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## Neural networks for small scale ORC optimization

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### Abstract

This study concerns a thermodynamic and technical optimization of a small scale Organic Rankine Cycle system for waste heat recovery applications. An Artificial Neural Network (ANN) has been used to develop a thermodynamic model to be used for the maximization of the production of power while keeping the size of the heat exchangers and hence the cost of the plant at its minimum. R1234yf has been selected as the working fluid. The results show that the use of ANN is promising in solving complex nonlinear optimization problems that arise in the field of thermodynamics.

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*Keywords:* Small scale ORC, optimization, artificial neural networks

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### 1. Introduction

Organic Rankine Cycle (ORC) technology is considered as a cost effective technology to produce electricity out of low grade thermal energy sources. Despite of the potential market available, ORCs for small scale applications still find it difficult to create their own market. One of the reasons is undoubtedly their excessive specific market price [€/kW] which leads to a high payback period. Another reason is that Small-Medium Enterprises, which are by far the customers that may profit the most from small scale ORCs, are not sufficiently aware of the potential savings this technology could lead to. As it is often the case, at the smaller scales additional design issues arise which raise the specific price €/kW even further, thus limiting the market potential of this technology.

Small scale ORCs represent a viable method to produce electricity from low grade heat sources. The ORC power plant receives as an input the thermal energy of the heat source and converts it into electricity by means of a thermodynamic cycle. The inherently low-temperature source leads inevitably to low recovery efficiencies, and therefore it is important for the designer to optimize both the parameters of the cycle and the working fluid, the two issues being of course intimately connected.

Researchers performed studies to optimize the ORC system from a technical and thermodynamic perspective. Quoilin et al. [1] performed a techno-economic analysis of an ORC for waste heat recovery applications. They stated that the optimization with respect to the performance and the optimization with respect to the cost and size of the

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system lead to the selection of different thermodynamic parameters. Cavazzini and Dal Toso [2] studied a commercial ORC system to recover the thermal energy of the exhaust gas of an internal combustion engine. They stated that the cost of the system was too high to guarantee its commercialization.

Machine learning techniques (MLT) are receiving interest in the optimization of ORC plants. The reason is twofold: firstly, the thermodynamic process is highly non-linear, which makes it impervious to design efficient optimization algorithms. Secondly, MLTs perform feature selection thus eliminating from the problem all these variables which do not affect significantly the objective/constraints functions. Among MLTs, artificial neural networks (ANNs) [3] have proved to perform well in energy optimization problems [4–6]. Bechtler et al. [7] modelled the dynamic behaviour of vapour compression liquid chillers using ANNs, claiming that they were able to identify all the characteristics of the process. Yilmaz et al. [8] used neural networks to optimize the efficiency of an ORC with internal regeneration. They derived mathematical expressions to calculate the efficiency of the ORC system for the fluids R410a and R407c. Rashidi et al. [9] used ANNs and artificial bees colony algorithms in the optimization of a Rankine cycle and of an ORC using R717 as the working fluid. The ORC using R717 as the working fluid resulted in higher performance than the Rankine Cycle. Literature lacks of works based on ANN to optimize the thermodynamic parameters of an ORC with respect to its performance and size. Several ANN models have been tested to assess the one with better generalization performance using the open source WEKA data mining software [10]. Hence, a code has been written which uses the chosen Neural Networks to define an optimization model with different performance criteria.

### Nomenclature

$\dot{m}$	mass flow rate [kg/s]
$p_{bottom}$	minimum pressure of the cycle [Pa]
$p_{top}$	maximum pressure of the cycle [Pa]
$\Delta T_{sh}$	working fluid's superheating rate [K]
$\eta$	efficiency [-]
$RD$	regeneration degree [-]
$T_i$	temperature at point $i$ [K]
$h_i$	enthalpy at point $i$ [J/kg]
$\Delta T$	minimum temperature difference [K]
$T_{limit}$	Maximum temperature for the working fluid [K]
$T_{limit,hs}$	Minimum temperature for the heat source [K]
$U_{turbine,in}$	inlet peripheral velocity [m/s]
$\psi$	head coefficient [-]
$Q_{turbine,out}$	volumetric flow rate of the working fluid at the turbine's outlet section [ $m^3/s$ ]
$x^*$	optimal value of $x$
$\omega_{turbine}$	rotational speed of the turbine [rpm]
$W_{turbine}$	Euler work of the turbine [J/kg]
$P$	power output [kW]

