Original citation:
Tran, Trung Hieu, French, Simon, Ashman, Rhys and Kent, Edward (2018) *Linepack planning models for gas transmission network under uncertainty.* European Journal of Operational Research .doi:10.1016/j.ejor.2018.01.033

Linepack Planning Models for Gas Transmission Network

Trung Hieu Tran[1]*, Simon French[1], Rhys Ashman[2], Edward Kent[2]

[1] *Department of Statistics, The University of Warwick, Coventry, CV4 7AL, The United Kingdom*

[2] *National Grid House, Warwick Technology Park, Warwick, CV34 6DA, The United Kingdom*

Abstract

Open market transient behaviours create challenges for National Grid, the UK gas transmission network operator, in meeting limits on pressure and linepack, i.e. the quantity of gas in the network. In this paper, four mixed-integer linear programming models are proposed for the optimal linepack planning to compensate for the fluctuation of gas demand. The first model minimises total deviation between planned and targeted linepacks such that all the customer's demand and other network constraints are satisfied. The second model determines actions, including timings, to minimise total cost for resolving the gas deficit. We then extend this to a third model to deal with the periodical supply shortfall in the gas transmission network, and a fourth model to investigate the impact of compressor failure on the linepack management. The efficiency of these models is investigated and validated using real case study data. Experimental results show that our models can produce the optimal linepack plans under certain scenarios that current tools at National Grid cannot achieve.

Keywords: gas network; linepack planning; mixed-integer linear programming; supply shortfall; compressor failure.

1 Introduction

Nowadays, despite moves towards new energy sources to replace traditional ones, e.g., coal, petroleum, and other liquid fuels, it is clear that natural gas will continue to play an essential role in the foreseeable future. Gas produces lower carbon dioxide, the main cause of global warming, and there are abundant reserves. In addition, natural gas has a primary role in electricity generation. According to the International Energy Outlook 2016, world demand for energy will grow by 48% between 2012

*Correspondence author.

Email addresses: t.h.tran@warwick.ac.uk; trunghieu.tran@nationalgrid.com.

and 2040, and fossil fuels account for more than three-quarters of this. Natural gas is the fastest-growing fossil fuel with global consumption increasing by 1.9% per year. Therefore, efficient and effective gas distribution networks are a critical requirement for gas operators. Gas transportation networks involve three major subsystems, namely the gathering system, the transmission system, and the distribution system. Unlike the gathering system (from oil-shores to terminals) and the distribution system (from off-takes to customers) that are characterised by low pressure, small diameter pipelines, the transmission system (from terminals to off-takes) is characterised by long, a large diameter pipelines operated with high pressures. The efficient performance of the gas transmission system thus poses a challenge in maintaining the safe regulation of pressure such that gas demand at off-takes is satisfied. Due to dynamically changing demands on a daily basis, regulating pressure while satisfying demand is extremely challenging. In meeting these challenges, use of the pipeline network for short-term gas storage using the compressibility of gas in pipelines, referred to as *linepack* becomes essential to deal with the transient demand of customers.

Before we present research relevant to linepack management, we survey some more general research on gas networks. Firstly, for literature reviews of optimisation problems related to gas networks, we refer to Zheng et al. (2010) and Rios-Mercado and Borraz-Sanchez (2015). Zheng et al. (2010) focus on three specific aspects (e.g., production, transportation and market) with six general problems: production scheduling, maximal recovery, network design, fuel cost minimisation, and regulated and deregulated market problems. Their survey discusses mathematical formulations and existing optimisation methods for the problems in detail. Rios-Mercado and Borraz-Sanchez (2015) present the relevant research works in the natural gas transport industry, studying: short-term storage, pipeline resistance and gas quality satisfaction, and fuel cost minimisation. For the empirical models, the theoretical foundations and the applications of long-term basis storage, readers can refer to Holland (2008), Zwitserloot and Radloff (2009), Neumann and Zachmann (2009) and Holland (2010). Studies on pipeline resistance and gas quality satisfaction, can be found in Foulds et al. (1992), Adhya et al. (1999), Misener and Floudas (2009) and Menon and Menon (2013). Fuel cost minimisation is discussed in Wu et al. (2000), Zhu et al. (2001), Herran-Gonzalez et al. (2009) and Woldeyohannes and Majid (2011). Although these surveys address applications of optimisation theory on the natural gas transmission and storage to satisfy contractual demands, there exists a very limited literature on efficient models and solution techniques for linepack problems. Linepack problems may be viewed from two perspectives: managerial and mathematical programming.

From the managerial perspective, for example, Welch et al. (1971) propose using other fuels to handle seasonal peak demands. Their objective function is the minimisation of a number of scheduled interruptions whenever gas flow breaks down. Their results show that large industrial contracts are one of major reasons for the peak demand. Contesse et al. (2005) study on the impact of changes in the gas industry regulatory system to demand fluctuations and propose contractual strategies based on the use of storage facilities to deal with the demand fluctuations. Two types of contracts are considered: (i) a sale customer contract on a supply uninterruptible basis where customers may pay a lower price for their gas supply shortened during the period of peak demand and (ii) a firm transportation contract where shippers are allowed to store a portion of total delivery capacity of the pipelines for their own use.

From the mathematical programming perspective, some approaches to linepack problems are offered in De Nevers and Day (1983), Carter et al. (2003), Krishnaswami et al. (2004), Abbaspour et al. (2007), Midthun (2007), Krishnaswami and Haugland (2008) and Borraz-Sanchez (2010). De Nevers and Day (1983) examine the ability of inventory of gas in pipeline networks to satisfy time-varying demands with supplies in a transient-state gas transmission system. The authors conduct a model on two dimensionless parameters for packing and drafting behaviours. Their model is limited in a single pipeline segment. Carter et al. (2003) present several control strategies to operate pipeline networks during the period of fluctuating demands. Their objective is to find the optimal linepack schedules under uncertain demand assumptions. As a result, a number of possible scenarios with specific schedules for modifying the set-point values of compressor stations are determined. Krishnaswami et al. (2004), and Abbaspour et al. (2007) later, introduce a systematic approach for pressure optimisation of the units of a compressor station to meet a specified linepack profile along transient, non-isothermal pipeline networks. The authors construct an implicit finite difference model for a flow capacity analysis, and then formulate a non-linear programming model in order to minimise the average fuel consumption rate of each compressor station over a given planning horizon. The non-linear model may be solved by applying a sequential unconstrained minimisation technique using a directed grid search method for unconstrained subproblems. However, due to the complexity of optimisation problem, this model is only capable for a linear (gun-barrel) pipeline network system with two compressor stations composed of three compressor units each. Midthun (2007) presents a short-term portfolio optimisation model for a large natural gas producer. The model considers spot market sales, production plans, storage management and fulfillment of long-term contracts. It aims to maximise profit for the gas producer based on actively using the storage capacity provided by the linepack of the pipelines. A stochastic model is also developed to investigate the problem under various scenarios (e.g., stochastic prices and take-or-pay quantities). A test case from the Norwegian continental shelf is used to demonstrate the performance of the models. However, these models are only applicable for a steady flow network (i.e., daily resolution), but not for a transient flow network (i.e., hourly resolution). Based on a concept of 'homogeneous' gas batches introduced in Carter et al. (2003), Krishnaswami and Haugland (2008) propose a mathematical formulation to deal with linepack levels for a pipeline network under steady state conditions. The authors assume that all gas sources supply the same gas quality. Hence, there are no quality constraints on the gas that is transported and delivered. They then use a number of batches (i.e., gas packages) inside the pipeline network for the future scheduled withdrawal. A blending process between the batches inside the pipeline network seems to be unrealistic unless a long lasting shortfall in downstream capacity takes place. Building on Krishnaswami and Haugland (2008), Borraz-Sanchez (2010) develop a mixed-integer non-linear programming model and a global optimizer-based mathematical programming method for large-scale gas transmission networks under steady state conditions. Borraz-Sanchez introduces 'heterogeneous' batches, i.e. gas packages of possibly different compositions, for a multiple-time period planning horizon, which allows for gas sources with different qualities. He assumes that no blending process among batches takes place inside the pipeline network, which is a rather common practice in the gas industry. Also, he assumes a specific gas quality at the sources, and satisfies the gas quality imposed at the terminals. Here, several gas streams of different composition may be blend at junction points of the network in order to meet the quality requirements. Furthermore, the model can keep track of energy content and gas quality to ensure that contract terms are met. A simple numerical example with two compressor stations (i.e., three identical units in each) on a linear topology network is used to validate the model. Once again, however, because of the complexity of optimisation problem the model encounters difficulties in dealing with real large-scale complex gas transmission networks.

The role of linepack management becomes more important when electricity generation and distribution is alongside gas supplies and transmission, optimising the gas and electric power systems in and integrated fashion. Chaudry et al. (2008) introduce a non-linear programming model for multi-time period combined gas and electricity network optimisation. The model considers the varying nature of gas flows, the gas storages and the power ramping characteristics of electricity generation units. The objective function is to minimise total costs associated with gas supplies, linepack management, gas storage operation, electricity generation and load shedding. The authors test the model's performance on the UK gas and electricity network in the steady-state condition (i.e., daily resolution). Two cases, i.e., loss of Bacton terminal without or with Rough storage facility, are investigated to demonstrate the consequences of failure to vital facilities on the combined network. In the model, although total non-electrical gas demand (e.g., residential, commercial and industrial) is much larger than total electrical gas demand, the non-electrical gas demand at off-take points is not concerned. Integrating the off-take points' gas demand may pose a challenge in the linepack management. In addition, the non-linear model cannot be solved optimally due to the lack of efficient optimisers. The existing optimisers only produce local optimisers . Moreover, since the computation time of this model is about 10 minutes, it may be much time-consuming for analysing all possible scenarios to determine the most important facility on the combined network. Qadrdan et al. (2010) extend the model to investigate the potential impact of large amounts of wind generation on the UK gas network in the transient-state condition (i.e., hourly resolution). The authors build three case studies: one case is based on the existing network and the other two cases are based on a hypothesized network in 2020 in which two distinct levels of wind generation (i.e., low and high wind periods) are considered. To reduce the network complexity and simulation run time, the authors simplify the UK gas and electricity network. The simplification of the gas network is based on gas terminals and storage facilities, while the simplification of the electricity network is based on the electrical bus-bar connection. Hence, the number of gas terminals, storage facilities and electrical buses is kept the same as the original network. The simulation results show that the UK gas transmission network system operator is incentivised to balance the volumes of gas delivered to and withdrawn from the system at the end of each day, known as the standard end of day gas linepack incentive.

