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Evaluation of Ultimate Strength of Reinforced Concrete Beams Strengthened with FRP Sheets under Torsion

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Abstract. The ultimate torque of reinforced concrete (RC) members strengthened with fiber reinforced polymer (FRP) sheets does not only depend on the torque of RC members, but also on the FRP contribution to the torque. For structural design, predicting the accurate torsional capacity of the strengthened beams is considerably important. Three existing models for calculating the ultimate torsional moment of RC beams and two existing models for computing the FRP contribution to the ultimate torque are described and combined. Based on an experimental database collected from existing literature, six combinations were discussed and evaluated from the calculative values compared with the experimental results. The comparison shows that the combination of ACI 318 and *fib* Bulletin 14 models (Group 2), as well as Chinese and Ghobarah models (Group 6), can reasonably and accurately predict the ultimate torque of beams strengthened with FRP sheet. Furthermore, the ultimate torque of six box-section beams strengthened with fully wrapping or U-wrap calculated by the Group 6 shows closely to the experimental results.

1 Introduction

Since the beginning of the present century, researchers have paid much attention to the torsional behavior of reinforced concrete (RC) beams strengthened with fiber-reinforced polymer (FRP) sheets by experimental investigation [1-3]; however, very few models exist to predict the ultimate strength of strengthened RC beams. The *fib* Bulletin 14[4] proposed equations to calculate the ultimate torsional moment of strengthened beams involving two typical failure patterns, namely, FRP fracture and debonding. Ghobarah [2] proposed a model of the torsional moment contributed by externally bonded FRP and assumed that the mean ultimate strain of fiber was approximately 0.003, as indicated by experimental records. These two models assumed that no interaction exists between the RC and FRP sheets, which contributed to the torsional capacity of the beam. Deifalla and Ghobarah [5] used the compression field theory to propose a series of equations to predict the complete behavior of strengthened RC beams with FRP. On the basis of the softened membrane model for torsion, Zojaji and Kabir [6] developed a computational procedure to predict the full torsional behavior of strengthened beams subjected to torsion. Although complete procedure analysis can accurately predict the torsional behavior, the models need a trial-and-error algorithm for iteration to calculate each point of the torsional moment versus the angle of twist curve. For obtaining the torsional capacity of beams strengthened with FRP in design, the full computational procedure is made to be considerably complicated but difficultly accessible. The use of available models to calculate the precise torque of

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strengthened beams does not only depend on the contribution of RC beam but also on the contribution of FRP. To determine which model for computing the torsional strength of RC beam combined with existing torsional contribution of wrapped FRP is more accurate in calculating the ultimate torque of strengthened beam with FRP, this study presents three models to compute the ultimate torque of RC beams and two models to calculate the FRP contribution for torque. On the basis of the experimental data collected from the literature, the combination of models and comparison of ratios are discussed and evaluated to identify the appropriate models for reasonably and accurately predicting the ultimate torque of strengthened beams.

2 Models of Torsional Strength of Reinforced Concrete Beams

2.1 Rahab model [7]

The model was proposed by hollow tube analogy, which was a simple method to predict the ultimate torque of RC beam subjected to torsion. This model showed that the torsional strength was correlated with the volume of the stirrup and longitudinal reinforcement of beams. The equation was deduced from the relationship between the ultimate torque and the ultimate shearing stress in the walls of the equivalent tube as follows:

$$T_{uc} = 0.67(A_c^2 / p_c)v_u \tag{1}$$

where T_{uc} is the ultimate torque of the RC beam section; A_c and p_c are the outer area and the perimeter of the beam cross section, respectively; and v_u is a nominal shear stress related to transverse and longitudinal steel and concrete compressive strength, which can be calculated from the curve based on the results of modified compression field theory[7].

