DEVELOPMENT AND ANALYSIS OF A PREDICTIVE CONTROL ALGORITHM FOR EMBEDDING IN A MICROPROCESSOR CONTROLLER

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ABSTRACT

The aim of this paper is to develop a predictive control algorithm and compare it with thermostat based control. The approach is based on modelling thermostat based control in EnergyPlus, and creating a predictive model using Fast Fourier Transform (FFT). Thermostat based control was modelled in EnergyPlus using a simple single zone building with 42% of south glazing. Predictive model was developed as an FFT response function to heat input, thus obtaining an inverse model and applying it several time steps ahead. The heating load and predictive percentage of dissatisfied of thermostat based control and of the predictive control cases were compared. The results show that in this particular study the predictive control approach leads to just over 20% reduction of energy consumption whilst keeping similar level of thermal comfort. In addition to investigating the modelling of predictive control using Fourier series, the aim of the research is to assess the feasibility of embedding this type of control algorithm in a microprocessor controller.

INTRODUCTION

Space heating in the majority of UK homes is controlled by on/off switching using room thermostats. This type of control stops supplying heat when the set temperature is reached. As heat input has a time-delayed effect on temperature rise, this type of control results in the room temperature still rising above the set temperature when the heating stops. Additionally, thermostatic control does not take into account solar gains until the set temperature is reached, by which time overheating will occur. This causes additional and unnecessary heat consumption and carbon emissions, whilst decreasing thermal comfort.

The aim of this paper is to compare thermostat based control with a predictive control algorithm in order to investigate the scale of comfort improvement and energy savings, and the feasibility of embedding this type of control algorithm in a microprocessor device for control of heating in homes. Thermostat based control was modelled in EnergyPlus using a simple box-building with 42% of south glazing. Predictive model was developed by first disaggregating the building response to external influences and heating system input, and subsequently by creating the corresponding FFT response functions. The resultant reverse model was applied several time steps ahead and compared with the model of thermostat based control.

Previous work

Model predictive control (MPC) has been under development over the past three decades. In this paper we overview only a few precedents relevant to this study.

Nygård Ferguson (1990) developed a predictive control algorithm based on anticipation of solar gains, and conducted a detailed study of predictive control of two identical offices in an experimental building over a winter period. One of the offices was controlled by a conventional controller, and the other by a computer running the predictive control algorithm. The test facility enabled the two controllers to be interchanged every two weeks in order to make a fair comparison between different control methods. There were over 40 sensors that monitored building performance every minute. In the office with the predictive controller, a 27% reduction of energy consumption was observed, together with improved thermal comfort. During sunny days the relative energy saving rose to 35%. At the same time a 29% lower rejection of solar gains was observed in the office with predictive controller, in comparison with 45% of solar gains rejection in the office with conventional controller.

Maciejowski (2002) demonstrated the advantage of MPC over other well established control methods, such as PID control. It is now well known that MPC tracks square wave and ramp wave fluctuations much better than PID control, in advance of signal changes, resulting in reduced control error. MPC compensates better for unexpected disturbance and is more robust to noise propagation, providing more stable control (Haugen, 2010).

As increasingly more powerful hardware used for control systems became capable of running more sophisticated software, numerous authors worked on predictive control in recent years with varying results. Cho and Zaheer-uddin (2003) used a model based on Fourier series in combination with TRNSYS and relied on weather predictions to create a predictive controller. They reported 10%-12% savings in winter months, increasing to 35% on a warm day, consistent with the findings by Nygård Ferguson (1990).

Eynard et al. (2013) conducted an experimental study of low energy tertiary buildings with intermittent occupancy. Predictive controllers, running either linear optimisation or a single simulation, were compared with PI controllers. Predictive controllers achieved energy savings between 12% and 16%, and a 4.5% improvement of thermal comfort.

Zhao et al. (2013) conducted a simulation study of predictive control using EnergyPlus and Matlab/Simulink and reported 18.9% of energy saving attributable to predictive control, whilst maintaining similar thermal comfort conditions.

The main difference between the body of other work and the study presented in this paper is in the way of capturing the near future building response using digital filters developed on the basis of Fourier transforms.