### Subscripts

$wf$	working fluid
$hs$	heat source
$cf$	cooling fluid

The paper is organized as follows. First, in Section 2 it is shown the mathematical model of the maximization of the power output of the ORC together with the minimization of the size of the system and the rotational speed of the expander, which have a direct impact on the specific cost of the plant. ANNs are presented in Section 3. Subsequently, an optimization of the model has been performed and the results are reported in Section 4. This study underlines the need for an integrated approach in which thermodynamic, technical and economic criteria are considered simultaneously in order to design an efficient and cost-effective system.

## 2. Mathematical model

Figure 1 shows the thermodynamic transformations of a regenerated ORC system on the Temperature-Entropy diagram. Table 1 provides a brief description of the processes that take place in each of the components of the system. The thermodynamic model of the ORC is introduced in section 2.1 while the optimization problem is introduced in section 2.2.

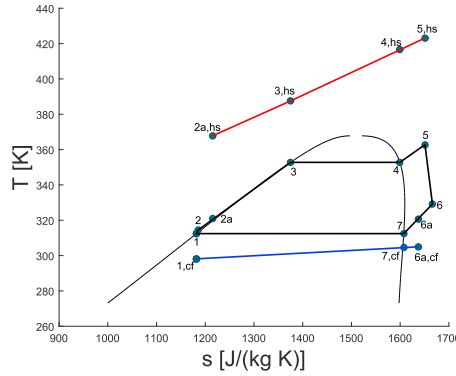


Fig. 1: T-s diagram of an ORC thermodynamic cycle.

Table 1: Transformations of the cycle.

Transformation	Component	Description
1 - 2	Pump	Fluid pressurized by the pump
2a - 3	Pre-heater	Pressurized fluid heating
3 - 4	Evaporator	Pressurized fluid vaporization
4 - 5	Super-heater	Pressurized fluid super-heating
5 - 6	Turbine	Expansion process
6 - 6a	Regenerator	Heat recovery to pre-heat the fluid at the pump outlet
6a - 1	Condenser	Fluid condensation

### 2.1. Thermodynamic model of the ORC

The thermodynamic model of the ORC has been derived using the commercial software package MATLAB [11]. The model is based on the mass conservation equation and the energy balance equation applied at each component of the system. Equations (1) to (8) are representative of the model implemented.

$$\dot{m}_{hs} = \text{const} \quad (1)$$

$$\dot{m}_{wf} = \text{const} \quad (2)$$

$$\dot{m}_{cf} = \text{const} \quad (3)$$

$$\dot{m}_{hs} \cdot (h_{5,hs} - h_{2a,hs}) = \dot{m}_{wf} \cdot (h_5 - h_{2a}) \quad (4)$$

$$P_{turbine} = \eta_{turbine} \cdot \dot{m}_{wf} \cdot (h_5 - h_6) \quad (5)$$

$$\dot{m}_{wf} \cdot (h_{6a} - h_1) = \dot{m}_{cf} \cdot (h_{6a,cf} - h_{1,cf}) \quad (6)$$

$$\dot{m}_{wf} \cdot (h_6 - h_{6a}) = \dot{m}_{wf} \cdot (h_{2a} - h_2) \quad (7)$$

$$P_{pump} = \dot{m}_{wf} \cdot (h_2 - h_1) / \eta_{pump} \quad (8)$$

The subscripts refer to the thermodynamic points depicted in Figure 1.

