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Comparison of forecasting performance between MODWT-GARCH$_{(1,1)}$ and MODWT-EGARCH$_{(1,1)}$ models: Evidence from African stock markets

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Abstract

Many researchers documented that if stock markets' returns series are significantly skewed, linear-GARCH$_{(1,1)}$ grossly underestimates the forecast values of the returns. However, this study showed that the linear Maximal Overlap Discreet Wavelet Transform MODWT-GARCH$_{(1,1)}$ actually gives an accurate forecast value of the returns. The study used the daily returns of four African countries' stock market indices for the period January 2, 2000, to December 31, 2014. The Maximal Overlap Discreet Wavelet Transform-GARCH$_{(1,1)}$ model and the Maximal Overlap Discreet Wavelet Transform-EGARCH$_{(1,1)}$ model are exhaustively compared. The results show that although both models fit the returns data well, the forecast produced by the Maximal Overlap Discreet Wavelet Transform-EGARCH$_{(1,1)}$ model actually underestimates the observed returns whereas the Maximal Overlap Discreet Wavelet Transform-GARCH$_{(1,1)}$ model generates an accurate forecast value of the observed returns.

Keywords: Volatility; Asset returns; MODWT; GARCH; EGARCH

1. Introduction

Stock market volatility is of essential concern, particularly to two major stake-holders. While the practitioner looks through his own lenses with the bird's-eye-view, he or she bothers himself or herself about the consequences of this behaviour on asset pricing and risk. Conversely, policy makers are burdened with the incidence of financial challenges and macroeconomic instability posed by the stock market phenomenon. Of optimum concern of these dual effects of stock market volatility, emanates predominantly from developing countries with infant stock markets, characterized by vulnerabilities. However, a plethora of stock markets studies seems to have chiefly been focused on developed and

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near-developed economies utilizing various models to offer empirical investigation to these effects. Consequently, due to the disparities inherent in different context, the application of a single-most model in unravelling the effects of stock market volatility may be contentious or inaccurate. This is because, developing economies may parade market indices not explicitly possessed by their superior developed counterparts. The quandary has been and continue to be how to adopt an efficient quantitative tool that possesses an apt measurement tendency that conscripts all the antecedents, based on a volatility model, universal to studying the volatility of stock markets in developing economies.

Available literature[1] originally developed the Autoregressive Conditional Heteroscedasticity Model (ARCH) to pre-empt the doubtful future nature of the Great Britain's national inflation rate. Engle painstakingly discovered by examining the time plot of the country's inflation rate that, large amount of changes is preceded by large amount of changes of the same or complementary sign magnitude and conversely, small amount of alterations are also predecessors of small amount alterations of the same or alternative sign magnitude. Resulting from this discovery, was an apt and important ideology called volatility clustering. The researcher, Engle, accurately estimated and assessed the clustering impacts by drawing his inspiration on the well-established pre-supposition of stable conditional returns mean value.

Even though Engle's research fortune has unearthed an important characteristic of volatility, still the other remaining behaviour of volatility were yet to be excavated by the instrumentality of his tool, the ARCH model. Bollerslev,[2] therefore developed a model that encompasses the ARCH model and called it the Generalized Autoregressive Conditional Heteroscedasticity model (GARCH). This development, stretches the ability of the ARCH model and captured the kurtosis nature of returns series and those of high frequency financial time series data in general.

The researchers[3–5] have unravelled a new paradigm of the GARCH model family that accounted for the fat tail skewed distribution nature of returns series. The respective researchers working independently at different times, have precisely called these group of newly found models the non-linear GARCH models. They are the Exponential GARCH (EGARCH); the Quadratic GARCH (QGARCH) and the Glosten, Jaganathan, Rankle (GJR) model. Amongst these models, the most successful and widely reported in empirical literature is the EGARCH model. However, wider empirical research literature has shown that its linear counterpart, the GARCH model is the most efficient tool for modelling returns series even if the returns series are significantly skewed.[6]

Apart from the kurtosis and skewness of returns series, an important factor that can shift the whole paradigm of stock markets' returns is “time”. This is because, the capital market comprises different players and separate decision makers acting independently at specific time horizons so as to maximize profit. Hence, they are interested in finding a tool of analysis that will guide them on the right time to enter and to exit the capital market with minimal risk of making losses. Fortunately, a tool by Refs. [7–11] called wavelet is apt for this urgent important task.

