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REVIEW

Convexity-preserving Bernstein–Bézier quartic scheme

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Abstract A C^1 convex surface data interpolation scheme is presented to preserve the shape of scattered data arranged over a triangular grid. Bernstein–Bézier quartic function is used for interpolation. Lower bound of the boundary and inner Bézier ordinates is determined to guarantee convexity of surface. The developed scheme is flexible and involves more relaxed constraints.

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1. Introduction

Shape preserving scattered data interpolation is always desirable in geometric modeling, visualization, engineering, sectional drawing, designing pipe systems in chemical plants, surgery, designing car bodies, ship hulls and airplane, geology, meteorology, etc. In general, ordinary interpolating techniques do not preserve the shapes of data. In this paper, we have developed a method to preserve the shape of scattered data when it is convex. Convexity is an important shape property and its applications are in designing of telecommunication system, nonlinear programming, engineering, optimization theory, parameter estimation, approximation theory [1–3].

In recent years, a good amount of work has been published on the shape preservation of univariate and bivariate convex data. It is very hard to detail all these existing schemes. Some of the noticeable contributions are reviewed here. Cai [4] presented a four-point ternary subdivision scheme for the convexity preservation of curve data. The parameters were constrained to preserve the convexity and the generated limit curve was C^2. Levin and Nadler [5] introduced a one parameter family of C^∞ algebraic curves and discussed its properties. The convexity-preserving scheme was developed using these curves to create a convex curve from a convex polygon. The method was further generalized to the convex-preserving C^∞ interpolation in R^3 by algebraic surfaces. Pan and Wang [6] proposed an automatic parametric convexity-preserving scheme for curve data. A family of interpolating spline curves with a shape parameter was introduced. The range of these parameters was determined for the convexity preservation of global and piecewise convex data points. Yong-juan and Guo-jin [7] developed a new trigonometric polynomial curve with a shape parameter for the convexity preservation of convex curve data. The trigonometric polynomial curve was obtained by the blending of parameterized polygon and trigonometric polynomial splines. This construction resulted in the automatic generation of trigonometric polynomial curves with $C^2(G^2)$ continuity. The range of the shape parameter was determined for the convexity preservation of curve data. Floater [8] defined convexity and rational convexity preservation of systems of functions and proved that total positivity and rational convexity preservation are equivalent. Roulier [9] introduced a data refinement scheme to preserve the shape of convex data arranged over the rectangular grid. The refined bivariate data could be interpolated by any standard surface interpolation technique. Iqbal [10] modified the bivariate interpolation scheme [9] and developed more relaxed constraints.

Lai [11] derived some sufficient conditions on the B-net of a multivariate Bernstein–Bézier polynomial to preserve the shape of convex data. In [11], author also discussed the sufficient conditions for the convexity of bivariate box spline surfaces. Lai [2] used bivariate C^1 cubic splines to preserve the shape of convex scattered data. In [2], convexity preserving interpolation problem was set as quadratically constrained quadratic programming problem. Quadratic programing problem was simplified to linearly constrained quadratic programming problem. Piah et al. [3] constructed a bivariate C^1 interpolant to preserve the shape of convex scattered data. The surfaces are comprised of cubic Bézier triangular patches and the sufficient conditions of convexity were derived as lower bounds of Bézier points. In a triangular patch where convexity is lost, the initial gradients at the data sites are modified so as to satisfy the sufficient conditions for convexity. Renka [12] developed a Fortran 77 software package for constructing a C^1 convex surface that interpolates arbitrarily distributed convex data. The set of nodal gradients were modified to make a convex surface from the convex nodal values and gradients. Schumaker and Speleers [13] constructed the sets of adequate linear conditions to ensure convexity of a triangle by Bernstein–Bézier method.

The study of this paper has proposed a C^1 convex scattered data interpolation scheme using Bernstein–Bézier quartic function. The Bernstein–Bézier quartic function has three inner, nine boundary and three vertex ordinates. The lower bounds of the inner and boundary Bézier ordinates are determined to preserve the convex shape of data. Since the Bernstein–Bézier quartic function has five more control points(Bézier ordinates) than cubic function [3], the convexity-preserving Bernstein–Bézier quartic scheme of this paper more accurately follows the convex shape of data as compared to [11]. In [2], the sufficient conditions for the convexity preservation of scattered data were in the form of system of inequalities with Bézier ordinates as parameters. The convexity preserving scheme of this papers has a unique lower bound for all the Bézier ordinates; thus, it is simple in implementation as compared to [11]. In [2], the convexity-preserving problem was transformed to a quadratic programming problem; thus, it is computationally expansive than the proposed convexity-preserving Bernstein–Bézier quartic scheme. The authors in [11] and [2] did not provide any numerical example of the developed convexity-preserving scheme.