Table 2: Fixed parameters of the problem.

parameter	value
$\Delta T$	10 K
$U_{turbine,in}$	400 m/s
$\psi_{turbine,in}$	1
$\omega_{turbine}^{max}$	30000 rpm
$W_{turbine}^{max}$	160 kJ/kg
$P_{cycle}^{target}$	$0.9 \cdot P_{cycle}^*$
$T_{limit}$	400 K
$T_{limit,hs}$	313.15 K
$\eta_{pump}$	0.8

## 2.2. Definition of the optimization problem

The aim of this study is twofold. Firstly, the maximization of the power output of the ORC system has been performed. Secondly, a supervised degradation of the performance in terms of power has been imposed and two other optimization problems have been considered corresponding to the minimization of the  $UA$  parameter and of the rotational speed of the turbine respectively. Each of these minimization problems has been solved considering the optimal value of the power output previously found to form an additional constraint. The minimization of the  $UA$  parameter ensures that the choice of the thermodynamic parameters of the ORC leads to a system that is as compact as possible. The reduction of the rotational speed of the turbine is crucial since it is directly related to the cost and the availability in the market of the electric generator to which the turbine is coupled [12]. In addition, a high rotational speed results in the reduction of the Mean Time Between Failure of the bearings. The minimization of the size of the system and that of the rotational speed of the expander have the effect to lower the specific cost of production of electricity. Notice that the parameter  $UA$  has been calculated in the model using the NTU-method [13] for a counter flow heat exchanger configuration while the expander considered in this specific application is a radial inflow turbine.

The optimization model (9) displays the three objective functions and the constraints considered in the analysis. Table 2 shows the values of the parameters which have been kept constant throughout the optimization process.

$$\begin{aligned}
 & \max_{x \in \Omega} (P_{turbine} - P_{pump}) = P_{cycle} \\
 & \min_{x \in \Omega} (UA_{evaporator} + UA_{condenser} + UA_{regenerator}) = UA_{sum} \\
 & \min_{x \in \Omega} \omega_{turbine} \\
 \text{subject to} & \quad T_i - T_{i,cf} \geq \Delta T, \quad i = 1, 7 \\
 & \quad T_{6a} - T_{6a,cf} \geq \Delta T \\
 & \quad T_{2a,hs} - T_{2a} \geq \Delta T \\
 & \quad T_{i,hs} - T_i \geq \Delta T, \quad i = 3, 5 \\
 & \quad T_6 - T_{2a} \geq \Delta T \\
 & \quad T_{6a} - T_2 \geq \Delta T \\
 & \quad T_5 \leq T_{limit} \\
 & \quad T_{2a,hs} \geq T_{limit,hs} \\
 & \quad T_7 \leq T_{6a} \leq T_6 \\
 & \quad T_2 \leq T_{2a} \leq T_3 \\
 & \quad W_{turbine} \leq W_{turbine}^{max} \\
 & \quad \omega_{turbine} \leq \omega_{turbine}^{max} \\
 & \quad P_{cycle} \geq P_{cycle}^{target}
 \end{aligned} \tag{9}$$

where  $W_{turbine}^{max}$  has been calculated using Equation (10):

$$W_{turbine}^{max} = U_{turbine,in}^2 \cdot \psi_{turbine,in} \cdot \quad (10)$$

Based on the calculations of the thermodynamic cycle and on the selection of an optimal value for the specific speed ( $n_s$ ) as defined by Balje [14],  $\omega_{turbine}$  has been calculated as:

$$\omega_{turbine} = n_s \cdot W_{turbine}^{0.75} / \sqrt{Q_{turbine,out}} \cdot \quad (11)$$

For the maximization of  $P_{cycle}$ , the last constraint is absent from the model of the problem.

The three problems formulated are global continuous nonlinear optimization problems, since all the functions (objectives and constraints) used in the optimization model (9) are nonlinear.

The vector  $\mathbf{x}$  in (9) represents the array of the decision variables, which are to be optimized:

$\mathbf{x} = (\dot{m}_{wf}, p_{bottom}, p_{top}, \Delta T_{sh}, \eta_{turbine}, RD)^T$ , while  $\Omega$  is the region defined by the bound constraints which limit the components of  $\mathbf{x}$ :  $\Omega := \{\mathbf{x} \in \mathbb{R}^6 : \mathbf{lb} \leq \mathbf{x} \leq \mathbf{ub}\}$ , where  $\mathbf{lb}$  and  $\mathbf{ub}$  are respectively the lower and upper bounds vectors and in this problem their values are defined as follows:  $\mathbf{lb} = (0.1, p_{min}, p_{min}, 0.1, 0.5, 0.01)^T$ ,  $\mathbf{ub} = (3.8, p_{crit}, p_{crit}, 50, 0.75, 0.99)^T$ .