Wavelet is a function that localizes stock market returns series in a time domain and as well as in a frequency domain and is subsequently deployed for the purposes of decomposing the returns series into basic primary functions, each conceiving different knowledge pertaining to the stock markets' returns series.[11] A number of tools have been deployed so as to gain access into the embedded statistical signals in a stock markets' returns as well as to filter and denoise it. However, researchers have consistently endorsed that the wavelet tool is superior to the others because it has the capacity to breakdown macroeconomic variables into their various time scale parcels.[11–15]

In the literature, different types of wavelets exist. These include the Discreet Wavelet Transform (DWT) and the Maximal Overlap Discreet Wavelet Transform (MODWT), among others. Empirically, the most successful type of wavelet reported in finance and economic studies is the Maximal Overlap Discreet Wavelet Transform (MODWT). This particular type of wavelet is utilized to analyse stock markets' returns series because of its courage of accommodating any sample size in addition to its non-sensitivity to the initial take up point of the series for the purposes of analysing the returns series.[7] Other phenomenon that makes it extremely expedient for researchers in the field of Finance and Economics to adopt the use of MODWT is its ability to give good and simple understanding of multiresolution properties of Finance and Economics data. Additionally, MODWT stock markets' returns series, has the ability to discover the structural breaks and extreme volatility clustering inherent in high frequency returns series. Also, the MODWT accurately realigns itself with the events in the original returns series. Finally, the MODWT returns series makes each time scale component independent of each other by dissolving the correlation structure between them.

An in-depth empirical study conducted by Gallegati[7] uses the Discreet Wavelet Transform (DWT) to study the monthly returns series of IBM stocks and discovered that there were chunks of swiftly alternating returns between successive length of the DWT wavelets coefficients w_1. Further probe into the future behaviour of high frequency financial time series conducted to purposely find a suitable tool that will exactly delve into the future with a high degree of accuracy was by Conejo et al.[16] Their study adopted and implemented tools of time series analysis; neural network and wavelet forecasting methodology using the PJM interconnection data of the day after today of some 24 market clearing prices. After accurately placing side by side the future forecast errors produced by the respective tools with the realization that the wavelet tool produces smaller forecasting errors, the research concluded with a strong appeal to researchers in the field of Finance and Economics and other fields of human endeavour using time series data to urgently consider the blending of wavelet transform and time series algorithms to further evaluate its predictive power.

In response to this, the research finding by Liu et al[17] that presumes future non-stationary wind speed using the instrumentality of wavelet Genetic Algorithm (GA)-Multi Layer Perceptron (MLP) and the wavelet Particle Swarm Optimization (PSO)-Multi Layer Perceptron (MLP), documented that, the wavelet parts of GA and PSO produce the accurate image of the future MLP.

Subsequently, based on overwhelming evidence of unparallelled excellent performance of wavelet methodology, the research by Tan et al[18] adopted and implemented the wavelet tool in conjunction with Autoregressive Integrated Moving Average (ARIMA) and the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model to accurately predict future prices. The outcome vindicated the wavelet methodology as a marvellous tool for prediction.

African stock markets offer a wider range of possibilities for foreign investors to make profit. However, weak institutions and bad investment climate are militating against it.[6]

Furthermore, the ability of market players; profit takers; policy makers; portfolio managers and spectators to make accurate judgement about the future behaviour of these markets is greatly hampered and jeopardize chiefly because of lack of basic infrastructures such as lack of good transportation network system; efficient and effective telecommunication system and a different mode of financial accounting; financial reporting and financial auditing.[6]

In order to augment the literature to cover the peculiarities of African stock markets, Ismail, Buba and Tumala, (2016) following (Gockan, 2000) methodology have assiduously sought by their unique algorithm the MODWT-$\text{GARCH}_{(1,1)}$ model and subsequently used it as the base model to compare the performance of the traditional linear $\text{GARCH}_{(1,1)}$ model. The authors concluded that, the MODWT-$\text{GARCH}_{(1,1)}$ model is the most suitable tool for modelling the returns of African stock markets. The chronicle of literature depicts significant studies utilizing the $\text{GARCH}_{(1,1)}$ model in unravelling the effect of returns series volatility. Conversely, it is academically inquisitive to attempt the comparison of the MODWT-$\text{GARCH}_{(1,1)}$ model and the MODWT-$\text{EGARCH}_{(1,1)}$ model to decipher the performance differentials amongst them, as well as the incidence of favorability in determining returns series volatility. This analysis drags in its trial a novelty, which previous research seems to leave fallow. Consequently, it is apt to address the crucial situation where by the MODWT-$\text{GARCH}_{(1,1)}$ model is used as the base model to evaluate the performance of its non-linear counterpart the MODWT-$\text{GARCH}_{(1,1)}$ model in order to fill this lacuna.

