LJMU Research Online

Maliene, V, Dixon-Gough, R and Malys, N

Dispersion of relative importance values contributes to the ranking uncertainty: sensitivity analysis of Multiple Criteria Decision-Making methods

http://researchonline.ljmu.ac.uk/id/eprint/8322/

Article

Dispersion of relative importance values contributes to the ranking uncertainty: sensitivity analysis of Multiple Criteria Decision-Making methods

Vida Maliene[1], Robert Dixon-Gough[2] and Naglis Malys[3,*]

[1]*Department of the Built Environment, The Built Environment and Sustainable Technologies Research Institute, Faculty of Engineering and Technology, Liverpool John Moores University, Byrom Street, Liverpool L3 3AF, United Kingdom*

[2]*Faculty of Environmental Engineering and Land Surveying, University of Agriculture in Kraków, 253c Balicka Street, Kraków 30-149, Poland*

[3]*BBSRC/EPSRC Synthetic Biology Research Centre (SBRC), School of Life Sciences, Centre for Biomolecular Sciences, University Park, University of Nottingham, Nottingham, NG7 2RD, United Kingdom*

**Corresponding author: Email: naglis.malys@nottingham.ac.uk*

Abstract

Multiple Criteria Decision-Making (MCDM) methods are widely used in research and industrial applications. These methods rely heavily on expert perceptions and are often sensitive to the assumptions made. The reliability and robustness of MCDM analysis can be further tested and verified by a computer simulation and sensitivity analysis. In order to address this, five different MCDM approaches, including Weighted Sum Model (WSM), Weighted Product Model (WPM), revised Analytic Hierarchy Process (rAHP), Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) and COmplex PRoportional ASsessment (COPRAS) are explored in the paper. Real data of the case study for assessing housing affordability are used for testing the robustness of alternative ranking and finding the most sensitive criteria to the change of criterion weight. We identify the most critical criteria for any and best ranking alternatives. The paper highlights the significance of sensitivity analysis in assessing the robustness and reliability of MCDM outcomes. Furthermore, randomly generated and model-based data sets are used to establish relationship between the dispersion of relative importance values of alternatives and ranking uncertainty. Our findings demonstrate that the dispersion of relative importance values of alternatives correlate with the Euclidian distances of aggregated values. We conclude that the dispersion of relative importance values contributes directly to the ranking uncertainty and can be used as a measure for identifying critical criteria.

Keywords: multiple criteria analysis, sensitivity analysis, robustness, data dispersion, housing affordability

1. Introduction

Decision-making determines the success and failure in many areas of human activity. It becomes a difficult task in the situation that requires handling large amounts of data and information. In the light of this problem, the Multi-Criteria Decision Analysis (MCDA) has widely emerged as an important domain of operations research aimed at assisting with the decision-making. Multiple Criteria Decision-Making (MCDM) methods have found their use in the evaluation of alternatives, involving many and often conflicting decision criteria, with the aim of establishing an optimal or best suited choice by ranking presented alternatives [1, 2].

The MCDA has also been extended to estimate a numerical solution, e.g. the prediction of a value [3, 4] and implemented to facilitate decisions in business, management, engineering and industry [5-8]. MCDM methods were identified as particularly suitable for decision-making in fields such as the urban regeneration [9], urban and housing market sustainability [10, 11], sustainable housing affordability [12], real estate valuation [3, 13], wind energy [14], waste and water resource management [15, 16], building site planning and materials [17], inventory classification [18], mining industry [19] and many others. For this purpose, a great variety of MCDM methodologies has been developed in last few decades [20-22]. It has also been recognised that none of the methods can be considered as the "best" or "most suitable" for a single or diverse problems and applications [20, 23].

MCDM problems can be divided into two categories, discrete and continuous [21]. The later usually involves more than one objective function, which are sought to be optimised simultaneously often using evolutionary algorithms and is named Multi-Objective Optimization (MOO) [24, 25]. Whereas, the former described as a multi-attribute discrete option often consists of a modest number of alternatives, often is aimed at achieving one objective and is termed Multi-Attribute Decision Making (MADM) [21, 26].

The key stages of MCDM methodologies include: 1) establishing the relevant criteria and alternatives; 2) determining the numerical values (weights) concerning the relative importance of the criteria; 3) determining the impacts of the alternative option on criteria; and 4) processing the numerical values in determining a ranking of each alternative [1].

The determination of the relative importance of different criteria is a complex problem and often becomes a very influential step in MCDA. The information for determining the weights of the relative importance for different criteria is difficult to obtain for the definition of weights itself and the precise input data are rarely if ever available [27]. Often, experts are employed in addressing this issue and to help determine the required parameters. Yet, the professional expert opinion and perception of the criteria importance can vary depending on the sector of industry and other similar factors [28]. Ambiguities in the input data cause variability (uncertainty) and imprecision of the results (output) [29]. It has been shown that different methods, if applied to the same problem, can return diverse results [30-36], suggesting that the choice of method may also contribute to the inconsistency of the output.

Therefore, depending on the MCDM method, the importance of the criteria can have a quite varied influence on the output of the analysis.

Whereas the usefulness of any model depends on the precision and reliability of its results [29], it is highly desirable to develop MCDM methods, which are less sensitive to the influence of the subjective assumptions made through the determination of the relative importance of the criteria, or to build strategies helping to assess the sensitivity of the model and the uncertainty of the result. To assist with this, the use of the sensitivity analysis has become widespread in many areas of sciences and is now widely acknowledged as a critical step in verifying the reliability and accuracy of the methodology. The sensitivity analysis is performed with a number of objectives: 1) to establish parameters, which require additional investigation to improve the knowledge base and thereby to reduce the result (output) variability (uncertainty); 2) to establish parameters that are insignificant and can be disregarded in the final model; 3) to determine data (inputs) that contribute most to the output uncertainty; 4) to identify parameters, which have most effect on the output; and 5) when the model is in operation, to calculate what the effect has on the change of a given input parameter on the output [37]. To date, a limited amount of research has been dedicated to the sensitivity analysis for MCDM models mostly reported in 1980s and 1990s [38-42]. Previous studies mainly explore the dependency of output on the input parameters by helping to identify criteria that are most sensitive to the change of weight and have significant impact on the MCDM outcomes. The statistics of input parameters has not been considered so far.

This paper focuses on five commonly used MCDM models, the Weighted Sum Model (WSM), the Weighted Product Model (WPM), the revised Analytic Hierarchy Process (rAHP), Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS), and Complex Proportional Assessment (COPRAS) [1, 30, 43-45]. By building on previous research [12, 46] and using real data from a case study on the assessment of housing affordability, the sensitivity analysis is performed identifying tolerable change of criteria weights and how change of criteria weights contribute to the ranking output of alternatives. The robustness and reliability of MCDM analysis results is evaluated. Moreover, the correlation between the dispersion of relative importance values of alternatives and either sensitivity of criteria, dissimilarity of aggregated values, or dissimilarity of ranking is investigated.

2. Materials and methods

2. 1. Data collection

The criteria were identified by conducting interviews and literature review as described previously [47]. A total of 18 decision criteria were utilized in the assessment of sustainable housing affordability, as an empirical case study. A questionnaire-centred survey was performed and criteria weights w_j were calculated as previously [12]. The criteria weights enabled to express the relative importance of the criteria (Table 1). Eleven residential housing areas (alternatives A_1 to A_{11}) in Liverpool, UK, were selected for empirical data collection the eighteen decision criteria. Openly available data from various sources as reported in [46] were used to calculate the relative importance a_{ij} for each alternative A_i in terms of criterion C_j. Obtained values are presented in Table 1.

The initial data collection process can be summarised in the following steps: 1) establishing criteria for the comprehensive assessment of the affordability of sustainable housing, which was achieved through the literature review and interviews with professionals; 2) determining criteria weights to reflect their importance, which were determined by experts; 3) selecting alternatives to be compared; and 4) computing relative importance values for each alternative.

Table 1. Initial matrix for MCDM

	Criterion$_j$	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}	C_{13}	C_{14}	C_{15}	C_{16}	C_{17}	C_{18}
	Cost (-), benefit (+)	-	-	+	+	+	-	+	+	+	+	+	+	+	-	+	+	-	-
	Weight, w_j	0.0689	0.0689	0.0634	0.0562	0.0515	0.0483	0.0586	0.0539	0.0546	0.0499	0.0545	0.0507	0.0436	0.0483	0.0602	0.0570	0.0634	0.0483
Alternative $_i$	A_1	0.922	0.833	1.000	0.667	1.000	0.556	1.000	1.000	1.000	1.000	1.000	1.000	0.667	0.655	0.922	1.000	1.000	0.226
	A_2	0.686	0.633	0.929	0.667	0.367	1.000	1.000	0.667	0.833	1.000	1.000	1.000	1.000	0.515	0.805	0.882	1.000	1.000
	A_3	0.961	1.000	0.286	0.333	0.933	0.289	1.000	0.500	1.000	0.333	1.000	1.000	0.500	0.032	0.782	0.809	1.000	0.051
	A_4	0.922	0.800	0.229	0.333	0.767	0.430	1.000	0.667	0.833	0.667	1.000	1.000	0.833	0.629	0.769	0.838	1.000	0.053
	A_5	0.961	0.933	0.586	0.333	0.900	0.304	1.000	0.833	0.833	0.667	1.000	0.833	0.833	0.086	0.883	0.779	1.000	0.032
	A_6	1.000	0.933	0.214	0.667	0.900	0.422	1.000	0.667	0.667	1.000	1.000	1.000	0.667	0.453	0.959	0.838	1.000	0.000
	A_7	0.784	0.800	0.429	0.667	0.833	0.415	0.667	0.667	0.667	0.333	1.000	1.000	0.833	0.416	1.000	0.941	1.000	0.398
	A_8	0.941	0.967	0.071	1.000	0.433	0.481	1.000	0.667	0.500	0.667	1.000	1.000	0.667	0.341	0.862	0.926	1.000	0.856
	A_9	0.706	1.000	0.786	1.000	0.367	1.000	1.000	0.833	0.833	1.000	1.000	1.000	0.833	0.279	0.810	0.971	1.000	0.960
	A_{10}	0.745	0.767	0.500	0.333	0.767	0.659	1.000	0.833	1.000	0.333	1.000	1.000	0.667	1.000	0.991	0.897	1.000	0.636
	A_{11}	0.863	0.867	0.503	0.600	0.727	0.556	0.967	0.733	0.817	0.700	1.000	0.983	0.750	0.441	0.878	0.888	1.000	0.421