2.2 ACI model [8]

The equation designed to calculate the ultimate torque of RC beams recommended by ACI 318-2011[8] is:

$$T_{uc} = \frac{2A_{cor}A_t f_{yv}}{s_t}\cot(\theta) \tag{2}$$

where A_{cor} is the gross area enclosed by shear flow path; A_t is the area of stirrup; f_{yv} is the yield strength of stirrup; S_t is the spacing of stirrups, and θ is the angle of the diagonal crack with respect to the horizontal axis of the beam. θ shall represent 45° for RC members and 37.5° for pre-stressed members. The equation assumed that the torsional resistance is afforded by mainly closed transverse reinforcement. The outside concrete of these stirrups is comparatively invalid.

2.3 Chinese code model [9]

The torsional strength of RC beams is obtained by the contribution of concrete tensile strength and the contribution of transverse and longitudinal reinforcement in the following models:

$$T_{uc} = 0.35 f_t W_t + 1.2\sqrt{\zeta} f_{yv} \frac{A_t A_{cor}}{s_t} \tag{3}$$

$$\zeta = \frac{f_y A_{sl} s_t}{f_{yv} A_t u_{cor}} \tag{4}$$

where f_t is the tensile strength of concrete; W_t is the plastic resistant moment of the cross section of torsional members; ζ is the strength ratio of longitudinal reinforcement to transverse reinforcement; f_y and A_{sl} are the yield strength and area of longitudinal steel, respectively; and u_{cor} is the perimeter of the center line of the shear flow. In the subsequent equations, W_t should be considered when a different cross section of members uses different equations to calculate the rectangular and box beams as follows:

$$W_t = \frac{b^2}{6}(3h - b) \quad \text{for rectangular beam} \tag{5}$$

$$W_t = \frac{b^2}{6}(3h - b) - \frac{(b-2t)^2}{6}\left[3(h-2t) - (b-2t)\right] \quad \text{for box beam} \tag{6}$$

where b, h, and t corresponding to the width, height, and web thickness of the box beam.

3 Models of FRP Contribution for Torsional Strength

3.1 *fib* model [4]

The externally bonded FRP to beam provides the contribution to the torsional capacity of beams, of which the contributions of full wrapping and U-jacketing wrapping are calculated as follows:

$$T_f = \frac{2E_f \varepsilon_f t_f w_f bh \cot(\theta)}{s_f} \quad \text{or} \quad T_f = \frac{E_f \varepsilon_f t_f w_f bh \cot(\theta)}{s_f} \tag{7}$$

For full wrapping with CFRP in fracture controls, effective strain is computed as follows:

$$\varepsilon_{fe} = 0.17\left(\frac{f_c^{2/3}}{E_f \rho_f}\right)^{0.3} \varepsilon_{fu} \tag{8}$$

For U-jacketing CFRP wrapping, the effective strain depended on fracture and peeling off, which is computed as follows:

$$\varepsilon_{fe} = \min\left[0.17\left(\frac{f_c^{2/3}}{E_f \rho_f}\right)^{0.3} \varepsilon_{fu}, 0.65\left(\frac{f_c^{2/3}}{E_f \rho_f}\right)^{0.56} \times 10^{-3}\right] \tag{9}$$

where T_f is the torsional strength of FRP contribution, E_f is Young's modulus of the FRP, ε_{fe} is the effective strain in the fiber, t_f is the thickness of FRP, s_f is the spacing of FRP strips, w_f is the width of FRP, f_c is the concrete compressive strength, and $p_f = 2t_f w_f / bs_f$, which represents the FRP reinforcement ratio.

3.2 Ghobarah model [2]

This model assumed that the average ultimate strain of fiber was approximately 0.003; thus, the equation was proposed by simplifying the *fib* model as torsional capacity of full and U-jacketing FRP contribution, which is computed as follows:

$$T_f = 0.006 w_f t_f E_f bh / s_f \quad \text{or} \quad T_f = 0.003 w_f t_f E_f bh / s_f \tag{10}$$