METHOD

The specific objectives of this study were: 1) to simulate a simple building controlled by a realistically modelled room thermostat, and 2) to create a predictive model using FFT.

Thermostat based control was modelled using logical functions, and implemented as an EMS code within EnergyPlus IDF file. Dynamic simulation was then carried out, and sub-hourly output of room temperature, which contained fluctuations resulting from the thermostat operation, was used to build a simplified dynamic model of the building with FFT.

Modelling thermostat based control

The first step was to model thermostat based control in a building simulation model. This was investigated and proved to be difficult with mainstream simulation tools, such as DesignBuilder, IES, TRNSYS, and standard features of EnergyPlus, as these tools keep the building at the set temperature during operating hours. Therefore, temperature fluctuations that occur in real buildings controlled by a room thermostat could not be replicated in the model using a standard approach to modelling a building.

The solution to use EMS scripting facility in EnergyPlus, running under EnergyPlus Runtime Language (Erl) was found to be promising. However, with minimum debugging facilities provided and the language still under development, coding in EMS proved to be hard but ultimately worthwhile task.

The thermostat functionality was implemented in EnergyPlus as shown in Figure 1.

The variable `SetTEmperatureAdjustment` was subsequently used in an EMS Actuator to override a standard schedule operating a fixed temperature setting, and this was altered between `Tmax` and `Tmax – deltaT` in the code, thus ensuring realistic room thermostat simulation. The results of this approach are shown in Figure 2 through to Figure 6, where there appears to be a realistic fluctuation of the zone air temperature within the dead-band of the set temperature.

```
EnergyManagementSystem:Program,
 Heating_Manager ,         !- Name
 SET deltaT = 2.0,         !- Program Line 1
 SET Tz = Tzone,           !- Program Line 2
 SET Tmax  = 19.0,         !- A3
 SET Tmin = Tmax - deltaT,!- A4
 IF (Tz > Tmax),           !- A5
  SET Heating_Status = 0, !- A6
  SET SetTEmperatureAdjustment = 5.0 !- A7
 ELSEIF (Tz < Tmin),       !- A8
  SET Heating_Status = 1, !- A9
  SET SetTEmperatureAdjustment=25.0, !- A10
 ENDIF;                    !- A11
```

Figure 1 Room thermostat code in EnergyPlus EMS

Creating a predictive model using Fourier transforms

Joseph Fourier demonstrated in 1822 in his book 'The Analytical Theory of Heat' that every periodic function can be represented with a combination of a series of sine and cosine waves with varying amplitudes and frequencies using a discrete transform (Discrete Fourier Transform or DFT):

$$f(x) = \frac{a_0}{2} + \sum_{k=1}^{n} (a_k \sin(kx) + b_k \cos(kx)) \qquad (1)$$

where

f(x) = periodic function of x

$a_{0,1,\ldots,k}$ = weighting factors

n = number of harmonics.

The DFT is computationally intensive and a breakthrough in its application was based on Danielson-Lanczos Lemma in the form of a Fast Fourier Transform or FFT (Weisstein, 2016). Instead of the computational intensity of the DFT being proportional to $O(N^2)$, the FFT computational intensity is proportional to $O(N\log_2 N)$. As Press et al. (2007) point out, the difference between the two methods for a large number of computations is immense, and can be equivalent to a difference between weeks of CPU time and seconds of CPU time. This makes the FFT attractive for simplified yet accurate dynamic modelling of buildings and creates opportunities for embedding such models in microprocessor control devices.

The basic premise for the method for predictive control in this study is to create a functional relationship between time series of driving functions, such as heat gains from solar radiation and zone heating input and conductive and convective heat losses on the one hand, and consequences, such as zone air temperature on the other hand. When these functional relationships are created using FFT, the resultant time series can be manipulated in a similar manner to simple arithmetic. On this basis the time series can be multiplied using a process of convolution, and divided using a process of deconvolution (Press, et al., 2007).