*The sign (+/-) indicates that a greater/lesser criterion value satisfies sustainable housing affordability

2.2. Sensitivity analysis

The sensitivity analysis was performed by applying two approaches. In the first, the analysis was performed to quantify the level of crosstalk between criteria and ranking. The methodological principles of this sensitivity analysis approach were described in [48]. By using this approach, the objective was to determine independently the effect of each criteria on the MCDM outcomes. The weight of each criterion was independently changed by 5% (small change) or 50% (large change) by increasing or decreasing it. The remaining criteria weights were kept unchanged. Obtained specific values of relative sensitivity coefficients indicate a number of changes in alternative ranking due to the change of criterion weight (5 or 50% increase/decrease). Inverse application of this approach enabled to determine a tolerable change of criteria weights as a percentage of allowed change (increase or decrease) for each criterion, which has no effect of alternative ranking result. This approach, in combination with the computer simulation, was used to study how the alteration of criteria weights contributes to the ranking of alternatives.

In the second sensitivity analysis approach, the "most critical criterion" was defined as the criterion C_j for which the smallest relative change (in percentage), denoted as D_j, in its weight value W_j must occur to alter the existing ranking of the alternatives [38]. The sensitivity coefficient of criterion C_j, was denoted as SC_j, and was used as a measure of the sensitivity to the change of criterion weight as follows:

$$SC_j = \frac{1}{D_j}, \quad j = \overline{1, n}.$$

(1)

For simulation purposes, the uniformly distributed pseudorandom numbers were generated using MATLAB function *rand*. Since the number of simulations depends on the quantity of input parameters, the limited number of alternatives (11) and defined number (12) of random number intervals were used to reduce computational costs.

2.3. MCDM models

Since there is no single method considered to be the most appropriate for all situations of decision-making, a large number of MCDM models and their derivatives have been developed [21] and the search for the "best" method continues. However, if applied to the same problem, different MCDM methods can deliver dissimilar results. As the method that is suitable for all types of decision-making has not been yet developed [20], the paradox of the selection of a suitable MSDM method continues [1]. The identification and selection of an appropriate MCDM method is therefore not a simple task and a considerable consideration must be given to the choice of method [20, 35, 49].

The affordability of sustainable housing in eleven residential areas was ranked by applying the five most common MCDM models, Weighted Sum Model (WSM), Weighted Product Model (WPM), revised Analytic Hierarchy Process (rAHP), Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) and COmplex PRoportional Assessment (COPRAS) [26, 30, 43, 45]. These were applied to the data from a real case study and provide the initial decision-making matrix (Table 1). Using each model, the aim was to determine the relative importance of each alternative (a_{ij}) in terms to each criterion and to compute the aggregated value for each alternative (A_i^{WSM}, A_i^{WPM}, A_i^{rAHP}, A_i^{TOPSIS}, A_i^{COPRAS}) establishing the overall ranking order of alternatives. Since input data (x_{ij}) often are expressed in different units of measurement (e.g. ratio, points, percentage, price), and in order to ensure that the data are non-dimensional, the data normalisation for WSM, rAHP, and COPRAS was performed following principles suggested in previous studies [50, 51] and as follows:

$$a_{ij} = \frac{x_{ij}}{\sum_{i=1}^{m} x_{ij}}; \ j = 1,2,3, \dots, n;$$

(2)

where, a_{ij} is a value of relative importance (weight) for the ith alternative in terms of the jth criterion. For WSM, this transformation allowed to replace multidimensional criteria, which are not permitted for use with this model, with the measures that are both numerical and comparable, and expressed in the same unit [51].

2.3.1. WSM

The WSM is one of the simplest MCDM methods. This method is based on weights given by the decision-maker. Each criterion is given a non-negative weight, and the alternatives are ranked by evaluating the weighted sum of the criteria [43]. The method involves adding together criteria values for each alternative and applying the individual criteria weights, which should be expressed as non-negative values. This is best expressed as a decision matrix:

$$\begin{array}{ccccc} & C_j & C_1 & C_2 & \dots & C_n \\ A_i & & (w_1 & w_2 & \dots & w_n) \\ A_1 & & a_{11} & a_{12} & \dots & a_{1n} \\ A_2 & & a_{21} & a_{22} & \dots & a_{2n} \\ & & \dots & \dots & \dots & \dots \\ A_m & & a_{m1} & a_{m2} & \dots & a_{mn} \end{array}$$

(3)

The decision matrix represents *m* alternatives (A_1 to A_m) in rows and *n* criteria (C_1 to C_n) in columns, where a_{ij} (i = 1,…,*m*; *j* = 1,…,*n*) indicates the relative importance of alternative A_i with respect to criterion C_j.

Following this, each criterion value is multiplied by its corresponding weight. The aggregated value for each alternative is calculated using the following formula:

$$A_i^{WSM} = \sum_{j=1}^{n} a_{ij} w_j \, ; \; i = 1,2,3, \dots, m;$$

(4)

The alternative with the highest value can be established as the best solution as follows:

$$A_{optimal}^{WSM} = \max_i \sum_{j=1}^{n} a_{ij} w_j \, ; \; i = 1,2,3, \dots, m;$$

(5)

where, the matrix *A* has data entries a_{ij} corresponding to the relative importance value of the *i*th alternative in terms of the *j*th criterion. A_i^{WSM} and $A_{optimal}^{WSM}$ are the WSM aggregated values for any and optimal alternatives, respectively, w_j is the weight (relative importance) of the *j*th criterion, and *m* and *n* are the numbers of alternatives and criteria, respectively.

2.3.2. WPM

The WPM is in principle similar to the WSM, except that it performs ranking of alternatives on the basis of a multiplicative measure instead of addition [1]. Therefore, when applying WPM in decision-making, the impact of zero values for the relative importance of criteria must be treated cautiously, since they might distort the multiplicative ranking measure. Similarly to the WSM, the WPM also requires cost criteria to be transformed into benefit ones prior to normalization. The aggregated value for each alternative is calculated by using the following formula:

$$A_i^{WPM} = \prod_{j=1}^{n} a_{ij}^{w_j};\ i = 1,2,3,\dots,m;$$

(6)

Then the alternative with the highest value can be selected as the best solution:

$$A_{optimal}^{WPM} = \max_i \prod_{j=1}^{n} a_{ij}^{w_j};\ i = 1,2,3,\dots,m;$$

(7)

where, A_i^{WSM} and $A_{optimal}^{WSM}$ are the WPM aggregated values for any and optimal alternative, respectively. Since WPM eliminates any units of measure, this method is considered as a dimensionless analysis and can be used for single and multi-dimensional problems [1].

2.3.3. rAHP

The rAHP employs, in a similar manner to the WSM and WPM, a matrix formulated from relative importance values of alternatives in terms of each criterion and the use of pair-wise comparisons, both to estimate criteria weights and to compare the alternatives with regard to the decision criteria [52]. However, instead of accumulated the sum of relative importance values of alternatives ($a_{1j}, \dots, a_{ij}$) being equal to 1, proposed in the original AHP [44], in the rAHP the relative importance value (a_{ij}) of the jth criterion is divided by maximum value of a_{ij} (maxa_{ij}) of the jth criterion [53] resulting in calculation of aggregated value using following formula:

$$A_i^{rAHP} = \sum_{j=1}^{n} \frac{a_{ij}}{\max_i a_{ij}} w_j;\ i = 1,2,3,\dots,m;$$

(8)

The alternative with the highest value can be established as the best solution as follows:

$$A_{optimal}^{rAHP} = \max_i \sum_{j=1}^{n} \frac{a_{ij}}{\max_i a_{ij}} w_j;\ i = 1,2,3,\dots,m;$$

(9)

The rAHP can use both benefit and cost criteria [54].

