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The Higgs mechanism for undergraduate students

Giovanni Organtini

Dip. di Fisica Sapienza Università di Roma - Piazzale Aldo Moro 5 - 00185 ROMA (Italy)

Abstract

The Higgs mechanism gives mass to particles as a result of the interaction between massless particles and a scalar field. In this version it is reformulated in a purely classical form, using a simple formalism suitable for undergraduate students. The need for the Higgs field is justified with arguments following from a review of the concept of energy and from special relativity.

While most of the popularisations of the Higgs mechanism relies on analogies with friction, the proposed explanation appears to be at the same time formally coherent and simple enough to be proposed to undergraduate students, the prerequisites being just the knowledge of the energy density of electric and magnetic fields.

Keywords: Higgs boson, energy, undergraduate student

1. Introduction

Prior to introduce the Higgs mechanism, let's review the concept of energy, as seen in most undergraduate courses, starting from the most familiar ones. Apart the kinetics term, a body of (gravitational) mass m in a gravitational field $\mathbf{G}$ has a potential energy [1]

$$U_G = m\mathcal{G}\,, \tag{1}$$

where $\mathcal{G}$ is some scalar function of the field and the coordinates, namely

$$\mathcal{G} = \int_\infty^r \mathbf{G}\cdot d\mathbf{r}\,. \tag{2}$$

For an electric charge q in an electrostatic field $\mathbf{E}$ we can write something very similar, i.e.

$$U_E = q\mathcal{E}\,, \tag{3}$$

where $\mathcal{E}$ (usually written as V in many textbooks) is the so called electrostatic potential, whose definition is pretty much the same as the gravitational one:

$$\mathcal{E} = \int_\infty^r \mathbf{E}\cdot d\mathbf{r}\,. \tag{4}$$

A current I in a small coil feels the effect of a magnetic field $\mathbf{B}$ and we can write that its energy U_B is

$$U_B = I\mathcal{B}\,, \tag{5}$$

where $\mathcal{B} = -S\,\hat{\mathbf{z}}\cdot\mathbf{B}$, S is the area of the coil and $\mathbf{z}$ a unit vector perpendicular to it. In general, the energy of a particle in a field can be written as a sum of a coupling constant (m, q or I) multiplied by some scalar function of the field and the coordinates ($\mathcal{G}$, $\mathcal{E}$ or $\mathcal{B}$). We are used to call *potential* such functions, apart for $\mathcal{B}$ for which, from now on, we adopt the same name.

Energy is also carried by the electromagnetic field. It is well known that, given a volume V, the electromagnetic energy contained within it is written as

$$U_a = V\,(u_E + u_B) = V\left(\frac{\epsilon_0}{2}\mathbf{E}\cdot\mathbf{E} + \frac{1}{2\mu_0}\mathbf{B}\cdot\mathbf{B}\right), \tag{6}$$

that can be interpreted as the energy due to the auto–interaction of each field with itself. There is no such term for gravitational fields, the reason being that even if the gravitational field would interact with itself, there is no measurable effect of it. In other words we can still add a term like

$$U'_a = V\frac{g}{2}\mathbf{G}\cdot\mathbf{G} \tag{7}$$

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with $g = 0$. In short, the energy contained in a given volume of the Universe filled with some field and particles can always be written as a sum of terms written as the product of a coupling constant depending on the particles and a *potential* (that, in turn, depends on the fields and coordinates) or as the scalar product of each field by itself. We can then state that in the absence of any interaction the energy density of a given volume of the Universe would be zero (apart an arbitrary constant, that is irrelevant for the dynamics).

In short, any term in the expression of the energy written as a coupling constant times a *potential* represents the interaction of a particle with a field, while those written as the square of a field represent the interaction of that field with itself.

2. Relativistic effect

According to the Einstein's special relativity, the energy contained in a volume with some field and N particles, each with (inertial) mass m_i at rest is the sum of the above terms plus the energy at rest of the particles $\sum_{i=1}^{N} m_i c^2$, c being the speed of light. The latter seems not to be in the form of the previous terms: it has not the form of a coupling constant times a *potential*, nor the form of a scalar product of two vector fields.

The energy at rest of a particle seems to be something very different with respect to classical terms. This fact is somewhat disturbing, because the energy of a particle in the Universe seems depending from some interaction with a field, while in this case just the existence of the particle is enough to give it some energy. In the following we try to rewrite the energy at rest of a particle as it comes from the interaction with a field.

For the sake of clarity, let's consider just one particle of mass m and charge q in a uniform electrostatic field $\mathbf{E}$ for which we can neglect gravitational interactions, i.e. a charged particle at rest in a capacitor. The energy contained in the volume V of the capacitor is then

$$U = q\mathcal{E} + \frac{\epsilon_0}{2}\mathbf{E}\cdot\mathbf{E} + mc^2 \,. \tag{8}$$

Our aim is to rewrite mc^2 in the form of a coupling constant times a *potential* plus a product of a field by itself.