The paper is fashioned in the following manner: While Section 2 gives the details of the materials and the methodology adopted in the research, the analysis of the data is presented in Section 3, and finally, the conclusion is given in Section 4.

2. Methodology

Four African countries stock markets' prices of NSE 20 (Kenya); All Share Price Index (Nigeria); FTSE/JSE100 (South Africa) and TUNNIDEX (Tunisia) for the period January 2, 2000 to December 31, 2014 were selected based on market development and data availability. Incidentally, the four countries, each falls within the four distinct regions of the African continent. The data was extracted and downloaded from the data stream of the Thomson Reuters. There is a negligible and insignificant variation in the length of the data of the respective countries because of government official holidays.

To fully understand the underlying nature, structure and characteristics of these African countries stock markets' level of volatility, their returns were calculated:

$$r_t = \ln p_t - \ln p_{t-1} \tag{1}$$

where p_t is the stock market's share price index at period t, p_{t-1} is the stock market's share price index at period $t-1$ and r_t is the respective countries daily stock markets' returns.

The use of the time series tool that has the $\text{ARMA}_{(p,q)}$ model representation in (2), was implemented on the original returns series and on the MODWT returns series by overfitting the (p,q) parameters. This is achieved by saintly adhering to the laid down iterative procedures of Box–Jenkins methodology of $\text{ARMA}_{(p,q)}$ modelling.

$$Y_t + \alpha_1 Y_{t-1} + \alpha_2 Y_{t-2} + \ldots + \alpha_p Y_{t-p} = C + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-1} + \ldots + \theta_q \varepsilon_{t-q} \quad (2)$$

where $\varepsilon_t \sim NIID(0,\sigma^2)$, i.e ε_t is normally, independently, and identically distributed as a random variable with mean 0 and variance σ^2.

$\alpha(L)Y_t$ is designated the AR component of the process (Y_t) and $\theta(L)\varepsilon_t$ is the MA component of the process (Y_t).

Also, the mathematical construction for $\text{GARCH}_{(1,1)}$ model given in (3) is used in modelling the original returns series and the MODWT returns series.

$$g_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1} \quad (3)$$

where,

β_1 measures the distance to which a present volatility shock goes into the future volatility.
$(\alpha_1 + \beta_1)$ measures the rate at which this effect dies in the future; and
g_{t-1} is the volatility at week $t-1$

The $\text{EGARCH}_{(1,1)}$ model is written as:

$$\log(g_t) = \alpha_0 + \sum_{i=1}^{p} \alpha_i \left[\frac{|\varepsilon_{t-i}|}{\sqrt{g_{t-1}}} - \sqrt{2/\pi} \right] + \gamma \frac{\varepsilon_{t-i}}{\sqrt{g_{t-i}}} + \sum_{j=1}^{q} \beta_j \log(g_{t-j}) \quad (4)$$

The constant parameters α_0, α_i, γ, and β_j (4) can take any positive or negative value. This is a direct contrast of the constants parameters in the GARCH model that must be absolutely positive. Conversely, the EGARCH model gives room for positive return shocks and negative return shocks to differ on the gravity of their impact on volatility.[4] The constant parameter γ is the chief causative agent of the asymmetry in volatility. In empirical studies, when $\gamma = 0$, then a positive return shock has the same effect on volatility as the negative return shock of the same amount. When$\gamma < 0$ then, a positive return shock actually reduces volatility and when $\gamma > 0$, a positive return shock increases volatility. A great deal of previous studies discovered that this coefficient is typically negative. It therefore, means that, positive return shocks generate less volatility than negative return shocks.[6]

We then use the MODWT to decompose the returns in to its w_1 component which actually represent the 2–4 days' time component of the original returns.[7] The MODWT w_1 wavelet coefficients is given by

$$r_{w_{j,l}} = \frac{wavelet_{(returns)}}{2^{j/2}}$$

where $r_{w_{j,l}}$ is the returns MODWT w_1 wavelet coefficients.

The computer programming algorithm formula that generated the MODWT w_1 is as given in (5).

$$w_1 = (r_{2k} - r_{2k-1}) \Big/ \sqrt{2}, \quad (5)$$

where w_1 is the MODWT of order 1.

$r_1,\cdots,r_n$ is the return series for $i = 1$, $i \le k$, $i{+}{+}$, $k = \frac{n}{2}$
$n =$ length of the return series.

