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On the Robustness of the Consumer Homogeneity Assumption with Respect to the Discount Factor for Remanufactured Products

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Abstract

The strategic closed-loop supply chains (CLSCs) literature makes the assumption that a consumer's willingness-to-pay (WTP) for a remanufactured product is a fraction of his/her WTP for the corresponding new product, and this fraction, called discount factor, is assumed to be constant among consumers. Recent empirical research challenges this assumption, by showing that there is considerable variability in discount factors among consumers. This paper considers a complex model in the CLSC literature: strategic remanufacturing under quality choice, and compares its solution under constant discount factors with the solution that assumes a probability distribution for the discount factors (which is analytically intractable and must be obtained numerically). We consider quality choice and remanufacturing for both monopoly and competitive cases. Overall, we find remarkable consistency between the results of the constant and variable discount factor models. Thus, we make a convincing argument that the constant discount factor assumption is robust and can be used due to its tractability.

Keywords: Strategic planning, remanufacturing, game theory, closed-loop supply chains

1. Introduction

Remanufacturing is the process of restoring a used product to a common aesthetic and operating standard (Lund, 1984). In many cases, products remanufactured by an original equipment manufacturer (OEM) are essentially like-new (e.g., Ferguson and Souza, 2010). However, the empirical closed-loop supply chain (CLSC) literature convincingly demonstrates that, in general, consumers' willingness-to-pay (WTP) for remanufactured products are lower than for new products, and this is

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reflected in lower prices for remanufactured products (for a review, see Souza, 2013). For example, Guide and Li (2010) auction new and remanufactured power tools and Internet routers on eBay, with fulfillment by the respective OEMs, and with the same warranty as new products. They find that the final auction price for remanufactured products is 15% lower than for their corresponding new counterparts. As another example, Subramanian and Subramanyam (2012) compare prices of new and remanufactured products at eBay, and find that price discounts for remanufactured products, after controlling for warranty, range between 15% and 40% for different types of consumer electronics. The drivers of this lower WTP have been investigated in the recent CLSC literature, and are related to lower *perceived* quality, as we further discuss in section 2.

Because consumers' WTP for remanufactured products are generally lower than their WTP for corresponding new products, OEMs that offer remanufactured products must carefully price new and remanufactured products to maximize profitability. This is because the addition of a remanufactured product to an OEM's product line has a positive market expansion effect, but also a negative cannibalization effect, and prices of remanufactured and new products determine the magnitude of these effects (see, e.g., Souza, 2013). Pricing is also critical for an OEM to deter competition from independent remanufacturers (Ferguson and Toktay, 2006). The literature has analyzed these and other strategic CLSC problems, as discussed in section 2, using analytic models. In these models, the classical approach to model a consumer's WTP for remanufactured products is to multiply her WTP for corresponding new products by a discount factor, as follows. Consumers are characterized by their WTP for a new product, denoted by θ; consumers are heterogeneous and $\theta \sim U[0,1]$. A customer of type θ has a WTP $\delta\theta$ for the remanufactured unit, where $\delta \leq 1$ is the *discount factor* (also called durability level by some authors), assumed *constant* across consumers. As shown in section 3, assuming that the discount factor δ is constant across consumers results in linear demand curves for new and remanufactured products, which is tractable for analyzing strategic CLSC problems.

Abbey et al. (2017), however, find that *consumers differ significantly in their discount factors for remanufactured products*. They empirically derive a probability distribution of discount factors for the iPhone 6, whose cumulative density function (cdf) is plotted in Figure 1, along with a truncated normal distribution with the same mean and standard deviation (the smooth curve in Figure 1), which demonstrates a good fit. They also assess the impact of this variable discount

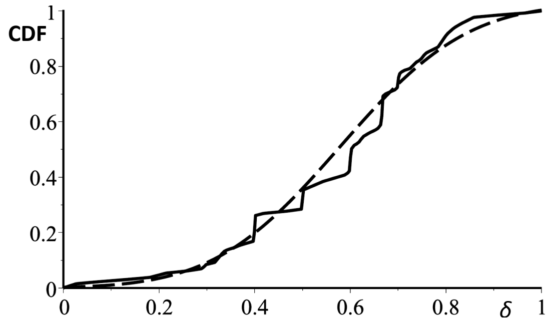

Figure 1: The empirical CDF and the corresponding CDF of the truncated normal distribution

factor finding on the simplest analytical model of the CLSC literature—whether a monopolist OEM should offer a remanufactured product, in addition to its new product. Their analysis considers the solutions under constant and variable discount factor assumptions. They show that the general insights provided by the model with constant discount factors hold under variable discount factors in this simple model. Thus, their paper provides initial evidence of the robustness of the constant discount factor assumption.

The major contribution of this paper is to show the robustness of the constant discount factor assumption for significantly more complex strategic models in the CLSC literature: the models—both monopoly and competition—where quality choice for the new product is also a decision variable by the firm(s), in addition to prices of new and remanufactured products. This is because, as we see in section 3.1, equation (1), a model with quality choice results in non-linear demand curves for new and remanufactured products, even under a constant discount factor assumption. Despite these analytical challenges, the quality choice model under constant discount factors has been analyzed by Atasu and Souza (2013) for the monopolist OEM, whereas competition against an independent remanufacturer (IR) has been analyzed by Örsdemir et al. (2014). We analyze quality choice models for both a monopolist OEM, and OEM competition against an IR under variable discount factors, and compare the findings against those with constant discount factors from Örsdemir et al. (2014). To further demonstrate robustness, we consider two significant extensions: (i) when there is a convex collection cost (for both monopoly and competition), and (ii) when both the OEM and IR offer remanufactured products, in addition to the new product by the OEM. There is some empirical evidence that unit collection costs increase in the quantity collected in some settings (Atasu et al., 2013); hence our proposed extension. Second, there are several industry scenarios where both the OEM and IR offer remanufactured products (Souza, 2013). The models with variable discount factors are analytically intractable—even the demand functions cannot be obtained in closed form, as we show in section 3.2, equations (7)-(8)—and must thus be solved numerically.

We find that the constant discount factor model is reasonably robust in generating valid qualitative insights in this complex setting. For example, there is consistency in how optimal and/or equilibrium prices and quality behave as a function of parameter values, such as unit costs, and mean discount factors. We find, of course, (relatively small) differences in the actual optimal values of prices and quality between constant and variable discount factor models. One additional insight brought by the variable discount model is that a firm (either OEM or IR) remanufactures under a much wider set of remanufacturing costs and discount factors, because for a given remanufactured product price that is less than the new product price, there is always a non-zero segment of consumers who are willing to buy the remanufactured product. This result holds for both monopoly and competition settings. Despite the differences in optimal and/or equilibrium prices between the two models, we remind the reader that the purpose of stylized models is not to provide actual decision support for firms in making pricing decisions, but rather to offer qualitative insights into strategic behavior. Thus, we provide convincing evidence that the tractable constant discount factor assumption is robust for use in stylized models for generating insights.

This paper is organized as follows. In section 2 we present a literature review. In section 3 we briefly describe the analytic model by Örsdemir et al. (2014) for both the monopolist OEM case, as well as when the OEM competes with an IR. We then describe how the models change under variable discount factors. In section 4, we describe our experimental design and the numerical solution approach. In section 5, we provide the numerical results for the monopolist OEM comparing both constant and variable discount factor models. In section 6, we provide the comparison for the case where the OEM competes with an IR. In section 7.1, we consider the extension with convex collection costs, for both monopoly, and OEM competition with an IR. In section 7.2, we consider the extension where both OEM and IR offer remanufactured products, in addition to the OEM's new product. We conclude in section 8.

2. Literature Review

OEMs remanufacture for such reasons as value recovery, product line extension, use for warranty fulfillment, and brand protection against independent remanufacturers (see, e.g., Atasu et al., 2010, Ferguson and Souza, 2010). These reasons led to a significant stream of normative CLSC research on an OEM's decision to remanufacture, including optimal pricing under competition (e.g., Majumder and Groenevelt, 2001, Debo et al., 2005, Ferguson and Toktay, 2006, Ferrer and Swaminathan, 2006), marketing strategies (e.g., Atasu et al., 2008, Oraiopoulos et al., 2012), impact of remanufacturing on new product quality and design (Atasu and Souza, 2013, Galbreth et al., 2013, Subramanian et al., 2013, Örsdemir et al., 2014), strategic product acquisition (Guide et al., 2003), and production capacity (Vorasayan and Ryan, 2006). This research assumes constant discount factors, which results in tractable linear demand curves as we see in the next section.

Recent behavioral CLSC research empirically scrutinizes this constant discount factor assumption. Ovchinnikov (2011) finds that WTP for remanufactured products decrease if remanufactured products are priced too low, due to a low-quality perception. Ovchinnikov et al. (2014) empirically derive demand functions, using conjoint analysis, that are not linear. Agrawal et al. (2015) show how remanufacturing by an OEM is perceived differently by consumers than remanufacturing by an IR. Abbey et al. (2015) use consumer experiments to obtain the attractiveness of remanufactured products as a function of price discounts, brand equity, and quality perceptions. For technology products, they find that under low discount levels the attractiveness of remanufactured products increases with an increasing discount level, confirming theoretical predictions. Yet, they also find that too high price discounts may decrease the attractiveness of remanufactured products, in line with Ovchinnikov (2011). Finally, Abbey et al. (2017) empirically derive the probability distribution of discount factors for a remanufactured iPhone 6.

This paper contributes to the CLSC literature by showing that the insights provided by the quality choice model of Örsdemir et al. (2014), which is arguably the most complex analytic model in the literature using the constant discount factor assumption, carry over to the (realistic) situation where discount factors are variable; we use the empirically-derived discount factor distribution of Abbey et al. (2017) as the basis of our analysis. In addition, we provide two significant extensions. First, we consider convex collection costs, which has been shown to hold empirically by Atasu et al. (2013) in some settings. Second, we consider the case where both the OEM and IR offer remanufactured products, in addition to the OEM's new product. This latter extension is analytically challenging even with constant discount factors. As a result, only simplified versions of it without quality as a decision variable have been analyzed analytically (e.g., Atasu et al., 2008, Ferrer and

Swaminathan, 2006), or the model has been analyzed numerically (Örsdemir et al., 2014).

3. Analytic model

3.1. Quality choice under constant discount factor for remanufactured products

We first briefly describe the benchmark quality choice model by Örsdemir et al. (2014); we refer to their paper for additional details. The notation is shown in Table 1. In their model, which is based on long-standing marketing literature (e.g., Mussa and Rosen, 1978, Moorthy, 1988), quality is modeled as a single, continuous decision variable s, which indicates a critical performance dimension for which consumers are willing to pay for, such as megapixels in a digital camera.