In the particular case used in this study, zone air temperature is driven by three forcing functions: site outdoor air temperature, direct solar radiation and zone heating rate. Hence the method is developed in the following steps:

1. Run the simulation model in heated mode and free-running mode in order to separate the building response to external forcing functions and heating system input;
2. Obtain FFT of the free-running zone air temperature;
3. Obtain FFT of a combination of outdoor air temperature and solar radiation added together;
4. Divide the transforms from 2 and 3 above, thus creating *FreeRunning/AmbTemp.Solar* digital filter;
5. Find a difference between thermostat controlled temperature and free-running temperature for each simulation time step and obtain FFT of the resultant time series;
6. Obtain FFT of the time series representing the total zone heating rate;
7. Divide the transforms from 6 and 5 above (in that order), thus creating *Heating rate/Heating response temperature* digital filter;
8. Obtain FFT of a combination of ambient air and solar radiation added together and multiply the filter from step 4 by this transform;
9. Obtain inverse FFT to the transform from step 8, thus obtaining simulated free running air temperature;
10. Find a difference between thermostat set temperature and simulated free running air temperature on a time step basis, representing the temperature rise that the heating plant needs to achieve in order to keep the zone air temperature at thermostat set temperature;
11. Obtain FFT of the resultant time series from step 10 and multiply the filter from step 7 by it;
12. Obtain inverse FFT of the resultant transform from step 11, thus obtaining simulated zone heating rate.
13. Calculate temperature difference between sequential time steps of simulated free running air temperature from step 9 as $\Delta Tfr = T_{fr}(t+1) - T_{fr}(t)$;
14. If $\Delta Tfr > 0$, set the simulated zone heating rate to zero, else use it as predicted in step 12.

Several additional details are required here to fully understand the above method. Firstly, the free-running zone air temperature and thermostat controlled air temperature are taken from one hour into the future with reference to the forcing functions. As the simulations are carried out in time steps of 10 minutes, this means that the time series from steps 2 and 5 are brought forward by six time steps before obtaining their respective FFT representations. For this reason, the digital filters from steps 4 and 7 contain a ‘memory of the future’. When these digital filters are multiplied in steps 8 and 11 by the corresponding Fourier transforms, the resultant time series are continuous predictions one hour ahead of the forcing functions.

Apart from steps 1, 10, 13 and 14, all other steps involve operations with complex numbers (i.e. numbers with real and imaginary part). Obtaining an FFT of a time series transfers the time series from the time domain into a frequency domain. Operations such as multiplication and division of FFT representations of the time series are carried out in the frequency domain. Inverse FFT operations transfer the time series back into the time domain.

The use of a combination of the outdoor air temperature and solar radiation added together, instead of the traditional sol-air temperature also needs to be explained here. The sol-air temperature, as defined by Jones (1973) is a traditional way of representing a combined influence of solar radiation and ambient air temperature on the rate of entry of heat on the external wall surface. So why not use it here? There are a number of reasons. Sol-air temperature does not deal with direct solar gain through glazing, as well as heat gains or losses through glazing arising from the temperature difference between internal air and ambient air. The influence of solar radiation and ambient air temperature on the building will occur at different diurnal frequencies. The consequent gradual propagation of heat in and out of the building will occur at frequencies further modulated by different time lag and decrement factor properties of different constructions. As the FFT transforms time series from a time domain into a frequency domain resulting in as many different frequencies as the number of data points in the time domain, this means that the multitude of frequencies of the heat transfer in and out of the building will be picked by the resonance with the multitude of frequencies generated by the FFT. This makes it possible to add together the outdoor air temperature and solar radiation added together, knowing that the FFT will effectively separate their influences due to different frequencies at which they occur.

SIMULATIONS AND RESULTS

Figure 2 shows the results of EnergyPlus simulation of a simple box building in Birmingham in December. As it can be seen from this figure, thermostat operation controls the zone air temperature between *Tset* and *Tset-dT*, which are 19 °C and 17 °C respectively, indicated by the corresponding horizontal lines.