3. Analysis of data

The presentation in Table 1 gives the descriptive statistics for the daily returns for the four African stock markets. The number of trading days are the sample size for the data. The mean returns for the four African countries' range from $3.2 \times 10^{-4}\%$ (Nigeria) to $2.0 \times 10^{-4}\%$ (Kenya). The statistic that measures volatility, in this case, the standard deviation, range from 0.007% (Tunisia) to 0.018% (South Africa). For the standard normal distribution, kurtosis (the fourth moment) should be positive three. Nevertheless, in this study, the kurtosis for all the four African countries' stock markets' returns, and even the kurtosis for the world stock market returns exceeds three. This implies that stock markets' returns are generally leptokurtic in nature. In addition, even though the skewness for standard normal distribution should be zero, the skewness for the four African stock markets is negative. This result shows that the lower tail of the distribution is fatter than the upper tail, which indicate that market losses are been witnessed more frequently than market profits. Also, the Ljung–Box Q-statistics at lag 24 show that for all four countries, the null

Table 1
Descriptive statistics for African stock markets.

Country	Sample size	Mean (%)	Standard deviation (%)	Skewness	Kurtosis	Q(24)[a]	Normality test[b]
Kenya	3913	2.0×10^{-4}	0.014	−0.22	686.20	141.79***	76081473***
Nigeria	3904	3.2×10^{-4}	0.014	−0.76	272.37	76.16***	11800377***
South Africa	3913	2.8×10^{-4}	0.018	−0.23	8.61	60.50***	5161.31***
Tunisia	3913	2.7×10^{-4}	0.007	−0.04	8.42	136.95***	4804.42***

[a] Ljung−Box Q statistics at lag 24. *** indicates significant at 1% level of significance.
[b] Normality of return series are tested by using Jarque−Bera statistics. *** indicates significance at 1% level of significance.

hypothesis of no autocorrelation is rejected at the $\alpha = 5\%$ level of significance. Additionally, based on the Jarque−Bera statistics, the null hypothesis of normality is rejected for all four countries stock markets' returns series.

The time plots of the four countries stock markets' returns time series are shown in Figs. 1−4.

The plots of the returns in Figs. 1−4 show that risk is associated to periods and it is randomly scattered with some degree of autocorrelation. The amplitudes of the returns vary over time as large or small chunks of changes are followed by large or small chunks of changes. This phenomenon is called volatility clustering and is one of the stylized facts of the financial times.

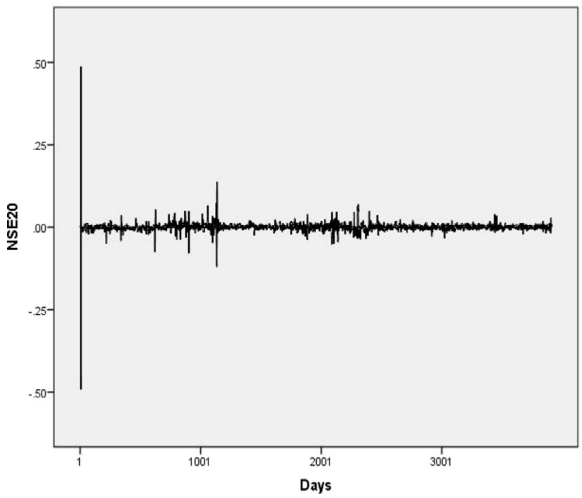

Fig. 1. Nairobi stock exchange daily returns.

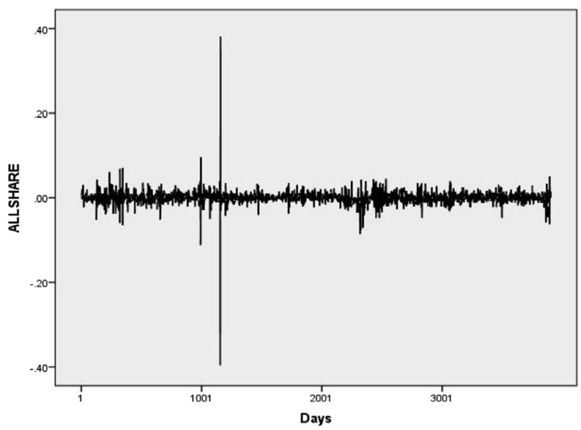

Fig. 2. Nigeria all share price index daily returns.