In other words, the hourly linepack monitoring and management is very important. The hourly non-electrical gas demand at off-take points then needs to be considered in the model, which is not really concerned in the model of Qadrdan et al. (2010). In addition, the impact of gas prices is neglected in their case studies, while the hourly change of gas prices plays an important role for the cost minimisation problem in a dynamic environment.

Keyaerts (2012) studies on the regulation of linepack flexibility and investigates the economic value and costs of linepack flexibility to balancing gas networks. In particular, the impact of its regulation on the competitive and non-competitive gas-market activities is assessed and the economic consequences of the trade-offs between the transport function and the flexibility function of the pipelines in the context of the European liberalizing gas market is analysed. In addition, the author explores the impact of the increasingly unpredictable electricity-generation system (e.g., wind power) on the balancing of the gas system. A mixed-integer non-linear programming model is formulated to minimise the balancing costs (e.g., operational costs, linepack costs, and procurement and dispatching costs) such that all the physical and technical constraints are satisfied. The model was verified for simple test cases (i.e., two periods, two nodes and one pipeline) for which analytical solutions could be computed. For the large and complex networks, the applicability and feasibility of the model has not been verified yet. Moreover, there is no optimizer that can guarantee for obtaining the global optimal solution of the non-linear model. Some other issues are also studied such as the effect of balancing-mechanism design and network regulation in multi-nation context (namely, cross-border balancing) and the cross-border procurement of balancing services.

Recently, researchers have started to consider other perspectives in the linepack operation and management. For example, Arvesen et al. (2013) study the benefit of using linepack as a short-term gas storage and how the linepack storage may offset the imbalance between the low flexibility of take-or-pay contracts and the high inherent flexibility of a gas-fired power plant. In particular, they investigate storage choices for a cycling power plant that faces volatile power prices while purchasing gas on a take-or-pay contract. Applying least-square Monte Carlo simulation, the results show that the option of linepack storage affects the plant significantly, especially during extreme price fluctuations. On the other hand, the volatility of power price and jump frequency are the major drivers for increasing the storage size. Chebouba (2015) addresses linepack management of the "GZ1 Hassi R'mell-Arzew" gas pipeline network with two objective functions: minimisation of the fuel consumption in compression stations and maximisation of gas linepack. A multi-objective decision-making technique is developed to find a balance between two objectives with an acceptable linepack management for the gas pipeline network, and a numerical method applied to analyze the flows in the pipeline network under transient isothermal conditions. The solver NSGA-II is then used for the multi-objective optimisation.

One of the causes of significant impacts on linepack operation and management is supply shortfall, which can be caused by human conflicts, natural disasters or unexpected disruptions (Carvalho et al., 2014). Carvalho et al. (2014) introduce a model to deal with network congestion on various geographical scales. They propose a resilient response strategy to energy shortages and evaluate its effectiveness in a variety of scenarios. As a result, with the fair distribution strategy Europe's gas supply network can be robust even to major supply disruptions. Olanrewaju et al. (2015) build a linear programming model to investigate the impact of the Ukraine transit capacity's loss on gas supply from Russia to Europe. The model is tested in two demand scenarios, e.g., a low-demand case and a high-demand case in the winter of 2014/2015. The results show that gas sources from inter-connectors, storages and liquefied natural gas import terminals compensate for the supply shortfall. To mitigate the effect of supply shortage, the author also considers increasing the capacities of selected pipelines within the Europe against enhancing the maximum storage withdrawal rates in southeast Europe. The comparison result concludes that the high storage withdrawal rates can obtain the lower demand curtailment than extending the inter-connector capacity in both scenarios.

In the other hand, the linepack operation and management poses challenge as compressor units are failed. These compressor unit failures affect to the capacity of compressor stations, which can cause the change of linepack planning, as well as actions and timings to minimise total cost for the gas deficit. Praks et al. (2017) develop a Monte Carlo simulation-based approach to investigate component disruptions (e.g., pipelines, terminals and compressor stations) in the European gas transmission network. The authors design a vulnerability identification algorithm to determine a combination of component failures leading to the most significant security of supply disruptions. In the simulation, they do not consider the operational configuration of compressor units in stations (i.e., serial, parallel or both). The simulation approach is also time-consuming as the number of components in the gas transmission network becomes significant. Moreover, it does not concern in the linepack operation and management in the gas transmission network.

In summary, several strands of research have been devoted to various perspectives on the linepack problem in the gas transmission network. However, how to produce an optimal linepack plan for a large-scale gas transmission network has not received attention in the literature. National Grid operates a complex and large-scale gas transmission network in the UK that includes pipelines, compressor stations, regulators, valves and other components (unlike the previous works of Chaudry et al. (2008) and Qadrdan et al. (2010), National Grid does not integrate storage facilities into the linepack management in the UK gas transmission network). They have experienced challenges in operating gas flow in order to meet targeted linepacks such that the regulation of pressure is maintained satisfactorily while meeting customer demand at off-take points. To address this issue, a network reduction technique is applied to transform the original network into a reduced network by aggregating sets of demand nodes among compressor stations into demand zones, which is different from the simplification of the combined gas and electricity network proposed by Qadrdan et al. (2010) as based on gas terminals, storage facilities and electrical bus connection. A mixed-integer linear programming (MILP) model is then built on this reduced network to find an optimal linepack plan for the aggregated zones. Unlike the above-mentioned works, the gas transmission time between zones is considered in the MILP model. The objective function is minimisation of total deviation between planned and targeted linepacks for the zones such that all constraints are satisfied. If the linepack targets cannot be met, we consider drawing commercial services into the model in which we buy additional gas at supply nodes to resolve gas deficits. A MILP model with commercial services is then developed in order to determine the time, location and amount of additional gas to minimise total cost for the linepack deficit. Our model is extended to deal with the linepack problem with supply shortfall and compressor failure for National Grid as well. Numerical experiments have carried out on a case study from National Grid to evaluate and validate the efficiency and effectiveness of our models.

The remaining of this paper is organised as follows. Section 2 describes the details of the UK gas transmission network, and how to transform the original network into an associated reduced network. Section 3 presents the MILP models for planning optimal linepack zones under four scenarios, namely without commercial services, with commercial services, with commercial services under supply shortfall, and with commercial services under compressor failure. The case study at National Grid used to evaluate these models is presented in Section 4. The computational results are discussed in Section 5. Finally, conclusions and future work are provided in Section 6.

2 The UK Gas Transmission Network

The UK gas transmission network consists of about 7,000 km pipes, 24 compressor stations, each of which comprises several compressor units in serial and/or parallel operation, 6 major terminals, 8 storage sites, more than 200 exit points, and other components (e.g., regulators and valves). Figure 1 shows the pipeline network to transmit gas from terminals to exit points. This complex, large-scale network poses many challenges to National Grid in meeting the demands of its customers. Modelling and optimizing such a large-scale network requires much effort. To overcome the issue, we apply a network reduction technique which was introduced by Rios-Mercado et al. (2002). Sets of supply and/or demand nodes bounded by compressor stations are aggregated into zones. In this case, we obtain 40 zones. Table 1 shows the list of compressor stations and their label. The list of aggregated zones and the information of zonal type (i.e., supply or demand) are provided in Table 2.

To model linepack planning problems at National Grid, we need to introduce some additional nodes in the associated reduced network. Figure 2 shows that green and red nodes represent supply and demand zones respectively. For each supply zone, there is a corresponding commercial service point (violet node) where the operator can buy additional gas to offset the gas deficit in the network; while for each demand zone, there is a corresponding linepack zone (blue node). The compressor stations are represented by orange nodes. Our reduced network then includes 9 supply zones (denoted by green nodes 1-9), 31 demand zones (denoted by red nodes 10-40), 24 compressor stations (denoted

Figure 1: The UK gas transmission network (National Grid source).

Table 1: List of compressor stations.

Label	Comp. Station	Label	Comp. Station	Label	Comp. Station
ABE	Aberdeen	CHU	Churchover	LOC	Lockerley
ALR	Alrewas	DIS	Diss	LON	Longtown
AVO	Avonbridge	FEL	Felindre	MOF	Moffat
AYL	Aylesbury	FER	St Fergus	PET	Peterborough
BIS	Bishop Auckland	HAT	Hatton	WAR	Warrington
CAM	Cambridge	HUN	Huntingdon	WIS	Wisbech
CAR/NEK	Carnforth/Nether Kellet	KIL	Kings Lynh	WOO	Wooler
CHE	Chelmsford	KIR	Kirriemuir	WOR	Wormington

Table 2: List of aggregated zones.

Zone	Compressor Stations	Type	Zone	Compressor Stations	Type
1	(•, FER)	Supply	21	(CAR/NEK, WAR)	Demand
2	(BIS, CAR/NEK, HAT)	Supply	22	(WAR, ALR)	Demand
3	(•, HAT)	Supply	23	(ALR, CHU)	Demand
4	(KIL, CAM, DIS)	Supply	24	(ALR, PET)	Demand
5	(CAM, CHE)	Supply	25	(HAT, PET)	Demand
6	(LOC, AYL)	Supply	26	(PET, CHU)	Demand
7	(•,FEL)	Supply	27	(PET, HUN)	Demand
8	(WAR, ALR)	Supply	28	(PET, WIS)	Demand
9	(LON, CAR/NEK)	Supply	29	(HAT, WIS, HUN)	Demand
10	(FER, ABE)	Demand	30	(WIS, KIL)	Demand
11	(ABE, WOO)	Demand	31	(KIL, CAM, DIS)	Demand
12	(ABE, KIR)	Demand	32	(DIS, CHE)	Demand
13	(KIR, AVO)	Demand	33	(CAM, CHE)	Demand
14	(AVO, MOF)	Demand	34	(HUN, CAM)	Demand
15	(AVO, WOO)	Demand	35	(HUN, AYL, CAM)	Demand
16	(MOF, LON)	Demand	36	(AYL, LOC)	Demand
17	(WOO, BIS)	Demand	37	(LOC, WOR)	Demand
18	(BIS, LAN)	Demand	38	(CHU, WOR)	Demand
19	(LON, CAR/NEK)	Demand	39	(WOR, FEL)	Demand
20	(BIS, CAR/NEK, HAT)	Demand	40	(•, FEL)	Demand

by orange nodes with labels of compressor stations), 31 linepack zones (denoted by blue nodes 41-71), and 9 commercial service points (denoted by violet nodes 72-80). The possible directions of gas flows among zones in the reduced network are described in Figure 2.