The remainder of the paper is organized as follows: In Section 2, the Bernstein–Bézier quartic function [14] is rewritten. In Section 3, constraints are also derived on the Bézier ordinates to interpolate convex scattered data as C^1 convex surface. The developed scheme of this paper is demonstrated graphically in Section 4. Finally Section 5 concludes the paper.

2. C^1 Bernstein–Bézier quartic triangular patch [14]

Let $T = \Delta V_1V_2V_3$ be a non-degenerate triangle, then any point $V = (x, y)$ of the triangle T can be expressed w.r.t the barycentric coordinates u, v and w as

$$V = uV_1 + vV_2 + wV_3, \quad u + v + w = 1 \quad u, v, w \geq 0, \tag{1}$$

where any point $V_i = (x_i, y_i)$, $i = 1, 2, 3$.

The Bernstein–Bézier quartic function $P(u, v, w)$ over a triangular patch is given by

$$\begin{aligned} P(u,v,w) = {} & u^4b_{400} + v^4b_{040} + w^4b_{004} + 4u^3vb_{310} + 4uv^3b_{130} \\ & + 4u^3wb_{301} + 4uw^3b_{103} + 4v^3wb_{031} + 4vw^3b_{013} \\ & + 6u^2v^2b_{220} + 6v^2w^2b_{022} + 6u^2w^2b_{202} \\ & + 12u^2vwb_{211} + 12uv^2wb_{121} + 12uvw^2b_{112}, \end{aligned} \tag{2}$$

Here b_{400}, b_{040} and b_{004} are the Bézier ordinates at the vertices. b_{310}, b_{130}, b_{301}, b_{103}, b_{013}, b_{031}, b_{220}, b_{022}, b_{202}, are the boundary Bézier ordinates and b_{211}, b_{121}, b_{112} are the inner Bézier ordinates.

The following values of the boundary Bézier ordinates b_{310}, b_{130}, b_{301}, b_{103}, b_{013} and b_{031} are given by [15].

$$\left.\begin{aligned} b_{310} &= b_{400} + \tfrac{1}{4}\{(x_2 - x_1)f_x(V_1) + (y_2 - y_1)f_y(V_1)\}, \\ b_{130} &= b_{040} - \tfrac{1}{4}\{(x_2 - x_1)f_x(V_2) + (y_2 - y_1)f_y(V_2)\}, \\ b_{031} &= b_{040} + \tfrac{1}{4}\{(x_3 - x_2)f_x(V_2) + (y_3 - y_2)f_y(V_2)\}, \\ b_{013} &= b_{004} - \tfrac{1}{4}\{(x_3 - x_2)f_x(V_3) + (y_3 - y_2)f_y(V_3)\}, \\ b_{103} &= b_{004} + \tfrac{1}{4}\{(x_1 - x_3)f_x(V_3) + (y_1 - y_3)f_y(V_3)\}, \\ b_{301} &= b_{400} - \tfrac{1}{4}\{(x_1 - x_3)f_x(V_1) + (y_1 - y_3)f_y(V_1)\}. \end{aligned}\right\} \tag{3}$$

The Bernstein–Bézier quartic function (2) is C^1 at the vertices of triangle for the values of Bézier ordinates given in the set of Eq. (3).

3. Sufficient conditions for convexity of Bernstein–Bézier quartic triangular patch

In this section, we have derived sufficient conditions on the Bézier ordinates of each triangular patch to form a convex surface.