2.3.4. TOPSIS

The TOPSIS is a model based on an aggregating function representing distances from the ideal solution [26]. TOPSIS approaches a MCDM problem by considering that the optimal alternative should have the shortest distance from the ideal solution and the greatest distance from the negative-ideal solution. The distances are computed for normalized and weighted data and are measured using the Euclidean metrics. TOPSIS can be applied to both benefit and cost criteria [55]. TOPSIS commences with the normalization of relative importance values of alternatives, which are calculated as following:

$$a_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}};\ j = 1,2,3, \dots, n;$$

(10)

where, x_{ij} and a_{ij} represent values of the ith alternative in terms of the jth criterion prior and after normalisation, respectively. In the next step, the values (v_{ij}) for weighted normalised decision matrix V are calculated:

$$v_{ij} = a_{ij} w_j;\ i = 1, \dots, m; j = 1, \dots, n;$$

(11)

In the third step, the positive ideal (best) (V^+) and negative-ideal (worst) [56] solutions are determined as follows:

$$V^+ = \{v_1^+, \dots, v_n^+\}, \qquad v_j^+ = \left\{\max_j(v_{ij})\ \text{ if } j \in J;\ \min_j(v_{ij})\ \text{ if } j \in J'\right\};$$

(12)

$$V^- = \{v_1^-, \dots, v_n^-\}, \qquad v_j^- = \left\{\min_j(v_{ij})\ \text{ if } j \in J;\ \max_j(v_{ij})\ \text{ if } j \in J'\right\};$$

(13)

where, J = {j = 1, 2, ..., N and j is associated with benefit criteria}; and J' = {j = 1, 2, ..., N and j is associated with cost/loss criteria}.

Then, separation measures (distances) (S_i^+ and S_i^-) from the positive ideal alternative (v_j^+) and negative ideal alternative (v_j^-) are calculated for each alternative using the n-dimensional Euclidean distance method as follows:

$$S_i^+ = \sqrt{\sum_{j=1}^{n} (v_j^+ - v_{ij})^2}\ ;\ i = 1, \dots, m;$$

(14)

$$S_i^- = \sqrt{\sum_{j=1}^{n} (v_j^- - v_{ij})^2}\ ;\ i = 1, \dots, m;$$

(15)

Finally, the relative closeness (aggregated value) of each alternative is calculated as A_i^{TOPSIS} and the ideal solution as $A_{optimal}^{TOPSIS}$:

$$A_i^{TOPSIS} = \frac{S_i^-}{\left(S_i^+ + S_i^-\right)};\ 0 \le A_i^{TOPSIS} \le 1;\ i = 1,2,3, \dots, m;$$

(16)

$$A_{optimal}^{TOPSIS} = \max_i A_i^{TOPSIS};\ i = 1,2,3, \dots, m;$$

(17)

It should be mentioned that TOPSIS does not consider the relative importance of the distances from the positive and negative ideal solution points [30]. The TOPSIS model uses squared terms in the evaluation of criteria and this should be emphasised. The consequence is that the most beneficial and most costly relative importance values in input data can have more of an impact on the final output, whereas average data points will have a lesser impact to the results if compared to the methods that do not utilise squared terms. Methods that utilise squared terms may not be particularly suitable where criteria values for different alternatives are similar, thus requiring further identification.

2.3.5. COPRAS

The COPRAS is, in principle, similar to the WSM. However, COPRAS allows for both benefit and cost criteria to be considered within the matrix [45].

After the normalization step as described in section 2.3, the sums of the weighted normalised criteria (S_i^+ and S_i^-) of the ith alternative are calculated as follows:S_i^-

$$S_i^+ = \sum_{j=1}^{n} a_{ij} w_j z^+;\ \ a_{ij} \geq 0;\ z^+ = 1;\ j = 1,2,3,\dots,n;$$

(18)

$$S_i^- = \sum_{j=1}^{n} a_{ij} w_j z^-;\ \ a_{ij} \leq 0\,;\ z^- = -1;\ j = 1,2,3,\dots,n;$$

(19

The aggregated value for each alternative is calculated using the following formula:

$$A_i^{COPRAS} = S_i^+ + \frac{\sum_{i=1}^{m} S_i^-}{S_i^- \sum_{i=1}^{m} \frac{1}{S_i^-}};\ i = 1,2,3,\dots,m;$$

(20)

The alternative with the highest value is established as the best solution:

$$A_{optimal}^{COPRAS} = \max_i \left(S_i^+ + \frac{\sum_{i=1}^{m} S_i^-}{S_i^- \sum_{i=1}^{m} \frac{1}{S_i^-}} \right);\ i = 1,2,3,\dots,m;$$

(21)

3. Results and discussion

3.1. Sensitivity analysis

In solving the MADM problem using MCDM models, the relative importance of the criteria (weights) is determined by decision-makers. They can employ experts to rank the criteria or to establish the importance of the criteria using other means. As a consequence, the relative importance of the criteria can be to some degree subjective, carry some level of uncertainty and, therefore, may shape the analysis of the results.

In order to investigate the sensitivity of the output to the uncertainty in the criteria weight, we used an example with parameters (data) acquired for the assessment of the affordability of sustainable housing [47]. Eighteen criteria and eleven alternatives (residential housing areas) were used to build a decision matrix as described in *Material and methods*. For MCDA, five MCDM methods such as WSM, WPM, rAHP, TOPSIS and COPRAS were applied. The expert opinion-determined values of criteria weights and the values of the alternatives were combined in the mathematical models of the MCDM methods described in subsections 2.3.1 to 2.3.5.

First, selected MCDM methods were investigated if they outputs are different with selected criteria and alternative inputs by comparing aggregated values and ranking of alternatives. Therefore, the aggregated values of alternatives (A_i) and their resulting ranking (rank) were determined (Table 2). All aggregated values and, with exception for A_1 and A_2, a majority of alternatives were ranked differently by five selected MCDM methods.

Table 2. Aggregated values (A_i) and ranking results (rank) using five different MCDM methods.

Alternative	WSM		WPM		rAHP		TOPSIS		COPRAS	
	A_i	rank	A_i	rank	A_i	rank	A_i	rank	A_i	rank
A_1	0.1042	1	0.1014	1	0.8782	1	0.5324	1	0.1032	1
A_2	0.0924	4	0.0000	11	0.7927	5	0.8632	9	0.0899	6
A_3	0.0881	8	0.0833	7	0.7612	9	0.9220	3	0.0925	4
A_4	0.0872	9	0.0842	6	0.7617	8	0.9131	8	0.0871	9
A_5	0.0964	2	0.0938	2	0.8167	2	0.7101	2	0.1006	2
A_6	0.0922	5	0.0890	4	0.7949	4	0.8342	4	0.0931	3
A_7	0.0899	7	0.0881	5	0.7748	7	0.9003	6	0.0893	8
A_8	0.0813	11	0.0724	10	0.7170	11	0.9388	10	0.0802	11
A_9	0.0933	3	0.0816	8	0.7961	3	0.7903	7	0.0919	5
A_{10}	0.0842	10	0.0747	9	0.7471	10	0.9305	11	0.0822	10
A_{11}	0.0909	6	0.0909	3	0.7840	6	0.8844	5	0.0899	7

Second, the pairwise correlation analysis between five MCDM methods was executed. Results showed that four methods, WSM, rAHP, TOPSIS and COPRAS, performed very similarly with a Pearson correlation coefficient ranging from 0.830 to 0.995 (Table 3). These findings extend and consolidate other studies, where WSM, AHP and TOPSIS [49], AHP and TOPSIS [57], and WSM and COPRAS [58] are compared. Previous research has suggested that it is important to use alternative MCDM methods in order to achieve reliable and viable ranking results. Although none of the MCDM approaches can significantly outclass other methods, the correlation analysis suggested that the COPRAS was most consistent with other models used for the assessment of sustainable housing affordability (Table 3). Whereas previous research proposed that the WSM model, which performs accurately and delivers satisfactory results for a majority of single-criteria based problems [59], can be used as a standard for evaluating MCDM methods [1], it was found to be second best according to the analysis.

Table 3. Correlation between alternative rankings computed using different MCDM methods.

	WSM	WPM	rAHP	TOPSIS	COPRAS
WSM	1.000	.201	.995	.861	.942
WPM	.201	1.000	.209	.410	.318
rAHP	.995	.209	1.000	.830	.922
TOPSIS	.861	.410	.830	1.000	.966
COPRAS	.942	.318	.922	.966	1.000

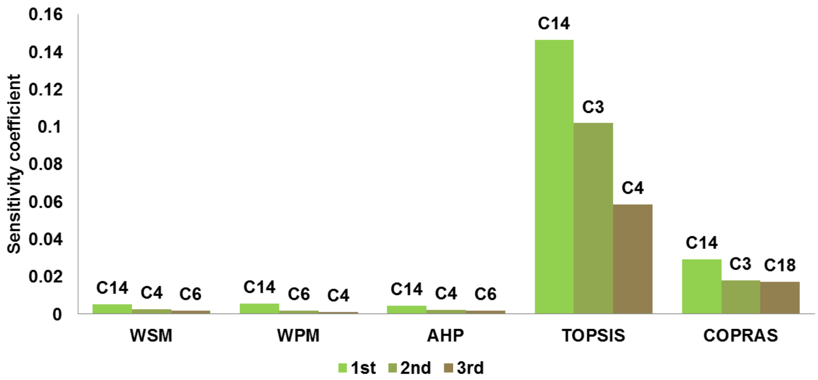
Note: similarity matrix is coloured as a heat-map that shows the level of correlation between ranking results. The most dissimilar rankings are coloured in light pink. MCDM method pairs with absolutely equal rankings have a Pearson correlation value equal to "1" and are in light blue.