Table 1: Notation

Parameters	
α	unit cost for the remanufactured product as a fraction of the new product's unit cost
β	scaling parameter in the unit cost
Decision variables	
p_n	price of the new product
p_r	price of the remanufactured product
s	quality level
Auxiliary variables	
θ	WTP for the new product with unit quality level
δ	discount factor for WTP for remanufactured products
q_n	quantity of new products
q_r	quantity of remanufactured products
θ_{ab}	a consumer type that is indifferent between alternatives a and b

Monopolist OEM. If the OEM provides a new product with quality s at price p_n, then the net utility that a consumer of type $\theta \sim U[0,1]$ derives from this product is $U_n(\theta) = \theta s - p_n$, where θ is interpreted as WTP of a consumer for a product with unit quality level. Thus, if only the new product is available the sales quantity given a unit market size is $q_n = 1 \cdot \Pr\{\theta s - p_n \geq 0\} = 1 - \frac{p_n}{s}$, where the last equality results from the uniform distribution for θ. A customer of type θ has a WTP $\delta\theta$ for the remanufactured unit, where $\delta \leq 1$ is the *discount factor*, assumed constant across consumers. Note that, as defined, a higher discount factor δ means that consumers consider the remanufactured product more similar to the new product. Thus, a consumer of type θ has a net utility for the remanufactured product priced at p_r equal to $U_r(\theta) = \delta\theta s - p_r$. Equating $U_n(\theta)$ and $U_r(\theta)$ results in the consumer $\theta_{rn} = \frac{p_n - p_r}{(1-\delta)s}$ that is indifferent between the two products. Likewise, setting $U_r(\theta) = 0$ results in the consumer $\theta_{zr} = \frac{p_r}{\delta s}$ that is indifferent between buying a remanufactured product and buying nothing. With the uniform distribution for θ, this yields:

$$q_n(p_n, p_r, s) = 1 - \frac{p_n - p_r}{(1-\delta)s} \quad \text{and} \quad q_r(p_n, p_r, s) = \frac{p_n - p_r}{(1-\delta)s} - \frac{p_r}{\delta s}. \tag{1}$$

Note that, if the quality s is normalized to one, as in a significant portion of the CLSC literature, then (1), which originate from the assumption of a constant discount factor δ across consumers, result in linear demand curves; this ensures tractability. When the quality choice s is a decision variable, then the demand curves are non-linear in s.

Following the literature on quality choice, the unit cost for the new product is convex increasing in the quality level, βs^2, whereas the unit cost for the remanufactured product, including the cost of collecting a used product, is a fraction α of the unit cost of the new product, $\alpha\beta s^2$. The monopolist OEM first selects the optimal quality of the product s^*. It then determines the prices of new and remanufactured products to optimize its profit in a single period. In the CLSC literature, one period is the length of the product's life with consumers, after which it must be remanufactured to be useful again. Considering a stationary setting, the period in the model can be thought of a period in an infinite planning horizon with identical periods. The price choices are constrained in such a way that the quantity of remanufactured products cannot exceed the quantity of new products because used products, or *cores*, are the key input for remanufacturing as follows. Since a new product lasts one period, a used product in a period corresponds to a new product sold in the previous period; considering the infinite-horizon setting with identical periods, q_n is the same in all periods, hence the constraint $q_r \leq q_n$. Another possibility for such core availability constraint would be to set $q_r \leq \tau q_n$, where $\tau < 1$ indicates that not all used products can be recovered. We use the constraint $q_r \leq q_n$ in order to directly compare our results with those of Örsdemir et al. (2014). Incorporating the additional parameter τ would increase the region where total remanufacturing (i.e., the core availability constraint is tight) is optimal, but would not change the key insights about the robustness of the constant discount factor model. To solve this problem, backward induction is used. First, the OEM solves the pricing optimization problem for a given quality level s:

$$\begin{aligned} \max_{p_n, p_r | s} \quad & \pi(p_n, p_r, s) = (p_n - \beta s^2) q_n(p_n, p_r, s) + (p_r - \alpha\beta s^2) q_r(p_n, p_r, s), \\ s.t. \quad & 0 \leq q_r(p_n, p_r, s) \leq q_n(p_n, p_r, s). \end{aligned} \tag{2}$$

Then, the OEM solves for the optimal quality level, given the optimal prices:

$$\max_{s \geq 0} \quad \pi(p_n^*(s), p_r^*(s), s), \tag{3}$$

where $p_n^*(s)$ and $p_r^*(s)$ are the optimal prices (as a function of the quality level s) found in (2). The closed-form solution to problem (1)-(3) can be found in Örsdemir et al. (2014).

Competition between OEM and IR. Now, the OEM offers only new products, whereas the IR offers a remanufactured version of the OEM's product. The OEM first chooses the quality level of the new product. Then, for a given quality level, the OEM and IR compete by choosing the new and remanufactured product prices, respectively, under the core availability constraint. The OEM and IR make simultaneous pricing moves, as assumed by Örsdemir et al. (2014), and this is also consistent with other competition models in the CLSC literature. This results in a game, which can be solved by backward induction, as follows. First, the OEM and IR solve the following pricing game:

$$\max_{p_n|p_r,s} \quad \pi_{OEM}(p_n, p_r, s) = (p_n - \beta s^2) \cdot q_n(p_n, p_r, s), \tag{4}$$
$$s.t. \quad q_n(p_n, p_r, s) \geq 0.$$
$$\max_{p_r|p_n,s} \quad \pi_{IR}(p_r|s, p_n) = (p_r - \alpha\beta s^2) \cdot q_r(p_n, p_r, s), \tag{5}$$
$$s.t. \quad 0 \leq q_r(p_n, p_r, s) \leq q_n(p_n, p_r, s).$$

The OEM then chooses the optimal quality s^* that maximizes its profit:

$$\max_{s \geq 0} \quad \pi_{OEM}(p_n^*(s), p_r^*(s), s). \tag{6}$$

The game comprised of (1) and (4)-(6) has a closed-form Nash equilibrium solution, given by Örsdemir et al. (2014). Although other equilibria are possible in a multi-period game, the equilibrium solution to the above model corresponds to a stationary equilibrium in an infinite-horizon setting with identical periods, which is standard in the CLSC literature (Souza, 2013).

3.2. Quality choice under variable discount factor for remanufactured products

Under a variable discount factor, δ is no longer constant, but has a probability distribution with cdf $F(\cdot)$, such as the one shown in Figure 1, with support $[0, 1]$; denote the probability density function of δ as $f(\cdot)$. A consumer can then be described as a point (θ, δ) in the respective plane. The starting point from the derivation of the demand functions is similar to the constant discount factor model. In particular, we equate the utility of buying a new product $U_n(\theta) = \theta s - p_n$ to the utility of buying a remanufactured product $U_r(\theta, \delta) = \delta\theta s - p_r$ to yield the indifference *curve* $\theta_{rn} = \frac{p_n - p_r}{(1-\delta)s}$. Likewise, we equate the utility of buying a remanufactured product $U_r(\theta, \delta)$ to the utility of buying nothing $U_z = 0$, yielding the indifference curve $\theta_{zr} = \frac{p_r}{\delta s}$. Finally, we equate the utility of buying a new product $U_n(\theta)$ to that of buying nothing U_z, yielding $\theta_{zn} = \frac{p_n}{s}$.

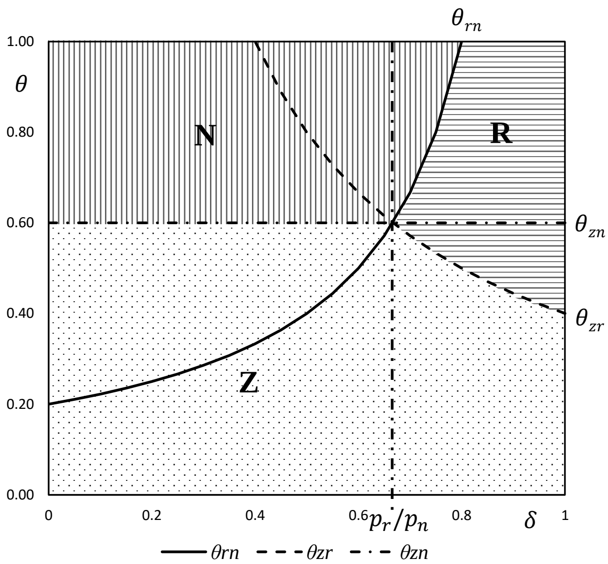

Figure 2: Consumer behavior depending in θ and δ

Figure 2 graphically depicts the consumer regions for buying new products (**N**), buying remanufactured products (**R**), and not buying (**Z**) in the plane (θ, δ). Each shaded region (e.g., **N**) characterizes the consumer types (θ, δ) who have higher utility for that alternative (i.e., the new product) than for the other alternatives (i.e., the remanufactured product, and nothing). As a result, demand functions for new and remanufactured products are the double integrals of the probability distributions of consumer types θ and δ over the respective regions, as follows:

$$q_n(p_n, p_r, s) = \int_0^{p_r/p_n} f(\delta)\,\mathrm{d}\delta \int_{\theta_{zn}}^{1} \mathrm{d}\theta + \int_{p_r/p_n}^{1-(p_n-p_r)/s} f(\delta) \int_{\theta_{rn}}^{1} \mathrm{d}\theta\,\mathrm{d}\delta, \tag{7}$$

$$q_r(p_n, p_r, s) = \int_{p_r/p_n}^{1} f(\delta) \int_{\theta_{zr}}^{1} \mathrm{d}\theta\,\mathrm{d}\delta - \int_{p_r/p_n}^{1-(p_n-p_r)/s} f(\delta) \int_{\theta_{rn}}^{1} \mathrm{d}\theta\,\mathrm{d}\delta. \tag{8}$$

The first term in (7) corresponds to the rectangle formed by θ values between θ_{zn} and 1, and δ values between 0 and $\frac{p_r}{p_n}$, where the latter is the intersection of the curves θ_{zn} and θ_{rn}. The second term in (7) corresponds to the area formed by θ values between the curve θ_{rn} and 1, and δ values between $\frac{p_r}{p_n}$ (the intersection of the curves θ_{zn} and θ_{rn}) and $1-\frac{p_n-p_r}{s}$, where the latter is the intersection of the curve θ_{rn} and $\theta = 1$. The first term in (8) corresponds to the area formed by θ values between the curve θ_{zr} and 1, and δ values between $\frac{p_r}{p_n}$ (the intersection of the curves θ_{zr} and θ_{rn}) and 1. The second term in (8) is the area related to the second term in (7).

Although a closed-form solution to (7) and (8) is possible for a simple continuous distribution of δ, such as the uniform, there is no closed-form solution when δ follows a (truncated) normal distribution, such as the one in Figure 1, which shows a good empirical fit.[1] Thus, the demand curves must be derived numerically: for each possible triple (p_n, p_r, s), the integrals in (7)-(8) are computed numerically to generate the corresponding $q_n(p_n, p_r, s)$ and $q_r(p_n, p_r, s)$.

Now, in the monopolist OEM case, the OEM solves the optimization model consisting of (7), (8) and (2)-(3). In the case where the OEM competes with the IR, the equilibrium is found by solving (7), (8), and (4)–(6). Given the fact that the demand functions themselves (7) and (8) cannot be found in closed-form, the respective solution and equilibrium need to be found numerically. We describe the experimental design and solution procedure for our numerical analysis next.

4. Experimental design and numerical solution approach

Experimental design. The numerical study consists of two parts, namely the case of the monopolist OEM, and the case of competition between the OEM and the IR. We consider a truncated normal distribution for the discount factor, following its good fit with the empirical discount factor distribution found by Abbey et al. (2017) for the iPhone 6, and shown in Figure 1. To analyze the impact of different values of $E[\delta]$, as well as the impact of different levels of variability in the distribution of δ, we consider five truncated normal distributions: truncated $N(0.3, 0.1^2)$, truncated $N(0.6, 0.2^2)$, truncated $N(0.3, 0.05^2)$, truncated $N(0.6, 0.1^2)$, and truncated $N(0.8, (\frac{0.8}{6})^2)$. The five curves provide three different levels of $E[\delta]$, namely 0.3, 0.6, and 0.8, and two levels of variability, with coefficients of variation (CV) equal to 1/3 and 1/6. In addition, we also consider the benchmark constant discount factor, that is, a CV of zero. Essentially, we use the minimum and maximum values of 0.3 and 0.8 for $E[\delta]$ in our design because they still allow us to use a truncated normal distribution with reasonable values of variability, considering that the distribution of discount factors has a support in [0,1]. In fact, for $E[\delta] = 0.8$, the CV value of 1/3 is not possible with a truncated normal distribution. We emphasize that the truncated normal distribution for δ has empirical support, as shown in Figure 1 (Abbey et al., 2017). Furthermore, we are not aware of any remanufactured product that sells with discounts higher than 70% relative to the price of a comparable new product; reported price discounts in the literature range between 15% and 55 % (Subramanian and Subramanyam, 2012, Hauser and Lund, 2003), and hence values for $E[\delta]$ below

[1] Abbey et al. (2017) show that even in case of a uniform δ distribution, demand functions are non-linear and optimal solutions for the simple monopolist OEM model *without quality choice* can only be derived numerically.