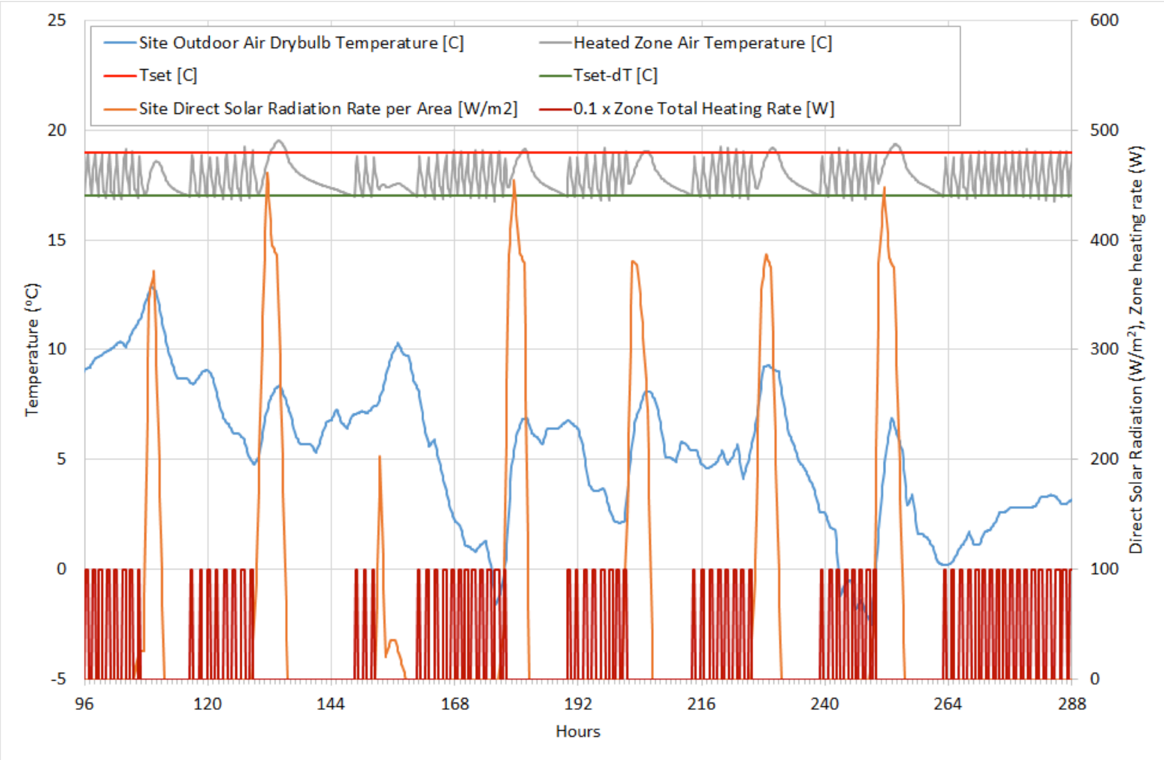

Figure 2 EnergyPlus simulation of a simple box building in December

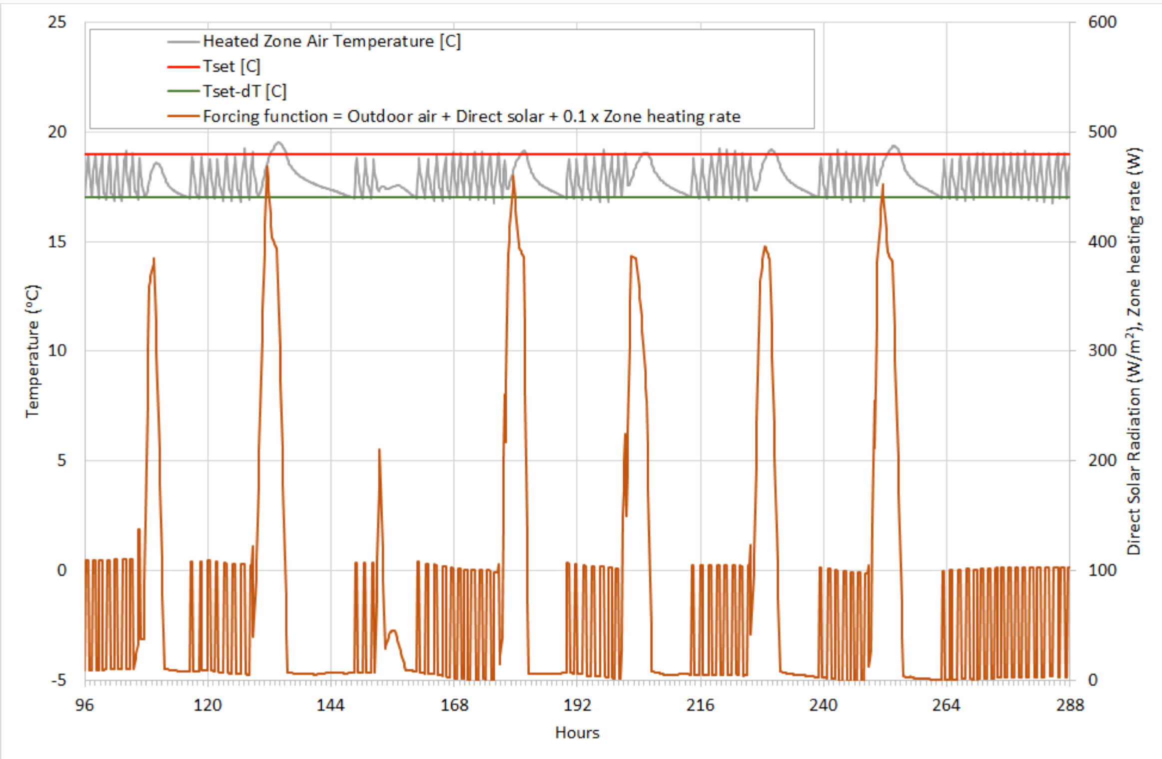

Figure 3 Simulated building response to three forcing functions added together

The figure also shows ambient air temperature, direct solar radiation and zone heating rate, all three being the forcing functions that cause the building response represented by the zone air temperature fluctuating between *Tset* and *Tset-dT*. In the next step the forcing functions are added together without modification in the case of ambient air temperature and direct solar radiation, and with a scaled down zone heating rate using the factor of 0.1, as shown in Figure 3.

In order to obtain the temperature rise that the heating plant achieved in the thermostat controlled case, a free-running simulation was carried out and the results are shown in Figure 4.