Next, as described in *Materials and methods* and by following approach described in [38], we determined the most critical criterion for each MCDM model. As shown in Figure 1A, the criterion C14 had the highest sensitivity coefficient for the best alternative in all MCDM models, whereas criteria C3, C4, C6 and C18 were identified as 2nd or 3rd amongst critical criteria for the best alternative. C14 was the most critical criterion for any alternative in the case of rAHP and positioned as 2nd and 3rd in WPM, TOPSIS and COPRAS (Figure 1B), whereas C18 and C3 were the most critical criteria for any alternative using WPM, TOPSIS and COPRAS (C18) and WSM (C3). As for the best alternative, C3, C6 and C18 were amongst the criteria positioned as 2nd and 3rd. In addition criterion C10 was identified as 2nd (WPM) and 3rd (rAHP) for any alternative.

Taken together, criteria C3, C4, C6, C10, C14, and C18 were identified as the most critical criteria for both the best and any alternative. These results revealed that the sensitivity of different criteria varies significantly across five selected MCDM methods. Moreover, the when different alternatives are taken into consideration, different criteria can be most critical, which highly depends on the MCDM method applied. Therefore, to reduce computational costs and especially when multiple MCDM approaches are used, other measures outside those that are provided by sensitivity analysis should be considered.

In order to quantify the level of crosstalk between criteria and ranking, the sensitivity analysis was performed. It was aimed at establishing how the ranking of alternatives (aggregated score A_i) change due to alteration of criteria weights (C_j). Table 4 shows the distribution of relative sensitivity coefficients. The specific value of the sensitivity coefficient denotes that a 5% (arbitrary small change) or 50% (arbitrary large change) increase or decrease of the criterion weight leads to single, double or multiple changes in the ranking of alternatives.

A



B

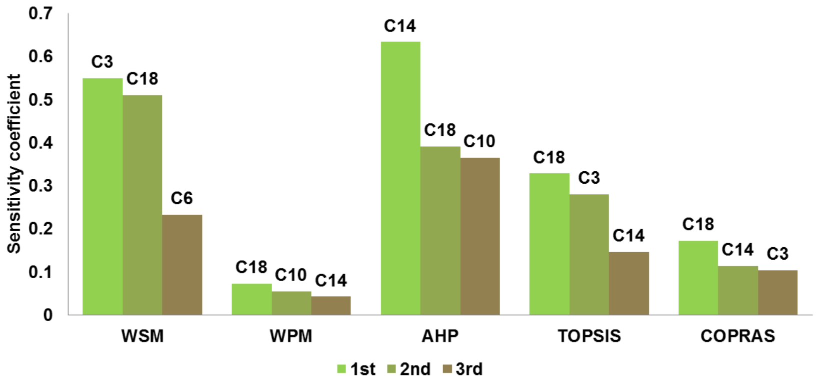


Figure 1. Most critical criteria. The bar chart compares sensitivity coefficients of the most critical criteria for best (A) and any alternatives (B) established using different MCDM models.

The robustness of the ranking output, expressed as a percentage of the tolerable change of the criterion weight, is summarised in Table 5. Results revealed that the WPM model had the lowest relative coefficients with the fewest changes in the ranking of alternatives and highest tolerance to the 5% (or 50%) alteration in criteria weights comparing to other four MCDM models. The simulated 5% change of criteria weights did not have any influence on the ranking of alternatives by using WPM and COPRAS. The MCDM output was absolutely robust to the changes of criterion C11 weight. This suggested that the exclusion of this criterion from the MCDM matrix and subsequent analysis had no effect on the ranking results and therefore can be eliminated or substituted with other criterion if required.

Table 4. Relative sensitivity coefficients calculated as a number of changes in the alternative ranking due to change of criteria weights.

Method	WSM				WPM				rAHP				TOPSIS				COPRAS			
Change of criterion weight	Decrease (%)		Increase (%)		Decrease (%)		Increase (%)		Decrease (%)		Increase (%)		Decrease (%)		Increase (%)		Decrease (%)		Increase (%)	
	5%	50%	5%	50%	5%	50%	5%	50%	5%	50%	5%	50%	5%	50%	5%	50%	5%	50%	5%	50%
C1	0	2	0	0	0	0	0	0	0	2	0	1	0	0	0	0	0	0	0	1
C2	0	1	0	1	0	0	0	0	1	1	0	3	0	0	0	0	0	1	0	1
C3	1	3	0	0	0	1	0	3	1	1	1	3	0	5	1	7	0	3	0	4
C4	0	2	0	1	0	0	0	1	0	2	0	0	0	2	0	4	0	1	0	3
C5	0	0	1	2	0	1	0	0	0	1	1	2	0	2	0	0	0	2	0	1
C6	0	0	1	2	0	1	0	0	0	1	1	2	0	2	0	2	0	3	0	2
C7	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0
C8	0	0	0	0	0	0	0	0	0	3	0	0	0	0	0	0	0	0	0	1
C9	0	1	0	0	0	0	0	0	0	1	0	2	0	0	0	0	0	0	0	1
C10	0	0	0	1	0	2	0	1	1	1	0	0	0	0	0	4	0	2	0	1
C11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
C12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
C13	0	1	0	0	0	1	0	0	1	2	0	2	0	0	0	0	0	0	0	1
C14	0	1	0	1	0	0	0	2	0	3	1	1	0	3	0	3	0	2	0	3
C15	0	0	0	1	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0
C16	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0
C17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
C18	0	1	1	2	0	3	0	0	1	2	1	1	2	8	0	5	0	4	0	3

Table 5. Tolerable change of criteria weights established through the sensitivity analysis.

Method	WSM				WPM				rAHP				TOPSIS				COPRAS			
Ranking status	no change	single change	no change	single change	no change	single change	no change	single change	no change	single change	no change	single change	no change	single change	no change	single change	no change	single change	no change	single change
Allowed change	Decrease (%)		Increase (%)		Decrease (%)		Increase (%)		Decrease (%)		Increase (%)		Decrease (%)		Increase (%)		Decrease (%)		Increase (%)	
C1	5	20	100	150	100	100	50	100	5	15	10	50	100	100	50	100	50	100	20	50
C2	5	100	20	50	50	100	50	400	1	90	10	10	100	100	50	50	20	100	20	50
C3	1	15	50	50	20	50	20	20	1	50	1	10	5	10	1	5	10	20	5	15
C4	20	20	20	50	100	100	20	50	5	15	50	50	15	20	15	15	20	50	5	20
C5	90	90	1	20	20	50	100	200	5	50	1	5	20	20	50	50	15	20	20	50
C6	50	90	1	20	20	50	100	200	5	50	1	5	10	10	20	20	10	20	15	20
C7	100	100	50	100	20	100	150	200	90	100	50	50	100	100	50	100	20	100	100	300
C8	50	90	50	150	50	100	50	150	5	10	50	150	100	100	50	50	50	100	20	100
C9	10	90	50	200	50	100	100	200	10	100	5	20	50	100	50	50	50	50	20	100
C10	50	50	20	100	15	20	20	50	1	50	50	100	50	100	10	10	10	15	10	50
C11	100	100	900	900	100	100	900	900	100	100	900	900	100	100	900	900	100	100	900	900
C12	100	100	300	400	100	100	400	400	100	100	200	200	100	100	400	900	100	100	900	900
C13	5	90	50	100	20	100	50	100	1	15	15	20	100	100	50	50	50	50	20	50
C14	20	50	20	50	50	50	20	20	10	20	1	50	5	15	5	20	5	20	5	20
C15	100	100	15	100	100	100	300	400	20	90	10	50	100	100	100	200	50	100	50	200
C16	50	100	200	300	100	100	150	150	15	20	50	150	100	100	300	400	100	100	50	150
C17	100	100	100	100	100	100	900	900	100	100	900	900	100	100	400	400	90	90	300	400
C18	20	50	1	10	10	20	50	150	1	20	1	50	1	1	5	20	5	10	15	20

The results of the sensitivity analysis including each individual criterion were summarised in Figure 2. Notably, changes of the most critical criteria for the best and any alternatives, i.e. C14, C18 and C3 (Figure 1), comprised a most important effect on the ranking of alternatives as shown in Figure 2. However, the relative sensitivity coefficients and tolerable change of criteria weights varied significantly across five MCDM methods. These results supported the notion that other than sensitivity analysis approach should be considered aiming to reduce computational costs.

Notably, criteria C14, C18 and C3 endured the highest dispersion of relative importance values of alternatives expressed as a coefficient of variation (CV). Followed analysis, as presented in Figure 3, confirmed a strong correlation between the coefficient of variation (dispersion) and number of changes in the ranking of alternatives.