A linepack zone operates as a temporary storage of gas to satisfy both customer demand in its zone and to resupply adjacent linepack zones as gas is taken from them. Gas demands on linepack zones arise both because demand may exceed supply over short time periods and because it takes time for gas from distant supply zones to transit the network to reach their assigned demand. Compressor stations direct the gas flows between linepack zones, and from supply zones into the network. If total demand is higher than total supply, i.e. there is a gas deficit that cannot be met within planned linepack targets, additional gas from commercial service points can be ordered. Decisions on the timing, location and quantities of gas bought depend on the price offered at each commercial service point. These decisions are critical for National Grid when supply deficits occur. If serious disruption to gas flows from supply zones (or compressor failure at stations) occurs and if there is a significant increase in demand within a zone, careful planning and use of linepack storage can minimise the use of commercial service points and save costs while still meeting demand.

3 Linepack Planning Models

We present four linepack planning optimisation models to support the management of gas volumes in linepack zones. The first model seeks an optimal linepack plan that minimises total deviation between planned and targeted linepack; the second model minimises the total cost of buying additional gas to offset the total deviation from planned linepack targets; the third model handles the linepack planning problem under supply shortfall; and the fourth model deals with the problem under impact of unexpected compressor failure. In these models, we assume that possible maximum velocity of gas at National Grid (e.g., 20 m/s) is used in solving the linepack planning problems, and that pressure limits can be ensured by appropriate capacity constraints.

3.1 Notation

To formulate the linepack planning problems, we introduce some notations of sets, decision variables, and parameters as follows (hereafter, million cubic meters is referred to as mcm).

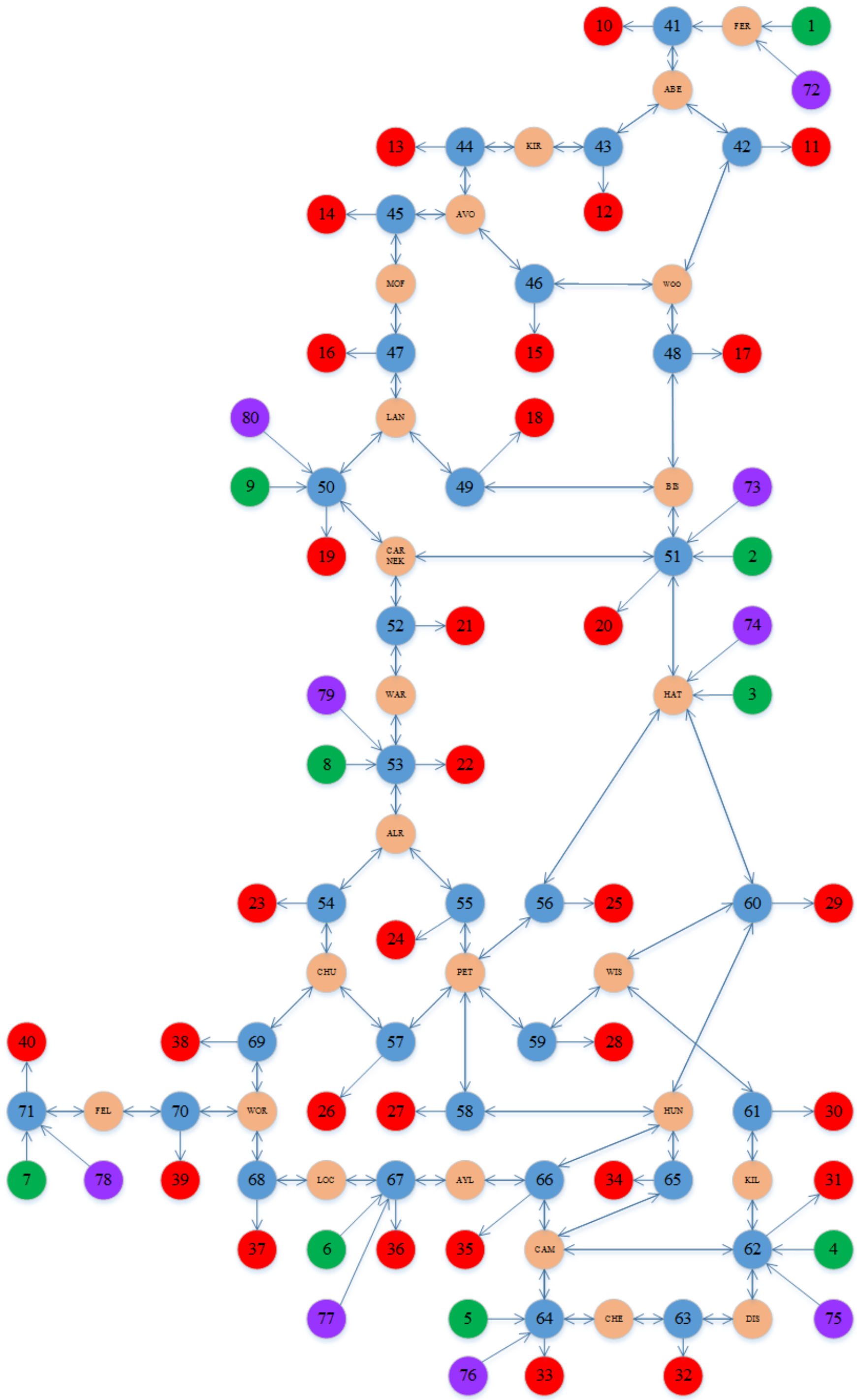

Figure 2: An associated reduced network for the UK gas transmission network.

Set:

T = set of discrete time in time horizon (e.g., $T = \{1, 2, .., |T|\}$)
S = set of supplies
D = set of demands
L = set of linepack zones (i.e., $|L| = |D|$)
C = set of compressor stations
I = set of commercial service points
N = set of all nodes (i.e., $N = S \cup D \cup L \cup C \cup I$)
$FS(i)$ = set of forward starts of node i
$BS(i)$ = set of backward starts of node i
A = set of all arcs
A_b = set of bi-directional arcs

Decision variables:

x_{ij}^t = flow rate (mcm per hour) from node i to j at time t
y_{ij}^t = binary variable w.r.t. flow rate x_{ij}^t
z_i = gas volume (mcm) deviation of planned and targeted linepacks in linepack zone i at final time
V_i^t = gas volume (mcm) in linepack zone or compressor station $i \in L \cup C$ at time t

Parameters:

d_{ij} = gas transmission time (hours) from node i to j
V_i^0 = initial gas volume (mcm) in linepack zone i
V_i^* = targeted gas volume (mcm) in linepack zone i at the end of time horizon
V_{min} = minimum gas volume (mcm) in linepack zone
V_{max} = maximum gas volume (mcm) in linepack zone
s_i^t = flow rate (mcm per hour) of supply i at time t
d_i^t = flow rate (mcm per hour) of demand i at time t

$$h_i^t = \begin{cases} s_i^t & \text{if } i \in S \\ -d_i^t & \text{if } i \in D \end{cases}, \forall t \in T$$

b_i = capacity (mcm) at compressor station $i \in C$ (mcm)
b_{max} = maximum capacity (mcm) of compressor stations (i.e., $\max\{b_i\}$)
$f_1(\cdot)$ = total gas volume (mcm) that does not meet linepack targets
$f_2(\cdot)$ = cost function (pounds per cubic meter) of buying additional gas to satisfy linepack targets
c_i^t = unit cost (pounds per cubic meter) to buy gas from commercial service point i at time t
$\triangle t$ = unit time (hour)

3.2 Linepack Model without Commercial Service

At National Grid, linepack management is used to meet the effects of the transient behaviour of customers. Linepack targets at the end of this day are determined using the forecast demands on the following days. Thus, if linepack targets are not met on one day, a cascade of loss may occur on next days. We developed the mathematical programming model below to address this issue.

[LPP1]

$$\min\ f_1(\cdot) = \sum_{i \in L} |V_i^{|T|} - V_i^*| \tag{1}$$

$$\textbf{s.t.} \sum_{j \in FS(i)} x_{ij}^t - \sum_{j \in BS(i)} x_{ji}^t = h_i^t \ \forall i \in S \cup D, t \in T, \tag{2}$$

$$V_i^t = V_i^{(t-1)} + \sum_{j \in BS(i)} x_{ji}^{(t-d_{ji})} \triangle t - \sum_{j \in FS(i)} x_{ij}^t \triangle t \ \forall i \in L \cup C, t \in T, \tag{3}$$

$$y_{ij}^t + y_{ji}^t \leq 1 \ \forall (i,j) \in A_b, t \in T, \tag{4}$$

$$x_{ij}^t \triangle t \leq b_{max} y_{ij}^t \ \forall (i,j) \in A_b, t \in T, \tag{5}$$

$$V_{min} \leq V_i^t \leq V_{max} \ \forall i \in L, t \in T, \tag{6}$$

$$0 \leq \sum_{j \in BS(i)} x_{ji}^t \leq b_i \ \forall i \in C, t \in T, \tag{7}$$

$$x_{ij}^t \geq 0, y_{ij}^t \in \{0,1\} \ \forall (i,j) \in A, t \in T. \tag{8}$$

The objective (1) is to minimise total absolute deviation between planned and targeted linepacks at the end of time horizon. Constraints (2) assure that the gas flow rate of supply and the demand of customers are met during the time horizon. Constraints (3) are used to compute gas volume in linepack zones and compressor stations at a given times. Constraints (4) allow only unidirectional flows between zones at any one time. Constraints (5) show that if a flow direction is not chosen ($y_{ij}^t = 0$) there is no flow rate on the direction ($x_{ij}^t = 0$); otherwise, the flow rate is less than the maximum capacity of compressor stations. Constraints (6)-(7) limit gas volumes in linepack zones and the capacity of compressor stations respectively. Constraints (8) define non-negative flow variables, and binary variables.

In the UK gas transmission network, the changes of mass flow rates between zones (represented by nodes) are controlled by pressure changes at compressor stations, and the changes of flow directions between zones are manipulated by regulators and valves. For the first issue, due to the safety of the gas transmission network, there are limits in the pressure changes, which leads to the limited capacity of flows through compressor stations as well as the limited gas volume in linepack zones. In our models, we put the limited gas volume constraints in linepack zones, i.e., constraints (6), and the flow capacity constraints through compressor stations, i.e., constraints (7), to remove the intervention of pressure. The lower bound and upper bound values in the constraints are pre-determined based on the pressure limits. We would like to avoid to solve directly the non-linear pressure equations in our models. Therefore, after we obtain the optimal flows for the linepack planning problem, we can derive the pressure values used at compressor stations. For the second issue, we use the set of bi-directional arcs A_b, and constraints (4)-(5) to model the changes of possible flow directions between zones that are manipulated by regulators and valves. In particular, the possible flow directions between zones are pre-identified and put the possible bidirectional flows into set A_b. Since there cannot exist bidirectional flows between zones at one time, we put constraints (4) into the model. The constraints are only valid to the arcs in set A_b. In addition, constraints (5) aim to set no flow rate for the non-chosen flow direction, and a limit of flow rate for the chosen flow direction.