0.3 are unlikely to happen in practice. We present mainly the results for $E[\delta] = 0.6$ (as the results for $E[\delta] = 0.3$ and $E[\delta] = 0.8$ are similar), except where we report on the behavior of the solution with respect to $E[\delta]$. We select $E[\delta] = 0.6$ to report most of the results, because it is similar to the value found by Abbey et al. (2017) for the remanufactured iPhone 6, which is $E[\delta] = 0.57$.

In terms of unit cost for the new product, we normalize $\beta = 1$, as in Örsdemir et al. (2014). For the remanufactured product's unit cost as a fraction α of the new product's unit cost, which is necessarily less than one, we consider a wide range of possible values: $\alpha \in \{0.1, 0.2, ..., 0.9\}$. Henceforth, we refer to α simply as the relative unit cost of the remanufactured product.

For each of the above eight distributions for δ (including the constant δ case), and the nine possible values of α, we analyze both the monopolist OEM and competition with IR cases. The experimental design thus yields $8 \cdot 9 \cdot 2 = 144$ experimental cells. For each cell, we compute the benchmark solution of Örsdemir et al. (2014) (under constant discount factors), and the numerical solution under variable discount factors.

Numerical solution approach for monopoly case. For any given value of the quality level s, and considering the well-behaved objective function (2), we approximate the optimal price combination (p_n^*, p_r^*) by combining an eight-directional local search procedure with a best-improvement strategy. A step size of 10^{-6} was used to ensure sufficient precision. The initial feasible solution was found with a grid search on a coarse 200x200 grid of prices. For determining the optimal quality level we then performed a grid search over the range of $s \in (0, 1)$ to ensure a precision of three digits.

Numerical solution approach for competition between OEM and IR. For a given quality level s, we find the equilibrium prices by applying a two-stage process. First we converted the game (4) and (5) into a matrix game on a coarse grid of prices p_n and p_r and solved it (see, e.g., Belleflamme and Peitz, 2015). Then, we iteratively changed p_n (to improve π_{OEM}) and p_r (to increase π_{IR}) by local search to better approximate the intersection point of the best-response functions. For this purpose, we applied a step size of 10^{-6}. We then determined the quality level s that maximizes π_{OEM} by performing a grid search over the range $s \in (0, 1)$ to ensure a precision of three digits.

5. Numerical results: monopolist OEM

In this section, we consider the monopolist OEM's case. We adopt the superscript "F" (for "forecast") to denote the values of variables obtained by solving the benchmark constant discount

Table 2: Production strategy regions

α	production strategy regions for $E[\delta]=0.3$ CV$=\frac{1}{3}$			$E[\delta]=0.6$			$E[\delta]=0.8$	
	$E[\delta]=0.3$ CV$=\frac{1}{3}$	CV$=\frac{1}{6}$	CV$=0$	CV$=\frac{1}{3}$	CV$=\frac{1}{6}$	CV$=0$	CV$=\frac{1}{6}$	CV$=0$
0.1	P	P	P	T	T	T	T	T
0.2	P	P	P	T	T	T	T	T
0.3	P	P	N	T	T	T	T	T
0.4	P	N	N	P	P	P	T	T
0.5	N	N	N	P	P	P	T	T
0.6	N	N	N	P	P	N	T	T
0.7	N	N	N	P	P	N	P	P
0.8	N	N	N	P	N	N	P	N
0.9	N	N	N	P	N	N	P	N

factor model, and the superscript "opt" to denote the variable value obtained by solving the model with variable discount factors. The following formatting rule is applicable to all figures in our paper: When "CV $=\frac{1}{6}$" appears in the legend, it denotes the corresponding results obtained from cases where the discount factor follows the truncated N(0.6, 0.1^2) distribution; "CV $=\frac{1}{3}$" indicates that the discount factor follows the truncated N(0.6, 0.2^2) distribution, and "Const. δ" indicates the benchmark constant discount factor model with $\delta = 0.6$. In addition, solutions to the variable discount factor model are plotted with solid lines while solutions to the benchmark constant discount factor model are plotted with dashed lines.

5.1. *Monopoly: Comparison between constant and variable discount factor solutions*

Let $(s^{F*}, p_n^{F*}, p_r^{F*})$ denote the solution for the benchmark model that assumes constant discount factors, and $(s^{opt*}, p_n^{opt*}, p_r^{opt*})$ denote the solution for the model with variable discount factors.

5.1.1. *Comparison of production strategy regions*

Table 2 displays the production strategy regions for both constant and variable discount factor models. We use "T", "P" and "N" to respectively denote the scenarios of total remanufacturing ($q_r^* = q_n^*$), partial remanufacturing ($q_r^* < q_n^*$), and no remanufacturing ($q_r^* = 0$). Under constant discount factors, Örsdemir et al. (2014) show that the monopolist OEM remanufactures if $\alpha < \delta$, which is shown in Table 2 in the CV$=0$ columns. Under variable discount factors, we defined no remanufacturing when $q_r < 0.0001$ which is a precision of our numerical results. From Table 2, it is clear that remanufacturing (either partial or total) is optimal for a wider range of cost values α under variable discount factors. For example, consider $E[\delta] = 0.6$. Under constant discount factors

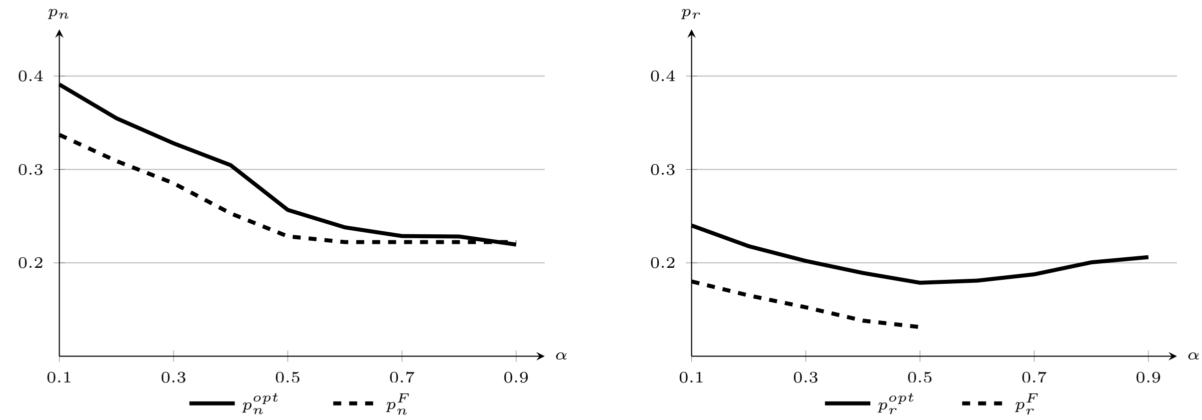

Figure 3: Impact of α on optimal prices p_n (left) and p_r (right) in the monopoly case ($E[\delta] = 0.6$, CV$= \frac{1}{3}$)

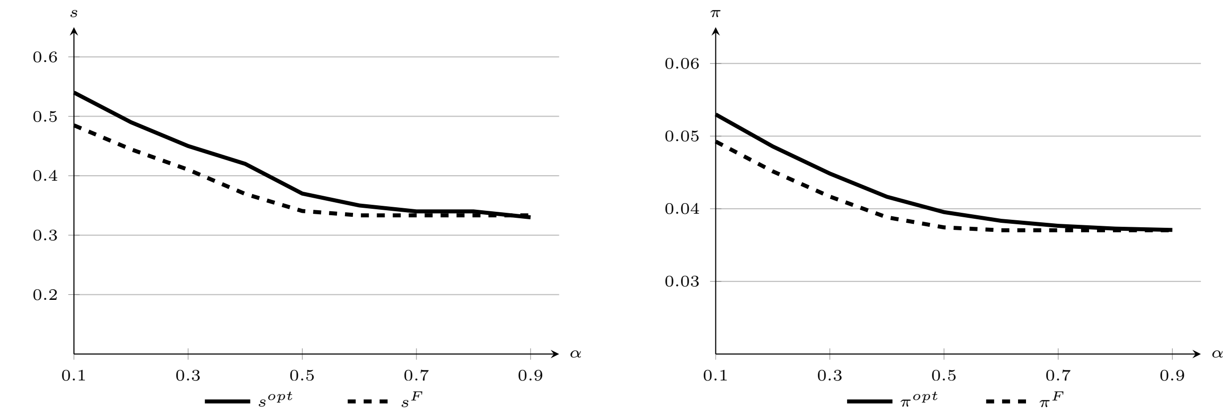

Figure 4: Impact of α on optimal quality level s (left) and profit π (right) in the monopoly case ($E[\delta] = 0.6$, CV$= \frac{1}{3}$)

(CV $= 0$), remanufacturing is not optimal for $\alpha \geq 0.6$, however, remanufacturing is always optimal when CV $= 1/3$.

5.1.2. Comparison of optimal values

Figure 3 plots the optimal prices at different levels of α for the two different models. The optimal price for the remanufactured product is always higher under variable discount factors than under the benchmark constant discount factor model. Note that in Figure 3 (right), there are no values for p_r^F plotted for $\alpha \geq 0.6$, because remanufacturing is not optimal under the benchmark model if $\alpha \geq \delta$. As a result, for $\alpha \geq 0.6$, the optimal new product prices are similar across models.

The optimal quality levels in Figure 4 (left) have a similar behavior as the new product prices, for both models. This is expected in the constant discount factor model, as Örsdemir et al. (2014) show that p_n^* is a quadratic function of s^*; we show here that this strong correlation between p_n^* and s^* also carries over to the variable discount factor model. Finally, Figure 4 (right) indicates that the profit π^{opt} decreases monotonically with α under variable discount factors, as remanufacturing always occurs in this case, and profits decrease as the volume of remanufacturing decreases due to higher unit costs. Under the benchmark constant discount factor model, the (forecasted) profit π^F decreases, as expected, with α until no-remanufacturing becomes optimal.

5.2. Comparison of profits achieved by the solutions proposed by the two models

In this subsection, we compare the actual profits that result from adopting the solutions from the two models. To find the impact of using the benchmark solution, computed under the assumption of constant discount factors, on actual (not forecast) profit, consider that the OEM (mistakenly) assumes constant discount factors, and uses the benchmark model to compute $(s^{F*}, p_n^{F*}, p_r^{F*})$. We then find the OEM's actual profit π^{RB} from using this solution in a world with variable discount factors. More precisely, π^{RB} is the profit computed using the solution $(s^{F*}, p_n^{F*}, p_r^{F*})$, but with the appropriate demand functions (7) and (8). The comparisons of the two solutions are displayed for all values of $E[\delta]$ and CV in Table 3, where we also include columns for the percent differences between the decision variables of the two models. That is, we also report on the percent deviations, defined as: $\Delta s^* = 100\% \cdot \frac{s^{F*} - s^{opt*}}{s^{opt*}}$, $\Delta p_n^* = 100\% \cdot \frac{p_n^{F*} - p_n^{opt*}}{p_n^{opt*}}$, $\Delta p_r^* = 100\% \cdot \frac{p_r^{F*} - p_r^{opt*}}{p_r^{opt*}}$, and $\Delta\Pi^* = 100\% \cdot \frac{\Pi^{RB*} - \Pi^{opt*}}{\Pi^{opt*}}$. Thus, the absolute value of $\Delta\Pi^*$ is the loss in profit when using the benchmark model as a heuristic to set optimal quality levels and prices. From Table 3 we point out the following: (i) The absolute percent differences in remanufactured product prices are almost always considerably higher than the absolute percent differences in new product prices; (ii) the absolute percent differences in quality levels move in the same direction as the percent differences in new product prices; and (iii) the absolute percent differences in profits almost always decrease in α and increase in $E[\delta]$. Overall, however, the results point to consistency between constant and variable discount factor models.

