98
8	MYX: 4715	25236.69	7.48	9.93	2.21	3.07	19.87
9	MYX: 5819	25679.56	15.25	91.07	0.00	5.75	11.65
10	MYX: 1082	16254.74	15.84	147.42	0.00	3.22	9.70
11	MYX: 5225	48757.88	4.02	21.95	1.59	6.64	64.41
12	MYX: 1961	27258.23	12.64	124.63	2.11	2.29	21.57
13	MYX: 5235	12727.60	7.91	20.88	1.89	9.40	13.57
14	MYX: 2445	24040.21	12.98	37.55	2.01	2.16	24.23
15	MYX: 1155	88041.22	13.57	75.72	0.00	4.69	12.43
16	MYX: 6012	49409.21	32.05	191.44	0.62	5.89	28.64
17	MYX: 3816	35263.97	8.40	31.49	2.05	3.79	17.80
18	MYX: 5183	50640.00	11.09	0.00	5.19	3.47	22.39
19	MYX: 5681	20445.28	10.51	10.40	1.12	0.63	40.41
20	MYX: 6033	42740.61	17.72	8.38	1.63	9.73	22.76
21	MYX: 4065	18232.99	5.65	3.29	2.48	4.93	19.81
22	MYX: 1295	73760.63	18.65	43.63	0.00	7.90	15.27
23	MYX: 1066	19128.88	11.47	69.21	0.00	2.97	9.79
24	MYX: 5218	14321.25	12.92	141.44	1.28	1.44	11.49
25	MYX: 4197	52670.62	11.18	39.62	2.02	1.20	16.94
26	MYX: 4863	24915.11	11.31	85.17	1.33	3.56	28.22
27	MYX: 5347	69303.55	16.51	74.69	1.49	1.62	10.97
28	MYX: 4588	11986.70	10.13	63.86	1.70	0.80	17.15
29	MYX: 4677	17270.39	11.30	233.44	2.14	0.90	9.74
30	MYX: 5246	13912.80	30.41	65.18	2.58	8.91	26.62

THANK YOU FOR YOUR COOPERATION

Appendix 1.4: Supervisor Email Confirming Application

University of Portsmouth

Abdul Yaakob <abdul.yaakob@port.ac.uk>

ethical review application from Abdul Yaakob

Alexander Gegov <alexander.gegov@port.ac.uk> 4 November 2016 at 19:47
To: ethics-tech - <ethics-tech@port.ac.uk>
Cc: Abdul Yaakob <abdul.yaakob@port.ac.uk>, Abdul Malek bin Yaakob <abd.malek@uum.edu.my>, Abdul Malek Yaakob <malek5877@gmail.com>, Alexander Gegov <alexander.gegov@port.ac.uk>

Dear colleagues,

I confirm that I have reviewed the ethical review application from my PhD student Abdul Yaakob and I am happy for it to be submitted by him for your consideration.

I would appreciate it if you could approve this application at your earliest convenience in order to allow the research to be started as soon as possible.

Kind regards,

Alex

--
Alexander Gegov, BSc, MSc, PhD, DSc
Reader in Computational Intelligence
University of Portsmouth
School of Computing
Buckingham Building
Portsmouth PO1 3HE
United Kingdom
Email: alexander.gegov@port.ac.uk
http://www.port.ac.uk/departments/academic/comp/staff/title,3828,en.html

Appendix 1.5: Confirmation from School of Computing Ethics Coordinator

University of Portsmouth

Abdul Yaakob <abdul.yaakob@port.ac.uk>

Ethics opinion letter - Multi Criteria Decision Making Methodology for Fuzzy Rule Based Systems and Networks Using Topsis

Philip Scott <philip.scott@port.ac.uk> 17 December 2016 at 00:32
To: Abdul Yaakob <abdul.yaakob@port.ac.uk>
Cc: Alexander Gegov <alexander.gegov@port.ac.uk>, Abdul Malek bin Yaakob <abd.malek@uum.edu.my>

Dear Abdul,

Thank you for confirming that you have implemented the guidance provided. Good luck with your research.

Regards,

Philip
[Quoted text hidden]
--
Dr Philip Scott
Senior Lecturer in Information Systems
School of Computing, University of Portsmouth
http://scottp.myweb.port.ac.uk
Vice-Chair (Events), BCS Health
http://www.bcs.org/health
Chair, HL7 UK
http://www.hl7.org.uk/

Appendix 1.6: Organizational Consent Letter

Computing and Mathematics

University of Portsmouth

Computing and Mathematics
University of Portsmouth
Lion Gate Building
Lion Terrace
Portsmouth PO1 3HF
United Kingdom

T: +44 (0)23 9284 6400
F: +44 (0)23 9284 6411

Date 19 Dec 2016

Dear Sir or Madam:

I am a student undertaking PhD in Computing at the University of Portsmouth. As part of my course I am undertaking a research study titled: Multi-criteria Decision Making Methodology for Fuzzy Rule Base Systems and Networks using TOPSIS supervise by Dr Alexander Gegov.