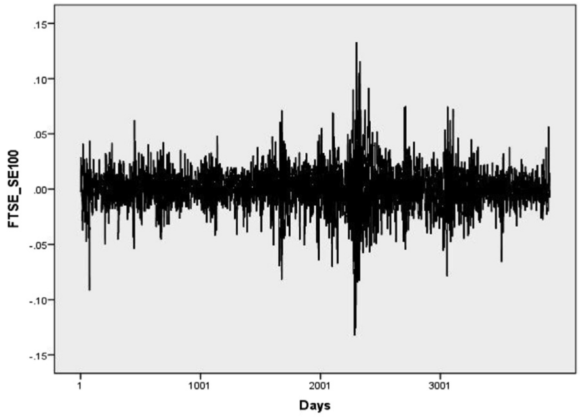

Fig. 3. FTSE/JSE100 daily returns.

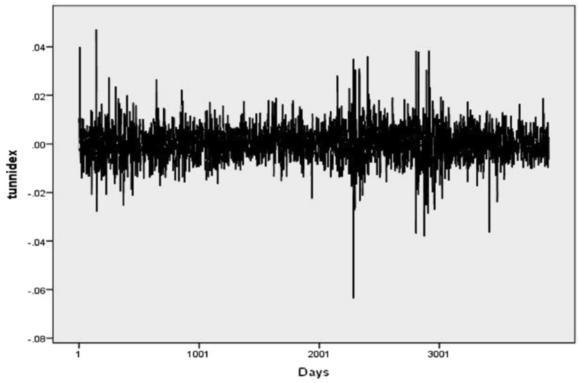

Fig. 4. TUNNIDEX daily returns.

The outcomes of estimating the parameters of the model given in Eq. (2) using the MODWT transformed return series, are reported in Table 2–6. The optimal lag lengths that best fits $AR_{(p)}$; $MA_{(q)}$ or $ARMA_{(p,q)}$ models for the MODWT transformed returns series for each of the four countries was obtained by systematically over fitting the values of p and q. The Akaike Information Criterion (AIC)[19–24] is used to decide on the best fitting model for each of the four countries MODWT transformed returns series. The AIC chooses the MODWT-$ARMA_{(0,1)}$ model as the best fitting model for Kenya, the MODWT-$ARMA_{(1,0)}$ model as the best fitting model for Nigeria, the MODWT-$ARMA_{(5,5)}$ as the best fitting model for South Africa, and the MODWT-$ARMA_{(4,4)}$ as the best fitting model for Tunisia.

Furthermore, the results of standard diagnostic residual checks for the AutoRegressive Conditional Heteroskedasticity (ARCH) effect for all the four countries best fitted MODWT-$ARMA_{(p,q)}$ model presented in Table 6, indicate that there is remaining ARCH effect in the residuals of all the best fitted models. This is because the null hypothesis of no remaining ARCH effect in the residuals of the best fitted MODWT-$ARMA_{(p,q)}$ models is rejected. This discovery, is a strong evidence of GARCH type of heteroskedasticity. Therefore, we fit the most widely used linear $GARCH_{(1,1)}$ model and the most widely used non-linear $EGARCH_{(1,1)}$ model to the MODWT transformed returns series.

All four countries parameter estimates of the MODWT-$GARCH_{(1,1)}$ models and the MODWT-$EGARCH_{(1,1)}$ models are reported in Table 7 and their AIC values are exhaustively compared. The AIC values of the MODWT-$GARCH_{(1,1)}$ models are lower than the AIC values of the MODWT-$EGARCH_{(1,1)}$ models. It therefore implies that the MODWT-$GARCH_{(1,1)}$ model have the capacity to capture the dynamic behaviour of the African countries emerging stock markets' returns more accurately. Furthermore, the good news is that the β parameters of the MODWT-$GARCH_{(1,1)}$ models for all four countries are positive and the $\alpha + \beta$ parameter values are each, less than one.

Table 2
Results of the Estimated MODWT-ARMA$_{(p,q)}$ for Kenya stock returns and their AIC values.