6. Numerical results: competition between the OEM and an IR

We now turn our attention to the numerical results for the case where the OEM competes with an IR: The OEM offers new products, whereas the IR offers remanufactured products.

6.1. Competition: Comparison between constant and variable discount factor solutions

Comparison of equilibrium regions. Under constant discount factors, Örsdemir et al. (2014) show the existence of four types of equilibria, depending on the ratio α/δ. Specifically, in decreasing

Table 3: Comparison of differences in decision making in the monopoly case

$E(\delta)$	CV	α	s^{opt*}	p_n^{opt*}	p_r^{opt*}	π^{opt*}	s^{F*}	p_n^{F*}	p_r^{F*}	π^{RB}	$\Delta s^*(\%)$	$\Delta p_n^*(\%)$	$\Delta p_r^*(\%)$	$\Delta\Pi^*(\%)$
0.3	1/3	0.1	0.37	0.25	0.08	0.038	0.38	0.26	0.06	0.038	2.4	3.3	-15.9	-2.0
0.3	1/3	0.2	0.34	0.23	0.08	0.037	0.34	0.23	0.06	0.036	-0.1	0.0	-25.4	-3.3
0.3	1/3	0.3	0.34	0.22	0.10	0.037	0.33	0.22	–	0.037	-0.5	-0.6	–	-0.2
0.3	1/3	0.4	0.33	0.22	0.11	0.037	0.33	0.22	–	0.037	-0.2	-0.2	–	> -0.1
0.6	1/3	0.1	0.53	0.39	0.24	0.053	0.49	0.34	0.18	0.040	-9.0	-12.9	-24.3	-24.1
0.6	1/3	0.2	0.49	0.36	0.22	0.049	0.44	0.31	0.17	0.037	-9.1	-13.1	-24.5	-24.1
0.6	1/3	0.3	0.45	0.33	0.20	0.045	0.41	0.29	0.15	0.034	-9.0	-13.0	-24.4	-24.1
0.6	1/3	0.4	0.42	0.30	0.19	0.042	0.37	0.25	0.14	0.034	-11.2	-15.9	-26.2	-19.2
0.6	1/3	0.5	0.38	0.26	0.18	0.040	0.34	0.23	0.13	0.036	-9.2	-12.5	-27.6	-9.6
0.6	1/3	0.6	0.35	0.24	0.18	0.038	0.33	0.22	–	0.037	-5.8	-8.0	–	-3.4
0.6	1/3	0.7	0.34	0.23	0.19	0.038	0.33	0.22	–	0.037	-2.5	-3.5	–	-1.6
0.6	1/3	0.8	0.34	0.23	0.20	0.037	0.33	0.22	–	0.037	-1.1	-1.5	–	-0.6
0.6	1/3	0.9	0.33	0.22	0.21	0.037	0.33	0.22	–	0.037	-0.2	-0.3	–	-0.1
0.3	1/6	0.1	0.37	0.25	0.07	0.038	0.38	0.26	0.06	0.038	2.1	3.1	-1.5	-0.2
0.3	1/6	0.2	0.34	0.23	0.07	0.037	0.34	0.23	0.06	0.037	0.5	0.8	-9.9	-0.6
0.3	1/6	0.3	0.33	0.22	0.08	0.037	0.33	0.22	–	0.037	-0.2	-0.2	–	> -0.1
0.6	1/6	0.1	0.50	0.36	0.20	0.049	0.49	0.34	0.18	0.043	-3.8	-5.8	-9.9	-12.0
0.6	1/6	0.2	0.46	0.33	0.18	0.045	0.44	0.31	0.17	0.040	-3.6	-5.5	-9.6	-12.0
0.6	1/6	0.3	0.43	0.30	0.17	0.042	0.41	0.29	0.15	0.037	-3.7	-5.7	-9.8	-12.0
0.6	1/6	0.4	0.38	0.27	0.16	0.039	0.37	0.25	0.14	0.038	-3.3	-4.9	-10.8	-3.2
0.6	1/6	0.5	0.35	0.24	0.15	0.038	0.34	0.23	0.13	0.037	-2.1	-2.9	-14.4	-2.2
0.6	1/6	0.6	0.34	0.23	0.16	0.037	0.33	0.22	–	0.037	-1.1	-1.4	–	-0.4
0.6	1/6	0.7	0.33	0.22	0.18	0.037	0.33	0.22	–	0.037	-0.2	-0.3	–	-0.1
0.8	1/6	0.1	0.59	0.41	0.34	0.061	0.55	0.37	0.28	0.047	-7.6	-10.2	-15.8	-23.1
0.8	1/6	0.2	0.53	0.37	0.30	0.056	0.50	0.34	0.26	0.043	-5.7	-8.5	-14.3	-23.1
0.8	1/6	0.3	0.49	0.34	0.28	0.052	0.46	0.31	0.24	0.040	-5.8	-7.6	-13.2	-23.1
0.8	1/6	0.4	0.45	0.31	0.25	0.048	0.43	0.29	0.22	0.037	-4.8	-6.6	-12.3	-23.1
0.8	1/6	0.5	0.42	0.29	0.24	0.045	0.40	0.27	0.21	0.035	-4.8	-6.9	-12.6	-23.1
0.8	1/6	0.6	0.40	0.28	0.23	0.042	0.38	0.26	0.19	0.032	-6.3	-8.9	-14.6	-23.1
0.8	1/6	0.7	0.37	0.26	0.21	0.040	0.34	0.23	0.18	0.034	-6.9	-10.1	-15.1	-15.0
0.8	1/6	0.8	0.35	0.24	0.21	0.038	0.33	0.22	–	0.037	-4.8	-6.7	–	-2.8
0.8	1/6	0.9	0.34	0.23	0.21	0.037	0.33	0.22	–	0.037	-2.0	-2.7	–	-0.7

order of α/δ, the equilibria are as follows. R1: the IR is not a threat and thus the OEM sets s^* as if it were a monopolist; R2: The OEM deters the IR from entering by setting s^* above the monopoly level; R3: The IR conducts partial remanufacturing ($q_r < q_n$); and R4: The IR conducts total remanufacturing ($q_r = q_n$). Table 4 displays the equilibria that occur, for both models. As the expected discount factor $E[\delta]$ decreases, or α increases, both models predict it to be more likely that the volume of remanufacturing by the IR decreases, from total to partial remanufacturing. As previously discussed for the monopoly case, variability in the discount factor δ makes remanufacturing more attractive (now, for the IR), and hence equilibrium regions R1 and R2 (no remanufacturing) take place less often under variable discount factors.

Table 4: Equilibrium regions

α	**Equilibrium regions** for							
	$E[\delta] = 0.3$			$E[\delta] = 0.6$			$E[\delta] = 0.8$	
	$CV = \frac{1}{3}$	$CV = \frac{1}{6}$	$CV = 0$	$CV = \frac{1}{3}$	$CV = \frac{1}{6}$	$CV = 0$	$CV = \frac{1}{6}$	$CV = 0$
0.1	R3	R3	R3	R4	R4	R4	R4	R4
0.2	R3	R3	R3	R4	R4	R4	R4	R4
0.3	R3	R3	R3	R4	R4	R3	R4	R4
0.4	R3	R3	R3	R4	R3	R3	R4	R4
0.5	R3	R2	R2	R3	R3	R3	R4	R4
0.6	R2	R2	R1	R3	R3	R3	R4	R3
0.7	R2	R2	R1	R3	R3	R3	R4	R3
0.8	R2	R2	R1	R3	R3	R2	R3	R3
0.9	R2	R1	R1	R3	R3	R2	R3	R2

Comparison of equilibrium values. We now present comparisons for the equilibrium values of the decision variables between variable and constant discount factors. Figure 5 plots the optimal quality levels as a function of the relative unit cost of the remanufactured product α for the two models. As discussed in the previous section, the equilibrium regions under constant discount factors change from R4 to R3 to R2 as α increases; thus the kinks in the curves. In addition, as shown by Örsdemir et al. (2014) under constant discount factors, the optimal quality level (also, new product price) increases in α in equilibrium region R3, decreases in α in region R2 (a higher quality level is necessary to deter the IR when remanufacturing is cheaper on a per unit basis), and it remains constant with α in regions R1 (IR is not a threat) and R4 (total remanufacturing by the IR). A similar pattern is observed under variable discount factors, except that equilibrium R2 does not take place.

Figure 6 plots the optimal prices as a function of the relative unit cost of the remanufactured product α for the two models. We observe from Figure 6 (left, bold lines) that, just like in the monopoly case, the optimal new product price follows the same pattern as quality level in Figure 5 (bold lines). The remanufactured product price also follows a similar trend as the new product price under both models. Figure 7 illustrates the corresponding equilibrium profits for the OEM (left) and IR (right) as a function of α for both models. Both models predict that the OEM's profit increases in α, while the IR's profit decreases in α, as a larger α makes the IR less competitive. Again, there is reasonable consistency between the constant and variable discount factor models.

6.2. *Comparison of profits achieved by the equilibria found by the two models*

Similarly to the analysis of section 5.2, we provide here an analysis of the impact of using the benchmark equilibrium, computed under the assumption of constant discount factors, on

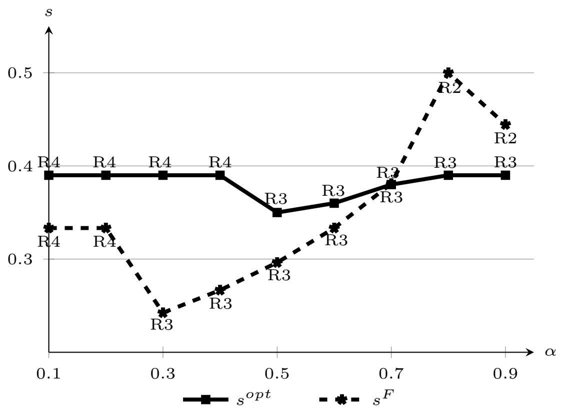

Figure 5: Impact of α on optimal quality level s in the competition case ($E[\delta] = 0.6$, CV$= \frac{1}{3}$)