This is a nonlinear model since absolute values are taken in the objective function. By letting $u_i = |V_i^{|T|} - V_i^*|$ and modifying additional constraints (10)-(12) for the auxiliary variables, we can linearise the model.

[LPP2]

$$\min\ f_1(\cdot) = \sum_{i \in L} u_i \tag{9}$$

$$\text{s.t. } (2) - (8),$$

$$u_i \geq V_i^{|T|} - V_i^* \ \forall i \in L, \tag{10}$$

$$u_i \geq -(V_i^{|T|} - V_i^*) \; \forall i \in L, \tag{11}$$

$$u_i \geq 0 \; \forall i \in L. \tag{12}$$

Then, we can solve the linear model with any commercial solver (e.g., CPLEX or GUROBI).

3.3 Linepack Model with Commercial Service

Although National Grid only manages the UK gas transmission network, the gas operator encourages suppliers/shippers to bring more gas onto the network due to the high priority of satisfying customer demand. In the case that imbalance of supply and demand occurs, commercial services are required. There are also some cases in which National Grid needs to buy additional gas, such as compressor fuel or unaccounted for gas (e.g., meter errors and theft). Thus, buying additional gas at commercial service points optimally in order to offset the deficit, to meet the current demands of customers and to achieve linepack targets during the coming days is necessary. There is a policy of buying additional gas at the end of gas day to offset the gas deficits. This policy cannot achieve the theoretical minimum cost since gas price changes occur throughout the time horizon and may be different at commercial service points. Thus, it is essential to consider the timing, location and quantities bought in achieving total minimum cost. The linepack planning model with commercial service is

[SLPP]

$$\min \; f_2(\cdot) = \sum_{t \in T} \sum_{i \in I} \sum_{j \in FS(i)} c_i^t x_{ij}^t \tag{13}$$

$$\text{s.t. } (2) - (8),$$

$$V_i^{|T|} = V_i^* \; \forall i \in L. \tag{14}$$

The objective function (13) is the minimisation of total cost of buying additional gas at commercial service points to meet the gas deficit of targeted linepacks as well as the demand of customers.

Constraints (14) assure satisfaction of linepack targets at the end of time horizon. Constraints (2)-(8) are described in the last subsection.

3.4 Linepack Model with Commercial Service under Supply Shortfall

Uncertainties relating to gas supply or unexpected failures at supply zones pose more challenges to satisfying customer demand and linepack targets. We developed a third linepack planning model to support the evaluation of the impact of supply shortfall,while minimising total cost. This model is a modification of the linepack planning model with commercial service. In particular, for $\forall i \in S, t \in T$ constraints (2) are transformed into

$$\sum_{j \in FS(i)} x_{ij}^t - \sum_{j \in BS(i)} x_{ji}^t = h_i^t(\xi_i^t), \tag{15}$$

where $h_i^t(\xi_i^t)$ represents the residual flow rate from supply zone i at time t under the impact of uncertainty or unexpected failure ξ_i^t.

This residual flow rate is computed by

$$h_i^t(\xi_i^t) = \begin{cases} s_i^t & \text{if } \xi_i^t = 0 \\ 0 & \text{otherwise} \end{cases}, \tag{16}$$

where $\xi_i^t \in \{0,1\}$. This means that if supply zone i is disrupted at time t ($\xi_i^t = 1$), there is no corresponding flow rate ($x_{ij}^t = 0 \, \forall j$); otherwise, there is flow rate s_i^t as given at the supply zone. In practice, the disruption may last for a period of time and occur several times during the time horizon T. After the period of disruption time, the flow rate at supply zone is recovered. We denote the recovery time of supply zone i by r_i.

The disruption event on supply zone i is assumed to follow Binomial distribution with failure probability p_i, and the disruptions may occur simultaneously at many supply zones. We have applied Monte Carlo simulation to determine total expected cost for the problem under supply shortfall. A number of scenarios are generated for the failure of supply zones based on a Binomial distribution and their failure probabilities. For supply zone i where failures occur, we generate randomly times t for the failure occurrence. The supply zone is then disrupted during r_i hours afterwards. The model can deal with any scenario of supply shortfall in the network. However, in the computational experiments, to evaluate the impact of supply shortfalls we assume that there is at most one disruption per day at supply zones. The model is then applied to solve all the scenarios to find the distribution of total cost for gas deficits. This provides some insight into critical supply and demand patterns, which helps develop appropriate investment policies for the future network.

3.5 Linepack Model with Commercial Service under Compressor Failure

In this section, we investigate the impact of unexpected compressor failure on the linepack planning problem with commercial service. Since compressor stations are comprised by a set of serial and/or parallel operational compressor units, failures of compressor units decrease the capacity of compressor stations. This leads to a significant change of linepack planning and management, as well as actions and timings to minimise total cost for resolving the gas deficit. We extend the linepack planning model with commercial service for solving the problem under unexpected compressor failure by transforming constraints (7) into

$$0 \leq \sum_{j \in BS(i)} x_{ji}^t \leq b_i^t(\zeta) \ \forall i \in C, t \in T, \tag{17}$$

where $b_i^t(\zeta)$ represents the uncertainty capacity of compressor station i at time t under the impact of unexpected compressor failure ζ. Since we investigate the gas transmission network under transient state (i.e., daily), the decreased capacity of compressor station due to the impact of compressor failure is assumed to be kept at the same value during the investigated horizon time. Then, constraints (17) can be described by

$$0 \leq \sum_{j \in BS(i)} x_{ji}^t \leq b_i(\zeta) \ \forall i \in C, t \in T. \tag{18}$$

A robust optimisation approach, based on the joint chance constraint programming, is developed to solve the linepack planning problem with commercial service under the impact of unexpected compressor failure. We let $z_i^t = \sum_{j \in BS(i)} x_{ji}^t$. Given that a confidence level $\alpha \in [0, 1]$, the minimum probability of occurring the event that $z_i^t \leq b_i(\zeta) \ \forall i \in C, t \in T$, we have a joint chance constraint programming as follows:

$$P\{z_i^t \leq b_i(\zeta), \forall i \in C, t \in T\} \geq \alpha;$$

corresponding to

$$\mathrm{Inf}_{P\in\mathbf{P}} P\{z_i^t \leq b_i(\zeta), \forall i \in C, t \in T\} \geq \alpha,$$

where $\mathbf{P}$ is the set of all probability distributions for random variable $b_i(\zeta)$ with known mean and variance (μ_i, σ_i^2).

Bonferroni's inequality leads to

$$\mathrm{Sup}_{P\in\mathbf{P}} P\{\cup_{i\in C, t\in T} z_i^t > b_i(\zeta)\} \leq 1-\alpha.$$

In addition, we have

$$P\{\cup_{i\in C, t\in T} z_i^t > b_i(\zeta)\} \leq \sum_{i\in C, t\in T} P\{z_i^t > b_i(\zeta)\} \ \forall P \in \mathbf{P}.$$

Set

$$\sum_{i\in C, t\in T} P\{z_i^t > b_i(\zeta)\} \leq 1-\alpha.$$

Let $1-\alpha = \epsilon$ (risk level), we have

$$\sum_{i\in C, t\in T} P\{z_i^t > b_i(\zeta)\} \leq \epsilon.$$

Let $\epsilon = \sum_{i\in C, t\in T} \epsilon_i^t$, we get

$$P\{z_i^t > b_i(\zeta)\} \leq \epsilon_i^t \ \forall i \in C, t \in T$$

$$\iff P\{z_i^t - b_i(\zeta) > 0\} \leq \epsilon_i^t \ \forall i \in C, t \in T$$

$$\iff P\{z_i^t \leq b_i(\zeta)\} \geq 1 - \epsilon_i^t \ \forall i \in C, t \in T$$

$$\iff \mathrm{Inf}_{P \in \mathbf{P}} P\{z_i^t \leq b_i(\zeta)\} \geq 1 - \epsilon_i^t \ \forall i \in C, t \in T$$

where

$$\sum_{i \in C, t \in T} \epsilon_i^t \leq 1 - \alpha.$$

We can set $\epsilon_i^t = \frac{1-\alpha}{|C|+|T|}$, then the joint chance constraint can be derived into

$$z_i^t \leq \mu_i + \sigma_i \sqrt{\frac{|C| + |T|}{1 - \alpha} - 1} \ \forall i \in C, t \in T.$$

Then, constraints (18) can be written by

$$\sum_{j \in BS(i)} x_{ji}^t \leq \mu_i + \sigma_i \sqrt{\frac{|C| + |T|}{1 - \alpha} - 1} \ \forall i \in C, t \in T. \tag{19}$$

Since constraints (19) are linear constraints, we can use any MILP solver for the linepack planning problem with commercial service under impact of unexpected compressor failure.

4 National Grid Case Study

We describe a National Grid case study used to validate the linepack planning models. Specifically, we investigate linepack management over a time horizon $T = 24$ (i.e. a gas day basis) in which the

Table 3: Length of pipeline among compressor stations in the network.

Comp. Station		Length	Comp. Station		Length
From	To	(km)	From	To	(km)
ABD	FER	69.15	CAM	AYL	45.20
ABD	KIR	75.64	CAM	CHE	94.77
ABD	WOO	139.66	CAM	DIS	80.43
ALR	WAR	132.30	CAM	HUN	59.85
ALR	PET	117.94	CHE	DIS	89.14
ALR	CHU	71.73	CHU	PET	79.06
AVO	KIR	135.25	CHU	WOR	139.57
AVO	WOO	148.26	FEL	WOR	320.80
AVO	LON	137.62	HAT	CAR/NEK	24.17
AVO	MOF	86.61	HAT	PET	82.51
AYL	HUN	96.02	HAT	HUN	73.36
AYL	LOC	113.38	HUN	PET	41.89
BIS	WOO	116.58	KIL	WIS	28.94
BIS	LON	105.73	LOC	WOR	190.16
BIS	CAR/NEK	133.78	MOF	LON	46.00
CAR/NEK	LON	111.35	PET	WIS	39.95
CAR/NEK	WAR	112.16			

starting time is at 6:00am. Gas flow rate is measured by million cubic meters per day (mcmd). We assume that the possible maximum gas velocity at National Grid (20 m/s or 72 km/h) is applied. The gas transmission time among zones is computed based on the length of pipeline network (Table 3).