Theorem 1. The Bernstein–Bézier quartic triangular patch $P(u, v, w)$, defined over the triangular domain, in (2), is convex in the direction $d = \lambda_1V_1 + \lambda_2V_2 + \lambda_3V_3$, with $\lambda_1 + \lambda_2 + \lambda_3 = 0$, if the boundary Bézier ordinates b_{310}, b_{130}, b_{301}, b_{103}, b_{013}, b_{031}, b_{220}, b_{022}, b_{202} and the inner Bézier ordinates and b_{211}, b_{121}, b_{112} satisfy the following constraint:

$$\begin{aligned} & b_{i,j,k} \geq -r_0, \ \text{where}(i,j,k) \\ & \quad \in \{(4,0,0),(0,4,0),(0,0,4)\}, i+j+k = 4. \end{aligned}$$

Proof. Let $\{(x_i, y_i, F_i), i = 1, 2, 3\}$ be the convex scattered data defined over a triangle $\Delta V_1V_2V_3$. Lai [2] defined the convex function in a certain direction $d = \lambda_1V_1 + \lambda_2V_2 + \lambda_3V_3$, $\lambda_1 + \lambda_2 + \lambda_3 = 0$ as

Definition 1. A function f is said to be strictly convex in a given direction d if there exists a positive number $\varepsilon > 0$ such that

$$D_d^2 f(x, y) \geq \varepsilon,$$

where $D_d f(x, y)$ denotes the directional derivative in the direction d.

The second order directional derivative $D_d^2P(u, v, w)$, in the direction $d = \lambda_1V_1 + \lambda_2V_2 + \lambda_3V_3, \lambda_1 + \lambda_2 + \lambda_3 = 0$ is

$$\begin{aligned} D_d^2P(u,v,w) = \frac{\partial^2 P}{\partial d^2} = {} & \lambda_1^2\frac{\partial^2 P}{\partial u^2} + 2\lambda_1\lambda_2\frac{\partial^2 P}{\partial u\partial v} + 2\lambda_1\lambda_3\frac{\partial^2 P}{\partial u\partial w} \\ & + \lambda_2^2\frac{\partial^2 P}{\partial v^2} + 2\lambda_2\lambda_3\frac{\partial^2 P}{\partial v\partial w} + \lambda_3^2\frac{\partial^2 P}{\partial w^2}. \end{aligned} \tag{4}$$

$$\begin{aligned} \text{Let } b_{400} = A, b_{040} = B, b_{004} = C, b_{310} &= b_{130} = b_{301} = b_{103} \\ &= b_{031} = b_{013} = b_{220} = b_{202} = b_{022} = b_{211} = b_{121} \\ &= b_{112} = -r, \text{where } r \geq 0. \end{aligned} \tag{5}$$

Putting values of Bézier ordinates from (5) in (2), (2) reduces to

$$\begin{aligned} P(u,v,w) = {} & u^4A + v^4B + w^4C - \big(4u^3v + 4uv^3 + 4u^3w + 4uw^3 \\ & +4v^3w + 4vw^3 + 6u^2v^2 + 6v^2w^2 + 6u^2w^2 + 12u^2vw \\ & +12uv^2w + 12uvw^2\big)r, \end{aligned} \tag{6}$$

Using the relation $(u + v + w)^4 = 1$, (6) is rewritten as

$$P(u,v,w) = u^4A + v^4B + w^4C - (1 - u^4 - v^4 - w^4)r. \tag{7}$$

Substituting the value of $P(u, v, w)$ from (7) in (4), we have

$$\begin{aligned} D_d^2P(u,v,w) = {} & \frac{\partial^2 P}{\partial d^2} \\ = {} & 12\{\lambda_1^2u^2(A+r) + \lambda_2^2v^2(B+r) + \lambda_3^2w^2(C+r)\}. \end{aligned} \tag{8}$$

Take $Q(u, v, w) = D_d^2P(u, v, w)$. If $r = 0$ then $Q(u, v, w) > 0$ provided $A > 0$, $B > 0$ and $C > 0$. We are interested in finding the minimum positive value of r for which Q is positive. At the minimum value Q satisfies the following constraints:

$$\frac{\partial Q}{\partial u} - \frac{\partial Q}{\partial v} = 0 \text{ and } \frac{\partial Q}{\partial u} - \frac{\partial Q}{\partial w} = 0. \tag{9}$$

Substituting the value of $Q(u, v, w)$ from (8) in (9) we obtain the following relations:

1. $\dfrac{\partial Q}{\partial u} = \dfrac{\partial Q}{\partial v}$

   $\lambda_1^2u(A+r) = \lambda_2^2v(B+r) \Rightarrow \dfrac{u}{v} = \dfrac{\lambda_2^2(B+r)}{\lambda_1^2(A+r)}.$
2. $\dfrac{\partial Q}{\partial v} = \dfrac{\partial Q}{\partial w}$

   $\lambda_2^2v(B+r) = \lambda_3^2w(C+r) \Rightarrow \dfrac{v}{w} = \dfrac{\lambda_3^2(C+r)}{\lambda_2^2(B+r)}.$

These computations assert the following values of u, v and w

$$u = \frac{1}{\lambda_1^2(A+r)}, v = \frac{1}{\lambda_2^2(B+r)}, w = \frac{1}{\lambda_3^2(C+r)}.$$

Moreover, $u + v + w = \dfrac{1}{\lambda_1^2(A+r)} + \dfrac{1}{\lambda_2^2(B+r)} + \dfrac{1}{\lambda_3^2(C+r)}.$

Table 1 A convex scattered data set.

x	−1.0000	−1.0000	−1.0000	−1.0000	−1.0000	−1.0000	−1.0000	−1.0000	−0.7500
y	−1.0000	−0.7500	−0.2500	0	0.2500	0.5000	0.7500	1.0000	−0.7500
z	2.5000	2.0625	1.5625	1.5000	1.5625	1.7500	2.0625	2.5000	1.6250
x	−0.7500	−0.7500	−0.7500	−0.7500	−0.5000	−0.5000	−0.5000	−0.5000	−0.2500
y	−0.2500	0.2500	0.5000	1.0000	−1.0000	−0.7500	0	1.0000	−0.7500
z	1.1250	1.1250	1.3125	2.0625	1.7500	1.3125	0.7500	1.7500	1.1250
x	−0.2500	−0.2500	−0.2500	−0.2500	−0.2500	0	0	0	0
y	−0.2500	0.2500	0.5000	0.7500	1.0000	−1.0000	−0.5000	−0.2500	0
z	0.6250	0.6250	0.8125	1.1250	1.5625	1.5000	0.7500	0.5625	0.5000
x	0	0	0.2500	0.2500	0.2500	0.2500	0.2500	0.2500	0.5000
y	0.7500	1.0000	−1.0000	−0.5000	−0.2500	0	0.7500	1.0000	−0.7500
z	1.0625	1.5000	1.5625	0.8125	0.6250	0.5625	1.1250	1.5625	1.3125
x	0.5000	0.5000	0.5000	0.7500	0.7500	0.7500	0.7500	0.7500	0.7500
y	−0.5000	−0.2500	1.0000	−1.0000	0.7500	−0.5000	0	0.2500	1.0000
z	1.0000	0.8125	1.7500	2.0625	1.6250	1.3125	1.0625	1.1250	2.0625
x	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	–	–
y	−1.0000	−0.7500	−0.5000	−0.2500	0	0.2500	1.0000	–	–
z	2.5000	2.0625	1.7500	1.5625	1.5000	1.5625	2.5000	–	–

$$u = \frac{u}{u+v+w} = \frac{\lambda_1^{-2}(A+r)^{-1}}{\lambda_1^{-2}(A+r)^{-1} + \lambda_2^{-2}(B+r)^{-1} + \lambda_3^{-2}(C+r)^{-1}}, \tag{10}$$

as $u + v + w = 1$.

Similarly,

$$v = \frac{\lambda_2^{-2}(B+r)^{-1}}{\lambda_1^{-2}(A+r)^{-1} + \lambda_2^{-2}(B+r)^{-1} + \lambda_3^{-2}(C+r)^{-1}}, \tag{11}$$

and $w = \dfrac{\lambda_3^{-2}(C+r)^{-1}}{\lambda_1^{-2}(A+r)^{-1} + \lambda_2^{-2}(B+r)^{-1} + \lambda_3^{-2}(C+r)^{-1}}$. (12)

Substituting the values of u, v and w from (10)–(12) in (8), (8) reduces to

$$Q(u,v,w) = \frac{12}{\lambda_1^{-2}(A+r)^{-1} + \lambda_2^{-2}(B+r)^{-1} + \lambda_3^{-2}(C+r)^{-1}}. \tag{13}$$