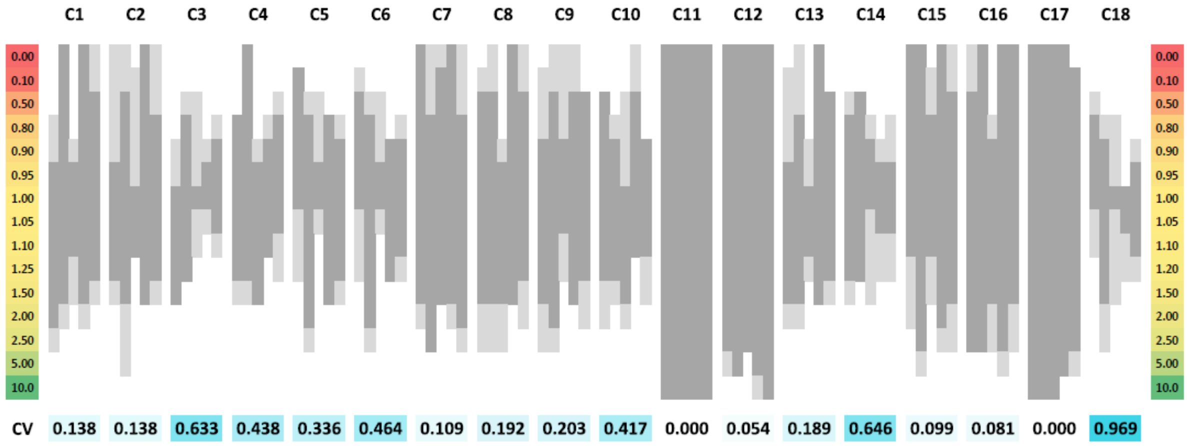


Figure 2. The sensitivity analysis of how the change of criterion weight affects the ranking of alternatives. The dark grey shading indicates the tolerable change of criteria weight, which is shown as folds of difference on the left and right panels. The light grey shading represents the range that contributes to a single change of alternatives. The abbreviations of criteria are shown on the top. The results for the five MSDM methods in each criterion panel are displayed in the following order: WSM (first from the left), WPM, rAHP, TOPSIS and COPRAS (last from the left). The coefficients of variation (dispersion) for each criterion is shown on the bottom.

3.2. Dispersion

The MCDM methods are multidimensional. They can incorporate a multiple conflicting criteria and consider all relevant aspects in a single evaluation process. Importantly, MCDM is capable of considering criteria of incommensurable units of measure (e.g. ratios, points, and percentages) and those of both benefit and cost. Due to the former, a wide dispersion in values of measurement, the calculated relative importance of the alternatives for individual criterion can span from 0 to 100 or higher. This can result in a precedent where the dispersion of values of alternatives a_{ij} can vary severely for different criteria with coefficient of variation (CV) values spanning from 0 to as high as 1 and in some extreme circumstances significantly above that value. Therefore, we investigated further the relationship between the distribution of relative importance values of alternatives (linear, polynomial and exponential), from now on denoted as *dispersion*, and aggregated values of alternatives or ranking results. For this purpose, two approaches were used.

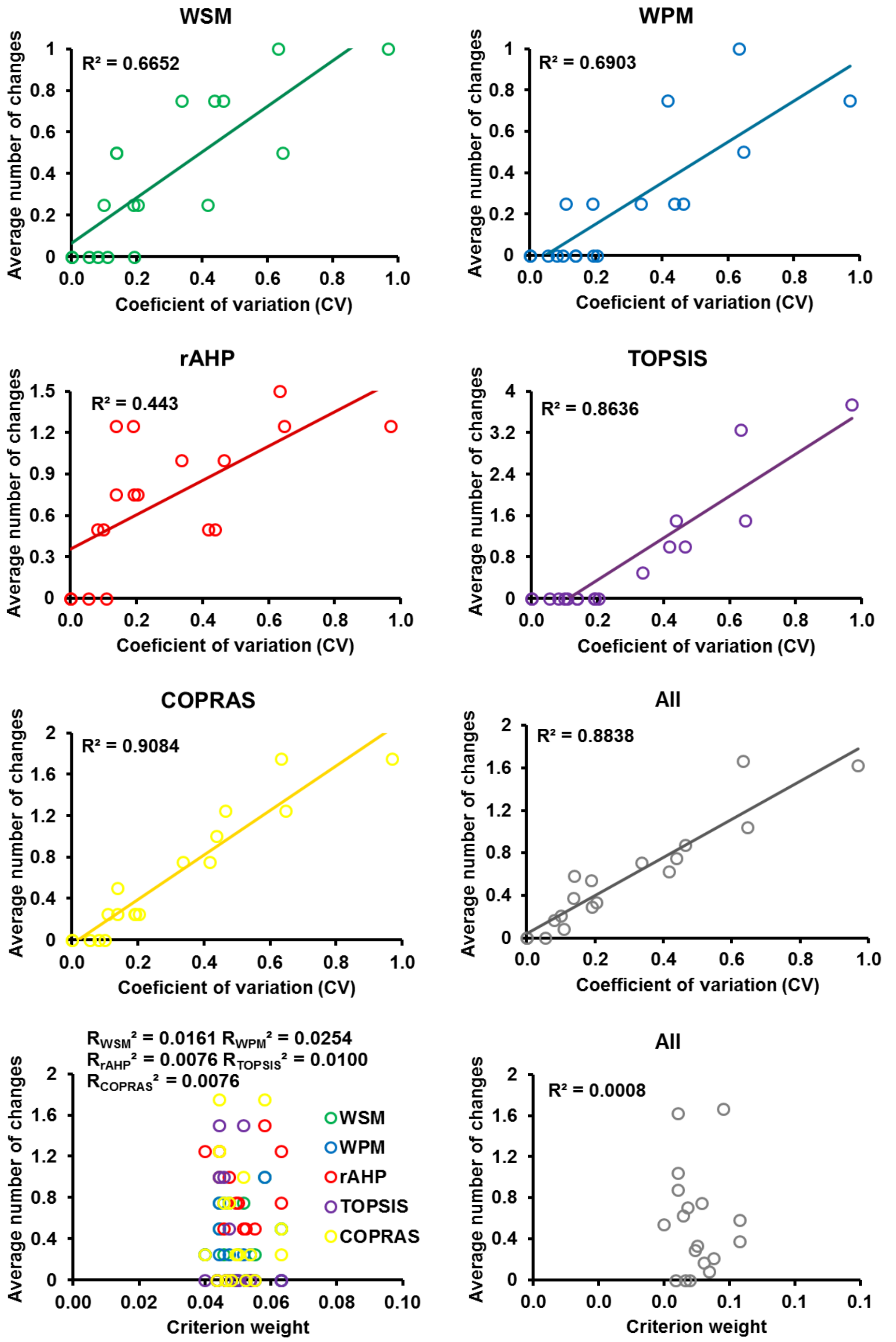


Figure 3. The correlation between the dispersion of relative importance values of alternatives (CV) and criteria sensitivity, measured as a number of changes in the alternative ranking. R^2 is coefficient of determination of the linear regression.

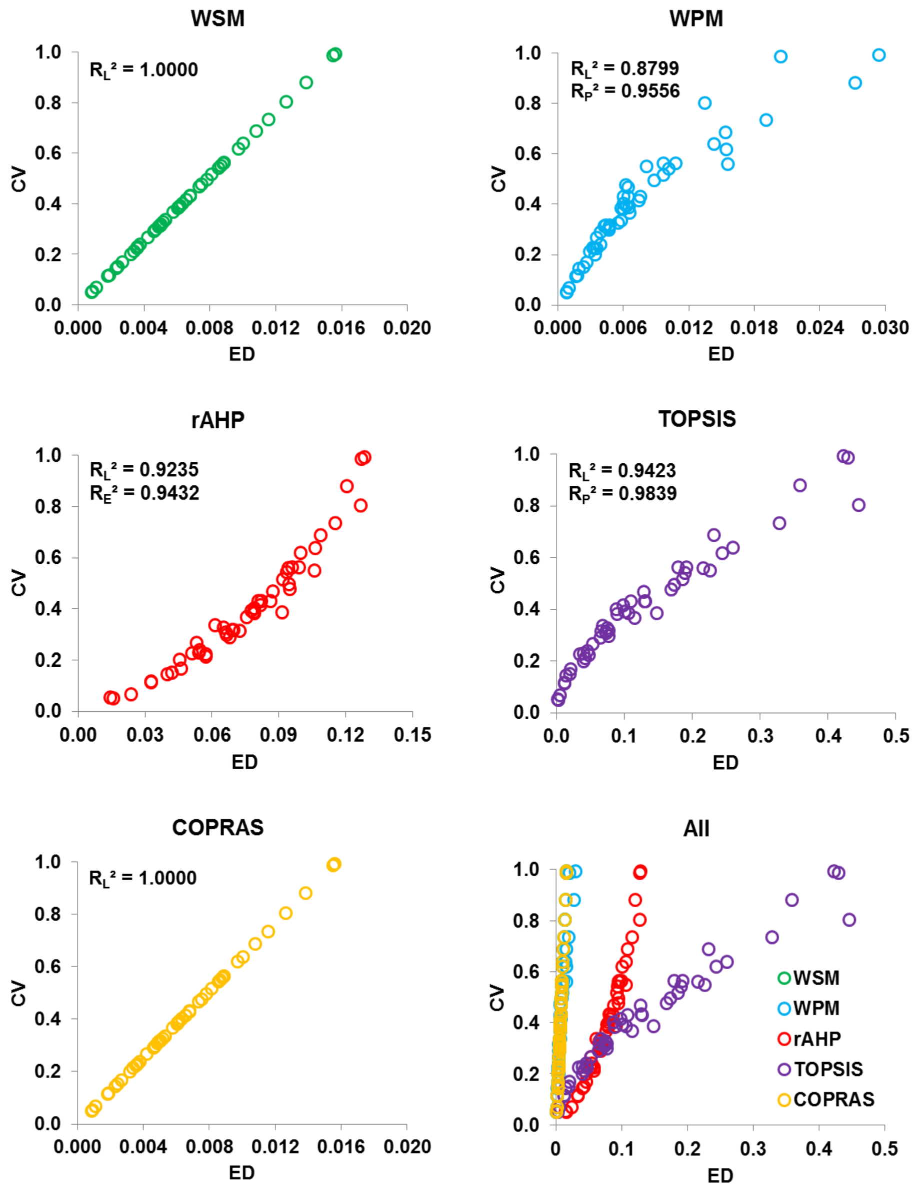


Figure 4. The correlation between the dispersion of relative importance values of alternatives (CV) and the dissimilarity of aggregated scores (Euclidian distance, ED). Random numbers were used as criterion C11 relative importance values of alternatives. R_L^2, R_P^2 and R_E^2 are coefficients of determination for linear, polynomial and exponential regressions, respectively.