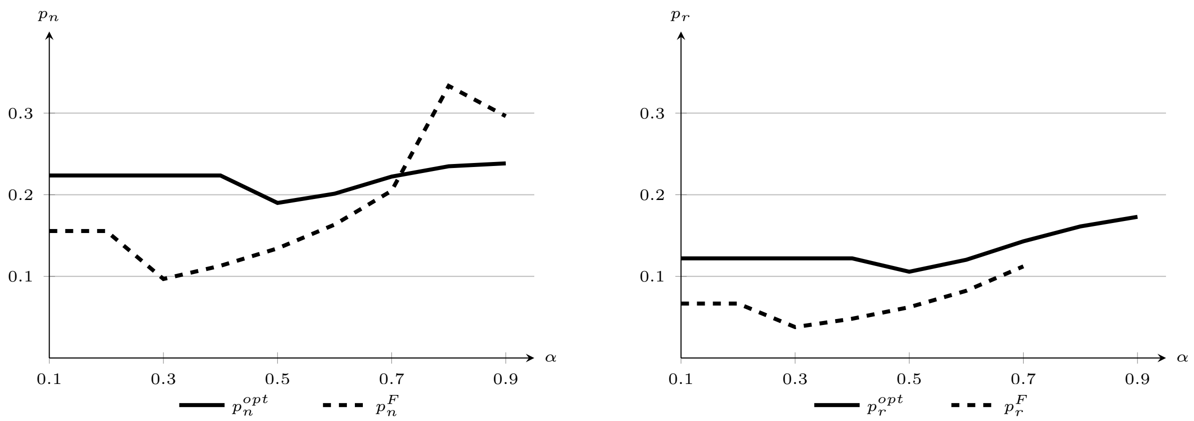

Figure 6: Impact of α on optimal prices p_n (left) and p_r (right) in the competition case ($E[\delta] = 0.6$, CV$= \frac{1}{3}$)

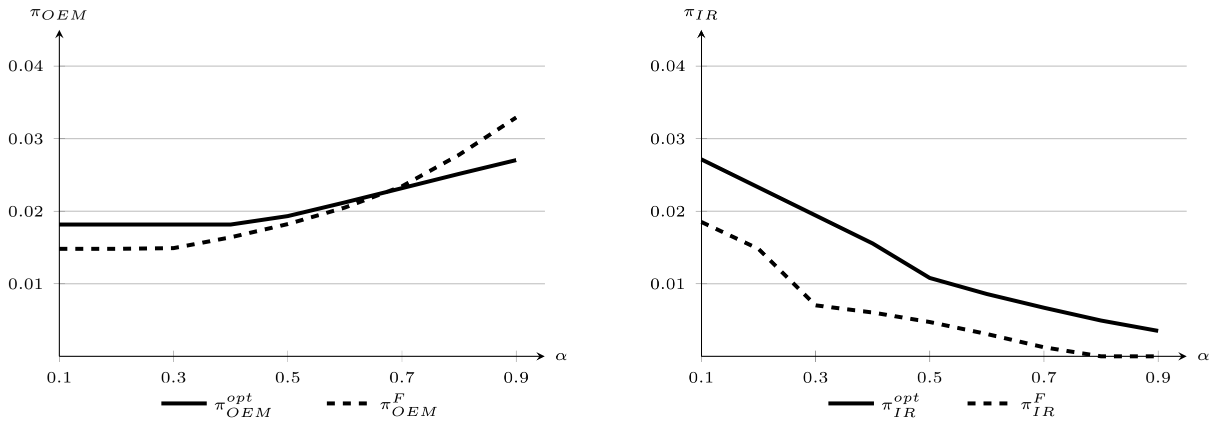

Figure 7: Impact of α on optimal OEM's (left)/IR's (right) profit in the competition case ($E[\delta]=0.6$, CV$= \frac{1}{3}$)

actual profits achieved by the OEM and IR. To that end, we compute the actual profit π_j^{RB}, $j \in \{OEM, IR\}$, that results from using the benchmark equilibrium values computed under constant discount factors $(s^{F*}, p_n^{F*}, p_r^{F*})$, but with the appropriate demand functions (with variable discount factors). We then compare these values against the equilibrium values under variable discount factors.

Table 5 displays the results of these comparisons for all levels of $E[\delta]$ and CV. We conclude the following. First, as $E[\delta]$ increases, the differences in prices, quality levels, and actual profits between the two equilibria increases. When consumers find remanufactured products more attractive, it becomes more important to correctly estimate the demand functions (through variable discount factors) to properly price new and remanufactured products. Second, the IR usually suffers the highest (percent) impact of incorrectly setting the prices, as its profits are lower than the OEM's, and hence are more sensitive to using the incorrect price. Third, there does not appear to be any pattern regarding the percent differences in prices, quality levels, and actual profits between the two models as the relative unit cost of the remanufactured product α increases. This is mostly due to the changes in equilibrium types (R1, R2, R3, and R4) as α increases, as discussed in the previous section.

7. Extensions

In this section we consider two model extensions to further strengthen the robustness of our results. First, we consider the case of convex collection cost. Second, we model a richer case of competition, where both the OEM and the IR can offer remanufactured products.

7.1. Convex collection cost

So far, we have abstracted from directly modeling collection costs; instead, such costs have been (indirectly) included in the unit remanufacturing cost α. In practice, any firm willing to remanufacture needs to exert costly effort to obtain the necessary cores. Collecting an extra unit may become more difficult with increasing volumes of collected cores, in some settings. As an example, a firm may have to turn to more remote consumer markets to obtain more cores when the local market has been exhausted. To model such settings, we consider an extra convex collection cost term $-\eta q_r^2$ in the objective function (2) for the monopoly case, and in (5) for the competition case, where η is a scaling parameter. This quadratic functional form is in line with the empirical observations by Atasu and Souza (2013) and Atasu et al. (2013) in some settings. We analyze both constant and variable discount factors numerically, using the same experimental design of section 4, with the additional collection cost parameter η in two levels: low ($\eta = 0.3$) and high ($\eta = 0.7$). We report here the results for an illustrative case below; other cases have similar behavior and insights.

Table 5: Comparison of differences in decision making in the competition case

$E(\delta)$	CV	α	s^{opt*}	p_n^{opt*}	p_r^{opt*}	π_{OEM}^{opt*}	π_{IR}^{opt*}	s^{F*}	p_n^{F*}	p_r^{F*}	π_{OEM}^{RB}	π_{IR}^{RB}	$\Delta s^*(\%)$	$\Delta p_n^*(\%)$	$\Delta p_r^*(\%)$	$\Delta\Pi_{OEM}^*(\%)$	$\Delta\Pi_{IR}^*(\%)$
0.3	1/3	0.1	0.29	0.16	0.03	0.026	0.006	0.29	0.16	0.03	0.025	0.006	0.6	1.6	-2.0	-1.1	2.6
0.3	1/3	0.2	0.31	0.17	0.04	0.028	0.004	0.31	0.18	0.04	0.027	0.004	0.4	0.7	-6.1	-2.9	-0.4
0.3	1/3	0.3	0.33	0.19	0.05	0.030	0.003	0.33	0.20	0.05	0.028	0.003	1.0	0.8	-8.9	-4.9	-8.3
0.3	1/3	0.4	0.35	0.22	0.07	0.032	0.002	0.36	0.22	0.06	0.030	0.001	2.6	2.0	-10.3	-6.8	-32.5
0.3	1/3	0.5	0.36	0.23	0.08	0.034	0.001	0.40	0.27	–	0.031	–	11.1	16.3	–	-9.3	–
0.3	1/3	0.6	0.36	0.23	–	0.035	–	0.33	0.22	–	0.032	–	-7.4	-5.2	–	-9.2	–
0.3	1/3	0.7	0.35	0.23	–	0.036	–	0.33	0.22	–	0.034	–	-4.8	-3.2	–	-4.8	–
0.3	1/3	0.8	0.34	0.22	–	0.037	–	0.33	0.22	–	0.036	–	-2.0	-0.8	–	-2.1	–
0.3	1/3	0.9	0.34	0.23	–	0.037	–	0.33	0.22	–	0.037	–	-2.0	-1.7	–	-0.8	–
0.6	1/3	0.1	0.39	0.22	0.12	0.018	0.027	0.33	0.16	0.07	0.012	0.015	-14.5	-30.4	-45.4	-31.9	-43.1
0.6	1/3	0.2	0.39	0.22	0.12	0.018	0.023	0.33	0.16	0.07	0.012	0.012	-14.5	-30.4	-45.4	-31.9	-46.9
0.6	1/3	0.3	0.39	0.22	0.12	0.018	0.019	0.24	0.10	0.04	0.012	0.007	-37.8	-56.7	-69.0	-32.1	-66.2
0.6	1/3	0.4	0.39	0.22	0.12	0.018	0.016	0.27	0.11	0.05	0.013	0.006	-31.6	-49.5	-60.6	-26.4	-59.6
0.6	1/3	0.5	0.35	0.19	0.11	0.019	0.011	0.30	0.13	0.06	0.015	0.006	-15.3	-29.3	-41.1	-24.7	-46.8
0.6	1/3	0.6	0.36	0.20	0.12	0.021	0.009	0.33	0.16	0.08	0.016	0.005	-7.4	-18.8	-31.6	-25.4	-46.0
0.6	1/3	0.7	0.38	0.22	0.14	0.023	0.007	0.38	0.20	0.11	0.017	0.003	0.3	-7.8	-21.5	-26.3	-59.9
0.6	1/3	0.8	0.39	0.23	0.16	0.025	0.005	0.50	0.33	–	0.017	–	28.2	41.9	–	-31.1	–
0.6	1/3	0.9	0.39	0.24	0.17	0.027	0.004	0.44	0.30	–	0.021	–	14.0	24.3	–	-24.1	–
0.3	1/6	0.1	0.29	0.16	0.03	0.026	0.006	0.29	0.16	0.03	0.026	0.006	0.6	1.2	2.3	0.6	1.8
0.3	1/6	0.2	0.31	0.17	0.04	0.028	0.004	0.31	0.18	0.04	0.028	0.004	0.4	0.8	0.9	0.2	1.2
0.3	1/6	0.3	0.34	0.20	0.05	0.030	0.002	0.33	0.20	0.05	0.030	0.002	-2.0	-2.9	-4.9	-0.9	-1.7
0.3	1/6	0.4	0.36	0.22	0.06	0.033	0.001	0.36	0.22	0.06	0.032	0.001	-0.3	-2.0	-5.9	-2.8	-16.4
0.3	1/6	0.5	0.36	0.23	–	0.035	–	0.40	0.27	–	0.033	–	11.1	15.2	–	-4.7	–
0.3	1/6	0.6	0.35	0.23	–	0.036	–	0.33	0.22	–	0.035	–	-4.8	-3.3	–	-4.4	–
0.3	1/6	0.7	0.34	0.23	–	0.037	–	0.33	0.22	–	0.037	–	-2.0	-1.5	–	-0.9	–
0.3	1/6	0.8	0.34	0.23	–	0.037	–	0.33	0.22	–	0.037	–	-2.0	-2.3	–	-0.1	–
0.3	1/6	0.9	0.33	0.22	–	0.037	–	0.33	0.22	–	0.037	–	1.0	1.3	–	0	–
0.6	1/6	0.1	0.38	0.20	0.09	0.015	0.023	0.33	0.16	0.07	0.013	0.017	-12.3	-20.6	-28.5	-10.4	-26.9
0.6	1/6	0.2	0.38	0.20	0.09	0.015	0.019	0.33	0.16	0.07	0.013	0.013	-12.3	-20.6	-28.5	-10.4	-28.4
0.6	1/6	0.3	0.38	0.20	0.09	0.015	0.015	0.24	0.10	0.04	0.014	0.007	-36.2	-50.6	-59.4	-9.3	-50.3
0.6	1/6	0.4	0.28	0.12	0.05	0.016	0.007	0.27	0.11	0.05	0.015	0.007	-4.8	-6.9	-10.6	-3.7	-5.9
0.6	1/6	0.5	0.31	0.14	0.07	0.018	0.006	0.30	0.13	0.06	0.017	0.005	-4.4	-7.2	-11.3	-5.5	-7.3
0.6	1/6	0.6	0.36	0.19	0.10	0.020	0.004	0.33	0.16	0.08	0.018	0.004	-7.4	-12.9	-17.9	-8.6	-15.5
0.6	1/6	0.7	0.39	0.22	0.13	0.023	0.003	0.38	0.20	0.11	0.020	0.002	-2.3	-6.8	-12.2	-12.5	-33.4
0.6	1/6	0.8	0.41	0.25	0.15	0.027	0.002	0.50	0.33	–	0.021	–	22.0	35.0	–	-20.6	–
0.6	1/6	0.9	0.41	0.26	0.17	0.030	0.001	0.44	0.30	–	0.025	–	8.4	15.8	–	-15.4	–
0.8	1/6	0.1	0.45	0.26	0.20	0.013	0.042	0.33	0.13	0.09	0.006	0.021	-25.9	-48.0	-55.5	-52.9	-50.5
0.8	1/6	0.2	0.45	0.26	0.20	0.013	0.038	0.33	0.13	0.09	0.006	0.018	-25.9	-48.0	-55.5	-52.9	-52.2
0.8	1/6	0.3	0.45	0.26	0.20	0.013	0.033	0.33	0.13	0.09	0.006	0.015	-25.9	-48.0	-55.5	-52.9	-54.4
0.8	1/6	0.4	0.45	0.26	0.20	0.013	0.028	0.33	0.13	0.09	0.006	0.012	-25.9	-48.0	-55.5	-52.9	-57.3
0.8	1/6	0.5	0.45	0.26	0.20	0.013	0.023	0.33	0.13	0.09	0.006	0.009	-25.9	-48.0	-55.5	-52.9	-61.4
0.8	1/6	0.6	0.45	0.26	0.20	0.013	0.018	0.22	0.07	0.04	0.006	0.004	-50.6	-73.5	-79.0	-52.7	-78.3
0.8	1/6	0.7	0.45	0.26	0.20	0.013	0.014	0.27	0.09	0.06	0.007	0.004	-40.7	-63.6	-68.9	-44.3	-71.1
0.8	1/6	0.8	0.41	0.22	0.17	0.014	0.009	0.33	0.14	0.10	0.008	0.003	-18.7	-36.6	-42.2	-39.1	-61.4
0.8	1/6	0.9	0.44	0.25	0.21	0.016	0.006	0.44	0.23	0.18	0.010	0.001	1.0	-6.6	-12.6	-39.3	-79.6

