Mathematical Modelling and Control of a Robotic Manipulator

A THESIS PRESENTED IN PARTIAL FULFILMENT OF THE
REQUIREMENTS OF THE DEGREE OF MASTERS OF
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NIGEL YEE

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Erratum

page 25, line 4

"$T(t)=D(q(t),\ddot{q}(t))+h(q(t),\dot{q}(t))+c(q(t))$"

change to

$T(t)=D(q(t))\ddot{q}(t)+h(q(t),\dot{q}(t))+c(q(t))$

page 26, after line 12

The pseudo inertia matrix is defined by

$$J_i = \begin{bmatrix} \dfrac{-\hat{I}_{ixx}+\hat{I}_{iyy}+\hat{I}_{izz}}{2} & \hat{H}_{ixy} & \hat{H}_{ixz} & m_i\hat{s}_{ix} \\ \hat{H}_{ixy} & \dfrac{-\hat{I}_{ixx}+\hat{I}_{iyy}+\hat{I}_{izz}}{2} & \hat{H}_{iyz} & m_i\hat{s}_{iy} \\ \hat{H}_{ixz} & \hat{H}_{iyz} & \dfrac{-\hat{I}_{ixx}+\hat{I}_{iyy}+\hat{I}_{izz}}{2} & m_i\hat{s}_{iz} \\ m_i\hat{s}_{ix} & m_i\hat{s}_{iy} & m_i\hat{s}_{iz} & m \end{bmatrix}$$

where $\hat{I}_{ixx}$ *is the moment of inertia about the x axis*

$\hat{I}_{iyy}$ *is the moment of inertia about the y axis*

$\hat{I}_{izz}$ *is the moment of inertia about the z axis*

$\hat{H}_{iyz}$ *is the product of inertia*

m_i *is the mass of link i*

$\hat{s}_{ix}$ *is the centre of mass of link i*

page 31, line 4

"$$\begin{bmatrix} \tau_i \\ \tau_i \\ \tau_i \end{bmatrix} = \begin{bmatrix} & A & \end{bmatrix} \begin{bmatrix} \ddot{\vartheta}_i \\ \ddot{\vartheta}_i \\ \ddot{\vartheta}_i \end{bmatrix} + \begin{bmatrix} & B & \end{bmatrix} \begin{bmatrix} \vartheta_{k,m} \\ \vartheta_{k,m} \\ \vartheta_{k,m} \end{bmatrix} + \begin{bmatrix} C \end{bmatrix} \quad i=1,2,\ldots,n$$

Where *A, B* and *C* were evaluated"

change to

$$\begin{bmatrix} \tau_i \\ \tau_i \\ \tau_i \end{bmatrix} = \begin{bmatrix} & D & \end{bmatrix} \begin{bmatrix} \ddot{\vartheta}_i \\ \ddot{\vartheta}_i \\ \ddot{\vartheta}_i \end{bmatrix} + \begin{bmatrix} & h & \end{bmatrix} \begin{bmatrix} \vartheta_{k,m} \\ \vartheta_{k,m} \\ \vartheta_{k,m} \end{bmatrix} + \begin{bmatrix} c \end{bmatrix} \quad i=1,2,\ldots,n$$

Where *D, h* and *c* were evaluated

page 37, line 12

"dI_i is an area change"

change to

dI_i is the change in the inner area radius.

page 42, line 14

"sin a"

change to

$\sin \alpha$

page 143, line 7

"A novel pneumatic actuator which provides a rotary action was developed and tested. A model was then developed for this."

change to

A novel pneumatic actuator which provides a rotary action was designed and a model developed for this.

Abstract

Control system engineering strives to alter a systems performance to suit the objectives of the user. This requires pre-requisite knowledge of the system behaviour. This is often in the form of mathematical models of the system. These models can then be used to simulate the system and obtain a sound understanding of the systems operation, these can then be used in controller design. Every real world physical system has its own unique characteristics. These must be modelled to develop a simulation of the system. The uniqueness of a real world system necessitates the use of experimental practices and procedures to obtain information about the system. This information is then used to form models representing the system. A simulation of the system can then be based on these models. In this project a robotic system comprising of a link structure, a pneumatic driving system and a valve regulating system, is investigated. Mathematical models describing each component of the robotic system are investigated.

Mathematical models describing the dynamic interactions of the link structure are developed and implemented in a fashion to facilitate control of the robot mechanism. The equations are in an explicit form which do not require the use of a numerical method for development of state space equations used in controller development.

The pneumatic muscle used as the desired actuator for the robot structure is analysed. Analytical models obtained from the available literature are examined and new models are developed to describe the characteristics of the pneumatic muscle.

A proto-type valve specially developed for supplying air to the pneumatic muscle is investigated. Experiments are conducted on this valve to characterise the valves behaviour. A model of the valves behaviour is then developed. A selection of controllers are then applied to the valve pneumatic muscle system.

The investigation of alternative actuation systems proposes the a new rotary pneumatic muscle design. Analytical models for the rotary pneumatic muscle are developed, a prototype is constructed as part of a feasibility study.