The degree of regeneration (RD) has been calculated using Equation (12):

$$RD = (h_6 - h_{6a}) / (h_6 - h_7) \cdot \quad (12)$$

Some of the pressures, temperatures and enthalpies of the ORC are not available in their analytical form and are calculated by means of a thermodynamic library. Hence the solution of the optimization model requires the use of derivative-free optimization methods, which are known to be not efficient and less effective in terms of the quality of the solution found [15].

This work proposes the use of a neural network model (see Section 3) to form an analytic approximation of both the objective functions and the constraints of the thermodynamic model of the ORC described above.

### 3. Neural networks

Learning machines are used as regression tools in all those cases in which examples of an unknown process are available but there is no known analytical model which correlates these quantities. Among learning machines, Artificial Neural networks are widely used for their simplicity, ease of implementation and for their capacity to achieve good performance in many different tasks [16,17]. At the beginning ANN were conceived to emulate the behaviour of the human brain, in the sense that they “learn” from examples, but the latest developed models of ANNs have little to share with their biological counterpart. When using ANN we suppose to have availability of training examples  $(\mathbf{x}^p, \bar{y}^p)$ , for  $p = 1, \dots, P$ , where  $\mathbf{x}^p \in \mathbb{R}^n$  represents the features of the input and  $\bar{y}^p \in \mathbb{R}$  the corresponding output. The aim of the ANN model is to define a function  $y : \mathbb{R}^n \rightarrow \mathbb{R}$  which is a good approximation of the unknown function underlying the process. The type of function and the parameters settings are defined through a learning procedure as briefly described below. The basic ANN architecture is composed by units (neurons) organized in layers forward connected with each other. A graph representation of an ANN can be seen in Figure 2 (a) where the nodes represent the neurons and the arcs represent the weighted synaptic connections among neurons. We distinguish some main elements:

- an input layer, that receives the training examples;
- an intermediate layer, called *hidden* layer;
- an output layer, which consists of a single neuron and produces the output of the network.

A single unit of the hidden layer is characterized by the activation function  $h$ , which acts as an on-off trigger on a weighted combination of the outputs of the neurons in the preceding layer, and a bias  $b_j$ ; each connection from input  $i$  to neuron  $j$  of the hidden layer is characterized by a weight  $w_i^j$  as shown in Figure 2 (b); the arc from the hidden neuron  $j$  to the output neuron are weighted by  $w_o^j$ .

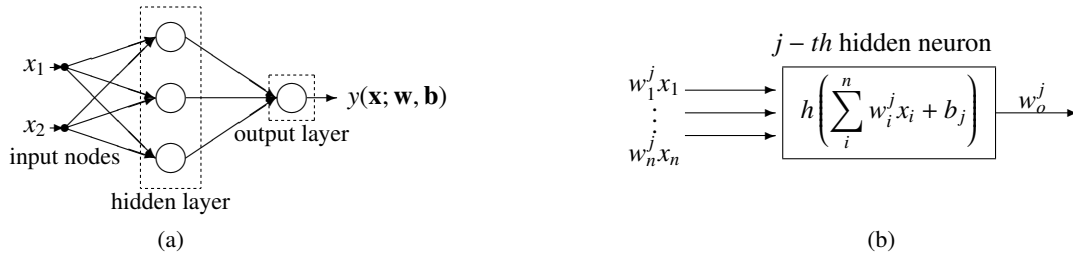


Fig. 2: (a) ANN with one hidden layer and one output; (b) internal structure of the  $j$ -th neuron of the hidden layer.

The number of neurons  $m$  in a hidden layer together with the activation function  $h$  of the neurons are user-dependent parameters. Once they are fixed, the output of the ANN network takes the following form:

$$y(\mathbf{x}; \mathbf{w}, \mathbf{b}) = \sum_j w_o^j h\left(\sum_i^n w_i^j x_i + b_j\right) + b_o ; \quad (13)$$

where we denote in short  $\mathbf{w} = \{w_o^j, w_i^j\}_{i=1, \dots, n, j=1, \dots, m}$  and  $\mathbf{b} = \{b_j, b_o\}_{j=1, \dots, m}$ .