Firstly, the relative importance values of alternatives for non-essential criterion C11 was consecutively substituted with randomly generated values, which were within the different dynamic ranges, computed as the ratio between the largest and smallest values, from 1 to 10^6, including 1 to 1.25, 1 to 1.5, 1 to 2, …, and 1 to 10^6. Using randomly generated relative importance values of alternatives, the simulations of MCDM were performed as described in *Material and methods*. The resulting aggregated values A_i and ranking results were subjected to a correlation analysis. Euclidian distances accounting for dissimilarity were plotted against the CV. All models demonstrated a very high level of positive correlation between the dispersion of relative importance values of alternatives for individual criterion, measured as CV, and the dissimilarity of aggregated values, calculated as Euclidian distances (Figure 4). For WSM and COPRAS, the correlation was absolute, with coefficient of determination R^2 = 1. Similarly, a significant correlation was also observed when ranking results were used instead of aggregated values (Figure 5).

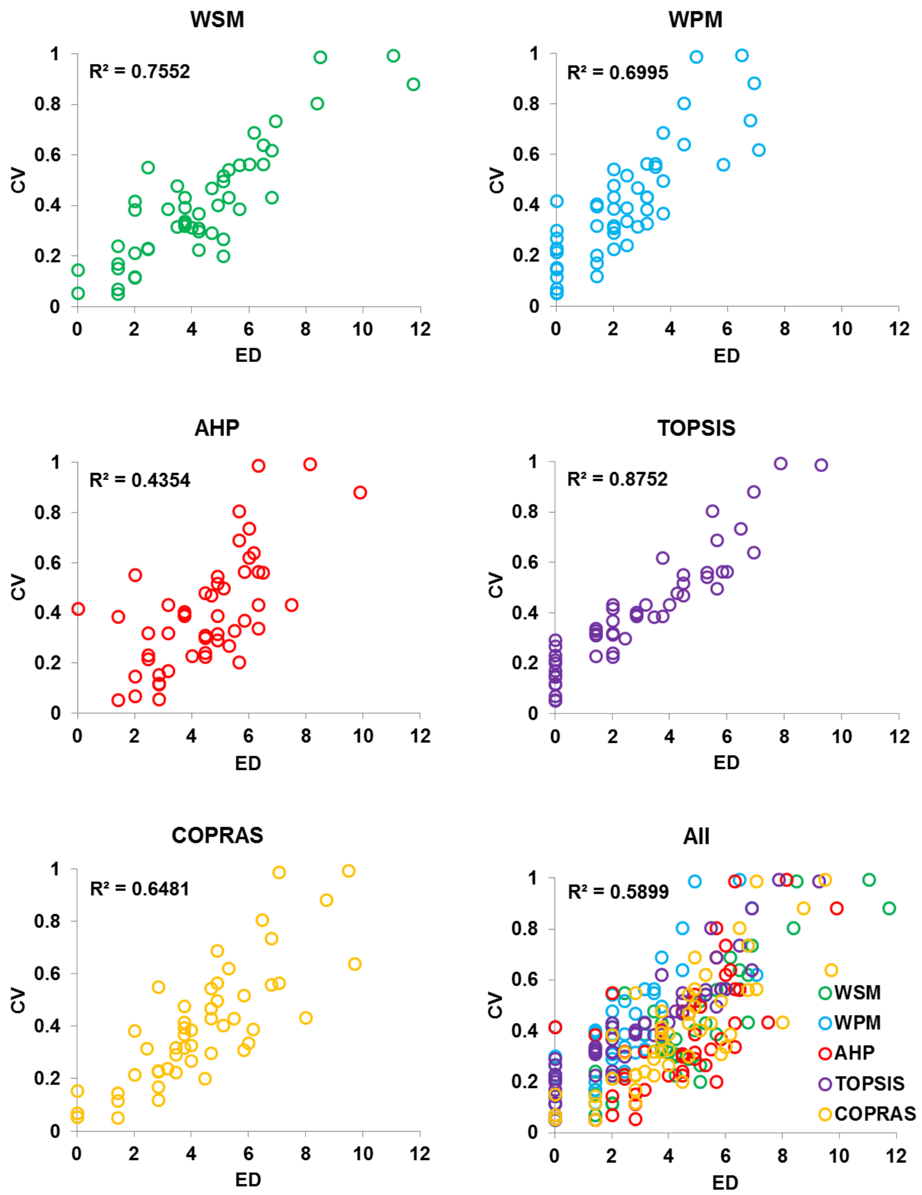


Figure 5. The correlation between the dispersion of relative importance values of alternatives (CV) and the dissimilarity of ranking (Euclidian distance, ED). Random numbers were used as criterion C11 relative importance values of alternatives. R^2 is coefficient of determination of the linear regression.

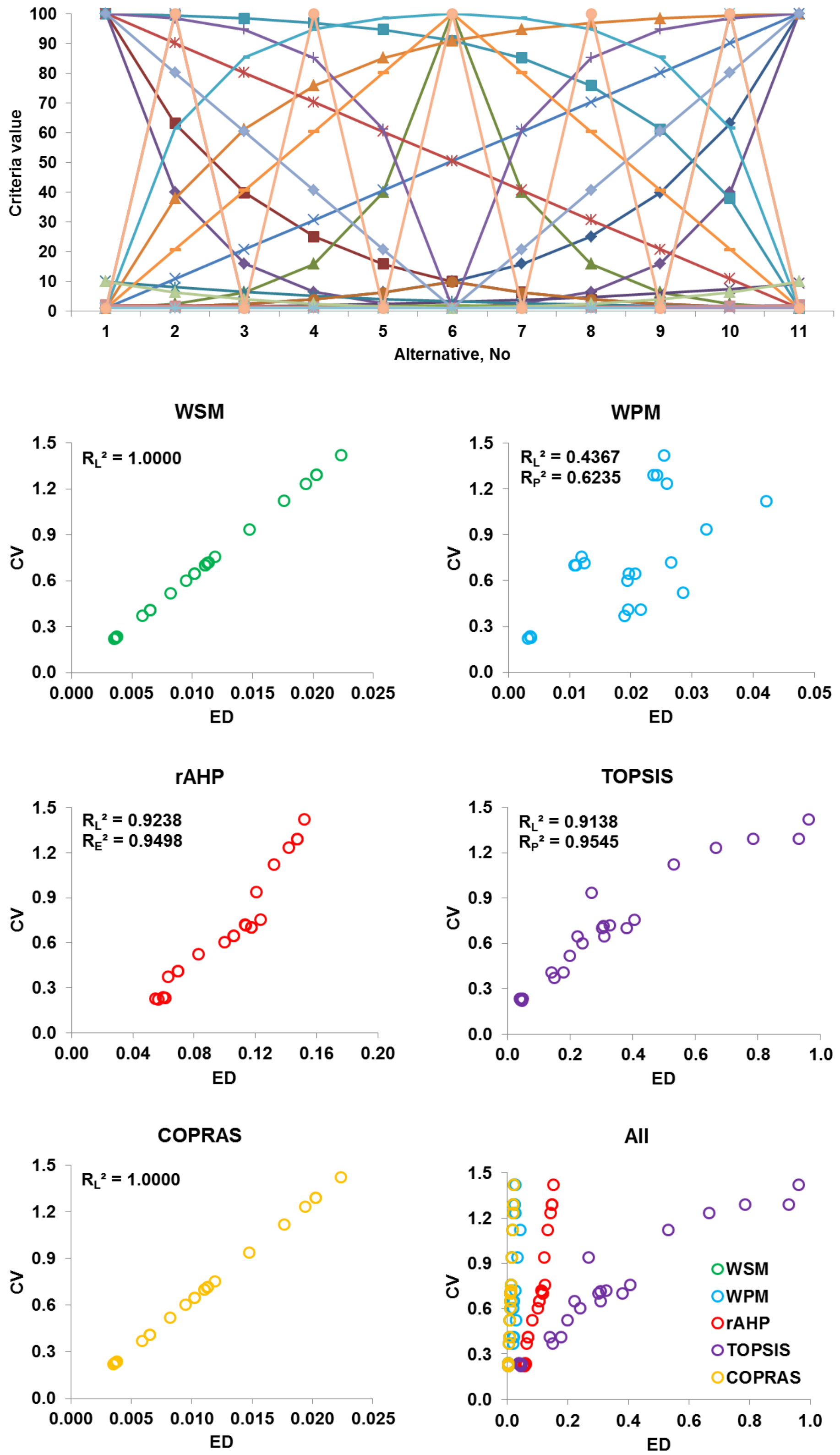


Figure 6. The correlation between the dispersion of relative importance values of alternatives (CV) and the dissimilarity of aggregated values (Euclidian distance, ED). Non-random numbers were used as criterion C11 relative importance values of alternatives. The top panel represents the dynamics of models that were used to generate non-random values. R_L^2, R_P^2 and R_E^2 are coefficients of determination for linear, polynomial and exponential regressions, respectively.