Acknowledgements

I would like to thank my supervisor Saied Nahavandi for his help and encouragement and Bob Chaplin my second supervisor for his contribution and technical support. I would like to thank Heather North for the invaluable time she spent editing and reviewing my work, Clive Marsh for organising my seminar as part of the Masters course. Thanks to Professor Don Barnes for his help in mathematical modelling. I would like to thank all the post-graduate students in the Department of the Production Technology for their support and encouragement and a special thanks to my grandmother.

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1. Preliminaries

1.1 Objectives

The control aspect of this research is aimed at getting the hand to manipulate and grasp objects. This is not trivial and requires the control of the system components through a series of steps.

- The system must select a grasp type to be used to grasp and/or manipulate the object.
- It must then select points on the object to which the end effectors are moved in order to form the grasp.
- A set of trajectories must be selected for each component in the hand system, which will allow the end effector to move to its desired location.
- The end effectors must then move to these locations in a co-ordinated manner.

Each of these steps are complex.

The method's used to achieve these steps must take into account the inherent properties of the system.

This industrial operating environment is dynamic and may contain other obstacles which operate within the same work-space as the end-effector. This is further complicated if these obstacles move in an unpredictable way.

The object that is to be grasped will contain surfaces. These are suspected to be rigid and will be a volume in the work space where the manipulator cannot pass through but must be navigated around.

The dynamics of the end-effector will change with changes in inertia of the object being grasped.

When the manipulator moves, the individual links will have accelerations and torque's; these will change the torque's applied to the other links in the manipulator. This is known as a coupled system [26].

The manipulator must be modelled and designed to follow a set trajectory robustly enough to cope with this dynamic operating environment.

1.2 Introduction

This report is a discussion of the theoretical aspects involved in the design of a robotic manipulator. Chapter one contains an introduction to the problem with a description of the system and a the list of steps involved in solving the control problem. Chapter two describes mathematical modelling techniques used to model link structures. Chapter three outlines types of actuation systems available and then describes the development of a novel pneumatic actuator. Chapter four is on the modelling of the braided pneumatic muscle. Chapter five investigates the characteristics of the air supply system delivering the air to the pneumatic muscle. Different control strategies are investigated for the muscle, valve control problem and finally Chapter six outlines conclusions and future work to be undertaken.

1.2.1 Description of the Robot Structure

The robotic hand design under consideration in this thesis is a system incorporating two fingers and two thumbs. Each finger will comprise three links with a total of four degrees of freedom. There will be two degrees of freedom about the base of the system (link number 1) with one degree of freedom in each of the other links. The thumb is similar with four degrees of freedom associated with a serial three link structure. This incorporates two degrees of freedom about the base link with one degree of freedom in each of the other links as shown in Figure 1-1.

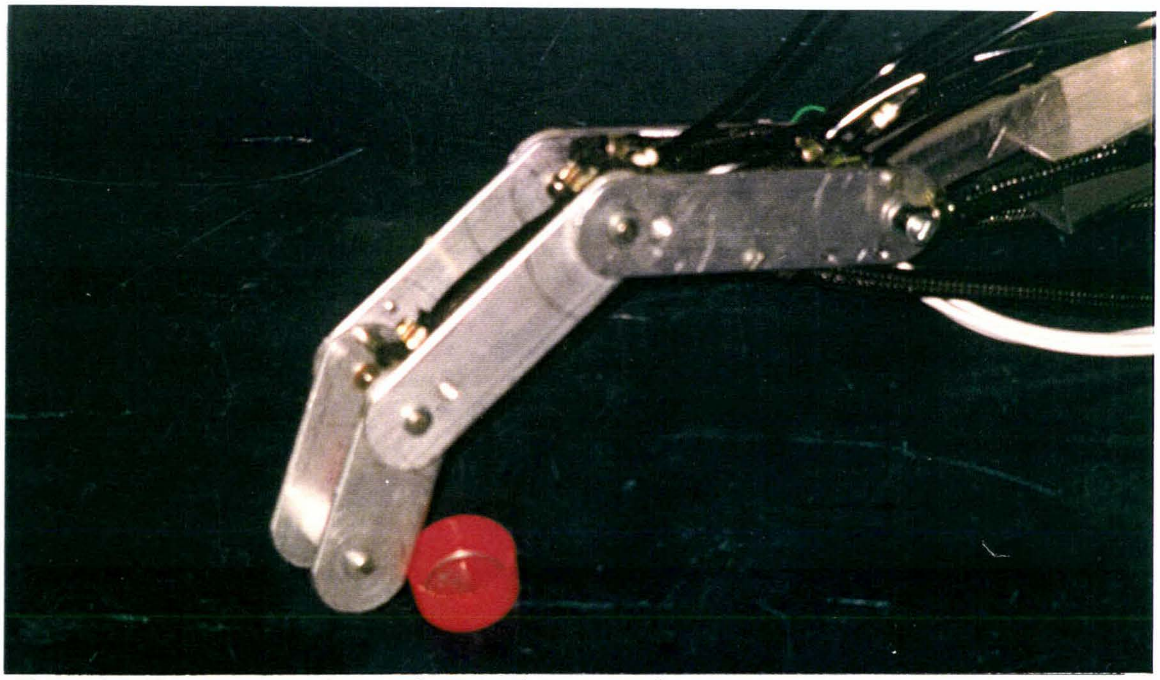

Figure 1-1 diagrammatic picture of a three link mechanism.