Hence, from (13) $Q(u, v, w) = 0$ if

$$\frac{12\lambda_1^2\lambda_2^2\lambda_3^2(A+r)(B+r)(C+r)}{\lambda_2^2\lambda_3^2(B+r)(C+r)+\lambda_1^2\lambda_3^2(A+r)(C+r)+\lambda_1^2\lambda_2^2(A+r)(B+r)} = 0,$$

or

$$(A+r)(B+r)(C+r) = 0. \tag{14}$$

The roots of (14) are $r = -A$, $r = -B$ and $r = -C$. Take $r_0 = \max(-A, -B, -C)$. □

Remark 1. The boundary Bézier ordinates defined in (3) may or may not satisfy the lower bound proposed in Theorem 1. To overcome this problem a parameter γ is introduced in (3) as follows:

$$b_{310} = b_{400} + \frac{\gamma}{4}\{(x_2 - x_1)f_x(V_1) + (y_2 - y_1)f_y(V_1)\}, b_{130} = b_{040} - \frac{\gamma}{4}\{(x_2 - x_1)f_x(V_2) + (y_2 - y_1)f_y(V_2)\},$$

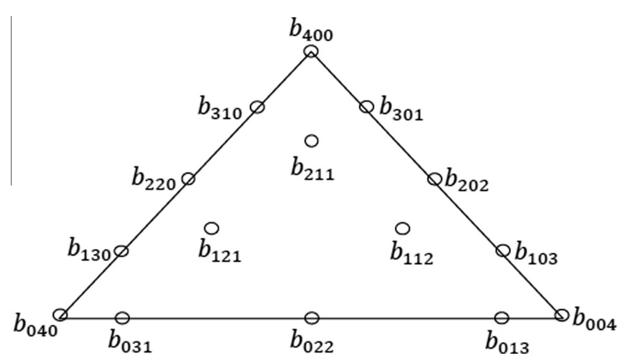

Figure 1 Locations of the Bézier ordinates of the Bernstein–Bezier quartic function defined over a triangle.

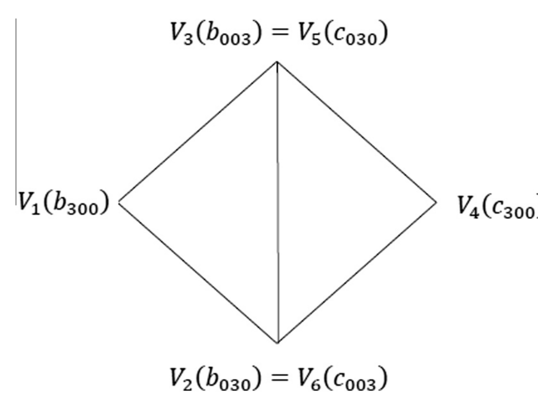

Figure 2 The adjacent triangles $T_1 = \Delta V_1V_2V_3$ and $T_2 = \Delta V_4V_5V_6$.

$$b_{031} = b_{040} + \frac{\gamma}{4}\{(x_3 - x_2)f_x(V_2) + (y_3 - y_2)f_y(V_2)\},$$
$$b_{013} = b_{004} - \frac{\gamma}{4}\{(x_3 - x_2)f_x(V_3) + (y_3 - y_2)f_y(V_3)\},$$

$$b_{103} = b_{004} + \frac{\gamma}{4}\{(x_1 - x_3)f_x(V_3) + (y_1 - y_3)f_y(V_3)\},$$
$$b_{301} = b_{400} - \frac{\gamma}{4}\{(x_1 - x_3)f_x(V_1) + (y_1 - y_3)f_y(V_1)\}.$$

For each of the Bézier ordinate(b_{310}, b_{130}, b_{301}, b_{103}, b_{013}, b_{031}), we are interested to choose $\gamma \in (0, 1)$ for which $b_{i,j,k} \geqslant -r_0$ or $b_{i,j,k} = b_{l00}(V) + \frac{\gamma}{4}D \geq -r_0$, $l = i + j + k$. b_{l00} is the Bézier ordinate at the vertex V and D is the directional derivative along the edge containing $b_{i,j,k}$ and b_{l00}.