All the parameters  $\mathbf{w}, \mathbf{b}$  that appear in (13) are “learned” through a *training* process which consists in the resolution of the minimization problem:

$$\min_{\mathbf{w}, \mathbf{b}} E(\mathbf{w}, \mathbf{b}) = \frac{1}{2} \sum_p^P \|\bar{y}_p - y(\mathbf{x}_p; \mathbf{w}, \mathbf{b})\|^2 . \quad (14)$$

In order to obtain an analytic expression for all the functions (objectives and constraints) in model (9), a training set has been constructed that is made up of pairs  $\mathbf{x}^p, \bar{y}^p$  obtained by sampling each input component of the vector  $\mathbf{x}$  over a grid restricted to box  $\Omega$ , defined in 2.2, and running the black-box thermodynamic model over all the samples to get the output samples  $\bar{y}^p$ .

These training data have been used to construct ANN models as implemented in WEKA. The trained ANNs in their analytical form have then been used into the model (9) to get a continuously differentiable nonlinear model that can be solved by using a standard constrained method for nonlinear programming [18].

#### 4. Results

The proposed model has been applied to the design of a regenerated ORC, considering R1234yf as the working fluid. The heat source considered in this work is pressurized water at the temperature of 413.15 K and its mass flow rate has been considered equal to 3.2 kg/s. The cooling fluid considered for this work is water at the temperature of 298.15 K. A multilayer perceptron (MLP) has been used and it has been trained and validated using a 10-fold cross-validation to improve performance of the network [3]. As activation function, the MLP implemented in WEKA uses an approximated sigmoid function  $h$  in the hidden layer. All other parameters were set to the default values, except for the number of neurons of the hidden layer, which was fixed to 20. The networks obtained with this choice of the parameters were characterized by a low validation error and a high correlation coefficient. Table 3 reports the means and the standard deviations of the correlation coefficient and the relative absolute error.

Table 3: Statistics relative to the 10-fold cross validation.

	mean	standard deviation
correlation coefficient	0.9995	0.0022
relative absolute error	0.7981%	0.8741%

The *active set* algorithm [19] implemented in the function `fmincon` of the MATLAB Optimization Toolbox [11] has been chosen to perform the optimization. As global optimization strategy a simple multi-start algorithm with 200

random starting points uniformly distributed in  $\Omega$  has been considered. As a first step, the problem of maximizing the power has been solved, obtaining as the optimal value for the power output of the ORC system ( $P_{cycle}^*$ ) 35.19 kW.

The aforementioned power output has been used to define a new constraint on the minimum value of the power output of the system that was set to  $0.9 \cdot P_{cycle}^*$ . After having added this new constraint, the optimization problems of the minimization of  $UA_{sum}$  and of  $\omega_{turbine}$  have been solved. The optimal solutions for the minimization problems performed are shown in Table 4, together with the value of the decision variables in both cases. The corresponding T-s diagrams can be seen in Figure 3.

Table 4: Optimal solutions and their values (in bold) of the maximization of  $P_{cycle}$  (first row), the minimization of  $UA_{sum}$  (second row) and  $\omega_{turbine}$  (third row). The problems have been solved using the ANNs and derivative-based optimization.

$\dot{m}_{wf}$ [kg/s]	$p_{bottom}$ [bar]	$p_{top}$ [bar]	$\Delta T_{sh}$ [K]	$\eta_{turb}$ [-]	$RD$ [-]	$P_{cycle}$ [kW]	$UA_{sum}$ [kW/K]	$\omega_{turbine}$ [rpm]	time [s]
3.24	11.73	33.82	10.81	0.75	0.3	<b>35.19</b>	53	30000	23.87
2.91	11.73	33.82	10.93	0.75	0.25	31.67	<b>44.15</b>	30000	106.56
3.51	11.75	28.79	5.32	0.75	0.15	31.67	55.34	<b>24298</b>	100.14