1.3 Background

The mechanical development of an intelligent robotic end effector at Massey University, initiated more research into robotic control [1]. An investigation was undertaken to source suitable controllers. It was quickly identified that a variety of control techniques have been applied to similar problems, many of these controllers require models to make them operational and to evaluate their performance [2]. This meant that it was necessary to model the robotic end effector in order to design the controller. The investigation then focused on the development of models for a controller for the robot end effector.

This was not a straight-forward task, involving the identification of existing models for the robotic end effector and adaptation of these to the particular robotic end effector under-development.

A design decision was made to use a new pneumatic technology for the actuation mechanism known as *The Pneumatic Muscle* [54]. Models describing this new technology are still in development. This necessitated the evaluation of existing models, to identify how well they applied to these Pneumatic Muscle's and the development of further models which better described the behaviour of the Pneumatic Muscle.

Models of the static and dynamic characteristics of the link structures were developed. These were obtained from available literature and applied to the design specifications of the structure being developed.

A valve was also designed to supply air to the pneumatic muscle. As the valve was specifically designed for the air supply application, no data was available on the valve characteristics for a model. Data was thus collected from the valve and a model of its characteristics was formed. This concluded the modelling of the robotic end effector, resulting in models of the three sub-systems of the robotic end effector;

- The model of the robotic structure.
- The model of the pneumatic muscle providing the actuating force for the link mechanism.
- The model of the valve providing the air to drive the pneumatic muscle.

Whilst looking at the existing actuation technology, an investigations into alternative actuation systems was undertaken. This resulted in the development of a novel pneumatic muscle design. This new muscle design provides a rotary action. An analytical model describing its behaviour was produced.

1.4 Background and Literature Review of the Mathematical Modelling of a Robotic Hand

1.4.1 Automatic Control

Automatic control has played a vital role in the advancement of engineering and science. In addition to its extreme importance in space-vehicle systems, missile-guidance systems, aircraft-autopiloting systems, robotic systems and the like, automatic control has become an integral part of modern manufacturing and industrial processes [3]. For example, automatic control is essential in the numerical control of machine tools in the manufacturing industries. It is also essential in such industrial operations as controlling pressure, temperature, humidity, viscosity, and flow in process industries [4].

Since advances in the theory and practice of automatic control provide the means for attaining optimal performance of dynamic systems, improving productivity of many routine repetitive manual operations, engineers and scientists must have a good understanding of this field.

1.4.2 Historical Review

The first significant work in automation and automatic control was James Watt's centrifugal governor for the speed of a steam engine in the eighteenth century. Other significant works in the early stages of development of control theory were due to Minorsky, Hazen, and Nyquist, amongst many others. In 1922 Minorsky worked on automatic controllers for steering ships and showed how stability could be determined from the differential equations describing the system. In 1932 Nyquist developed a relatively simple procedure for determining the stability of closed loop systems on the basis of open-loop response to steady-state sinusoidal inputs. In 1934 Hazen, who introduced the term *servomechanisms* for position control systems, discussed the design of relay servomechanisms capable of closely following a changing input.

During the decade of the 1940s, frequency-response methods made it possible for engineers to design a linear closed-loop control system that satisfied performance requirements. From the end of the 1940s to the early 1950s, the root locus method due to Evans was fully developed.

The frequency-response and root-locus methods, which are the core of classical control theory, lead to systems that are stable and satisfy a set of more of less arbitrary performance requirements. Such systems are, in general, acceptable but not optimal in any meaningful sense. Since the late 1950s, the emphasis in control design problems has been shifted from the design of one of many systems that works, to the design of one optimal system in some meaningful sense.

As modern plants with many inputs and outputs become more and more complex, the description of a modern control system requires a large number of equations, Classical control theory, which deals only with single-input, single output systems, becomes powerless for multiple-input, multiple-output systems [5]. Since the 1960s the availability of digital computers made possible time-domain analysis of complex systems, modern control theory based on time-domain analysis and synthesis using state variables, has been developed to cope with the increased complexity of modern plants and the stringent requirements on accuracy, weight, and cost in military, space and industrial applications.

Recent developments in modern control theory include optimal control of both deterministic and stochastic systems, as well as the adaptive and learning control of complex systems. Now that digital computers have become cheaper and more compact, they are used as integral parts in these control systems. Recent applications of modern control theory include such non-engineering systems as biological, biomedical, economic, and socio-economic systems.

1.4.3 An Overview of Robotic Control Systems

Industrial robots are frequently used in industry to improve productivity and quality. The robot can handle monotonous jobs as well as complex jobs without error in operation. The robot can work in an environment intolerable to human operators. For example, it can work in extreme temperatures (both high and low), high- or low- pressure environments, under water or in space exploration [6].

The industrial robot must handle mechanical parts that may have particular shapes and weights. Hence, it must have at least an arm, a wrist and maybe a hand. It must have sufficient power to perform the task and the capability for at least limited mobility. In fact, some of today's robots are able to move freely by themselves in a limited space in a factory [7].

The industrial robot must have some sensory devices. In some more primitive robots, microswitches are installed in the arms as sensory devices. The robot first touches an object and then, through the microswitches, confirms the existence of the object in space and proceeds in the next step to grasp it.