Gas flow rates into supply zones and extracted from demand zones during the time horizon T are shown in Tables 4-5. It can be seen that a deficit of gas exists at the beginning of time horizon since supply is less than demand. Linepack zones then provide gas to offset the deficit of gas until supply is greater than demand. Figure 3 shows the pattern of national supply and demand (i.e., sum of gas flow rates), along with the accumulated deficit of gas volume. The deficit occurs at the beginning of gas day and increases till time $t = 16$. At time $t = 17$ when supply starts to be greater than demand, the deficit starts to decrease. To offset the deficit, the operator has directed gas volume from some linepack zones to the unsatisfied demand zones. How to transmit gas to satisfy customers' demand and linepack targets at the end of time horizon is a critical question for National Grid. Table 6 shows initial, targeted, minimum and maximum values of linepack zones. Here, the initial values are the remaining gas volume of the previous day, while the target values represent the forecasting results of supply and demand on next days. The minimum values are considered as safety stock of linepack zones, while the maximum values are applied to assure that pressure limits are not breached in the network.

If the requirements cannot be met, it is possible to buy additional gas from commercial service points. The gas price changes at the commercial service points during the time horizon, thus there

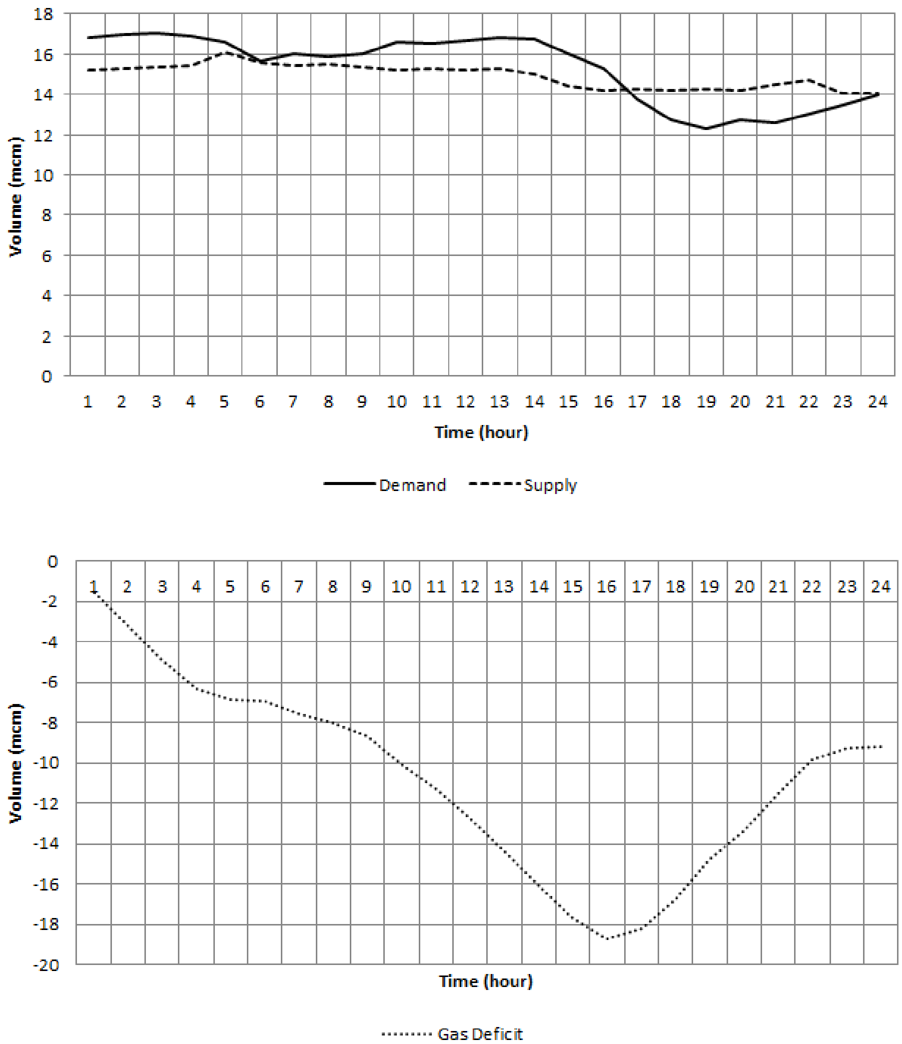

Figure 3: National supply and demand pattern.

are questions of where, when and what quantities of gas to buy. In the case study, we used the standard gas price (18p per cubic meter). Figure 4 provides the changes in gas prices at commercial service points over the 24 hours on a percentage of the standard gas price.

We generated the probabilities of supply shortfalls at terminals from historical data (years 2013-2015). Table 7 presents failure probabilities and recovery times at supply zones. It can be seen that the high probabilities of failure occur at supply zones 1, 4 and 7, while low probabilities of failure occur at supply zones 5, 6 and 8. The longest recovery time is at supply zone 9 (12 hours), while the shortest recovery time is at supply zone 8 (4 hours). The disruption at supply zones with the high probability of failure and/or the long recovery time may cause significant challenges to planning linepack.

Table 8 describes mean and variance of capacity for each compressor station. These data are used

Table 4: Data of supply zones (mcm).

Zone	Time																							
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	3.11	3.07	3.07	3.14	3.17	3.21	3.29	3.30	3.30	3.28	3.28	3.27	3.28	3.22	3.23	3.22	3.22	3.22	3.23	3.19	3.18	3.19	3.11	3.32
2	1.53	1.55	1.59	1.61	1.97	1.59	1.65	1.66	1.62	1.65	1.63	1.66	1.62	1.65	1.36	1.37	1.42	1.42	1.44	1.41	1.39	1.38	1.35	1.45
3	5.24	5.25	5.24	5.21	5.24	5.23	4.91	4.93	4.92	4.72	4.75	4.71	4.67	4.35	4.06	3.77	3.78	3.75	3.77	3.76	4.46	4.69	4.99	4.93
4	2.63	2.64	2.62	2.63	2.62	2.48	2.49	2.51	2.49	2.48	2.47	2.50	2.50	2.50	2.52	2.55	2.56	2.56	2.67	2.69	2.34	2.33	2.32	2.32
5	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
6	0.29	0.29	0.29	0.29	0.30	0.29	0.29	0.30	0.28	0.28	0.28	0.27	0.28	0.28	0.26	0.26	0.26	0.26	0.26	0.26	0.24	0.24	0.22	0.26
7	1.33	1.32	1.35	1.38	1.36	1.37	1.37	1.38	1.38	1.37	1.48	1.46	1.54	1.65	1.65	1.65	1.64	1.65	1.62	1.54	1.54	1.53	1.52	1.33
8	0.87	0.86	0.90	0.87	1.16	1.13	1.14	1.13	1.13	1.14	1.11	1.06	1.09	1.10	1.08	1.09	1.09	1.07	1.04	1.05	1.03	1.06	0.28	0.16
9	0.25	0.31	0.29	0.28	0.30	0.29	0.29	0.27	0.28	0.29	0.28	0.26	0.30	0.29	0.27	0.29	0.29	0.29	0.26	0.29	0.29	0.27	0.29	0.29

Table 5: Data of demand zones (mcm).

Zone	Time																							
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
10	0.25	0.21	0.21	0.21	0.21	0.22	0.22	0.22	0.23	0.24	0.24	0.23	0.22	0.21	0.20	0.20	0.21	0.20	0.20	0.17	0.16	0.16	0.17	0.20
11	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
12	0.02	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.02	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
13	0.13	0.16	0.31	0.30	0.28	0.28	0.28	0.28	0.28	0.29	0.30	0.30	0.30	0.34	0.29	0.27	0.29	0.24	0.09	0.09	0.09	0.09	0.09	0.10
14	0.04	0.04	0.03	0.03	0.03	0.03	0.03	0.02	0.02	0.03	0.04	0.04	0.04	0.03	0.03	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.02
15	0.88	0.86	0.79	0.80	0.74	0.72	0.71	0.71	0.75	0.76	0.81	0.83	0.86	0.86	0.82	0.71	0.63	0.50	0.56	0.51	0.63	0.65	0.65	0.78
16	0.27	0.11	0.11	0.11	0.11	0.10	0.22	0.23	0.26	0.26	0.11	0.10	0.08	0.08	0.08	0.12	0.09	0.11	0.11	0.21	0.27	0.29	0.29	0.08
17	0.32	0.32	0.28	0.29	0.30	0.28	0.28	0.29	0.29	0.30	0.34	0.34	0.34	0.33	0.31	0.24	0.18	0.15	0.15	0.13	0.16	0.19	0.24	0.19
18	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
19	0.45	0.42	0.42	0.44	0.43	0.38	0.39	0.36	0.38	0.44	0.35	0.36	0.46	0.52	0.50	0.48	0.44	0.41	0.41	0.42	0.30	0.38	0.36	0.43
20	2.22	2.37	2.37	2.43	2.30	2.31	2.36	2.35	2.29	2.36	2.26	2.27	2.31	2.29	2.23	2.27	2.13	1.95	1.94	1.88	1.82	1.91	1.94	1.91
21	0.72	0.58	0.59	0.58	0.56	0.54	0.61	0.64	0.59	0.60	0.62	0.61	0.61	0.60	0.60	0.59	0.52	0.50	0.42	0.31	0.31	0.31	0.31	0.36
22	1.82	1.87	1.88	1.87	1.86	1.90	1.87	1.81	1.85	1.96	1.98	1.90	1.94	1.93	1.87	1.74	1.61	1.62	1.53	1.50	1.50	1.54	1.55	1.52
23	0.24	0.23	0.21	0.23	0.22	0.19	0.22	0.23	0.22	0.22	0.22	0.21	0.21	0.20	0.21	0.24	0.27	0.28	0.26	0.27	0.23	0.26	0.28	0.20
24	0.24	0.24	0.23	0.22	0.23	0.22	0.22	0.22	0.22	0.24	0.24	0.24	0.24	0.23	0.23	0.22	0.16	0.15	0.15	0.16	0.17	0.19	0.20	0.20
25	0.21	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.30	0.29	0.27	0.31	0.32	0.30	0.31	0.31	0.28	0.16	0.16	0.17	0.16	0.16	0.16	0.16
26	0.06	0.06	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.06	0.06	0.06	0.06	0.06	0.05	0.05	0.04	0.04	0.03	0.04	0.04	0.05	0.05	0.05
27	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
28	0.08	0.08	0.06	0.05	0.05	0.05	0.05	0.04	0.05	0.06	0.08	0.08	0.08	0.07	0.06	0.05	0.03	0.02	0.02	0.02	0.02	0.02	0.03	0.06
29	0.32	0.33	0.33	0.34	0.30	0.29	0.27	0.27	0.28	0.33	0.35	0.35	0.34	0.33	0.33	0.33	0.25	0.23	0.25	0.24	0.23	0.19	0.18	0.23
30	0.04	0.04	0.03	0.03	0.03	0.02	0.02	0.03	0.03	0.03	0.04	0.04	0.04	0.04	0.03	0.03	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.03
31	1.32	1.32	1.28	1.26	1.25	0.38	0.38	0.37	0.39	0.26	0.28	0.28	0.28	0.38	0.37	0.35	0.20	0.18	0.18	0.95	0.92	0.94	1.09	1.26
32	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.22	0.22	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.13	0.15	0.22	0.21
33	2.20	2.31	2.45	2.33	2.20	2.11	2.17	2.17	2.22	2.41	2.36	2.39	2.38	2.35	2.12	2.04	1.62	1.51	1.52	1.37	1.43	1.64	1.72	1.85
34	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04
35	1.35	1.34	1.35	1.35	1.34	1.36	1.36	1.40	1.40	1.41	1.42	1.53	1.54	1.46	1.41	1.37	1.33	1.32	1.13	1.19	1.21	1.20	1.24	1.27
36	0.83	0.83	0.83	0.84	1.01	1.04	1.07	1.04	1.11	1.13	1.15	1.24	1.24	1.24	1.19	1.09	1.04	0.99	0.99	0.96	0.95	0.86	0.84	0.90
37	1.04	1.08	1.06	1.05	1.02	1.08	1.08	1.02	1.01	1.05	1.13	1.10	1.09	1.08	1.00	0.97	0.82	0.79	0.86	0.76	0.74	0.74	0.74	0.81
38	0.32	0.31	0.30	0.28	0.29	0.29	0.30	0.29	0.30	0.31	0.34	0.33	0.33	0.31	0.30	0.29	0.33	0.33	0.30	0.31	0.30	0.30	0.31	0.32
39	0.67	0.73	0.72	0.70	0.71	0.75	0.76	0.74	0.74	0.78	0.79	0.76	0.76	0.74	0.70	0.65	0.61	0.57	0.57	0.55	0.55	0.54	0.53	0.56
40	0.41	0.41	0.40	0.40	0.40	0.40	0.40	0.40	0.40	0.40	0.40	0.40	0.40	0.40	0.40	0.40	0.39	0.23	0.23	0.23	0.23	0.23	0.23	0.23