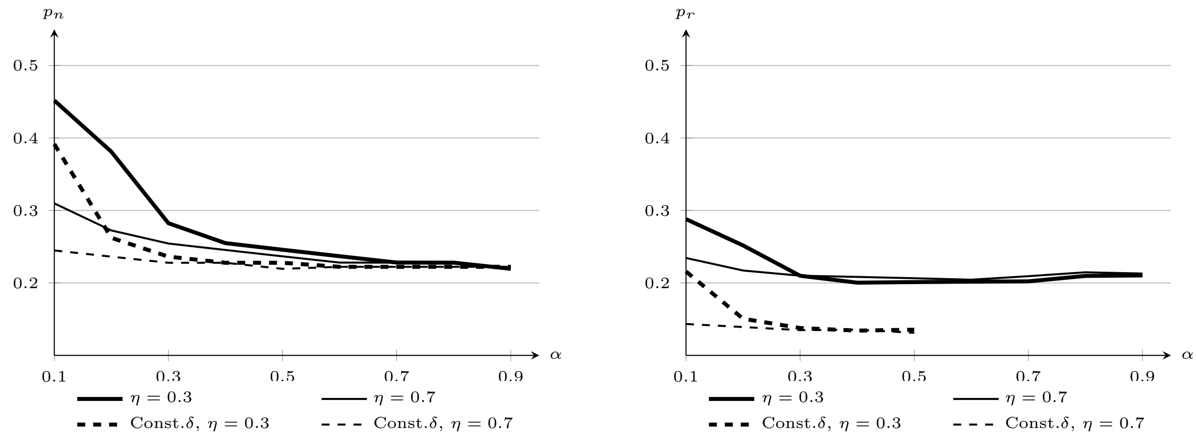

Figure 8: Impact of α on optimal prices p_n (left) and p_r (right) in the monopoly case with convex collection costs ($E[\delta] = 0.6$, CV $= \frac{1}{3}$)

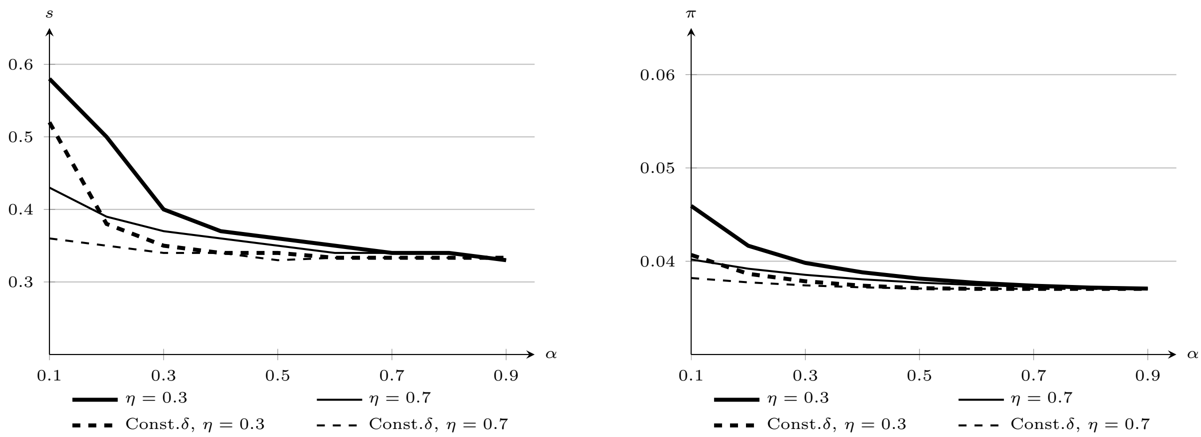

Figure 9: Impact of α on optimal quality level s (left) and profit π (right) in the monopoly case with convex collection costs ($E[\delta] = 0.6$, CV $= \frac{1}{3}$)

Monopoly case results. Figures 8 and 9 replicate the findings from Figures 3 and 4 for the monopolist OEM under the two levels of collection cost. Essentially, all the findings for p_n, p_r, s and π are structurally identical to those already discussed in Section 5.1.2 for both constant and variable discount factor models; as a result we omit the full set of results for brevity.

Competition case results. We now consider the setting where the OEM competes with an IR under convex collection costs. Figures 10 and 11 show the impact of α on the equilibrium decisions s, and (p_n, p_r), respectively, under constant and variable discount factors, for two values of η, low and high. It is clear that the equilibrium decisions move in the same direction for both constant and variable discount factor models, thus confirming the robustness of the constant discount factor model. We now turn to specific insights brought upon by adding convex collection costs.

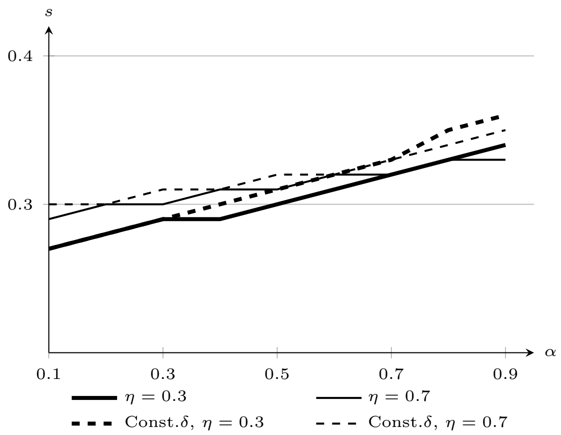

Figure 10: Impact of α on equilibrium quality level s in the competition case with convex collection costs ($E[\delta] = 0.6$, CV $= \frac{1}{3}$)

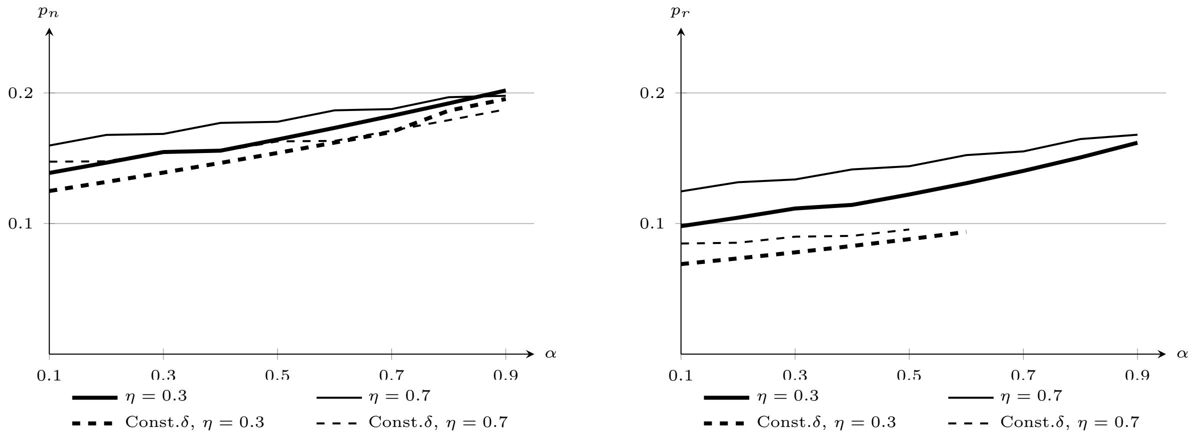

Figure 11: Impact of α on equilibrium prices p_n (left) and p_r (right) in the competition case with convex collection costs ($E[\delta] = 0.6$, CV $= \frac{1}{3}$)

In our previous analysis without convex collection costs, Figure 5 shows that the behavior of the equilibrium quality level s is non-monotonic with respect to the unit relative remanufacturing cost α, because there is a change in equilibrium regions from total to partial remanufacturing as α increases, under both variable and constant discount factor models. In Figure 10, where there are convex collection costs, quality (weakly) increases in α for both constant and variable discount factor models. The reason is, with convex collection costs, the total remanufacturing scenario ($q_r = q_n$) disappears for both constant and variable discount factor models. Because collection cost increases with q_r at an increasing rate, higher levels of remanufacturing become less and less attractive for the IR. Furthermore, in the analysis without convex collection costs, the equilibrium new product price (Figure 6, left) changes in the same direction as the equilibrium quality level (Figure 5) under

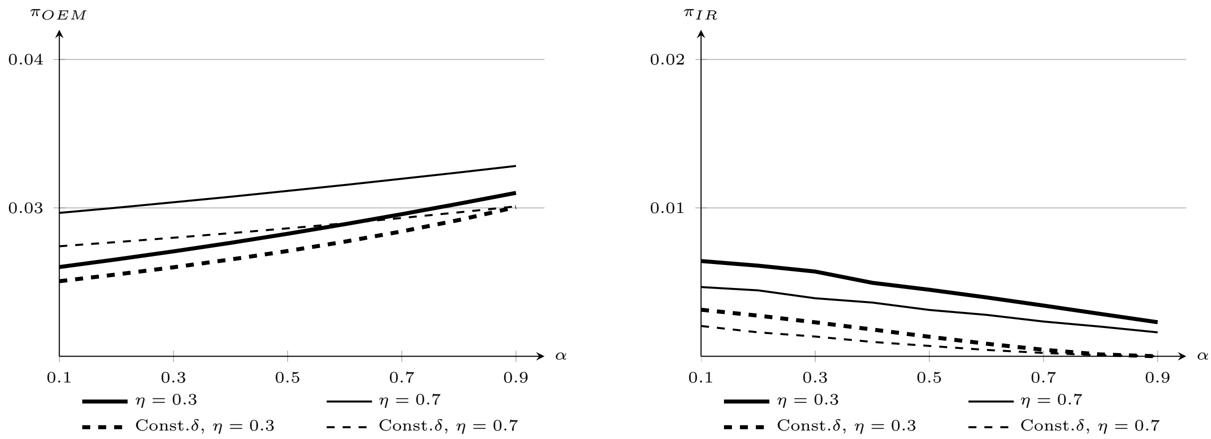

Figure 12: Impact of α on equilibrium OEM's profit π_{OEM} (left) and IR's profit π_{IR} (right) in the competition case with convex collection costs ($E[\delta] = 0.6$, CV $= \frac{1}{3}$)

both constant and variable discount factors. This also happens under convex collection costs, as shown in Figures 10 and 11 (left).