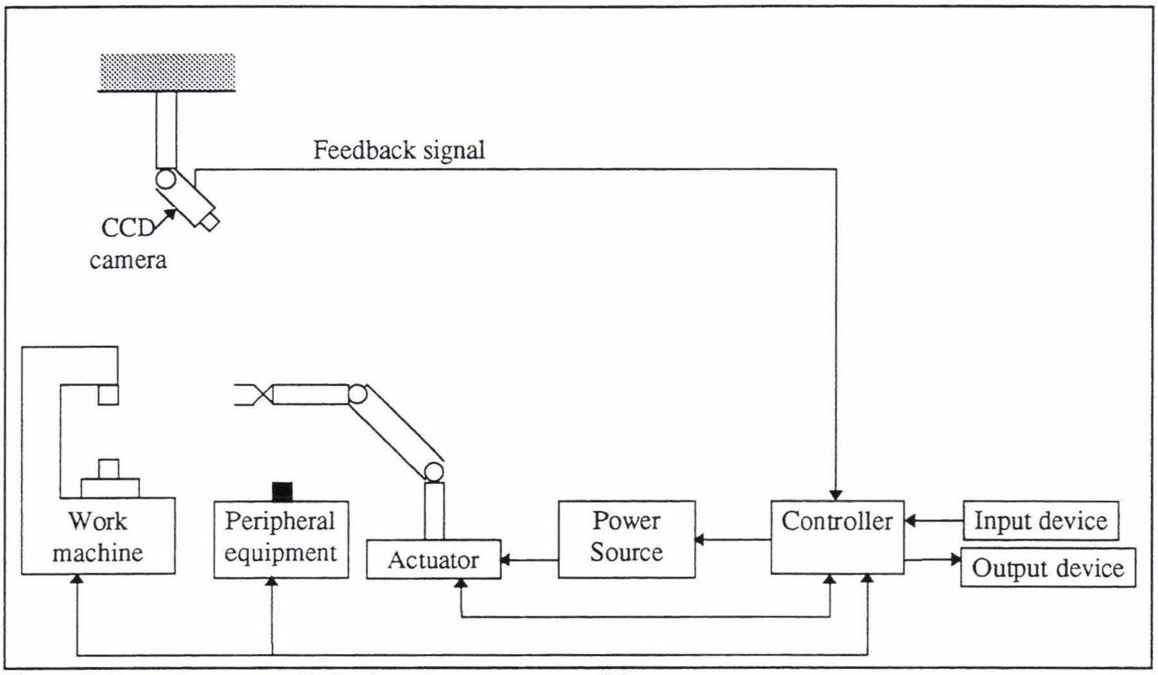

Figure 1-2 A robot controlled using a pattern recognition process.

In a high-level robot, an optical means (such as a CCD camera system) may be used to scan the background of the object. It recognises the pattern and determines the presence and orientation of the object. A computer is necessary to process signals in the pattern-recognition process (see Figure 1-2). In some applications, the computerised robot's control system recognises the presence and orientation of each mechanical part by a pattern recognition process that consists of reading the code numbers attached to it. Then the robot picks up the part and moves it to an appropriate place for assembling, and there it assembles several parts into a component. Normally a well-programmed digital computer acts as a controller.

1.4.4 The Arrangement for a One Arm Controlled Robot

Figure 1-3 shows a schematic diagram for a simplified version of a robot arm control system. The diagram shows a straight-line motion controlled version of the robot arm. A straight line motion is a 1 degree of freedom motion. The actual robot arm has 3 degrees of freedom, that is up-and-down motion, forward-and-backward motion and left-and-right motion. The wrist attached to the end of the arm also has 3 degrees of freedom and the hand has 1 degree of freedom (grasp motion). Altogether the robot arm system has 7 degrees of freedom. Additional degrees of freedom are required if the robot body must move on a plane. In general, robot hands may be interchangable parts: a different type of grasping device can be attached to the wrist to serve as a hand to handle each different type of object.