Table 6: Data of minimum, maximum, initial and targeted values of linepack zones (mcm).

Zone	Min	Max	V_i^0	V_i^*	Zone	Min	Max	V_i^0	V_i^*
41	19.69	20.26	20.25	19.81	57	3.47	3.64	3.64	3.59
42	18.53	19.08	18.99	18.73	58	1.74	1.94	1.94	1.79
43	9.16	9.77	9.77	9.18	59	2.52	2.65	2.65	2.58
44	18.53	18.77	18.77	18.54	60	8.90	9.57	9.57	9.15
45	8.84	9.54	9.50	8.90	61	2.48	2.60	2.60	2.53
46	3.50	3.91	3.91	3.91	62	20.18	21.39	21.39	20.60
47	4.44	4.81	4.81	4.51	63	2.84	3.22	3.22	3.04
48	8.60	9.04	9.02	8.86	64	10.68	12.30	12.30	11.81
49	4.69	5.03	5.03	4.81	65	2.35	2.57	2.54	2.56
50	13.77	14.49	14.02	14.04	66	12.11	13.63	13.61	12.96
51	54.90	60.30	58.69	56.71	67	9.39	11.25	11.25	10.44
52	12.22	12.72	12.58	12.39	68	10.26	11.75	11.13	11.75
53	13.79	14.30	14.24	13.92	69	5.68	5.91	5.85	5.90
54	2.14	2.25	2.25	2.19	70	24.01	25.57	25.19	25.57
55	4.48	4.82	4.82	4.66	71	9.72	10.39	10.08	10.39
56	13.43	14.24	14.24	13.82					

Table 7: Data of supply shortfall.

Supply Zone	Failure Probability (%)	Recovery Time (hour)
1	14.50	11
2	7.25	10
3	2.29	10
4	23.66	11
5	0.67	9
6	0.29	10
7	13.00	6
8	0.10	4
9	7.54	12

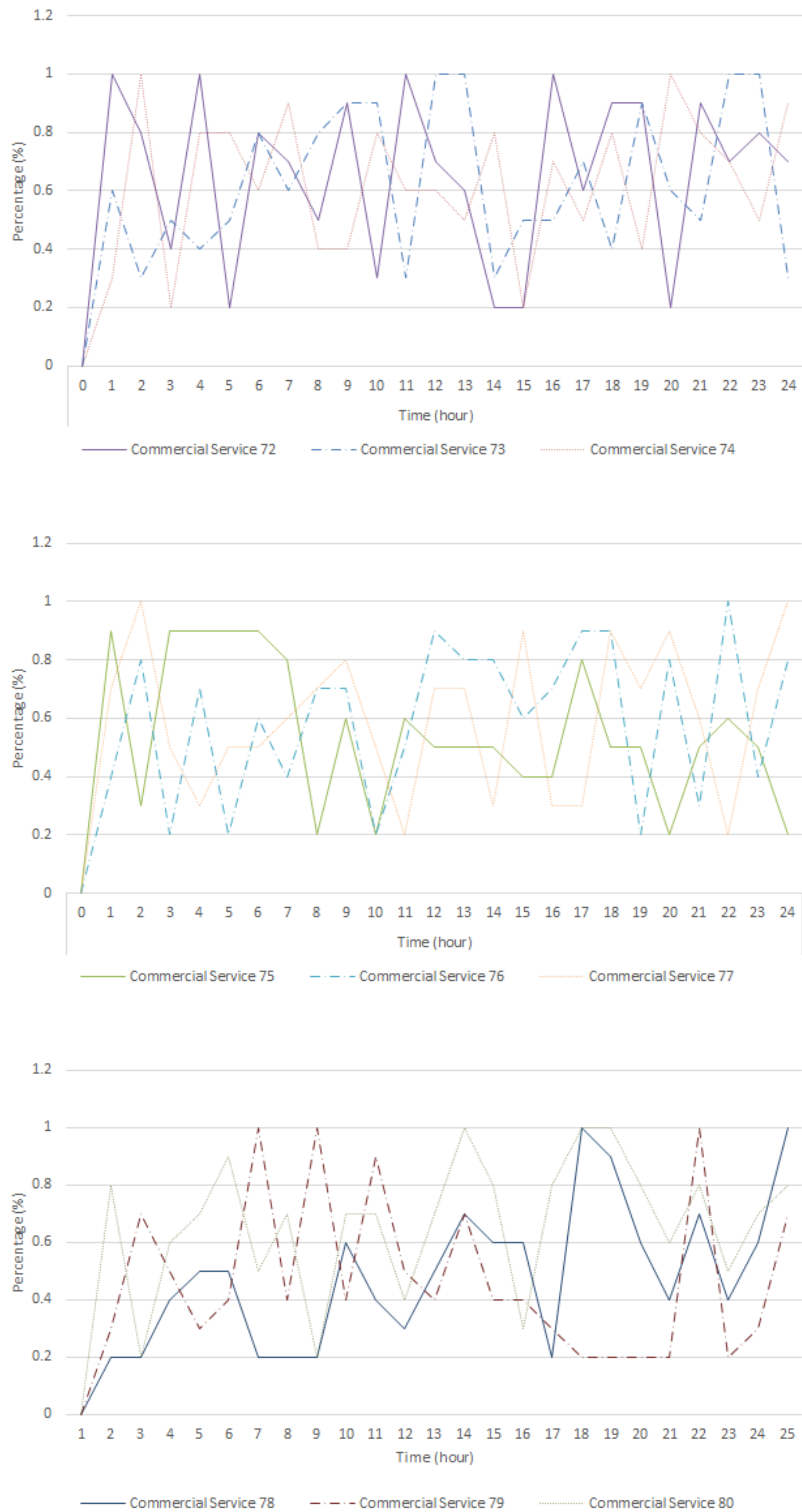

Figure 4: Gas price at commercial service points.

to test the robust optimisation approach for solving the linepack planning problem with commercial services under the impact of compressor failure. In particular, they are input into constraints (19) to approximate the capacity of compressor stations.

Table 8: Data of mean and variance of compressor station's capacity (mcm).

Compressor	Capacity		Compressor	Capacity	
	μ	σ^2		μ	σ^2
FER	80.00	1.00	CHU	36.67	1.00
ABD	125.00	1.00	PET	71.00	1.00
KIR	66.50	1.00	WIS	21.67	1.00
AVO	81.67	1.00	HUN	53.33	1.00
WOO	40.00	1.00	KIL	60.67	1.00
MOF	41.33	1.00	DIS	29.67	1.00
LON	51.25	1.00	CHE	28.67	1.00
CAR/NEK	84.00	1.00	CAM	32.00	1.00
BIS	66.67	1.00	AYL	40.00	1.00
WAR	53.33	1.00	LOC	16.00	1.00
ALR	46.67	1.00	WOR	56.67	1.00
HAT	65.00	1.00	FEL	72.33	1.00

5 Computational Results

We applied the linepack planning models to solve this case study. These models were implemented in Visual C++ and solved with the IBM ILOG CPLEX version 12.5 callable library. The computational experiments were run on a Microsoft Windows 7 Enterprise PC with an Intel Core i7-3770 processor (3.40 GHz per chip) and 24 GB of RAM.

We first solved the linepack planning problem without commercial service, supply shortfalls, or compressor failure. The result shows that two linepack targets cannot be met at the end of day (e.g., 0.09 mcm at zone 49, North of England, and 1.05 mcm at zone 68, South West of England). Figure 5 presents the best linepack plan to achieve the objective value. According to this, National Grid should control gas volume in linepack zone at possible minimum level, which may assure the safety regulatory of gas pressure for customers. The minimum level also plays a role as safety stock in inventory to assure that customers' demands in the zone are always satisfied. When any customer's demand increases suddenly, an amount of gas is ordered to transmit into that zone. We set a maximum level of gas volume in linepack zone to assure that the pressure limit is not breached. In addition, based on the linepack profiles of zones the gas transmission network operators at National Grid can determine the timings and the gas volume that needs to be delivered into the zones. Then, they can operate efficiently the corresponding compressor stations. For example, looking at the linepack profile of zone 41, National Grid does not operate the compressor station at St Fergus until time 5. At time 5, the compressor station is operated to deliver gas volume (0.10 mcm) from St

Fergus terminal to zone 41. Next, the compressor station is in a non-operational state until time 11. At time 11, the compressor station is secondly operated to deliver gas volume (0.55 mcm) from St Fergus terminal to zone 41. Then, the same amount of gas volume is delivered at times 12, 18 and 20. In addition, depending on the gas transmission time between zones, the compressor station can be operated earlier to meet the delivery time. For example, if it takes 1 hour to deliver gas from St Fergus to zone 41, National Grid has to operate the compressor station 1 hour before the linepack need at the zone. Before these models, National Grid did not have efficient tools for planning linepack with given targets. National Grid mainly relies on operator's experience and the simulation result from SIMONE software (the simulation and optimisation of gas transport and distribution) to control gas volume in linepack zones to obtain the targets. Since National Grid cannot set up the given targets for linepack zones in the software, they could not often obtain the given targets. A linepack profile provided by SIMONE software for the case study at National Grid is shown in Figure 6. With the linepack profile, National Grid missed total gas volume 2.21 mcm as compared with the given targets.