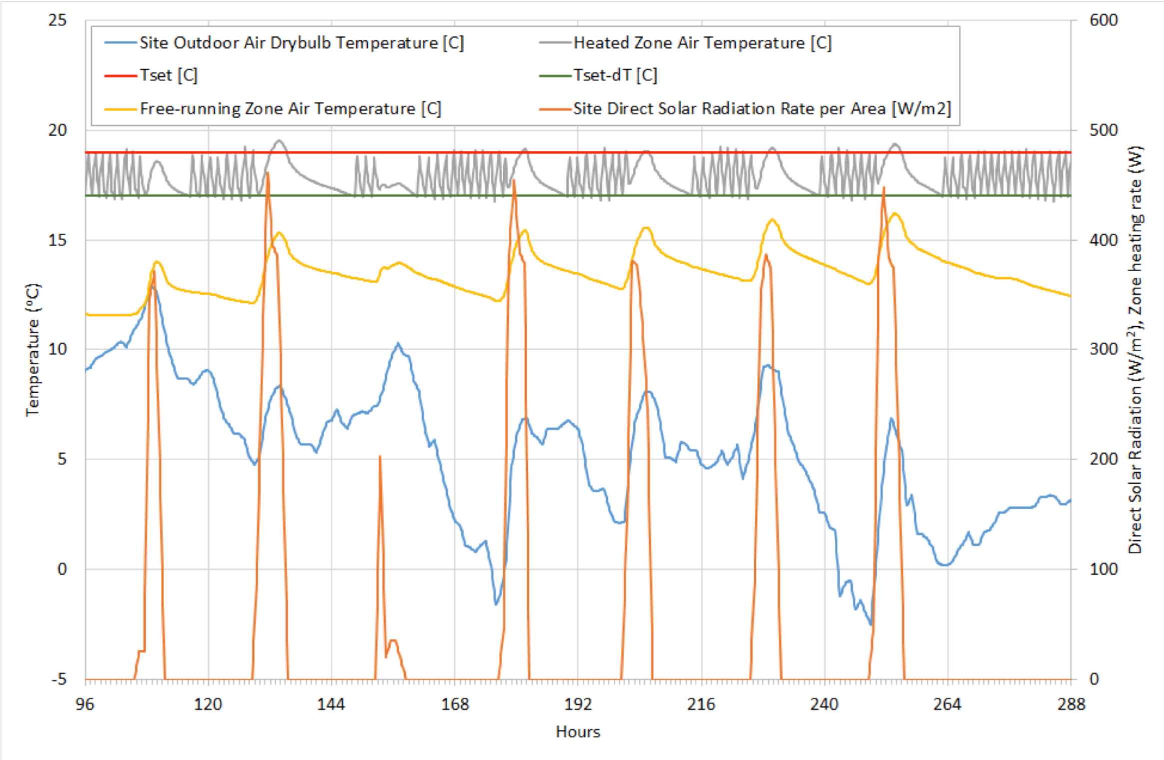

Figure 4 Results of free-running EnergyPlus simulation

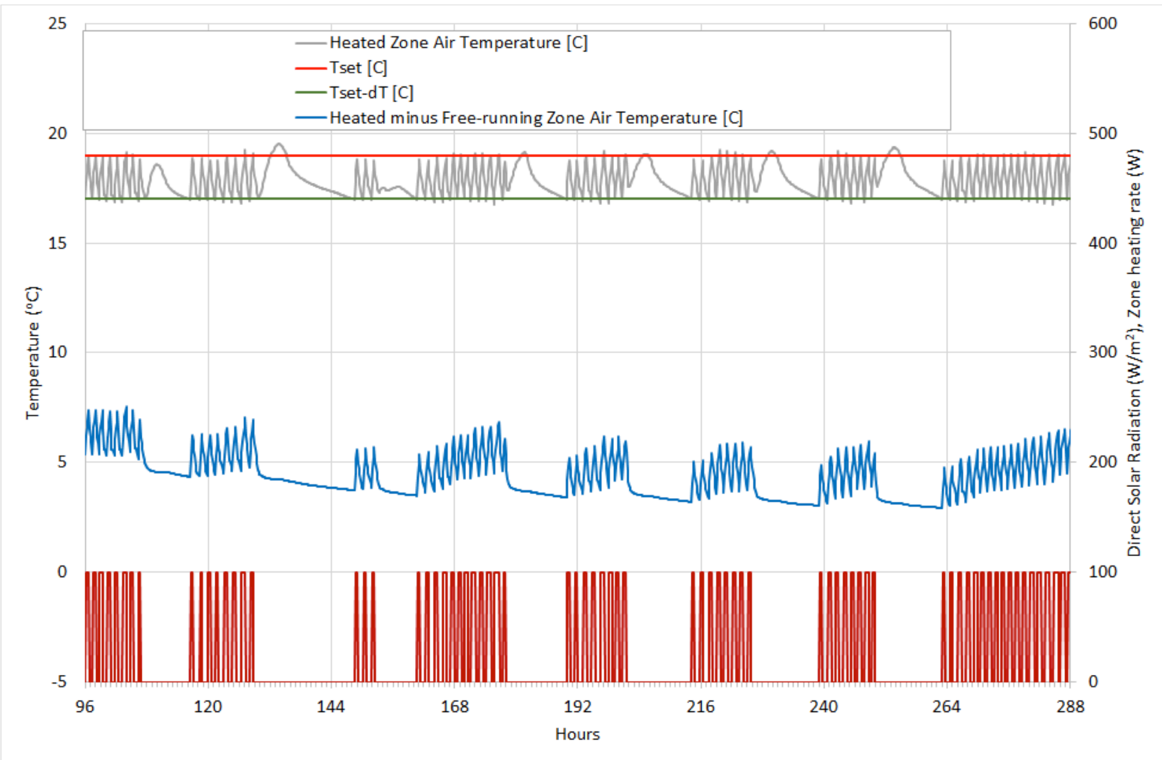

Figure 5 Air temperature rise achieved by the heating plant separated from the overall zone air temperature

The free-running zone air temperature obtained in this way was subsequently subtracted from the thermostat controlled zone temperature, and the result is shown in Figure 5, where both zone heating rate and the temperature rise achieved by the heating plant.

Finally, Figure 6 shows the predicted time series of the free-running temperature, the temperature rise to be achieved by the heating plant, and the predicted zone heating rate required to achieve this temperature rise. The free-running prediction is shown twice in this Figure: aligned with the prediction start time, (time step 120) where it appears to be out of sync with the actual free-running temperature because it is brought forward by 60 minutes; and aligned with the time step to which it relates, where it appears to be completely synchronised with the predicted free-running temperature.