The equation shows that that the angle of the diagonal crack with respect to the horizontal axis of the beam θ is at 45° for RC members, and the equation is only applicable for the vertical fiber wrap of transverse fiber with respect to the longitudinal axis of the beam. For U-jacketing FRP torsional contribution, the torque is equal to half of full wrapping. The total torsional moment of the beam strengthened with FRP can be computed by combining the torsional contribution of the RC beam and the contribution of the externally bonded FRP as follows:

$$T_u = T_{uc} + T_f \tag{11}$$

4 Database Collection

A full database consisting of 28 experimental beams (Table 1) was utilized to compare the theoretical torsional moment of FRP strengthened beams with the experimental results. The database was collected from the existing literature [1, 2, 10-12], which included different cross-sections, such as rectangular and box beams, strengthening configuration of full wrapping with strips or continuous sheets and U-shaped beams, and beams without internal stirrups. The values of the database were obtained from 6 beams with box cross section and 22 beams with rectangular cross-section. In the database, 5 beams strengthened with U-jacketing strips or continuous sheets, 6 beams strengthened with complete wrapping, and 17 beams wrapped with full strips were clearly identified. 6 rectangular beams without transverse reinforcement were also included in the database. The properties of the strengthened RC beams such as cross section dimension, concrete compressive strength, stirrup and longitudinal steel properties, FRP strengthening configuration, and FRP properties are listed in Table 1.

5 Comparisons and Discussions

Assuming that no correlation exists in the contribution of the torsional moment between RC beam and FRP wrapping [2], the total ultimate torque should be computed by two-part addition. Therefore, the discussion above described the equations to calculate the torque of non-strengthened beam and those for calculating the torsional contribution of FRP wrapping. The experimental ultimate torque for calculating ratios, average values, and standard deviations through six combinations are listed in Table 2. As shown in the table, Group 5 gives the poorest calculation among the experimental results, exhibiting the highest average and standard deviation values. Therefore, the combination of Group 5 is inappropriate in calculating the ultimate torque of strengthened beams. The ACI and *fib* models (Group 2), as well as the Chinese and Ghobarah models(Group 6), present good results in average ratios, which are 1.097 and 1.099, respectively; however, their standard deviations are higher than those of other groups, except Group 5. Almost all numbers calculated from Group 1 and Group 3 are noticeably greater than the experimental numbers, excluding the results of several beams, showing that the combinations have remarkably overestimated the calculation of the ultimate torque. The experimental ultimate torsional values of strengthened beams from the literature[2] are all remarkably lower than the computational values in all combinations. Although Group 3 presents the lowest standard deviation 0.156, it also displays the same case that most computed values with Group 1 and Group 4 are overestimated. Groups 1 and 4 of the Rahal model, combined with two equations of FRP torsional contribution calculating the strengthened box beams, show twice the experimental values probably because the area of the box beams used in the equation is not subtracted a hollow part.

According to all the group comparisons, Groups 2 and 6 showed relatively reasonable and close results compared with the experimental values, which are particularly dotted in Figure 1. Two groups display the same case with the ratio distributed between two lines, but the ratio is very close to the line of T_{exp} / T_{cal} = 1. The calculation values are fully conservative at the high level of the torsional moment, but at the low level, some values are conservative and some are overestimated. Removed the highly overestimated calculative values of the literature [2], Group 2 and Group 6 can more accurately estimate the ultimate torque of strengthened beams. However, the combination of Chinese model and Ghobarah model (Group 6) can predict the ultimate strength of RC box-section strengthened with fully wrapping or U-wrap precisely which can be clearly found from the results of six strengthened box-section beams in Table 2.

Table 1. Details of beams collected from the literature and tested by the authors.