Next, we solved the linepack planning problem with commercial service but without supply shortfall or compressor failure. The results show that to offset the gas deficit in linepack zones we needs to buy additional gas at commercial service points. We obtain the optimal linepack plan with total minimum cost £41,400 for buying the additional gas. In particular, we should buy 0.45 mcm of commercial service point 78 (Milford Haven) at time 14, and 0.69 mcm of commercial service point 79 (Burton Point) at time 8. As compared with the current policy at National Grid, i.e., the gas deficit is offset at the end of gas day, thus the average gas price is used to buy additional gas, our model suggests savings of £123,120 - £41,400 = £81,720 are possible.

For the linepack planning problem with commercial service under supply shortfall, we generate 1,000 scenarios based on the failure probability and the recovery time of supply zones in Table 7, using Monte Carlo simulation with Binomial distribution for success/failure probability at supply zones. We assume that at most one failure occurs for each supply zone, but that many supply zones may fail simultaneously during the time horizon. Our model was applied to solve these scenarios to find the total expected cost of buying additional gas for this problem under supply shortfall. Figure 7 shows the histogram of total cost obtained. In particular, the total minimum cost is £41,400, the total maximum cost £2,840,400, the total expected cost £404,095 and the deviation £576,360. In 52.2% of the scenarios the total minimum cost was obtained. Confidence levels of 5% and 95% on the total cost are £41,400 and £1,699,200 respectively. The total computation time was 290 seconds. We therefore expect reasonable computation times for larger case studies.

Finally, we solved the linepack planning problem with commercial service under the impact of compressor failure for two cases, i.e., without and with supply shortfall. In the case with the impact of supply shortfall, we also generated 1,000 scenarios as described above. Table 9 presents the computational results with a range of various confidence levels $\alpha = 0.7, 0.75, .., 0.99$ for these two cases. We did not solve the case study with $\alpha = 1.00$ to avoid overflow issues with $(1 - \alpha)$

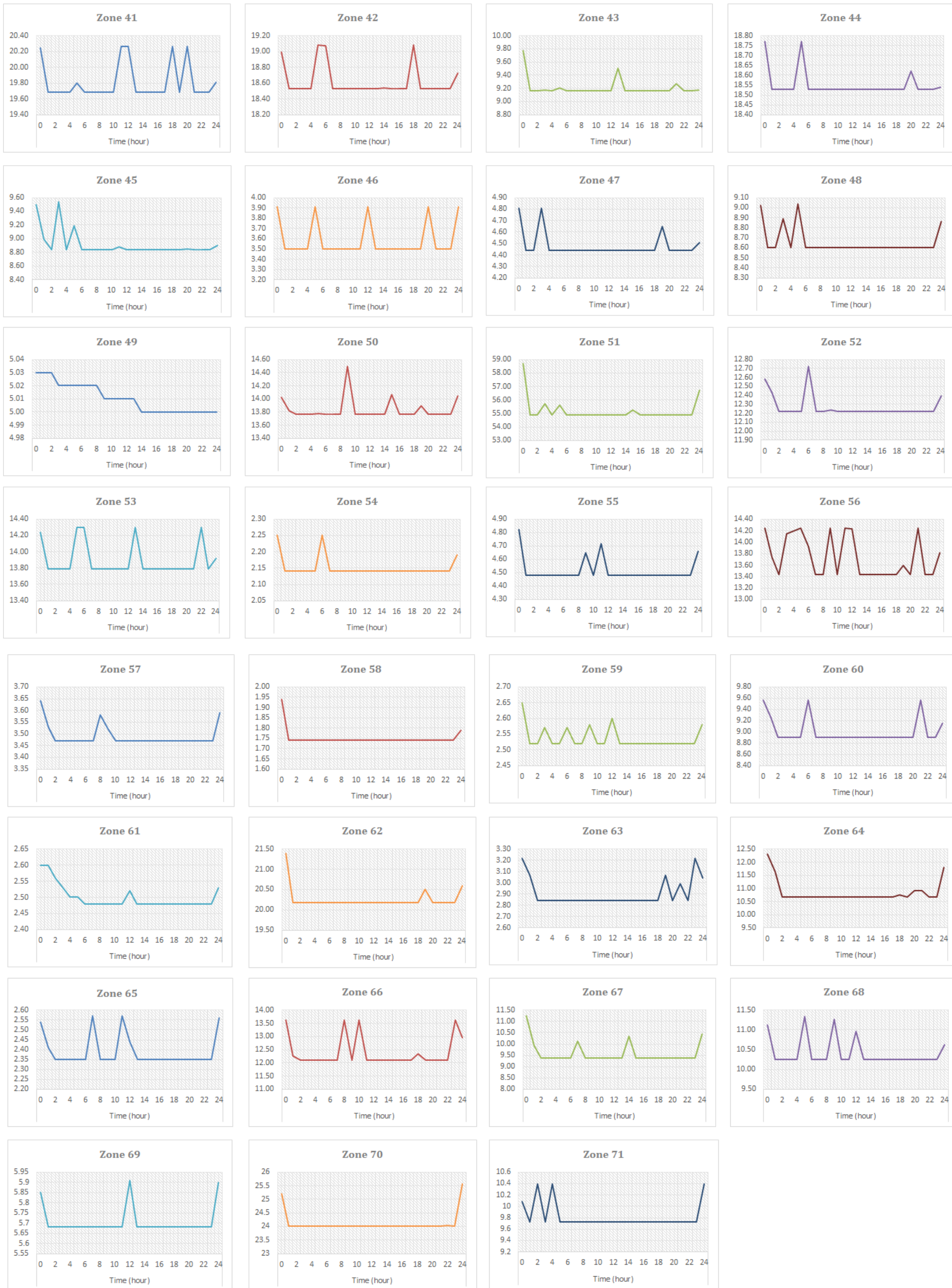

Figure 5: Linepack profile for the solution of our model.

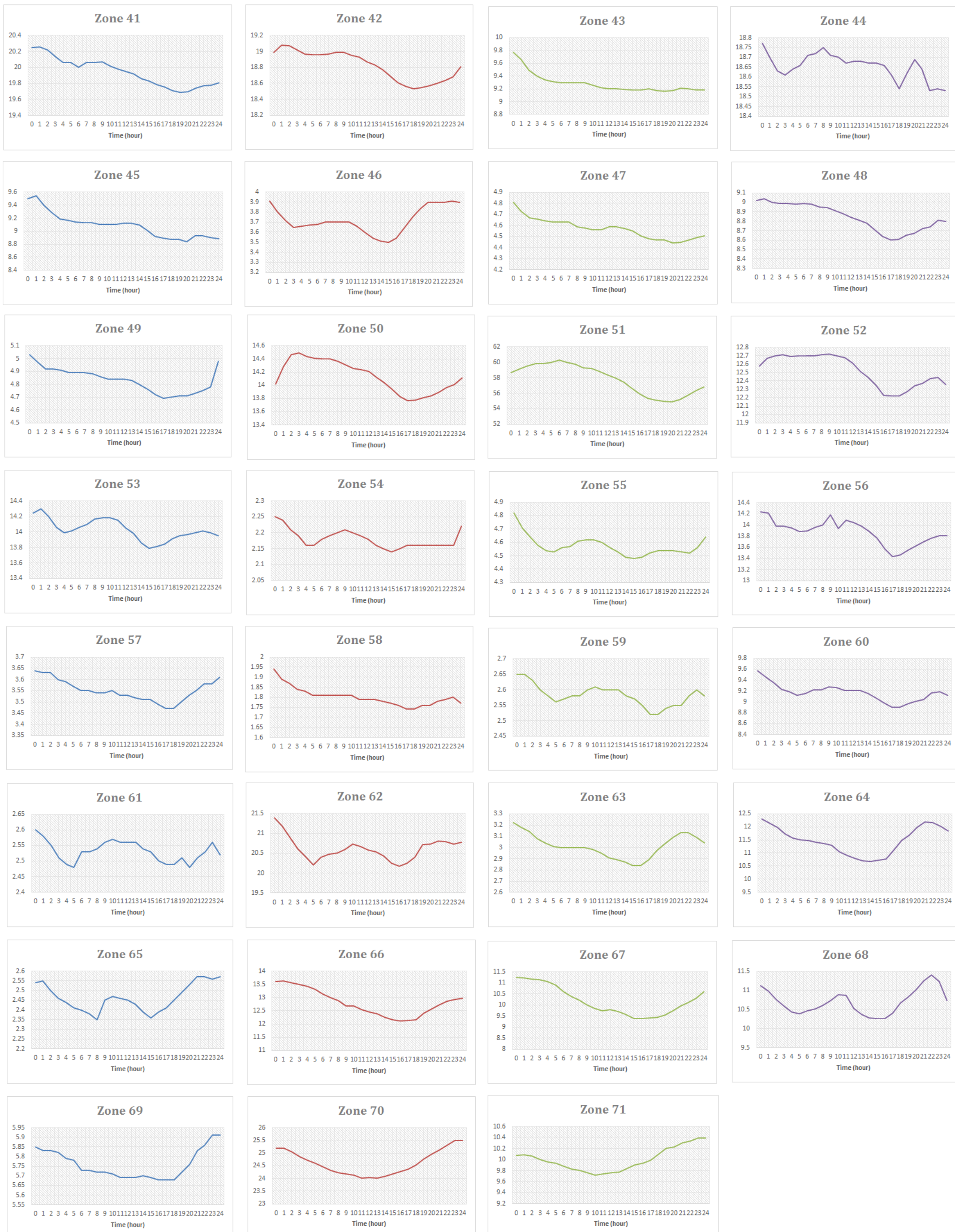

Figure 6: A linepack profile provided by SIMONE software for the case study at National Grid.