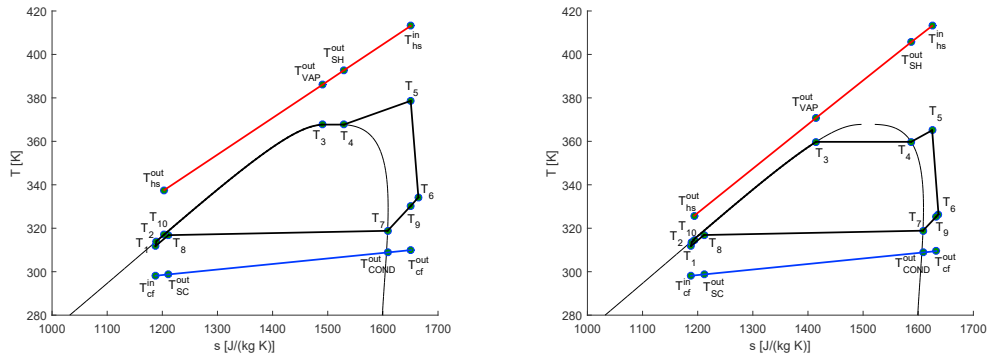


Fig. 3: T-s diagram of the optimal solution of: (a) the minimization of  $UA_{sum}$ ; (b) the minimization of  $\omega_{turbine}$ .

As it is possible to notice from the results in Table 4, the reduction in the rotational speed of the turbine is obtained by means of a reduction of the cycle top pressure. The minimization of the  $UA$  parameter implies the maximization of the efficiency of the cycle, which is obtained for high cycle top pressure. In both cases, the super-heating rate results to be low. Table 5 shows the results obtained with a derivative-free algorithm coupled with the original thermodynamic model coded in MATLAB, without using ANNs. For this optimization, the software NOMAD 3.7.2 [20] was used, interfaced with MATLAB using OPTI Toolbox 2.24 [21].

Table 5: Optimal solutions and their values (in bold) of the maximization of  $P_{cycle}$  (first row), the minimization of  $UA_{sum}$  (second row) and  $\omega_{turbine}$  (third row). The problems have been solved using the original model coded in MATLAB and derivative-free optimization.

$\dot{m}_{wf}$ [kg/s]	$p_{bottom}$ [bar]	$p_{top}$ [bar]	$\Delta T_{sh}$ [K]	$\eta_{turb}$ [-]	$RD$ [-]	$P_{cycle}$ [kW]	$UA_{sum}$ [kW/K]	$\omega_{turbine}$ [rpm]	time [s]
3.11	11.74	33.72	14.26	0.75	0.001	<b>36.65</b>	54.3	30000	343.45
3.01	12.28	33.72	12.97	0.75	0.2	32.99	<b>45.06</b>	30000	470.49
3.4	11.74	29.87	7.14	0.75	0.001	32.98	61.96	<b>25646</b>	465.83

The results obtained using the derivative-free algorithm are in line with the ones obtained using the ANNs. The discrepancy in the values of the results obtained using the two different optimization methods is related to the high nonlinearity of the problem and to the level of approximation of ANNs. Also, the time required to solve the optimization problem using ANNs is substantially lower than that of a classic nonlinear optimization algorithm, making ANNs promising for the solution of complex thermo-economic optimization problems, which are typical of the ORC optimization.

## 5. Conclusions

The global optimization of a regenerated ORC system approximated with ANNs has been presented. The networks were trained using WEKA, with a training set generated from a thermodynamic model coded in MATLAB. The ANN model was applied to the design of a regenerated ORC system. The obtained results showed that neural networks have been capable of well approximating all the functions of the problem. This work demonstrates that the application of ANNs to the resolution of ORC optimization problems represents a promising alternative to the classical optimization methods, allowing for the use of derivative-based methods that substantially reduce the computational time required for the optimization. The results show that the maximum power output that can be extracted from the heat source is 35.19 kW while the minimum values obtained for the rotational speed and the  $UA$  parameter are respectively 24298 rpm and 44.15 kW/K.

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