Ref.	Beams	Section (mm)	*fc* (MPa)	Ø* (mm)	S_s (mm)	f_{yv} & f_y (MPa)	t_f & layer (mm)	w_f (mm)	s_f (mm)	E_f (GPa)
Zhang et al. [1]	L2	150x250	16.75	6.5,10	120	256,446	0.111	60	120	235
	L3	150x250	16.75	6.5,10	120	256,446	0.111	60	120	235
	L5	150x250	18.72	6.5,10	120	256,446	0.111	60	120	235
	L6	150x250	18.72	6.5,10	120	256,446	0.111	60	150	235
	L7	150x250	18.72	6.5,10	120	256,446	0.111	60	150	235
	L10	150x250	16.75	6.5,10	120	256,446	0.111	60	120	235
Ameli et al. [11]	CFE	150x350	39	6,16	80	251,502	0.165	1[d]	1[d]	244
	CFE2	150x350	39	6,16	80	251,502	0.165	1	1	244
	CFS	150x350	39	6,16	80	251,502	0.165	100	200	244
	CJS[c]	150x350	39	6,16	80	251,502	0.165	100	200	244
	CJE[c]	150x350	39	6,16	80	251,502	0.165	1	1	244
Ghobarah et al. [2]	C1	150x350	37	6.32,15	70	457,409	0.165	1	1	235
	C2	150x350	37	6.32,15	70	457,409	0.165	100	200	235
	C4	150x350	37	6.32,15	70	457,409	0.165	250	325	235
	C5	150x350	37	6.32,15	70	457,409	0.165	100	250	235
Chalioris [12]	Ra-F1	100x200	27.5	0,8		0,560	0.111	1	1	230
	Ra-F2	100x200	27.5	0,8		0,560	0.112	1	1	230
	Ra-FS1502	100x200	27.5	0,8		0,560	0.112	150	300	230
	Rb-F1	150x300	28.5	0,8		0,560	0.111	1	1	230
	Rb-F2001	150x300	28.5	0,8		0,560	0.111	200	400	230
	Rb-F3001	150x300	28.5	0,8		0,560	0.111	300	600	230
Hii et al. [10]	FS050D2	350x500	56.4	6,25	125	426,395	0.176	50	175	240
	FH075D1	350x500x50[b]	48.9	6,25	125	426,395	0.176	50	262.5	240
	FH050D2	350x500x50[b]	52.8	6,25	125	426,395	0.176	50	175	240
	FH050D1	350x500x50[b]	56.4	6,25	125	426,395	0.176	50	175	240
Authors[a]	TBS[c]	400x350x50[b]	40.37	6,16	100	372,502	0.117	100	200	230
	TBSL1[c]	400x350x50[b]	35.03	6,16	100	372,502	0.117	100	200	230
	TBSL2[c]	400x350x50[b]	37.57	6,16	100	372,502	0.117	100	200	230

[a] beams were tested by authors; [b] for box beams;[c] U-jacketing wrapping; [d] for FRP complete wrapping.
*refer to the stirrup and longitudinal steel diameters.

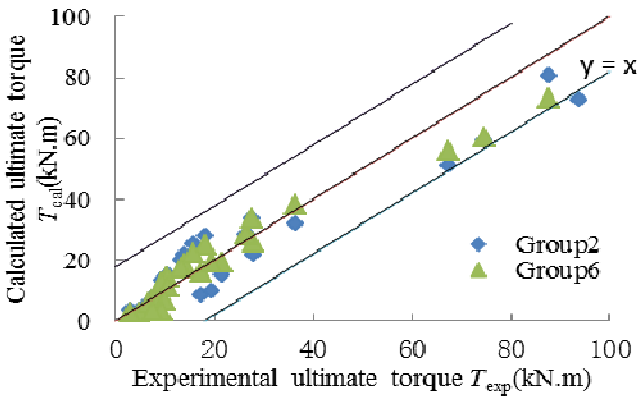

Figure 1. Group 2, 6 with experimental results comparison.

Table 2. Comparison of experimental results with calculated results with different model combinations.