In Figure 6 (right), without convex collection costs, the IR does not remanufacture for $\alpha > 0.7$, whereas in Figure 11 (right), with convex collection costs, the IR does not remanufacture for $\alpha > 0.6$ (0.5) for $\eta = 0.3$ (0.7), under constant discount factors. The convex collection cost structure makes it less attractive for the IR to remanufacture. Under variable discount factors, however, Figures 6 and 11 indicate that the IR always remanufacture in this example. Again, variable discount factors make remanufacturing more attractive, which is in line with the insights from section 6.1, obtained without convex collection costs. Finally, Figure 12 confirms that, as the unit relative remanufacturing cost α increases, the OEM's profit increases, whereas the IR's profit decreases, under both constant and variable discount factors. In sum, the overal robustness of the constant discount factor model is confirmed under convex collection costs, for both monopoly and competition.

7.2. Extended competition: Remanufacturing by both OEM and IR

In this section, we extend our base competition model to also allow the OEM to remanufacture its own product and thus compete with the IR for remanufactured products. Given its analytical difficulty, this case has only been numerically analyzed by Örsdemir et al. (2014) under constant discount factors. In terms of notation in this section, the only change with respect to Table 1 is that the subscript r now refers to the remanufactured product by the OEM, whereas the subscript i

now refers to the remanufactured product by the IR. So, for example, p_i is the IR's remanufactured product's price. There is also a new parameter to differentiate consumer's WTP for the IR's remanufactured product relative to the OEM's, as we discuss next.

7.2.1. Demand functions

As before, a consumer of type (θ, δ) has a WTP for the new product θ, and has a WTP for the OEM's remanufactured product $\delta\theta$. That same consumer type, however, has a WTP for the IR's remanufactured product equal to $\gamma\theta\delta$, where $\gamma \in [0,1]$ is assumed to be *constant* across consumers. We thus assume that there is heterogeneity in WTP in consumers towards remanufactured products in general (given by δ), as before, and that remanufacturing by an IR implies an additional fixed discount on their WTP for remanufactured products. There is some empirical support for consumers preferring products remanufactured by an OEM as opposed to an IR. For example, Subramanian and Subramanyam (2012) show that remanufactured products sold by an OEM-authorized reseller command higher average prices than products remanufactured by an IR. To the best of our knowledge, however, there is no empirically-derived distribution of discount factors for both OEM and IRs, hence our simplifying assumption that γ is constant across consumers. This assumption also ensures *some* tractability in deriving demand curves as follows.

A consumer of type (θ, δ) derives net utilities $U_n = \theta s - p_n$ from a new product, $U_r = \delta\theta s - p_r$ from an OEM's remanufactured product, $U_i = \gamma\delta\theta s - p_i$ from an IR's remnuanufactured product, and $U_z = 0$ from buying nothing. Similarly to section 3.2, equating the net utilities from each pair of alternatives yields indifference curves (now six, as opposed to three): $\theta_{rn} = \frac{p_n - p_r}{(1-\delta)s}$, $\theta_{zr} = \frac{p_r}{\delta s}$, $\theta_{zn} = \frac{p_n}{s}$, $\theta_{in} = \frac{p_n - p_i}{(1-\gamma\delta)s}$, $\theta_{ir} = \frac{p_r - p_i}{(1-\gamma)\delta s}$, and $\theta_{zi} = \frac{p_i}{\gamma\delta s}$.

Demand functions under constant discount factors δ. First, consider the case where δ is constant across consumers. A consumer purchases a new product if $\theta > \max\left\{\frac{p_n - p_r}{(1-\delta)s}, \frac{p_n}{s}, \frac{p_n - p_i}{(1-\gamma\delta)s}\right\}$, the OEM's remanufactured product if $\max\left\{\frac{p_r}{\delta s}, \frac{p_r - p_i}{(1-\gamma)\delta s}\right\} < \theta < \frac{p_n - p_r}{(1-\delta)s}$, the IR's remanufactured product if $\frac{p_i}{\gamma\delta s} < \theta < \min\left\{\frac{p_n - p_i}{(1-\gamma\delta)s}, \frac{p_r - p_i}{(1-\gamma)\delta s}\right\}$, and nothing if $\theta < \min\left\{\frac{p_r}{\delta s}, \frac{p_n}{s}, \frac{p_i}{\gamma\delta s}\right\}$. An algebraic manipulation, similar to what was used to derive (1), results in the demand functions, which are omitted for brevity (details available upon request).

Demand functions under variable discount factors δ. Now, consider the case where δ is variable across consumers. Figure 13 graphically depicts the consumer regions for buying new products

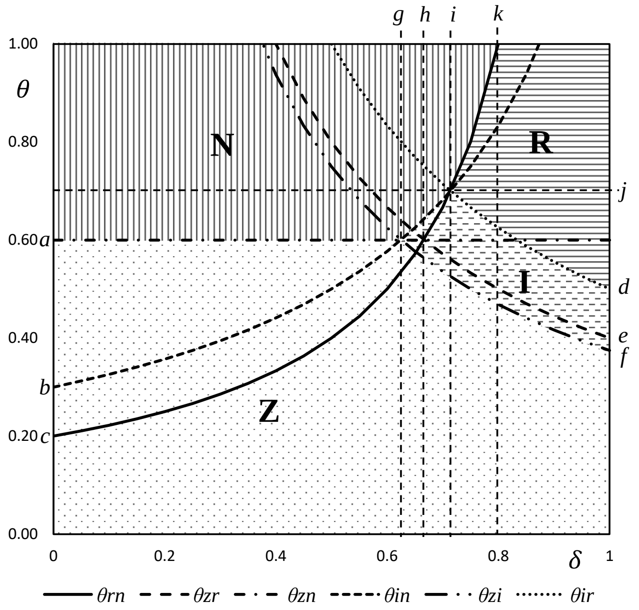

Figure 13: Consumer behavior in the extended competition case depending in θ and δ ($\gamma = 0.8$, $s = 1$, $p_n = 0.6$, $p_r = 0.4$, $p_i = 0.3$)

Table 6: The values of a-k in Figure 13.

$a = \theta_{zn}(0) = \frac{p_n}{s}$	$d = \theta_{ir}(1) = \frac{p_r - p_i}{(1-\gamma)s}$	$g = \frac{p_i}{\gamma p_n}$	$j = \frac{p_n}{s} + \frac{\gamma p_r - p_i}{(1-\gamma)s}$
$b = \theta_{in}(0) = \frac{p_n - p_i}{s}$	$e = \theta_{zr}(1) = \frac{p_r}{s}$	$h = p_r/p_n$	$k = 1 - \frac{p_n - p_r}{s}$
$c = \theta_{rn}(0) = \frac{p_n - p_r}{s}$	$f = \theta_{zi}(1) = \frac{p_i}{\gamma s}$	$i = \frac{p_r - p_i}{(1-\gamma)p_n + \gamma p_r - p_i}$	

(**N**), buying remanufactured products from the OEM (**R**) and the IR (**I**), and not buying (**Z**) in the plane (θ, δ). The values of a-k in Figure 13 are given in Table 6.

Under reasonable assumptions ($p_n > p_r > p_i$) it holds that $a > b > c$. Further on, if $p_i \geq \gamma p_r$ then $d \leq f$, the IR does not remanufacture ($q_i = 0$), and demand functions (7)-(8) apply (Case 1). If $p_i < \gamma p_r$ it holds that $d > e > f$, $g < h < i < k$, and $j > a$ (all functions and intersection points are ordered as in Figure 13) and the IR does remanufacture (Case 2). In Case 2, demand functions (using abbreviations from Table 6) become:

$$q_n(p_n, p_r, p_i, s) = \int_0^g f(\delta)\,\mathrm{d}\delta \int_a^1 \mathrm{d}\theta + \int_g^i f(\delta) \int_{\theta_{in}}^1 \mathrm{d}\theta\,\mathrm{d}\delta + \int_i^k f(\delta) \int_{\theta_{rn}}^1 \mathrm{d}\theta\,\mathrm{d}\delta, \tag{9}$$

$$q_r(p_n, p_r, p_i, s) = \int_i^1 f(\delta) \int_{\theta_{ir}}^1 \mathrm{d}\theta\,\mathrm{d}\delta - \int_i^k f(\delta) \int_{\theta_{rn}}^1 \mathrm{d}\theta\,\mathrm{d}\delta, \tag{10}$$

$$q_i(p_n, p_r, p_i, s) = \int_g^i f(\delta) \int_{\theta_{zi}}^{\theta_{in}} \mathrm{d}\theta\,\mathrm{d}\delta + \int_i^1 f(\delta) \int_{\theta_{zi}}^{\theta_{ir}} \mathrm{d}\theta\,\mathrm{d}\delta. \tag{11}$$

Again, we assume the supports of θ and δ to be in the range [0,1], or else truncated accordingly.

To model the strength of each party in core collection, we add a new parameter ρ to represent the fraction of total cores that are available to the OEM, so that $1-\rho$ is the fraction available to the IR. That is, $q_r(p_n, p_r, p_i, s) \leq \rho q_n(p_n, p_r, p_i, s)$, and $q_i(p_n, p_r, p_i, s) \leq (1-\rho) q_n(p_n, p_r, p_i, s)$. Thus, if $\rho > 0.5$ ($\rho < 0.5$), the OEM (IR) has an advantage in core collection. The game to be solved is similar to the game stated in (4)-(6), except that the demand functions are given by (9)-(11), the OEM's profit function has an additional term to represent its net revenues from selling its remanufactured product $q_r(p_r - \alpha\beta s^2)$, subject to its core availability constraint $q_r \leq \rho q_n$; finally, the IR's profit function is the net revenues from selling its remanufactured product $q_i(p_i - \alpha\beta s^2)$ subject to its core availability constraint $q_i \leq (1-\rho) q_n$. Here, we make the assumption, for simplicity of presentation and space purposes, that both OEM and IR have the same remanufacturing costs.

Numerical solution approach and experimental design. Under either constant or variable discount factors, the model needs to be solved numerically by a procedure similar to that described in section 4, with the additional complication that now the second-stage game between the OEM and IR has an *additional* decision variable for the OEM, p_r. This significantly increases computational time, as the search space for an equilibrium is now three-dimensional, as opposed to two-dimensional. In terms of experimental design, we use the same parameter values described in section 4, with the addition of two new parameters, γ, and ρ. For γ, we consider low and high values $\gamma \in \{0.6, 0.9\}$; the low value of $\gamma = 0.6$ represents a case of weak competition (values of γ lower than 0.6 are not interesting as the IR is not able to compete with the OEM), whereas the high value $\gamma = 0.9$ represents the case of a stronger IR. For the core availability parameter ρ, we consider three levels to represent scenarios where the OEM (IR) has an advantage in core collection $\rho = \frac{2}{3}$ ($\rho = \frac{1}{3}$), and where the OEM and IR are in equal position ($\rho = \frac{1}{2}$).