Thermal comfort was calculated and compared for the thermostat controlled and predictively controlled cases. In the thermostat controlled case, the average PPD was 8.8% and the maximum PPD was 12.3%. In the predictively controlled case where the target temperature was *Tset*, the average PPD was 6.25% and the maximum PPD was 7.17.

Energy consumption in the predictive controlled case where the temperature target was set to the average of zone air temperature fluctuations between T_{set} and T_{set}-dT was 9% lower than in the thermostat controlled case.

Following these initial results, the scope of analysis was extended to the entire year using the same approach. Energy consumption in this extended period in the predictive control case was 79.96% of energy consumption in the thermostat controlled case, thus making energy consumption 20.04% lower in the predictive control case.

ANALYSIS AND DISCUSSION

The method reported in this paper is in its initial stage and it is developed as a prof of concept of predictive control using FFT. It appears to predict near future performance of the building accurately, whilst resulting in substantial energy savings and a small improvement in thermal comfort.

In a similar manner as the approach by Nygård Ferguson (1990), the predictive model in this study also uses anticipation of solar gains to reduce energy consumption. The Fourier filter (the response function) that encapsulates the relationship between a short term future of the free-running zone air temperature and a combination of ambient air temperature and solar radiation makes it possible for the model to check the near future temperature gradient caused by these external forcing functions and reduce the heating input accordingly.