My study is to develop decision-making methodology for fuzzy rule based systems and networks using TOPSIS, basically this methodology is to rank alternative based on experts' opinion on alternative performance. For the validation purposes of proposed methods a case study of stock selection is conducted.

Prior to undertaking the study I need your agreement/consent to approach the staff within your organisation to take part in the study. They have been identified as a potential respondent by their capacity as financial expert. I will recruit people to the study via questionnaire. I hope to recruit 3 numbers of participants as an expert.

I can assure you that I will make every effort to ensure the study does not disrupt the working environment or student lectures in any way and any data collected will remain confidential. I am received ethical approval for the study from the University of Portsmouth, Faculty of Technology Ethics Committee.

If you wish to learn more about this letter please contact my supervisor or me at alexander.gegov@port.ac.uk.

Yours Sincerely

Abdul Malek Yaakob
abd.malek@uum.edu.my

Appendix 1.7: Participants Consent

University of Portsmouth

Abdul Yaakob <abdul.yaakob@port.ac.uk>

Your Opinion is Important

sharifah Ahmad <sharifah.ahmadia@gmail.com> 15 December 2016 at 10:33
To: Abdul Yaakob <abdul.yaakob@port.ac.uk>

Dear Abdul,

Many thanks for your email.
Yes, I am happy to provide my opinion regarding your study.

Best wishes
Sherry
[Quoted text hidden]

University of Portsmouth

Abdul Yaakob <abdul.yaakob@port.ac.uk>

Your Opinion is Important

adam \adli <shamsul.adlin@gmail.com> 15 December 2016 at 11:17
To: Abdul Yaakob <abdul.yaakob@port.ac.uk>

Hi Abdul,

It is sound good for me. Absolutely, I agree to provide information and good luck in your study.

Kind Regards
Sham

On Sun, Dec 11, 2016 at 12:22 PM, Abdul Yaakob <abdul.yaakob@port.ac.uk> wrote:
[Quoted text hidden]

Abdul Yaakob <abdul.yaakob@port.ac.uk>

Your Opinion is Important

azmi.saaban@yahoo.com <azmi.saaban@yahoo.com> 15 December 2016 at 12:18
Reply-To: azmi.saaban@yahoo.com
To: Abdul Yaakob <abdul.yaakob@port.ac.uk>

Dear Mr. Yaakob,

It is my pleasure to accept your invitation to provide information in your study.

Wishes
Azmi
[Quoted text hidden]

APPENDIX 2

Appendix 2.1: University of Portsmouth, Research and Inovation News-Summer 2016

Graduate School update

MRes science success

Congratulations to two of our MRes Science students – Ryan Williams and Tom Thorp – who have recently been awarded NERC (Natural Environment Research Council) studentships to go on to complete their PhD's at Reading and Leeds respectively. Ryan will be using satellite data and models to examine climate change, and Tom will be examining aerosol transport in the atmosphere. Both Ryan and Tom are part of the Environmental Processes and Change research group within the Department of Geography. NERC funds postgraduate training that sustains the flow of top talent and skills in the UK science business so these are fantastic achievements and a huge credit to their supervisors and the excellent research culture in both the Geography department and Faculty of Science.

Our MRes courses aim to help develop our students into researchers, giving them the knowledge and skills they need to develop their research further by going on to undertake a PhD or to pursue a research career, so we are pleased to hear that our students are developing into successful early career researchers. Ryan said 'I thought the MRes Science course was great in helping to bridge the gap between undergraduate and PhD-level study. It has given me time to refine my research interests and has ultimately helped me to decide on a research project for my PhD. Furthermore, gaining a Master's degree qualification undoubtedly supported my application for a PhD studentship to fund my research – studentships are highly competitive nowadays and so it is important to be in a strong position when you apply'. Congratulations to Tom, Ryan and the supervisors and staff who help to deliver the successful MRes course.

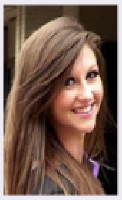
Join the new Postgraduate research Google + Community!

The postgraduate research student forum is a University-wide online platform on Google+ to enable all PGR's to stay connected, share ideas, news, research advice and issues. PGR's and supervisors can also use this platform to advertise faculty events and find out more about what is going on across the University and at the Graduate School. We will also use the forum to help set up social events and additional workshops and lunchtime seminars so we encourage you to join the group to help provide ideas for extra training sessions you might need and to keep up to date with everything going on across the university.