ARIMA	AIC	Serial correlation		ARCH-effect		Model significance
		F-stat	P-value	F-stat	P-value	
1,1,0	−5.82	3.94	0.00	5.26	0.00	Significant
2,1,0	−5.82	129.12	0.00	5.91	0.00	Significant
0,1,1	−5.83	0.49	0.98	6.06	0.00	Significant
0,1,2	−5.82	2.01	0.00	6.04	0.00	Not-significant
1,1,1	−5.83	9.93	0.00	6.44	0.00	Not-significant
1,1,2	−6.07	37.52	0.00	3.54	0.00	Significant
2,1,1	−5.87	123.37	0.00	6.66	0.00	Not-significant
2,1,2	−6.19	50.67	0.00	4.21	0.00	Significant
3,1,2	−6.70	8.00	0.00	2.06	0.00	Significant
2,1,3	−6.28	44.46	0.00	5.03	0.00	Significant
3,1,3	−6.74	4.52	0.00	2.51	0.00	Significant
4,1,3	−6.78	1.12	0.31	2.44	0.00	Not-significant
3,1,4	−6.75	4.13	0.00	2.79	0.00	Significant
4,1,4	−6.78	0.13	0.88	2.44	0.00	Not-significant
5,1,4	−6.78	0.12	0.89	2.44	0.00	Not-significant
4,1,5	−6.78	0.13	0.88	2.44	0.00	Not-significant
5,1,5	−6.78	0.20	0.82	2.44	0.00	Not-significant

Table 3
Results of the Estimated MODWT-ARMA$_{(p,q)}$ for South Africa stock returns and their AIC values.

ARIMA	AIC	Serial correlation		ARCH-effect		Model significance
		F-stat	P-value	F-stat	P-value	
1,1,0	−5.18	2.56	0.00	35.31	0.00	Not-significant
2,1,0	−5.18	2.30	0.00	33.00	0.00	Significant
0,1,1	−5.18	2.51	0.00	35.30	0.00	Not-significant
0,1,2	−5.18	2.29	0.00	32.79	0.00	Significant
1,1,1	−5.18	2.55	0.00	35.12	0.00	Not-significant
1,1,2	−5.19	1.95	0.00	32.49	0.00	Significant
2,1,1	−5.19	1.95	0.00	32.37	0.00	Significant
2,1,2	−5.19	1.82	0.01	32.36	0.00	Significant
3,1,2	−5.19	1.82	0.01	32.65	0.00	Not-significant
2,1,3	−5.19	1.89	0.01	32.87	0.00	Significant
3,1,3	−5.19	1.64	0.03	30.49	0.00	Significant
4,1,3	−5.19	1.66	0.02	30.59	0.00	Not-significant
3,1,4	−5.19	1.64	0.03	30.63	0.00	Not-significant
4,1,4	−5.18	1.81	0.01	32.18	0.00	Not-significant
5,1,4	−5.19	1.61	0.03	31.18	0.00	Significant
4,1,5	−5.19	1.81	0.01	32.71	0.00	Significant
5,1,5	−5.19	1.41	0.09	29.89	0.00	significant

4. Forecasting

Before deciding on the linear or the non-linear MODWT-GARCH models, the true unconditional volatility in Eq. (7) below should be calculated in order to ascertain the extent of forecasting abilities of the MODWT-GARCH$_{(1,1)}$ model and the MODWT-EGARCH$_{(1,1)}$ model.[6,25]

$$\tau^2 = \left(r_t - \widehat{r}_t\right)^2 \tag{7}$$

where τ^2 is the unconditional volatility, r_t is the actual monthly return for month t, and $\widehat{r}_t$ is the expected return for month t. The method of moving average is used to calculate the expected return.[26]

Table 4
Results of the Estimated MODWT-ARMA$_{(p,q)}$ for Nigeria stock returns and their AIC values.

ARIMA	AIC	Serial correlation		ARCH-effect		Model significance
		F-stat	P-value	F-stat	P-value	
1,1,0	−5.82	0.93	0.56	35.21	0.00	Significant
2,1,0	−5.78	3.77	0.00	54.00	0.00	Not-significant
0,1,1	−5.82	0.67	0.88	32.58	0.00	Significant
0,1,2	−5.78	3.71	0.00	54.02	0.00	Not-significant
1,1,1	−5.82	0.65	0.90	32.88	0.00	Significant
1,1,2	−5.82	0.60	0.94	32.98	0.00	Significant
2,1,1	−5.82	0.67	0.89	33.29	0.00	Significant
2,1,2	−5.82	0.65	0.90	32.97	0.00	Not-significant
3,1,2	−5.82	0.63	0.92	32.87	0.00	Not-significant
2,1,3	−5.82	0.66	0.89	33.11	0.00	Significant
3,1,3	−5.81	0.63	0.92	32.96	0.00	Not-significant
4,1,3	−5.82	0.48	0.99	32.56	0.00	Significant
3,1,4	−5.81	0.61	0.93	32.67	0.00	Not-significant
4,1,4	−5.82	0.47	0.99	32.44	0.00	Not-significant
5,1,4	−5.82	0.52	0.97	32.81	0.00	Significant
4,1,5	−5.82	0.45	0.99	32.97	0.00	Significant
5,1,5	−5.82	0.73	0.83	33.02	0.00	significant

Table 5
Results of the Estimated MODWT-ARMA$_{(p,q)}$ for Tunisia stock returns and their AIC values.