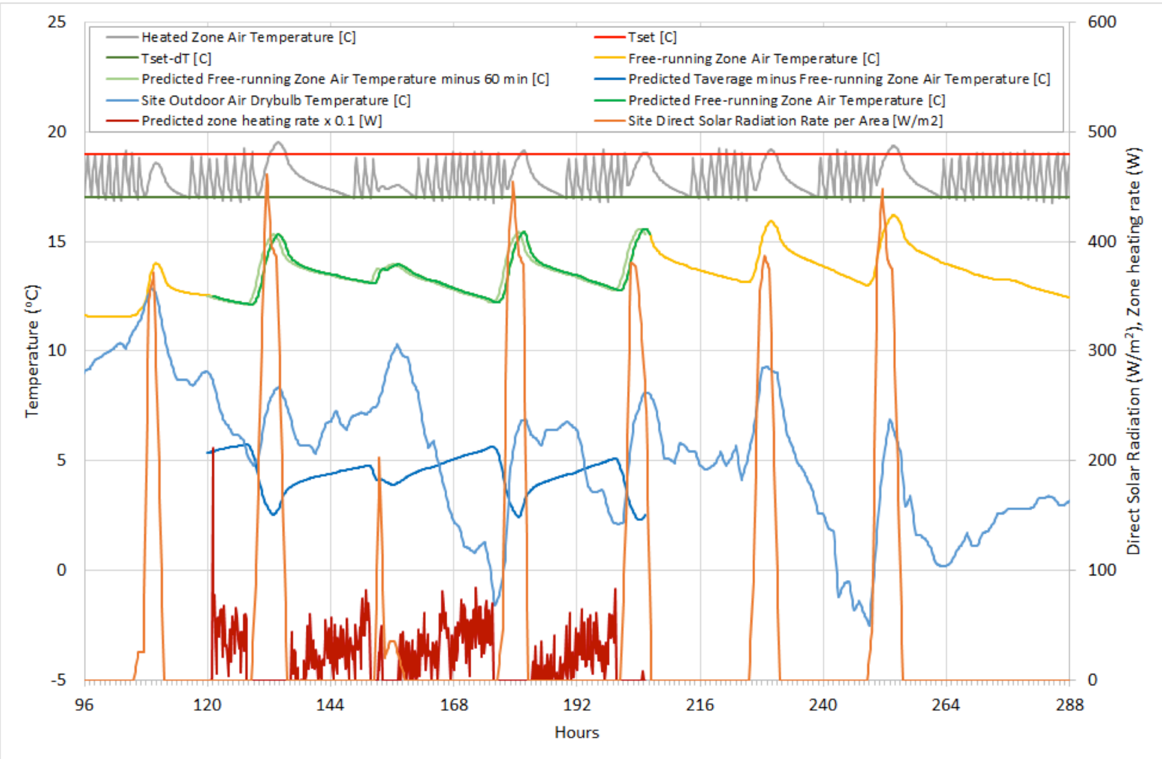

Figure 6 Predicted time series by the FFT model

The method of predictive modelling of a building described in this paper produces results that are almost identical to the thermostat controlled case in EnergyPlus. Root mean square error between thermostat controlled and predictively controlled case was found to be RMSE = 0.000000003 °C over the period of entire year in 10-minute time steps. This result is consistent with other similar FFT based modelling applications by Jankovic (2013, 2014).

One of the questions arising from our industrial collaboration is whether this type of predictive algorithm can be implemented in a software programmable device for controlling heating and cooling in homes, and whether the predictive aspect of control can be improved to deliver higher energy savings.

The predictive control analysis in this study was initially carried out over 512 time steps of 10 minutes each (just over 85 hours) in December. The energy savings of 9% over that period were compatible with the more substantial savings over a longer period influenced by higher solar gains, as reported by Nygård Ferguson (1990) and Cho and Zaheer-uddin (2003). As energy savings in predictive control increase with solar gain, this study was extended to annual analysis in anticipation of higher savings. This turned out to be the case, and annual energy savings were found to be just over 20%.

Although Fourier transforms have high computational intensity of $O(N^2)$ in a DFT mode, the FFT mode has considerably reduced computational intensity of $O(N\log_2 N)$, as well as high accuracy. This makes the model potentially suitable for running in small scale control devices, and future work will investigate the application of this algorithm in a control device named i-Magine (Figure 7).

Figure 7 i-Magine home heating controller (with permission from InteSys Ltd, http://i-magine.world)

Buildings are driven by periodic forcing functions, such as diurnal fluctuations of ambient air temperature, solar radiation, and fluctuations of the heat input from the heating system. These inputs are further modulated by the properties of building materials, reducing the source frequencies and thus increasing the multitude of resultant frequencies of heat transfer fluctuations through building elements.