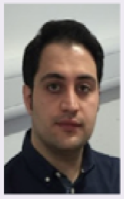
To join visit
http://tiny.cc/pgrcommunity

Student success stories

Abdul Malek Yaakob – a final year PhD student from the School of Computing has become the first and main author of a research article entitled 'FN-TOPSIS: Fuzzy Networks for Ranking Traded Equities'. This article has been recently accepted for publication in the special issue of the IEEE Transaction of Fuzzy Systems on Fuzzy Techniques in Financial Modelling and Simulation. The article disseminates research results from Abdul's PhD thesis. The idea of submitting his research results to this prestigious journal came to him after attending a Graduate School Development workshop on Writing a Journal Article, Book Chapter and Research Monograph led by his first supervisor, Dr Alexander Gegov. Dr Gegov advised and encouraged Abdul to disseminate his research results further by submitting an article to the special issue of the journal.



Danielle Norman, a second-year PhD researcher in the Centre for Studies in Literature, was recently awarded external funding from the British Association for Victorian Studies (BAVS). Founded in 2000, BAVS is a multidisciplinary organisation dedicated to the advancement and dissemination of knowledge about the Victorian period. It has over 600 members based in the UK and beyond drawn both from the academic community and the general public. Danielle submitted an Events Funding bid in November 2015 requesting support from BAVS for CSL's annual postgraduate conference (for which she was the lead-organiser), entitled 'All Things Victorian: Exploring Materiality and the Material Object'. The funding bid for the conference, which took place on March 19 2016, was generously granted by BAVS and resulted in reduced registration fees for all thirty postgraduate delegates.



Congratulations to **Farzad Arabikhan**, a final year PhD student in the School of Computing who has been awarded a grant for 2,500 euros from an established EU funding organisation – COST, for his recent research proposal on the role of Intelligent Transportation Systems in Transit. Farzad will be collaborating with Dr Ariane Dupont-Keiffer from the University Paris 1 Pantheon Sorbonne, France. Farzad has successfully now received a number of grants across the course of his studies for his research related activities (approximately £10,000 in total).

Appendix 2.2: Faculty of Technology Faculty research Conference 2016 (First Place - Best Paper Award)


LUCEM SEQUAMUR

UNIVERSITY of PORTSMOUTH

Faculty of Technology

Faculty Research Conference - 7 June 2016

Paper Prize - First place

Awarded to

Abdul Malek Yaakob

by Prof Djamel Ait-Boudaoud

Dean of Faculty

"Recognising and Rewarding Excellence in our Postgraduate Research Community"

FORM UPR16

Research Ethics Review Checklist

University of Portsmouth

Please include this completed form as an appendix to your thesis (see the Postgraduate Research Student Handbook for more information

Postgraduate Research Student (PGRS) Information		**Student ID:**	693690
PGRS Name:	Abdul Malek Yaakob		
Department:	Computing	**First Supervisor:**	Alexander Gegov
Start Date: (or progression date for Prof Doc students)	1 June 2013		

Study Mode and Route:						
	Part-time	☐	MPhil	☐	MD	☐
	Full-time	☒	PhD	☒	Professional Doctorate	☐

Title of Thesis:	Multi Criteria Decision Making Methodology for Fuzzy Rule Based Systems and Networks using TOPSIS
Thesis Word Count: (excluding ancillary data)	42350 words

If you are unsure about any of the following, please contact the local representative on your Faculty Ethics Committee for advice. Please note that it is your responsibility to follow the University's Ethics Policy and any relevant University, academic or professional guidelines in the conduct of your study

Although the Ethics Committee may have given your study a favourable opinion, the final responsibility for the ethical conduct of this work lies with the researcher(s).

UKRIO Finished Research Checklist:
(If you would like to know more about the checklist, please see your Faculty or Departmental Ethics Committee rep or see the online version of the full checklist at: http://www.ukrio.org/what-we-do/code-of-practice-for-research/)

Question		
a) Have all of your research and findings been reported accurately, honestly and within a reasonable time frame?	YES NO	☒ ☐
b) Have all contributions to knowledge been acknowledged?	YES NO	☒ ☐
c) Have you complied with all agreements relating to intellectual property, publication and authorship?	YES NO	☒ ☐
d) Has your research data been retained in a secure and accessible form and will it remain so for the required duration?	YES NO	☒ ☐
e) Does your research comply with all legal, ethical, and contractual requirements?	YES NO	☒ ☐

UPR16 – August 2015

Candidate Statement:			
I have considered the ethical dimensions of the above named research project, and have successfully obtained the necessary ethical approval(s)			
Ethical review number(s) from Faculty Ethics Committee (or from NRES/SCREC):			AY1
If you have *not* submitted your work for ethical review, and/or you have answered 'No' to one or more of questions a) to e), please explain below why this is so:			
Signed ***(PGRS)*****:**			**Date:** 25 July 2017