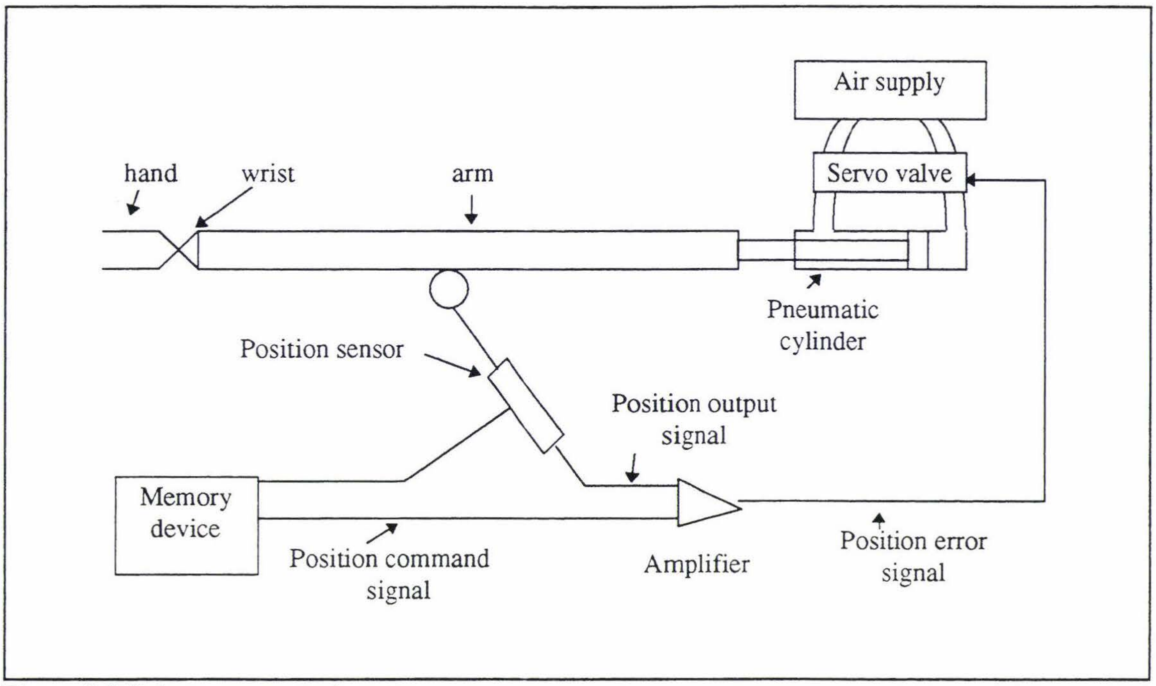

Figure 1-3 Robot arm control system.

A servo system may be used to position the arm and wrist. Since the robot arm motion frequently requires speed and power, hydraulic pressure or pneumatic pressure is used as the source of power. For medium power requirements dc motors may be used, and for small power requirements stepper motors may be used.

1.5 A Review of some Control Schemes used in Robotic Manipulators

1.5.1 Classically Based Controllers

Classical controllers are those which involve PID, PI, P or PD control.

For a classical controller with a classical control action, the controller produces signals based on the difference between the reference inputs R(s) and the output C(s). The output of the controller U(s), is determined from the error signal E(s), by one of the four schemes, shown in Equation 1-1 to Equation 1-4.

$$u(t) = k_p\, e(t)$$ **Equation 1-1**

Proportional action

$$u(t) = k_p\, e(t) + k_p\, T_d \frac{de(t)}{dt}$$ **Equation 1-2**

Proportional Derivative action

$$u(t) = k_p\, e(t) + k_p \int_0^t e(t)\, dt$$ **Equation 1-3**

Proportional Integral action

$$u(t) = k_p e(t) + k_p \int_0^t e(t)\, dt + k_p\, T_d \frac{de(t)}{dt}$$ **Equation 1-4**

Proportional Integral and Derivative action (PID)

The control action can be shown in Figure 1-4.

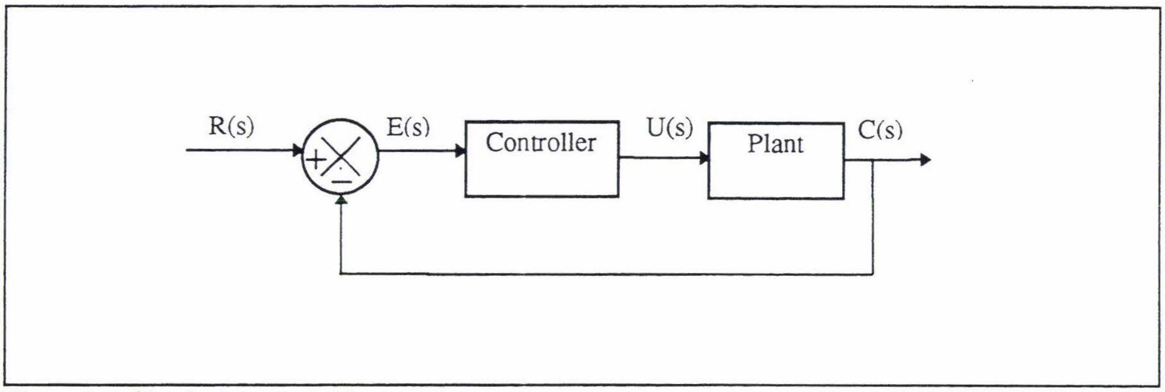

Figure 1-4 Classical control scheme.
R(s) is the reference input, U(s) is the controller output, C(s) is the output variable, E(s) is the error signal.

The conventional way of tuning a classical controller is known as the Ziegler Nichols criteria [5]. This requires the development of a system, with a proportional only controller incorporated into the system. Then the gain on the proportional controller (k_p) is increased until the system begins to exhibit sustained oscillations. The controller parameters are then based on this proportional controller value, known as the critical gain (k_{cr}), which causes the sustained oscillations.

It should be noted that this technique usually employ's simulations of the system [8,9,10].

1.6 Model Based Control

This technique provides a useful method of specifying system performance by means of a model that will produce the desired output for a given input. The model is often a mathematical model simulated on a computer. In a model based control system, the output of the model and that of the plant are compared and the difference is used to generate the control signals. The basic layout for model based control schemes is shown in Figure 1-5.

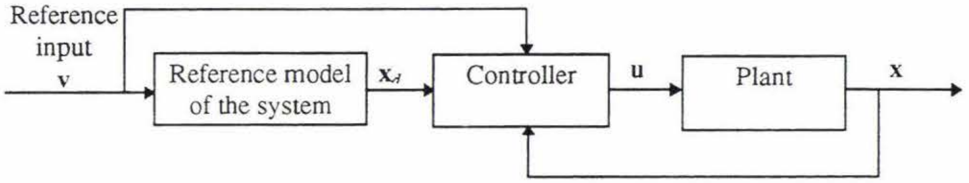

Figure 1-5 Model based control schematic.