The FFT generates a spectrum of frequencies equal to the number of points of the source time series. These frequencies sample the heat transfer in and out of the building, detecting the resonance between corresponding frequencies on both sides. This approach therefore enables easy separation of frequencies of the external influences on the building, making the modelling task robust and flexible.

CONCLUSION

A method of predictive control that captures the relationship between forcing functions acting upon the building, such as solar radiation, ambient air temperature and heating energy input on the one hand, and a near future resultant internal air temperature on the other hand, has been developed using inverse modelling with the FFT.

This study was initially carried out over a short time period in December, achieving 9% of energy savings in the predictive control case in comparison with thermostat controlled case. Anticipating higher energy savings over a wider range of solar gains, the study was extended to the entire year. The results showed that the predictive control approach led to accurate prediction of the near future performance and to annual energy savings of just over 20% together with a small improvement in thermal comfort.

As the FFT transforms time series from time domain to frequency domain using as many frequencies as the number of data points in the time series, it is capable of capturing different frequencies of building heat inputs, both from the forcing functions, and from frequencies further modulated by the building materials. This enables the model do deal with the driving functions, such as ambient air temperature and solar radiation, combined by a simple addition, due to different frequencies of these driving functions being separated by the FFT process.

This scope of this research will be extended to a wider range of buildings and climate conditions in order to investigate opportunities for higher energy savings. The future work will also include an embedded hardware implementation.

NAMENCLATURE

CPU,	Central Processing Unit;
DFT,	Discrete Fourier Transform;
EMS,	Energy Management System scripting facility in EnergyPlus;
Erl,	EnergyPlus Runtime Language;
FFT,	Fast Fourier Transform;
IDF,	EnergyPlus Input Data File;
MPC,	Model Predictive Control;
PID,	Proportional-Integral-Derivative control;
PI,	Proportional-Integral control;

PPD,	Predicted Percentage of Dissatisfied;
$RMSE$,	Root Mean Square Error;
T_{set},	Thermostat set temperature;
$T_{set} - dT$,	Zone air temperature within the dead-band of the set temperature;

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