3. Introducing the Higgs field

Suppose that there exists a **scalar** field S (the reason for the field to be a scalar is explained below)[1], interacting with the particle with a coupling constant a and with itself as

$$U_S = a\mathcal{S} + \frac{b}{2}S\cdot S \,, \tag{9}$$

where $\mathcal{S}$ is, as usual, some function of the field and the coordinates. Suppose, in contrast with other fields, that the minimum energy for the auto–interaction term is attained for a non null value of S: in other words, we are supposing that the energy of an empty space may be larger with respect to the energy contained in a volume V *filled* with some field S. Then, contrary to the above fields, the Higgs field S assumes its smallest energy density when $S \neq 0$, i.e. when $S = H_0$, hence we can always write $S = H_0 + H$ and, correspondingly, $\mathcal{S} = \mathcal{H}_0 + \mathcal{H}$. Substituting the expressions of $\mathcal{H}$ and H into eq. (9) we obtain

$$U_S = a\left(\mathcal{H}_0 + \mathcal{H}_1\right) + \frac{b}{2}\left(H_0 + H\right)^2 \,. \tag{10}$$

Expanding the parenthesis we find

$$U_H = a\mathcal{H}_0 + a\mathcal{H}_1 + \frac{b}{2}H_0^2 + \frac{b}{2}H^2 + bH_0H \,. \tag{11}$$

Let's now analyse each term individually. In analogy with equations (1), (3) and (5), terms composed by a constant times a *potential* (i.e. a scalar function of fields and coordinates) represent the interaction of a particle with a field. Hence, terms like $a\mathcal{H}_0$ and $a\mathcal{H}$ represents the interaction of a particle with a field H_0 and H, respectively, the coupling constant a depending on the nature of the particle.

However, $\mathcal{H}_0$ is a universal constant, then $a\mathcal{H}_0$ is a constant that depends only on the particle interacting with the minimum value attained by the Higgs potential. If $a\mathcal{H}_0 = mc^2$ the energy at rest of a particle can be interpreted as a dynamical effect due to the interaction of the particle with the Higgs field. As a consequence, any particle coupling with the Higgs field gets then a mass just because it interacts with that field.

On the other hand, $a\mathcal{H}$ represents the interaction of the same particle with some Higgs field in excess with respect to the minimum one. The presence of this term tells us that particles with mass must interact with some extra Higgs field, if any, and it should be possible to detect the presence of such a field looking at the dynamics of the particle. In fact, since energy is conserved, any

[1] In fact, what follows can be further simplified assuming that S is a vector as the other fields, without changing the conclusions. The need for keeping S a scalar is only to preserve the similarity with the quantum Higgs field.

variation of the $a\mathcal{H}$ contribution to the energy results in a change of the particle's speed.

Terms in which appears the square of the Higgs field represent, analogously to terms proportional to $\mathbf{E}^2$ or $\mathbf{B}^2$, the self–interaction of the field. The first, $\frac{b}{2}H_0^2$ has again a constant value that, this time, depends only on universal constants (the minimum value of the Higgs potential and its self–coupling constant). As a consequence such a term is a bare constant that can be ignored since it does not gives rise to any dynamical observable effect.

The term $\frac{b}{2}H^2$ represents, in fact, the auto–interaction of the Higgs field in excess with respect to the minimum. That means that, if some extra Higgs field is present in a region of the Universe, the volume containing the excess field contains some energy in excess with respect to the electromagnetic one.

The latter term bH_0H represents the mass of the Higgs field (the Higgs boson in quantum mechanics). In fact it represents the interaction of a Higgs field H with the constant minimum potential $\mathcal{H}_0$ vie a coupling constant b depending on the Higgs field itself. In a volume containing N particles, the energy at rest of those particles is in fact $aN\mathcal{H}_0$. Similarly, in a volume containing a given amount H of (extra) Higgs field, the energy at rest for that field is $nH\mathcal{H}_0$ (here the intensity H of the field plays the same role of N for particles).

4. On the nature of the Higgs field

The reason for the field $S = H_0 + H$ to be a scalar is the following: the potential of all other fields, as a function of the field, is a parabola that gets its minimum value when the field vanishes. The state with the minimum possible energy is the vacuum state that coincides with an empty state. In order for the Higgs field to give mass to particles, its potential must have a minimum value $\mathcal{H}_0$ for $S = H_0 > 0$. In this case the vacuum state (i.e. the state with the minimum possible energy) is different from an empty state: the latter has an energy greater with respect to that of a state in which some Higgs field is present.

In vacuum all directions in space should be equivalent. As a consequence the field H cannot be a vector field, otherwise it may provide some preferential direction in space as the direction of the vectorial sum of all the fields present in a given region. If the field were a scalar, on the contrary, the vacuum state is isotropic. That's why the Higgs field should be considered as a scalar field.

To make the Higgs potential such that its minimum $\mathcal{H}_0$ is obtained for $H = H_0 > 0$ it is enough to assume that its auto–interaction is of the form

$$U_S\,(\text{auto}) = \frac{b}{2}S^2 + \frac{c}{2}S^4\,. \tag{12}$$

In fact the minimum for such an energy can be found for

$$S = -\frac{b}{2c}\,. \tag{13}$$

If the signs of b and c is not the same $S > 0$. This form does not change the way in which we interpret the energy of matter and fields. Adding a term proportional to S^4 is consistent with the assumption that the energy is something that comes from interactions: in this case the interaction is just a little bit different from that of electromagnetic fields.

In fact the Higgs mechanism [2] was formulated to explain why intermediate vector bosons have a large mass, in contrast with photons that have a null mass. The model outlined above can even be formulated in such a way to include such an explanation that, however, goes beyond the purpose of this paper. For a detailed discussion, refer to [3].

References

[1] For all the formulas of Section 1 and 2 any physics textbook is a valid reference.
[2] P. Higgs, "Broken Symmetries and the Masses of Gauge Bosons", 1964 Phys. Rev. Lett. 13 (16): 508
[3] G. Organtini, "Unveiling the Higgs mechanism to students", 2012 Eur. J. Phys. 33 1397