X_d is the output of the model, u is the output from the controller, x is the output from the plant, v is the reference input.

1.6.1 Pole Placement Controller Design

Pole placement control uses a mathematical model of the plant to be controlled. Controller values are assigned according to desired pole locations [5]. Pole placement requires the feedback of all state variables as shown in Figure 1-6, therefore it becomes necessary for all state variables be available for feedback.

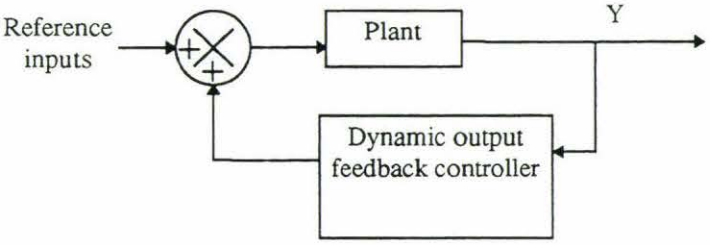

Figure 1-6 Diagram of closed loop control system using pole placement.

In the above diagram it was assumed that all the states were directly measurable. If this is not the case then an observer must be incorporated into the control diagram. A state observer estimates the state variables based on measurements of the output and control variables. The design of a state observer requires a model of how the plant operates. Using this model and outputs from the plant, estimates of the states are made without being able to physically measure them as shown in Figure 1-7.

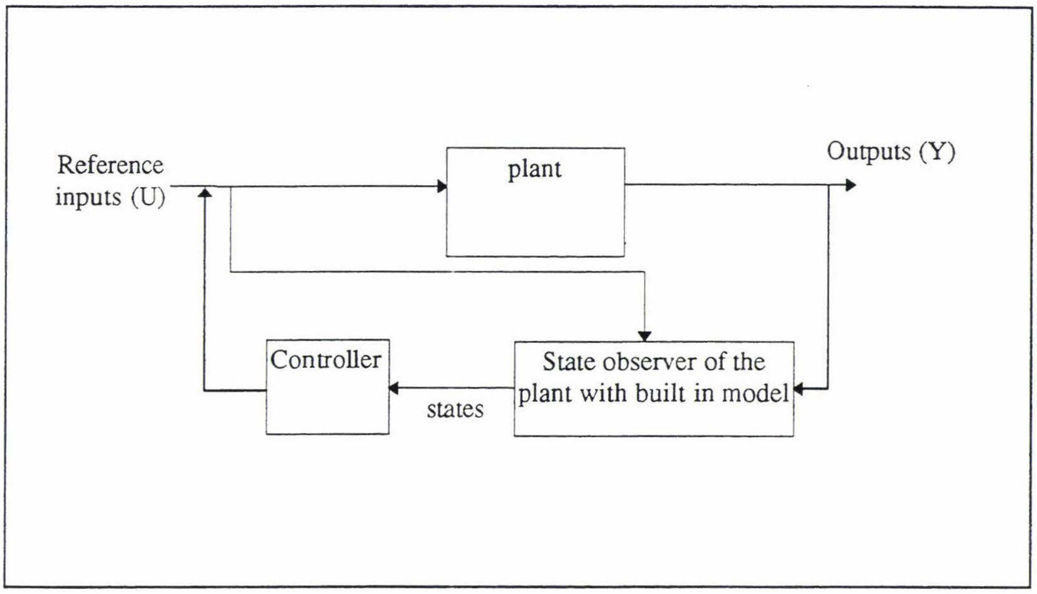

Figure 1-7 Observer schematic for a model based controller.

This is the schematic used to estimate the value of the states which are otherwise unmeasurable.

Model based control algorithms have been applied to robot applications by various researchers [11,12,13].

1.6.2 Feed Forward Control

Feed forward control is a method of dealing with the effects of disturbances in a control scheme.

If disturbances are measurable, for example the increase in air flow from a valve, then feedforward control is a useful method of concealing their effects on the system output. Feed forward control is defined in Ogata [14] as *'Control of undesirable effects of measurable disturbances by approximately compensating for them before they* *materialise'*. This is advantageous because generally in a usual feedback control system the corrective action starts only after the output has been affected.

Feed forward control can minimise the transient error, but since feed forward control is open-loop control, there are limitations to its functional accuracy. Feed forward control will not cancel the effects of unmeasurable disturbances under normal operating conditions. It is therefore necessary that a feed forward control system include a feedback loop.

Feed forward control minimises the transient error caused by measurable disturbances while feedback control compensates for any imperfections in the functioning of the feed forward control and provides corrections for measurable disturbances, Figure 1-8 shows a feed forward schematic.