If more than one triangle is incident at vertex V, then γ is calculated for all such triangles. The least of values of γ is the most plausible choice of γ.

$$b_{i,j,k}|_t = b_{l00}(V) + \frac{\gamma_t}{4}D_t \geqslant -r_0, t = 1, 2, 3, \ldots, s,$$

$$\gamma = \min\{\gamma_t, t = 1, 2, 3, \ldots, s\}.$$

Here s is the number of triangles incident at the vertex V.

3.1. C^1 continuity condition for the Bernstein–Bézier quartic triangular patch

Given two Bernstein–Bézier quartic triangular patches $P_1(u, v, w)$ and $P_2(u, v, w)$, having vertices $b_{i,j,k}$ and $c_{i,j,k}$ defined over the triangles $T_1 = \Delta V_1V_2V_3$ and $T_2 = \Delta V_4V_5V_6$ respectively. The necessary and sufficient conditions for C^1 continuity of these Bernstein–Bézier triangular patches along the edge $V_2V_3 = V_6V_5$ given by [16] are

$$c_{103} = ub_{130} + vb_{040} + wb_{031}, \quad (15)$$

$$c_{112} = ub_{121} + vb_{031} + wb_{022}, \quad (16)$$

$$c_{121} = ub_{112} + vb_{031} + wb_{022}, \quad (17)$$

$$c_{130} = ub_{103} + vb_{013} + wb_{004}. \quad (18)$$

Due to the gradient based estimation of Bézier ordinates b_{310}, b_{130}, b_{301}, b_{103}, b_{013} and b_{031}, the Eqs. (15) and (18) are automatically satisfied. The inner and boundary Bézier ordinates b_{022}, b_{121} and b_{112} are estimated from (16) and (17) provided they satisfy the lower bound $b_{i,j,k} \geqslant -r_0$ to ensure convex surface through convex data. Similarly, the C^1 continuity is established along the remaining edges of the triangle.

4. Demonstration

In this Section, the convexity preserving scheme developed in Section 3 is tested for the convex scattered data generated from the convex function $F(x, y) = x^2 + y^2 + 0.5$, $(x, y) \in [-1, 1] \times [-1, 1]$. The generated convex scattered data set is given in Table 1 (see Figs. 1 and 2).

In Fig. 3, the domain of the convex scattered data set of Table 1 is triangulated by the Delaunay triangulation scheme.

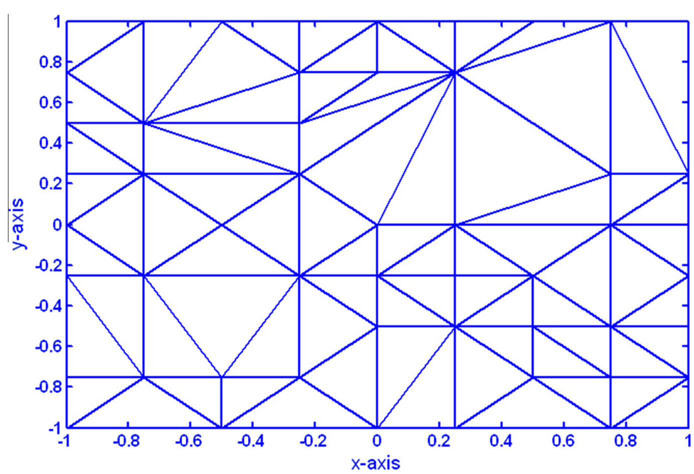

Figure 3 Triangulation of the domain for convex data of Table 1.

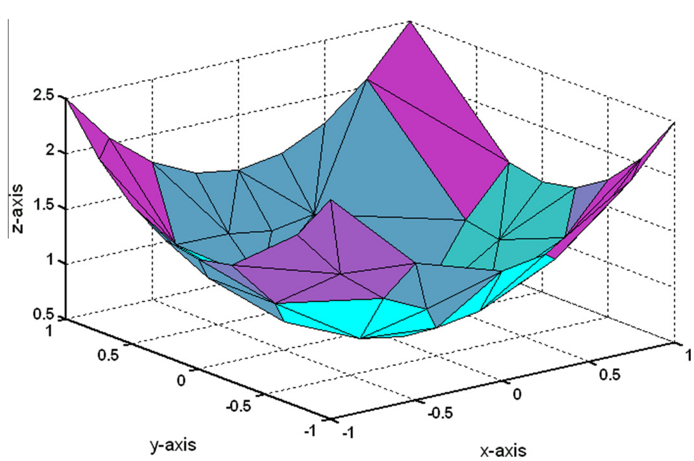

Figure 4 Linear interpolation of the convex data of Table 1.