Beams	Section (mm)	Ultimate Torque, T_{exp} (kN.m)	Group 1	Group 2	Group 3	Group 4	Group 5	Group 6
				T_{exp} / T_{cal}			T_{exp} / T_{cal}	
L2	150x250	8.21	0.775	0.974	0.752	1.018	1.391	0.979
L3	150x250	10.23	0.605	0.693	0.593	0.734	0.869	0.718
L5	150x250	10.53	0.615	0.701	0.598	0.759	0.895	0.733
L6	150x250	9.09	0.923	1.174	0.879	1.225	1.711	1.149
L7	150x250	9.55	0.621	0.719	0.602	0.788	0.954	0.758
L10	150x250	10.51	0.75	0.888	0.734	0.956	1.19	0.929
CFE	150x350	28	0.996	1.287	0.945	1.157	1.572	1.09
CFE2	150x350	36.5	0.948	1.136	0.913	0.99	1.197	0.951
CFS	150x350	21.7	0.998	1.412	0.934	1.215	1.89	1.121
CJS	150x350	17.4	1.161	2.022	1.056	1.184	2.093	1.075
CJE	150x350	19.5	1.203	1.982	1.101	1.092	1.698	1.007
C1	150x350	18.1	0.568	0.651	0.632	0.643	0.751	0.726
C2	150x350	14.1	0.547	0.65	0.626	0.639	0.784	0.749
C4	150x350	15.83	0.542	0.63	0.61	0.624	0.744	0.716
C5	150x350	13.4	0.54	0.66	0.632	0.63	0.799	0.759
Ra-F(1)	100x200	4.87	1.024	1.29	1.031	1.212	1.604	1.604
Ra-F(2)	100x200	6.65	0.935	1.084	0.939	0.943	1.095	1.095
Ra-FS150(2)	100x200	3.02	0.635	0.8	0.639	0.751	0.995	0.995
Rb-F(1)	150x300	10.05	0.767	1.04	0.778	0.979	1.471	1.471
Rb-F200(1)	150x300	9.315	0.993	1.566	1.012	1.359	2.727	2.727
Rb-F300(1)	150x300	7.52	0.801	1.264	0.817	1.097	2.202	2.202
FS050D2	350x500	93.8	1.069	1.291	0.699	1.58	2.12	0.886
FH075D1	350x500x50	67.5	0.475	1.566	0.942	0.534	2.468	1.208
FH050D2	350x500x50	74.8	0.467	1.451	0.932	0.534	2.369	1.24
FH050D1	350x500x50	87.7	0.462	1.207	0.864	0.543	1.982	1.199
TBS	400x350x50	26.67	0.423	0.886	0.74	0.484	1.201	0.948
TBSL1	400x350x50	26.52	0.428	0.902	0.744	0.485	1.194	0.932
TBSL2	400x350x50	27.56	0.41	0.8	0.678	0.454	0.989	0.809
Average Ratio			0.739	1.097	0.801	0.879	1.463	1.099
Standard Deviation			0.241	0.381	0.156	0.3	0.571	0.443

* Note:

Group 1 - (Rahal)Eq.1 + (fib 14)Eq.7
Group 2 - (ACI)Eq.2 + (fib 14)Eq.7
Group 3 - (China)Eq.3 + (fib 14)Eq.7
Group 4 - (Rahal)Eq.1 + (Ghobarah)Eq.10
Group 5 - (ACI)Eq.2 + (Ghobarah)Eq.10
Group 6 - (China)Eq.3 + (Ghobarah)Eq.10

6 Summary

Three models for calculating the torsional strength of RC beams and two models for computing FRP torsional contribution to strengthened members have been described. Assuming that no relationship exists between these two parts, the total ultimate torque of strengthened members can be obtained by the torsional strength of the RC beams and the torque of the FRP contribution. Six groups were combined using different models from two parts. Group 5 presents the highest mean and standard deviation values, which makes the combination inappropriate to calculate the ultimate torque. Although Group 1 and Group 3 reveal the lowest standard deviation, the most calculated results are more overestimated than the experimental values the same case to Group 4. The combination of ACI 318 and *fib* Bulletin 14 models (Group 2), as well as the combination of Chinese and Ghobarah models (Group 6), predict relatively reasonable and accurate results compared with the experimental values. For prediction of the ultimate torque of RC box-section beams strengthened with FRP completely wrapping or U-wrap, the results computed by Group 6 are in good agreement with the experimental values.

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