7.2.2. Results

Similarly to Figures 5, 6, and 7 from section 6.1, we use a representative case to illustrate the difference between equilibria for constant discount factors (superscript "F") and variable discount factors (superscript "opt"). They are shown in Figure 14 for the equilibrium prices p_n, p_r, and p_i, and in Figure 15 for the equilibrium quality level s, and profits, respectively. OEM's (IR's) variables are shown in bold (regular) lines. Figure 14 (left) shows that both constant and variable

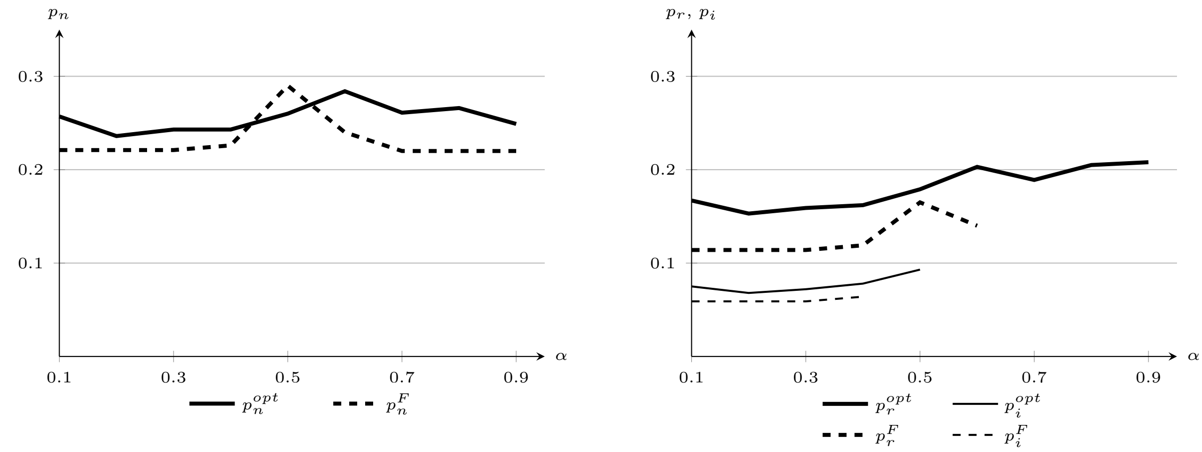

Figure 14: Impact of α on optimal prices p_n (left) and p_r and p_i (right) in the extended competition case ($E[\delta] = 0.6$, CV$= \frac{1}{3}$, $\gamma = 0.6$, and $\rho = \frac{1}{3}$)

discount factor models predict that α has little impact on new product prices, except when there is a change in the equilibrium region (from total to partial remanufacturing as α increases).

Figure 14 (right) shows that both constant and variable discount factor models predict that remanufactured product prices typically increase in α, with a potential decrease when there is a change from total to partial remanufacturing. Both constant and variable discount factor models also predict that the IR stops remanufacturing at a lower value of α than the OEM, due to valuation disadvantage $\gamma < 1$ in the eyes of consumers. Finally, in line with all previous findings, remanufacturing is more attractive under a wider range of α values under variable than under constant discount factors. Figure 15 (right) shows that the IR's profit decreases monotonically in α under both constant and variable discount factor models. Both constant and variable discount factor models also predict that the OEM's profit may increase in α at high enough values of α when the IR is being driven out of the market. In sum, the constant discount factor model demonstrates its reasonable robustness in this extended competition case as well.

Considering that the full analysis of such an extended competition model is beyond the scope of this paper (as our objective is to compare constant and variable discount factor solutions), we provide a quick analysis of the most impactful parameter values on this extended competition model through regression analyses as follows. We consider a restricted set of results from our experimental design, where we only have low and high values of each parameter: $E[\delta] \in \{0.3, 0.6\}$, CV $\in \{\frac{1}{6}, \frac{1}{3}\}$, $\gamma \in \{0.6, 0.9\}$, $\rho \in \{\frac{1}{3}, \frac{2}{3}\}$, and $\alpha \in \{0.3, 0.7\}$. This results in 32 different experimental cells in a full-factorial fashion, with solutions for both constant and variable discount factors. We then run

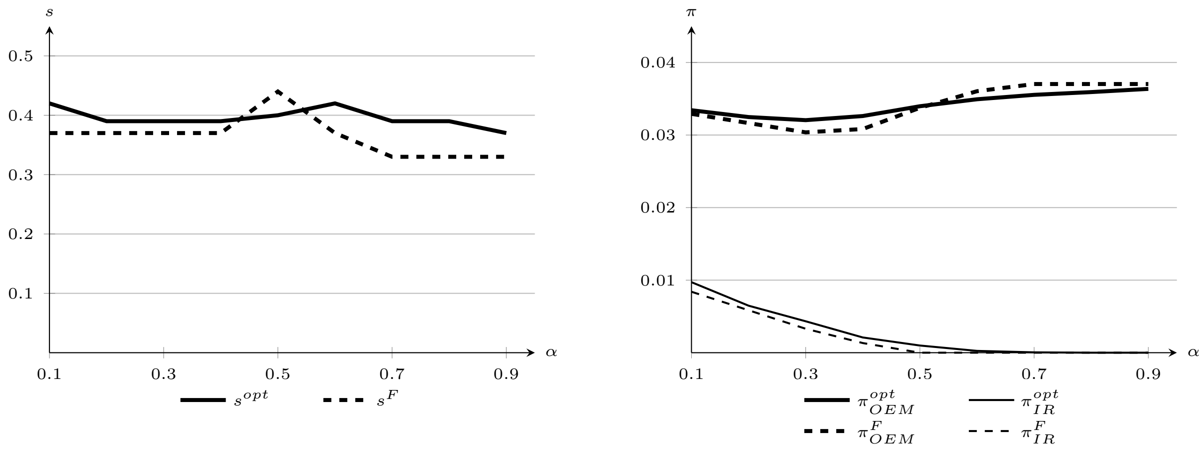

Figure 15: Impact of α on optimal quality level s (left) and π_{OEM} and π_{IR} (right) in the extended competition case ($E[\delta] = 0.6$, CV$= \frac{1}{3}$, $\gamma = 0.6$, and $\rho = \frac{1}{3}$)

six different regressions, where in each regression the dependent variable is a particular equilibrium value of the variable discount factor model (e.g., s^{opt*}), and the independent variables are the five parameters $E[\delta]$, CV, γ, ρ, and α. Despite the small sample size, the relative magnitude of the t-statistics for the respective regression coefficients in this high-low experimental design provides a measure of that variable's impact on the dependent variable (Wagner, 1980). Although a larger sample size may improve the accuracy of the regression coefficients (Kelley and Maxwell, 2003), our objective here is not to estimate the regression coefficients. Rather, our focus is on the *relative* impact of each parameter on the variable of interest, which is captured by the magnitude of a coefficient's t-statistic (not the coefficient itself) relative to those of other parameters, considering the simple high-low experimental design (Wagner, 1980). Likewise, we conduct five additional regressions where the dependent variables are now the the difference between constant and variable discount factor solutions as defined in section 5.2 (e.g., $\Delta s^* = 100\% \cdot \frac{s^{F*} - s^{opt*}}{s^{opt*}}$ is a dependent variable in a regression). We do not conduct a regression for $\Delta\Pi^*_{IR}$, as in some cases $\Pi^F_{IR} = 0$, which renders $\Delta\Pi^*_{IR}$ meaningless. The results are displayed in Table 7, where t-values for the intercepts are not displayed.

For s^{opt*}, the most impactful parameter is $E[\delta]$ (t-value = 1.61), although the impact is not significant at p $<$ 0.1 in this restricted 32-cell design, because the variation in equilibrium values of s^{opt*} is small across the design. For p_n^{opt*}, the most significant (negative) impact is from γ (t-value = -2.28), as a stronger IR (higher γ) means that the OEM must reduce its new product price to better compete with the IR. For p_r^{opt*}, the most significant impact is from $E(\delta)$ (t-value =7,

Table 7: Values of t-statistics for variable coefficients in regressions (one regression per column)

Variable	s^{opt*}	p_n^{opt*}	p_r^{opt*}	p_i^{opt*}	π_{OEM}^{opt*}	π_{IR}^{opt*}	$\Delta s^*(\%)$	$\Delta p_n^*(\%)$	$\Delta p_r^*(\%)$	$\Delta p_i^*(\%)$	$\Delta\Pi_{OEM}^*(\%)$
$E[\delta]$	1.61	0.20	7.00**	5.48**	-4.04**	4.01**	0.69	0.47	1.60	2.11**	-2.80**
CV	1.00	1.14	1.96*	1.26	-0.49	0.55	-0.39	-0.42	-0.96	-0.68	-1.35
γ	-1.17	-2.28**	-4.92**	1.49	-4.77**	1.87*	0.73	0.83	2.22**	1.66	-1.00
ρ	-1.00	-0.54	-0.86	-0.37	2.52**	-1.35	2.04*	2.20**	2.16**	2.28**	2.25**
α	-1.35	-0.44	4.58**	4.70**	1.23	-3.64**	0.88	0.88	-0.90	-0.77	5.28**

[*] p<0.1; [**] p<0.05

positive as a larger expected WTP means a larger price), followed by γ (t-value = -4.92), then α (t-value = 4.58), and CV (t-value = 1.96, positive as a larger CV means the OEM can price the remanufactured product higher, in line with the results of section 5.1). For p_i^{opt*}, the most significant impact is again from $E(\delta)$ (t-value=5.48), followed by α (t-value = 4.70, again positive as a larger cost means a larger price). The OEM's profit π_{OEM}^{opt*} has its highest (negative) impact from γ (t-value = -4.77), followed by $E(\delta)$ (t-value=-4.04), ρ (t-value = 2.52, positive as a higher ρ means a higher availability of cores for the OEM). The IR's profit, on the other hand, is mostly (negatively) impacted by $E(\delta)$ (t-value=4.01), followed by α (t-value = -3.64), γ (t-value = 1.87, as a higher WTP for IR remanufactured products increases IR profit). In terms of the deviations Δ of the equilibrium values between constant and variable discount factors, the most impactful parameter is ρ, which has a high positive t-value in all five regressions; this means that a higher value of ρ increases the deviation between constant and variable discount factor equilibrium values. All other parameters do not explain the differences, except for γ in Δp_r^*, and α in $\Delta\Pi_{OEM}^*$. In these cases, a higher γ (α) means a relative higher remanufactured product price (OEM's profit) forecasted by the constant discount factor model compared to the variable discount factor model.

8. Conclusion

This paper has studied the robustness of the constant discount factor assumption for remanufactured products—an assumption widely used in the prescriptive CLSC literature, particularly in strategic-level, stylized models—considering recent empirical challenges to it. In particular, we take as an input the approximately normal probability distribution of discount factors empirically derived by Abbey et al. (2017) for the iPhone 6, and use it in what is arguably the most complex stylized models in the CLSC literature: quality choice with remanufacturing, under monopoly and competitive settings. This problem has been studied by Atasu and Souza (2013) and Örsdemir et al. (2014) under constant discount factors. We also provide two significant extensions: convex collection costs, and an extended competition case, where both OEM and IR can offer remanufactured products, in addition to new products by the OEM. The quality choice model with variable discount factors is intractable and must be solved numerically.

Overall, we find reasonable consistency in the results between the variable and constant discount factor models, for both monopoly and competitive cases. We believe this paper makes a convincing argument that the constant discount factor assumption is appropriate for use in stylized strategic models involving remanufacturing. The variable discount factor model brings additional insights: (i) the remanufacturing firm optimally prices the remanufactured product higher than the price suggested under constant discount factors; (ii) a higher variability in the distribution of discount factors makes remanufacturing more attractive in a wider cost range, as a larger variability implies that there is a larger fraction of customers with WTP higher than the mean; and consequently (iii) profits are higher for the remanufacturing firm when the variability in discount factors is higher.

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