ARIMA	AIC	Serial correlation		ARCH-effect		Model significance
		F-stat	P-value	F-stat	P-value	
1,1,0	−7.22	1.51	0,05	5.37	0.00	Not-significant
2,1,0	−7.22	1.54	0.05	5.26	0.00	Not-significant
0,1,1	−7.22	1.44	0.08	5.38	0.00	Not-significant
0,1,2	−7.22	1.50	0.06	5.27	0.00	Not-significant
1,1,1	−7.22	1.40	0.08	5.21	0.00	Not-significant
1,1,2	−7.22	1.51	0.05	5.31	0.00	Not-significant
2,1,1	−7.22	1.53	0.05	5.37	0.00	Not-significant
2,1,2	−7.22	1.45	0.08	5.27	0.00	Not-significant
3,1,2	−7.22	1.43	0.08	5.33	0.00	Not-significant
2,1,3	−7.22	1.45	0.07	5.24	0.00	Not-significant
3,1,3	−7.22	1.37	0.11	5.50	0.00	Not-significant
4,1,3	−7.22	1.51	0.05	5.22	0.00	Not-significant
3,1,4	−7.22	1.48	0.06	5.25	0.00	Not-significant
4,1,4	−7.23	1.28	0.16	5.65	0.00	Significant
5,1,4	−7.22	1.25	0.18	5.44	0.00	Not-significant
4,1,5	−7.23	1.36	0.11	5.48	0.00	Not-significant
5,1,5	−7.22	1.55	0.04	5.43	0.00	Not-significant

Table 6
Results of the Estimated MODWT-ARMA$_{(p,q)}$ and test for ARCH-effect.

Country	Model	AIC[a]	ARCH test
Kenya	ARMA$_{(0,1)}$	−5.83	6.06***
Nigeria	ARMA$_{(1,0)}$	−5.82	35.21***
South Africa	ARMA$_{(5,5)}$	−5.19	29.89***
Tunisia	ARMA$_{(4,4)}$	−7.23	5.68***

Note: *** denote significant at 1% level of significance.
[a] Akaike Information Criterion.

Table 7
Results of the Estimated MODWT-GARCH$_{(1,1)}$ and MODWT-EGARCH$_{(1,1)}$ Parameters and their AIC values.

Country	Parameter estimates			Parameter estimates	AIC[a] values	
	MODWT-GARCH(1,1)			MODWT-EGARCH$_{(1,1)}$	MODWT-GARCH$_{(1,1)}$	MODWT-EGARCH$_{(1,1)}$
	α_0	α_1	β_1	γ		
Kenya	1.41×10^{-5} (25.24)***	0.25 (18.03)***	0.49 (31.20)***	0.035 (5.25)***	−7.15	−7.16
Nigeria	8.90×10^{-6} (7.27)***	0.51 (13.17)***	0.62 (27.77)***	0.11 (19.55)***	−6.38	−6.10
South Africa	9.20×10^{-6} (4.76)***	0.10 (8.08)***	0.86 (49.23)***	−0.035 (−3.43)***	−5.47	−5.46
Tunisia	5.21×10^{-7} (3.19)***	0.05 (10.32)***	0.94 (123.96)***	−0.0026 (0.75)***	−7.34	−7.34

Note:*** denote significant at a 1% level of significance.
[a] Akaike Information Criterion.

Table 8
Five days forecast average error terms.

Country	MODWT-GARCH$_{(1,1)}$	MODWT-EGARCH$_{(1,1)}$
Kenya	−0.649	0.423
Nigeria	−1.137	15.002
South Africa	−0.969	−0.686
Tunisia	−0.989	−0.897

To ascertain the extent of the forecasting abilities of the MODWT-GARCH$_{(1,1)}$ model and the MODWT-EGARCH$_{(1,1)}$ model and to decide on which model best fits the African stock markets data, the average of five periods ahead out-of-sample forecast for the returns series are obtained.

Following Ref.,[6] the one period ahead forecasting errors for the MODWT-GARCH$_{(1,1)}$ model and the MODWT-EGARCH$_{(1,1)}$ model are obtained from Eq. (8):

$$\eta_{t+1} = \tau^2 - g_{t+1} \tag{8}$$

where η_{t+1} is the forecasting error of the MODWT-GARCH$_{(1,1)}$ model and the MODWT-EGARCH$_{(1,1)}$ model, and g_{t+1} is the forecasted variance as generated by Eqs. (3) and (4).