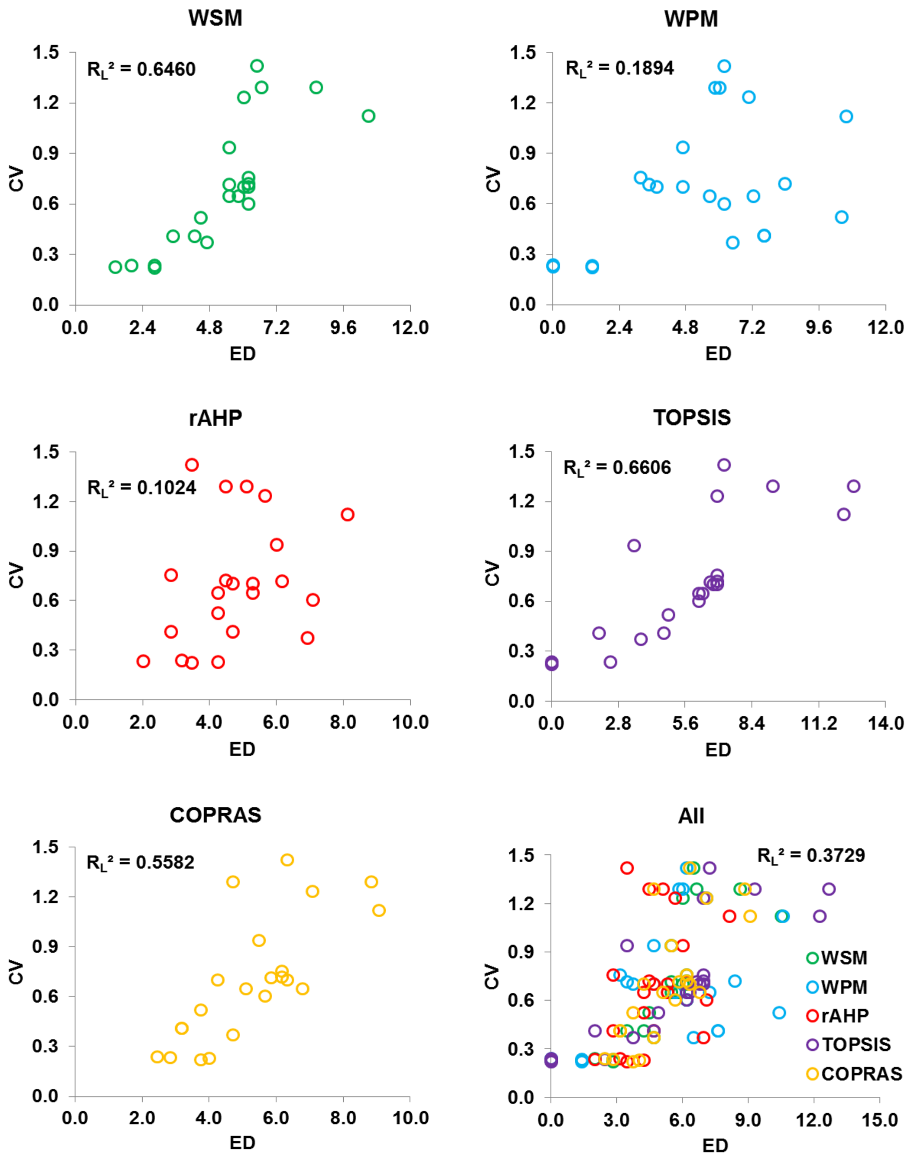


Figure 7. The correlation between the dispersion of relative importance values of alternatives (CV) and the dissimilarity of ranking (Euclidian distance, ED). Nom-random numbers were used as criterion C11 relative importance values of alternatives. R_L^2, R_P^2 and R_E^2 are coefficients of determination for linear, polynomial and exponential regressions, respectively.

Secondly, the non-random relative importance values of alternatives were generated using a variety of well-defined distribution models, including linear, exponential growth and decay, and geometric growth and decay, which contained maxima values different in the scale and position amongst eleven alternatives (Figure 6, top panel). Again, using a non-random distributions of relative importance values of alternatives, the WSM and COPRAS demonstrated absolute correlation between the dispersion of relative importance values and the dissimilarity of aggregated values (Figure 6). The WPM showed a significantly lower R^2 value. The WSM, TOPSIS and COPRAS demonstrated a significant correlation between the dispersion of relative importance values and the dissimilarity of ranking (Figure 7).

Altogether, the results demonstrated that the dispersion of relative importance values of alternatives strongly correlates with dissimilarity of aggregated values, which is one of the key parameters for ranking uncertainty. The dispersion can be used as an alternative measure for identification of most and least sensitive criteria, potential criteria ranking and is applicable for at least four MCDM methods including WSM, rAHP, TOPSIS and COPRAS.

4. Conclusions

The ranking output in the MCDM heavily depends on a nature of criteria that are used in the analysis and most notably on a distribution of the weighting amongst criteria. In addition, the factor that the criteria weights are usually established on the basis of expert perception should be taken into consideration, which can, to some extent, be subjective and may vary accordingly. Therefore, the effect of a possible deviation of the criteria relative importance values of alternatives should be evaluated.

Commonly, for the purpose of MCDM, the information about alternatives is derived in qualitative and quantitative forms. Then, qualitative data have to be transformed into measurable numerical values [60]. Both types of data can have a very different level of statistical dispersion. We used real data, collected for the assessment of the affordability of sustainable housing, to investigate how the data dispersion and distribution can influence alternative ranking using the five MCDM models including WSM, WPM, rAHP, TOPSIS and COPRAS. The sensitivity analyses of these different MCDM methodologies confirmed an earlier view that alternative MCDM methods should be used for a thorough and, most significantly, a critical assessment of decision-making. The criteria sensitivity differed amongst MCDM methods and amongst criteria themselves.

Here, we proposed that those criteria, of which the relative importance values of alternatives are distributed across a broad range of values, should be treated with particular care. For this purpose, the ranking susceptibility to the change of criteria weight can be assessed analytically using the sensitivity analysis. Then, the prospective ranking results must be taken into consideration. To verify the ranking results, alternative MCDM methods, in particular those that transform relative importance values of alternatives using constrained functions, as for example preference functions in PROMETHEE [61], can also be considered.

Finally, in this study, the sensitivity of MCDM methods to the change of weight of the decision criterion was assessed and the link between the ranking uncertainty and the dispersion of relative importance values of alternatives was established. Importantly, it was demonstrated that the criteria sensitivity and resulting uncertainty in the ranking output strongly correlates with the dispersion of relative importance values of alternatives for at least four MCDM methods including WSM, rAHP, TOPSIS and COPRAS. Therefore, the dispersion of relative importance values of alternatives can be used as a measure for identification of critical criteria.