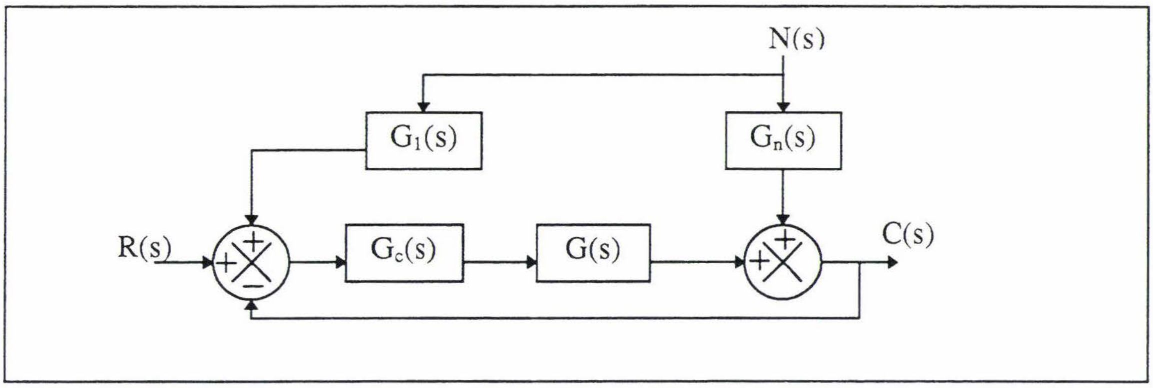

Figure 1-8 Generalised feed forward controller.

R(s) is the reference signal, N(s) is the noise term, C(s) is the output of the system.
The controller transfer function $G_c(s)$ is designed to satisfy the required system specifications in the absence of the disturbances. The disturbance feed forward transfer function $G_1(s)$ is determined such that the effects of N(s) are eliminated in the output C(s).

$G_1(s)$ is determined in general by.

$$\mathbf{G_1(s)} = \frac{-G_n(S)}{G_c(S)\,G(S)} \qquad \textbf{Equation 1-5}$$

The procedure for determining $G_c(s)$ requires the use of a system model for the design of the controller and for simulation. Likewise a model of the system is used for determining $G_1(s)$ and simulating the performance of the system with the feed forward controller $G_1(s)$, incorporated into the system. The feed forward technique has been utilised by numerous researchers [15,16].

1.6.3 Adaptive Position Control

An adaptive control system is one that continuously and automatically measures the dynamic characteristics (such as the transfer function or state equations) of the plant, compares them with the desired dynamic characteristics, and uses the difference to vary adjustable system parameters (usually controller characteristics) or to generate an actuating signal so that optimal performance can be maintained regardless of the environmental changes. Alternatively, such a system may continuously measure its own performance according to a given performance index and modify, if necessary, its own parameters so as to maintain optimal performance regardless of the environmental changes.

An adaptive controller consists of the following three functions;

- Identification of dynamic characteristics of the plant.
- Decision making based on the identification of the plant.
- Modification or actuation based on the decision made.

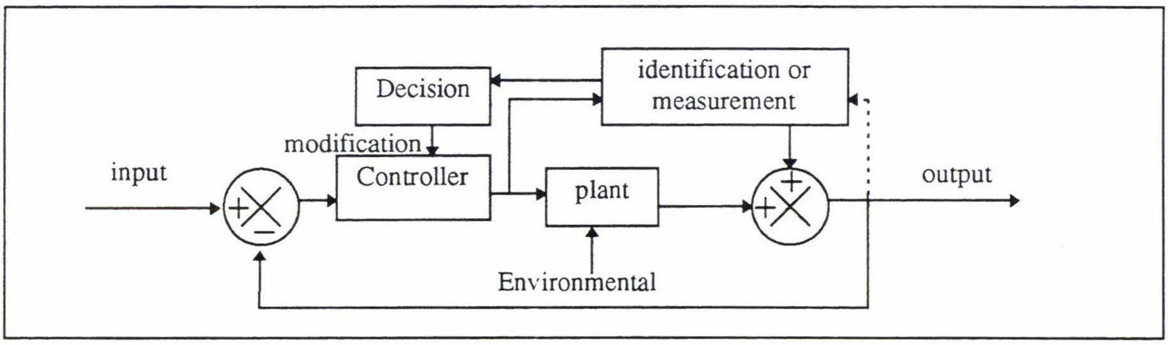

Figure 1-9 Adaptive control scheme.

A block diagram of an adaptive control system is shown in Figure 1-9. In this system the plant characteristics are identified and compared with the optimal then a decision is made on the findings as to how the actuating signal should be modified. Since the plant is identified within the system itself, adjustment of the parameters is a closed-loop operation.

Adaptive control provides an intelligent method of controlling a robot manipulator. It can successfully deal with changes in the robot system and operating environment while still maintaining a reasonable operating performance. There are many different adaptive control algorithms available.

Design of adaptive controllers requires knowledge of the plant dynamics. The general procedure for obtaining these dynamics is by the use of a model. This model is used in the design of a controller to set the initial controller parameters. This model is then implemented in the plant and the controller parameters are updated in order to obtain optimal performance of the plant [17,18,19].

1.7 Modelling Process

There is a broad spectrum of components which are used in control. These may be electromechanical, hydraulic, pneumatic, electronic and so on. In control engineering we replace such devices or components by their mathematical models, rather than dealing with hardware devices.

To obtain such adequate mathematical models of a physical component is one of the most important problems in control engineering.

A mathematical model must represent the essential aspects of a physical component. The predictions of the system behaviour based on the mathematical model must be reasonably accurate, as they will be used to design the controller.

1.7.1 Controller Design Steps

Given a system (in most cases its dynamics are unalterable), we must first choose appropriate actuators and sensors. Then using the previously developed mathematical models, we design a controller such that the closed-loop system will satisfy the given specifications. The controller designed is the solution to the mathematical version of the design problem.