The one period-ahead forecast variance of the fifth to the last day of the MODWT returns series would be found by running the regressions using the two models as in Eqs. (3) and (4) by employing the MODWT returns data from the first day to the sixth day and obtaining the constant parameters. The constant parameters are therefore entered into Eqs. (3) and (4) of the two different models, and as a result, each of the forecast variances is found. Similarly, the forecast variances for the fourth day are obtained by running the regressions using the two models that employ the data from the second day to the fifth to the last day to obtain the constant parameters. This procedure is followed in obtaining the forecast variances for the fifth to last day until the forecast variance of the last day is obtained.[6]

In Table 8, the five days forecast average errors are obtained from Eq. (8). The results indicate that for all four African countries, the MODWT-GARCH$_{(1,1)}$ model produces smaller forecasting errors than the MODWT-EGARCH$_{(1,1)}$ model.

5. Conclusion

In this paper, both the MODWT-GARCH$_{(1,1)}$ model and the MODWT-EGARCH$_{(1,1)}$ model were applied to four African stock markets returns series. In the comparisons in Tables 7 and 8 for all four African countries, the MODWT-GARCH$_{(1,1)}$ model produced better results than the MODWT-EGARCH$_{(1,1)}$ model. There is also within-sample evidence that the conditional estimates of the MODWT-GARCH$_{(1,1)}$ model outperform the conditional estimates of the MODWT-EGARCH$_{(1,1)}$ model. The out-of-sample evidence proves that daily volatilities are better predicted with the MODWT-GARCH$_{(1,1)}$ model.

Ismail, Audu and Tumala, (2016) developed an algorithm to model the volatility of four African countries stock markets' returns series using the MODWT-GARCH$_{(1,1)}$ model and subsequently used the model as a based model to evaluate the performance of the linear $GARCH_{(1,1)}$ model. In contrast, this study used the $MODWT\text{-}GARCH_{(1,1)}$ model and the $MODWT\text{-}EGARCH_{(1,1)}$ model to evaluate the volatility of the returns series of the four African countries stock markets.

The empirical studies by Ismail, Audu and Tumala, (2016) proved that although the returns series are substantially skewed, the linear $MODWT\text{-}GARCH_{(1,1)}$ model is of extreme importance in modelling the returns series volatility. Similarly, in this study the $MODWT\text{-}GARCH_{(1,1)}$ model is still excellently good in capturing the stylized facts of returns series volatility despite using the non-linear $MODWT\text{-}EGARCH_{(1,1)}$ model that naturally, is believed to capture the skewness. This result resonates with Gokcan (2000).

Additionally, the result of this research showed that the $MODWT\text{-}GARCH_{(1,1)}$ model generates more excellent results than that of the non-linear $MODWT\text{-}EGARCH_{(1,1)}$. This result agrees with Gokcan (2000) that discovered and established that the linear $GARCH_{(1,1)}$ model is superior than the non-linear $EGARCH_{(1,1)}$ model when the forecast results produced by both models are compared. Therefore, the within sample conditional estimates of the linear $MODWT\text{-}GARCH_{(1,1)}$ model is better and more superior than the within sample conditional estimates of both the linear $GARCH_{(1,1)}$ model and the non-linear $MODWT\text{-}EGARCH_{(1,1)}$ model respectively.

In conclusion, the $MODWT\text{-}GARCH_{(1,1)}$ model is more superior to the linear $GARCH_{(1,1)}$ and the non-linear $MODWT\text{-}EGARCH_{(1,1)}$ model because of its ability to accurately mimic the present returns series in the future. This is evident from the smaller out-of-sample forecast values produced by the $MODWT\text{-}GARCH_{(1,1)}$ model.

Dwelling on the results of this study and that of Ismail, Audu and Tumala, (2016) there is a justification that the linear $MODWT\text{-}GARCH_{(1,1)}$ model gives accurate forecast volatility of African countries stock market more than the linear $GARCH_{(1,1)}$ model. Of great importance, is that the linear $MODWT\text{-}GARCH_{(1,1)}$ model exceeds the non-linear $MODWT\text{-}EGARCH_{(1,1)}$ model in given the accurate 5 days average forecast values of the stock markets' returns of African countries.

Conflicts of interest

All authors have none to declare.

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