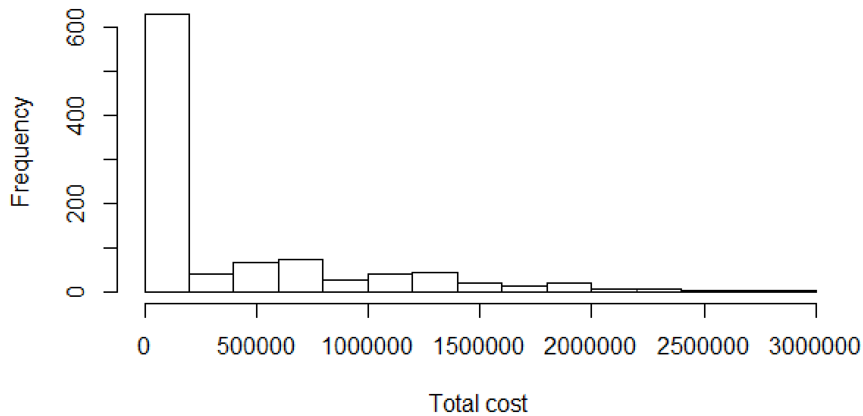

Figure 7: Histogram of total cost for the case study under supply shortfall.

in the denominator of constraints (19). The results show that decreasing confidence level α (i.e., increasing the probability of compressor failure) makes National Grid spending more total cost for resolving the gas deficit. In particular, if the probability of compressor failure is 1%, the total minimum cost is £41,400, the total maximum cost £2,840,400, the total expected cost £404,095 and the deviation £576,360 (which is similar to the result of the problem under supply shortfall above). If the probability of compressor failure is 30%, the total minimum cost is £1,690,200, the total maximum cost £4,489,200, the total expected cost £2,007,626 and the deviation £482,238. It can be seen a significant increasing of total cost for the gas deficit. The total average computation time was 425 seconds, which again shows the capability of our model for solving larger case studies.

In summary, our models can solve linepack planning problems without/with commercial service under the impact of supply shortfall and compressor failure that are encountered in National Grid. The models support the operator in achieving the optimal linepack plan to satisfy customers' demands and linepack targets such that total cost for offsetting the gas deficit is minimised. The solutions obtained compared favourably current tools/policies at National Grid.

6 Conclusions and Future Work

Although the linepack planning problem plays an important role in the gas transmission network under transient state, it has not received appropriate attention. In the paper, we proposed four MILP models to support the National Grid in obtaining optimal linepack management under various scenarios. The models are constructed on a reduced network of UK gas transmission network in

Table 9: Results of total cost (£) for the case study under the impact of compressor failure.

Confidence level (%)	w/o supply shortfall	w/ supply shortfall			
	Total cost	Minimum	Maximum	Average	Deviation
99	41,400	41,400	2,840,400	404,095	576,312
95	1,049,400	1,049,400	3,848,400	1,384,430	515,416
90	1,366,200	1,366,200	4,165,200	1,692,560	498,209
85	1,506,600	1,506,600	4,305,600	1,829,126	491,055
80	1,591,200	1,591,200	4,390,200	1,911,096	486,678
75	1,647,000	1,647,000	4,447,800	1,966,021	484,461
70	1,690,200	1,690,200	4,489,200	2,007,626	482,238

which we group supply and demand nodes among compressor stations into a single zone to reduce complexity. The first model solves the linepack planning problem without commercial service in which linepack targets at the end of time horizon are given. Our model can find the optimal linepack plan that minimises total deviation between planned and target linepacks. To offset the gas deficit for meeting the target linepacks as well as the demand of customers, we introduce the linepack planing model with commercial service that determines time, location and gas quantity to be bought to minimise total cost. In the case of unexpected, random supply loss, we build the third model to evaluate the impact of supply shortfall and search the optimal linepack plan to mitigate the loss. This model can also be used as a filter tool to find the critical supply and demand patterns for National Grid to do comprehensive analysis of investment policies. In addition, in the case of unexpected, random compressor failure, we construct the four model to investigate its impact on the linepack planning management. All four models are validated on the case study with actual data at National Grid. The computational results show the effectiveness of our models in solving the linepack planning problems that the current tools at National Grid cannot achieve.

We believe that these models can be extended to solve the linepack planning problems with linepack targets set up at several different times during time horizon. This problem is especially important for gas transmission network operators when considering longer time horizons. If they consider linepack management for more than 1 week, referred to as middle or long term forecasting, then several linepack targets need to be considered and our model needs to be extended for this. In the present models, we assume that gas prices are known perfectly. Imperfect knowledge of prices would decisions in timing, location and the amount of gas bought to minimise total cost for resolving the gas deficit, which leads to a change of the cost savings. Hence, a stochastic programming model to study the uncertainty of gas prices is essential for future research work. In addition, an extended model with the impact of that both the market players and the transmission system operator compete for purchase or sale of gas on the gas prices is a possibility of future research work.

Acknowledgment

We would like to thank our colleagues at National Grid and the University of Warwick for many helpful conversations. The National Transmission System Constraint Modelling has been funded by National Grid under contract No. NIA_NGGT0022.

References

M. Abbaspour, P. Krishnaswami, and K.S. Chapman. Transient optimization in natural gas compressor stations for linepack operation. *Transactions of the ASME*, 129(4):314–324, 2007.

N. Adhya, M. Tawarmalani, and N.V. Sahinidis. A lagrangian approach to the pooling problem. *Industrial and Engineering Chemistry Research*, 38(5):1956–1972, 1999.

O. Arvesen, V. Medbo, S.E. Fleten, A. Tomasgard, and S. Westgaard. Linepack storage valuation under price uncertainty. *Energy*, 52:155–164, 2013.

C. Borraz-Sanchez. Optimization methods for pipeline transportation of natural gas, 2010.

R.G. Carter, H.H. Rachford, and Jr. Advantica. Optimizing linepack management to hedge against future load uncertainty. pages 1–19. Pipeline Simulation Interest Group, 2003.

R. Carvalho, L. Buzna, F. Bono, M. Masera, D.K. Arrowsmith, and D. Helbing. Resilience of natural gas networks during conflicts, crises and disruptions. *PLOS ONE (doi:10.1371/journal.pone.0090265)*, 9(3):1–9, 2014.

M. Chaudry, N. Jenkins, and G. Strbac. Multi-time period combined gas and electricity network optimisation. *Electric Power Systems Research*, 78(7):1265 – 1279, 2008.

A. Chebouba. Multiobjective optimization of linepack management of gas pipeline system. *Journal of Physics: Conference Series*, 574(1):1–4, 2015.

L. Contesse, J.C. Ferrer, and S. Maturana. A mixed-integer programming model for gas purchase and transportation. *Annals of Operations Research*, 139(1):39–63, 2005.

N. De Nevers and A. Day. Packing and drafting in natural gas pipelines. *Journal of Petroleum Technology*, 35(3):655–658, 1983.

L.R. Foulds, D. Haugland, and K. Jornsten. A bilinear approach to the pooling problem. *Optimization*, 24(1-2):165–180, 1992.

A. Herran-Gonzalez, J.M. De La Cruz, B. De Andres-Toro, and J.L. Risco-Martin. Modelling and simulation of a gas distribution pipeline network. *Applied Mathematical Modelling*, 33(3):1584–1600, 2009.

A. Holland. *Applications and Innovations in Intelligent Systems XV.* Springer, 2008.

A. Holland. *Artificial Intelligence and Cognitive Science.* Springer, 2010.

N. Keyaerts. Gas balancing and line-pack flexibility, 2012.

P. Krishnaswami and D. Haugland. Linepack management for improved regularity in pipeline gas transportation networks. In S. Martorell, C. Guedes-Soares, and J. Barnett, editors, *Safety, reliability and risk analysis: theory, methods and applications*, pages 2963–2969. CRC Press, 2008.

P. Krishnaswami, K.S. Chapman, and M. Abbaspour. Compressor station optimization for linepack maintenance. In *Proceedings of the 36th PSIG annual meeting.* Palm Springs, 2004.

E.S. Menon and P.S. Menon. *Gas Pipeline Hydraulics.* Trafford, 2013.

K.T. Midthun. Optimization models for liberalized natural gas markets, 2007.

R. Misener and C.A. Floudas. A review of advances for the pooling problem: Modeling, global optimization, and computational studies. *Applied and Computational Mathematics*, 8(1):3–22, 2009.

A. Neumann and G. Zachmann. *The Economics of Natural Gas Storage: A European Perspective.* Springer-Verlag, 2009.

O.T. Olanrewaju, M. Chaudry, M. Qadrdan, J. Wu, and N. Jenkins. Vulnerability assessment of the european natural gas supply. *Proceedings of the Institution of Civil Engineers - Energy*, 168 (1):5–15, 2015.

P. Praks, V. Kopustinskas, and Masera M. Monte-carlo-based reliability and vulnerability assessment of a natural gas transmission system due to random network component failures. *Sustainable and Resilient Infrastructure*, 2(3):97–107, 2017.

M. Qadrdan, M. Chaudry, J. Wu, N. Jenkins, and J. Ekanayake. Impact of a large penetration of wind generation on the gb gas network. *Energy Policy*, 38(10):5684 – 5695, 2010.

R.Z. Rios-Mercado and C. Borraz-Sanchez. Optimization problems in natural gas transportation systems: A state-of-the-art review. *Applied Energy*, 147(1):536–555, 2015.

R.Z. Rios-Mercado, S. Wu, L.R. Scott, and E.A. Boyd. A reduction technique for natural gas transmission network optimization problems. *Annals of Operations Research*, 117(1):217–234, 2002.

T.H. Welch, J.G. Smith, J.P. Rix, and R.D. Reader. Meeting seasonal peak demands for natural gas. *Journal of Operational Research Society*, 22(1):93–106, 1971.

A.D. Woldeyohannes and M.A.A. Majid. Simulation model for natural gas transmission pipeline network system. *Simulation Modelling Practice and Theory*, 19(1):196–212, 2011.

S. Wu, R.Z. Rios-Mercado, E.A. Boyd, and L.R. Scott. Model relaxations for the fuel cost minimization of steady-state gas pipeline networks. *Mathematical and Computer Modelling*, 31(2-3): 197–220, 2000.

Q.P. Zheng, S. Rebennack, N.A. Iliadis, and P.M. Pardalos. *Optimization models in the natural gas industry.* Springer-Verlag, 2010.

G.Y. Zhu, M.A. Henson, and L. Megan. Dynamic modelling and linear model predictive control of gas pipeline networks. *Journal of Process Control*, 11(2):129–148, 2001.

R. Zwitserloot and A. Radloff. *Handbook Utility Management.* Springer-Verlag, 2009.