After a mathematical design has been completed, the control engineer simulates the model on a computer to test the behaviour of the resulting system in its response to various signals and disturbances. The system is then modified and analysed until satisfactory responses are obtained. This process of design and analysis is repeated until a satisfactory system is obtained. Then a prototype physical system can be constructed [5].

Note that this process of constructing a prototype is the reverse process to modelling. The prototype is a physical system that represents the mathematical model with reasonable accuracy. Once the prototype has been built, the engineer tests it to see whether or not it is satisfactory. If it is, the design is complete. If not, the prototype must be modified and tested. This process continues until the prototype is completely satisfactory.

1.8 Development of Mathematical Models of the System

Mathematical modelling is used in the development of control systems to;

1. Simulate the system and then analyse the performance of the control system investigated.
2. Obtain models which are incorporation directly into certain control system schematics that is, model based control.

The modelling process used for a model based control in robot manipulators as suggested by An, Atkinson et al [20] is;

- Motor models for joint torque control.
- Kinematic models of link lengths and of locations of joints and axes.
- Inertial models of mass, centre of mass, and moment of inertia for loads and links.

This author's strategy also cites the need for separate modelling of motor characteristics, kinematic parameters and inertial parameters.

Motion control consists of ;

- Obtaining dynamic models of the manipulator.
- Using these models to determine control laws or strategies to achieve the desired system response and performance [21].

1.9 Overview of Models Describing the Robotic System

The actual modelling of the robotic mechanism will contains three parts. These will be models which describe the main parts of the physical system. These are;

1. The robotic structure must be modelled. The robotic structure is that which moves and carries out the operations of the robot.
2. The actuation mechanism must be modelled. This is the part of the system which supplies the force to the structure. As the pneumatic muscle was chosen as the actuation mechanism this must be modelled.

3. The power supply to the actuation mechanism must be modelled. As a pneumatic mechanism was chosen, and this employed valves to regulate the actuation force to the structure the valves must be modelled.

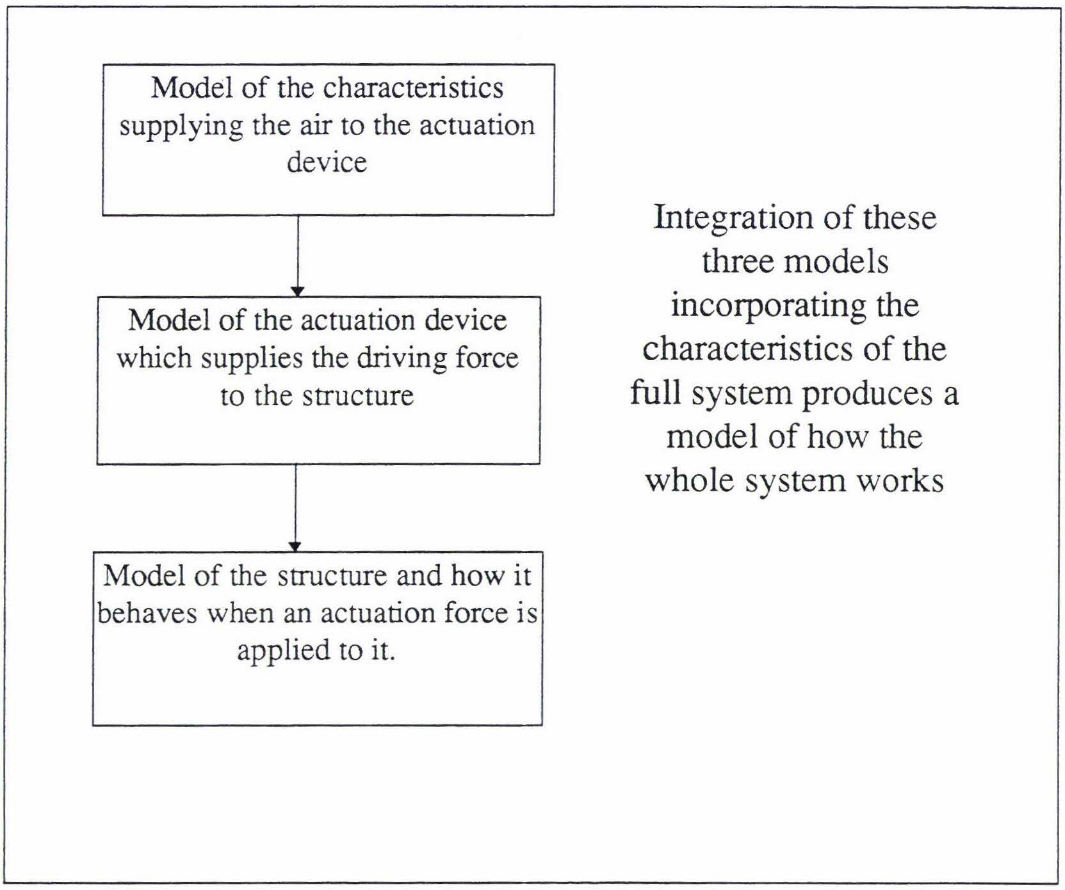

Figure 1-10 Overview of system modelling.

Once the full system has been modelled, the system can then be simulated and different control strategies investigated.