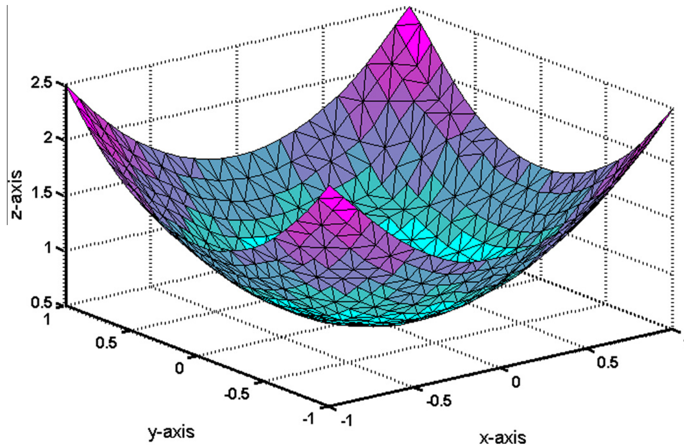

Figure 5 The Bernstein–Bézier quartic function.

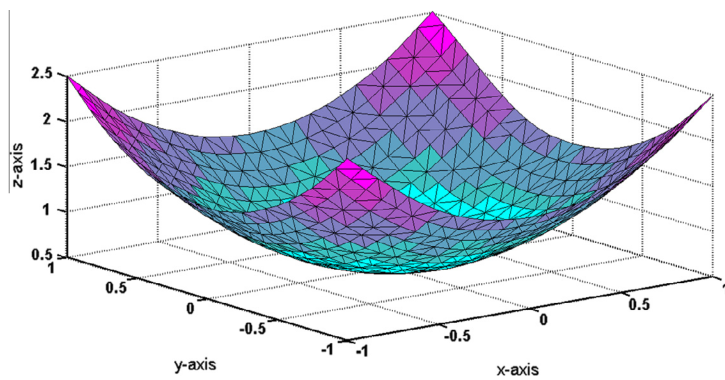

Figure 6 The convex surface generated from the numerical scheme of [3].

The linear interpolation of the data set of Table 1 is given by Fig. 4. Finally, the convex data is interpolated by the convexity preserving scheme developed in Section 3 with $d = (1, 1)$ and the interpolated convex surface is shown in Fig. 5.

The graphical results in Fig. 3–5 are obtained by the MATLAB software. The CPU time for the implementation of above mentioned developed convexity preserving scheme for the data set of Table 1 is 4.213 s. Moreover, the proposed scheme of this paper has CPU time less than [11] and [2] but greater than [3].

Fig. 6 is generated by the convexity-preserving scheme developed in [3]. The convexity-preserving Bernstein–Bézier quartic scheme developed in Section 3 has 15 control points, while the numerical scheme of [3] provides 10 control points. Thus the Bernstein–Bézier quartic scheme has more chances of convex shape preservation without the adjustment of derivatives.

5. Conclusion

In this study, lower bound ($b_{i,j,k} \geqslant -r_0$) of the boundary and inner Bézier ordinates of Bernstein–Bézier quartic interpolant is determined to ensure convex surface through convex scattered data. The Bézier ordinates b_{310}, b_{130}, b_{301}, b_{103}, b_{013} and b_{031} are estimated by C^1 continuity at the vertices. These estimated values b_{310}, b_{130}, b_{301}, b_{103}, b_{013} and b_{031} may or may satisfy the derived lower bound for convexity. As a remedy, parameter is introduced in the definition of b_{310}, b_{130}, b_{301}, b_{103}, b_{013}, b_{031}. The Bézier ordinates b_{220}, b_{022}, b_{202}, b_{211}, b_{121} and b_{112} are computed by guaranteeing C^1 continuity along the edges and convexity of surface ($b_{i,j,k} \geqslant -r_0$). The developed scheme of this paper involves more Bézier ordinates as compared to [3], hence more flexible. The developed constraints of convexity preservation are more relaxed than [2,11].

In this paper shape-preserving scheme is developed for convex scattered data. The authors are keen to develop shape-preserving schemes for monotone and positive data in the subsequent papers.

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