References

[1] E. Triantaphyllou, S.H. Mann, An examination of the effectiveness of multi-dimensional decision-making methods: A decision-making paradox, Decision Support Systems, 5 (1989) 303-312.
[2] B. Roy, Decision-aid and decision-making, European Journal of Operational Research, 45 (1990) 324-331.
[3] V. Maliene, Specialised property valuation: Multiple criteria decision analysis, Journal of Retail and Leisure Property, 9 (2011) 443-450.
[4] V. Maliene, A. Kaklauskas, E.K. Zavadskas, Application of method of multiple criteria analysis for establishing value, Statyba (Journal of Civil Engineering and Management), 6 (2000) 295-306.
[5] J. Lee, H. Cho, Y.S. Kim, Assessing business impacts of agility criterion and order allocation strategy in multi-criteria supplier selection, Expert Systems with Applications, 42 (2015) 1136-1148.
[6] J. Antuchevičiene, E.K. Zavadskas, A. Zakarevičius, Multiple criteria construction management decisions considering relations between criteria, Technological and Economic Development of Economy, 16 (2010) 109-125.
[7] R. Lahdelma, P. Salminen, J. Hokkanen, Using multicriteria methods in environmental planning and management, Environmental Management, 26 (2000) 595-605.
[8] S. Chakraborty, E.K. Zavadskas, Applications of WASPAS method in manufacturing decision making, Informatica (Netherlands), 25 (2014) 1-20.
[9] M.G. Riera Pérez, E. Rey, A multi-criteria approach to compare urban renewal scenarios for an existing neighborhood. Case study in Lausanne (Switzerland), Building and Environment, 65 (2013) 58-70.
[10] W.W. Kropp, J.K. Lein, Assessing the geographic expression of urban sustainability: A scenario based approach incorporating spatial multicriteria decision analysis, Sustainability, 4 (2012) 2348-2365.
[11] T. Nuuter, I. Lill, L. Tupenaite, Comparison of housing market sustainability in European countries based on multiple criteria assessment, Land Use Policy, 42 (2015) 642-651.
[12] E. Mulliner, N. Malys, V. Maliene, Comparative analysis of MCDM methods for the assessment of sustainable housing affordability, Omega (United Kingdom), 59 (2016) 146-156.
[13] V. Maliene, Valuation of commercial premises using a multiple criteria decision-making method, International Journal of Strategic Property Management, 5 (2001) 87-98.
[14] F. Cavallaro, L. Ciraolo, A multicriteria approach to evaluate wind energy plants on an Italian island, Energy Policy, 33 (2005) 235-244.
[15] J. Hokkanen, P. Salminen, Choosing a solid waste management system using multicriteria decision analysis, European Journal of Operational Research, 98 (1997) 19-36.
[16] S. Hajkowicz, A. Higgins, A comparison of multiple criteria analysis techniques for water resource management, European Journal of Operational Research, 184 (2008) 255-265.
[17] J.S. Jeong, L. García-Moruno, J. Hernández-Blanco, A site planning approach for rural buildings into a landscape using a spatial multi-criteria decision analysis methodology, Land Use Policy, 32 (2013) 108-118.
[18] S. Kiriş, Multi-criteria inventory classification by using a fuzzy Analytic Network Process (ANP) approach, Informatica (Netherlands), 24 (2013) 199-217.
[19] D. Stanujkic, N. Magdalinovic, D. Milanovic, S. Magdalinovic, G. Popovic, An efficient and simple multiple criteria model for a grinding circuit selection based on MOORA method, Informatica (Netherlands), 25 (2014) 73-93.
[20] B. Roy, R. Słowinski, Questions guiding the choice of a multicriteria decision aiding method, EURO Journal on Decision Processes, 1 (2013) 69-97.
[21] E.K. Zavadskas, Z. Turskis, S. Kildiene, State of art surveys of overviews on MCDM/MADM methods, Technological and Economic Development of Economy, 20 (2014) 165-179.
[22] J. Rezaei, Best-worst multi-criteria decision-making method, Omega (United Kingdom), 53 (2015) 49-57.

[23] E. Triantaphyllou, Multi-Criteria Decision Making Methods: A Comparative Study, Springer, US, 2000.
[24] E. Zitzler, L. Thiele, Multiobjective optimization using evolutionary algorithms - A comparative case study, in: Lecture Notes in Computer Science (including subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics), 1998, pp. 292-301.
[25] M. Ravber, M. Mernik, M. Črepinšek, The impact of Quality Indicators on the rating of Multi-objective Evolutionary Algorithms, Applied Soft Computing, 55 (2017) 265-275.
[26] C.L. Hwang, K. Yoon, Multiple Attribute Decision Making: Methods and Applications A State-of-the-Art Survey, Springer Berlin Heidelberg, 1981.
[27] B. Mareschal, Weight stability intervals in multicriteria decision aid, European Journal of Operational Research, 33 (1988) 54-64.
[28] E. Mulliner, V. Maliene, An Analysis of Professional Perceptions of Criteria Contributing to Sustainable Housing Affordability, Sustainability, 7 (2015) 248-270.
[29] R.L. Iman, J.C. Helton, An investigation of uncertainty and sensitivity analysis techniques for computer models, Risk Analysis, 8 (1988) 71-90.
[30] S. Opricovic, G.H. Tzeng, Compromise solution by MCDM methods: A comparative analysis of VIKOR and TOPSIS, European Journal of Operational Research, 156 (2004) 445-455.
[31] J. Antucheviciene, A. Zakarevicius, E.K. Zavadskas, Measuring congruence of ranking results applying particular MCDM methods, Informatica, 22 (2011) 319-338.
[32] M.T. Chu, J. Shyu, G.H. Tzeng, R. Khosla, Comparison among three analytical methods for knowledge communities group-decision analysis, Expert Systems with Applications, 33 (2007) 1011-1024.
[33] L. Anojkumar, M. Ilangkumaran, V. Sasirekha, Comparative analysis of MCDM methods for pipe material selection in sugar industry, Expert Systems with Applications, 41 (2014) 2964-2980.
[34] R. Lourenzutti, R.A. Krohling, The Hellinger distance in Multicriteria Decision Making: An illustration to the TOPSIS and TODIM methods, Expert Systems with Applications, 41 (2014) 4414-4421.
[35] A. Guitouni, J.M. Martel, Tentative guidelines to help choosing an appropriate MCDA method, European Journal of Operational Research, 109 (1998) 501-521.
[36] E.K. Zavadskas, Z. Turskis, Multiple criteria decision making (MCDM) methods in economics: An overview, Technological and Economic Development of Economy, 17 (2011) 397-427.
[37] D.M. Hamby, A review of techniques for parameter sensitivity analysis of environmental models, Environmental Monitoring and Assessment, 32 (1994) 135-154.
[38] E. Triantaphyllou, A. Sánchez, A sensitivity analysis approach for some deterministic multi-criteria decision-making methods, Decision Sciences, 28 (1997) 151-185.
[39] C.H. Antunes, J.N. Clímaco, Sensitivity analysis in MCDM using the weight space, Operations Research Letters, 12 (1992) 187-196.
[40] C.A. Bana E Costa, A methodology for sensitivity analysis in three-criteria problems: A case study in municipal management, European Journal of Operational Research, 33 (1988) 159-173.
[41] D. Rios Insua, S. French, Framework for sensitivity analysis in discrete multi-objective decision-making, European Journal of Operational Research, 54 (1991) 176-190.
[42] W.T.M. Wolters, B. Mareschal, Novel types of sensitivity analysis for additive MCDM methods, European Journal of Operational Research, 81 (1995) 281-290.
[43] P.C. Fishburn, Conjoint measurement in utility theory with incomplete product sets, Journal of Mathematical Psychology, 4 (1967) 104-119.
[44] T.L. Saaty, Fundamentals of Decision Making and Priority Theory With the Analytic Hierarchy Process, RWS Publications, Pittsburgh, 1994.
[45] E.K. Zavadskas, A. Kaklauskas, V. Sarka, The new method of multicriteria complex proportional assessment of projects, Technological and Economic Development of Economy, 1 (1994) 131-139.
[46] E. Mulliner, K. Smallbone, V. Maliene, An assessment of sustainable housing affordability using a multiple criteria decision making method, Omega (United Kingdom), 41 (2013) 270-279.

[47] E. Mulliner, V. Maliene, Criteria for Sustainable Housing Affordability, in: D. Cygas, K.D. Froehner (Eds.) 8th International Conference on Environmental Engineering, Vilnius Gediminas Technical University Press Technika, Vilnius, Lithuania, 2011, pp. 966–973.
[48] O. Davies, P. Mendes, K. Smallbone, N. Malys, Characterisation of multiple substrate-specific (d)ITP/(d)XTPase and modelling of deaminated purine nucleotide metabolism, BMB Reports, 45 (2012) 259-264.
[49] S.H. Zanakis, A. Solomon, N. Wishart, S. Dublish, Multi-attribute decision making: A simulation comparison of select methods, European Journal of Operational Research, 107 (1998) 507-529.
[50] N. Banaitiene, A. Banaitis, A. Kaklauskas, E.K. Zavadskas, Evaluating the life cycle of a building: A multivariant and multiple criteria approach, Omega, 36 (2008) 429-441.
[51] E. Triantaphyllou, K. Baig, The impact of aggregating benefit and cost criteria in four MCDA methods, IEEE Transactions on Engineering Management, 52 (2005) 213-226.
[52] V. Belton, T. Stewart, Multiple criteria decision analysis: an integrated approach. , Springer, US, 2002.
[53] V. Belton, T. Gear, On a short-coming of Saaty's method of analytic hierarchies, Omega, 11 (1983) 228-230.
[54] I. Millet, B. Schoner, Incorporating negative values into the Analytic Hierarchy Process, Computers and Operations Research, 32 (2005) 3163-3173.
[55] M. Behzadian, S. Khanmohammadi Otaghsara, M. Yazdani, J. Ignatius, A state-of the-art survey of TOPSIS applications, Expert Systems with Applications, 39 (2012) 13051-13069.
[56] M. Bojesen, L. Boerboom, H. Skov-Petersen, Towards a sustainable capacity expansion of the Danish biogas sector, Land Use Policy, 42 (2015) 264-277.
[57] S. Alireza, A.P. Seyedeh, Comparing fuzzy AHP and fuzzy TOPSIS for evaluation of business intelligence vendors, Decision Science Letters, 4 (2015) 137-164.
[58] V. Podvezko, The comparative analysis of MCDA methods SAW and COPRAS, Engineering Economics, 22 (2011) 134-146.
[59] K. Mela, T. Tiainen, M. Heinisuo, Comparative study of multiple criteria decision making methods for building design, Advanced Engineering Informatics, 26 (2012) 716-726.
[60] C.L. Hwang, M.J. Lin, Group Decision Making under Multiple Criteria: Methods and Applications, Springer-Verlag Berlin Heidelberg, 1987.
[61] J.P. Brans, P. Vincke, A Preference Ranking Organisation Method: The PROMETHEE Method for Multiple Criteria Decision Making, Management Science 31 (1985) 647-656.